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Modeling of Non- Newtonian Fluid for Blood Flow in Stenosed Arteries; A Comparative Study

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I. Introduction

The blood flow through arteries with stenosis is considered as non-Newtonian Misra, J.C *et al* 1993. Non- Newtonian flow is the flow that does not obey the Newtonian relationship between shear stress and shear rate. In non-Newtonian flow the slope of shear stress versus shear rate curve is not constant and so it is not a straight line but a carve and the viscosity of fluid decreases with increasing of shear rate. Blood flow problems are more complicated than the problems of fluid flow in engineering for some reasons like unusual curvature of blood vessels, unusual large numbers of branches and unusual fluid properties of blood.

Blood flow characteristics in arteries can be altered significantly by arterial disease, such as stenosis. The abnormal and unnatural growth in the arterial wall that develops at various location of the cardiovascular system under diseased conditions is called stenosis. This can cause serious circulatory disorders by reducing of occluding the blood supply. For instance, stenosis in the arteries supplying blood to brain can bring about cerebral strokes. Likewise in coronary arteries it can cause myocardial infarction leading to the heart failure Musad (2012). The effect of severe stenosis calcified atherosclerotic vessel is less elastic than a healthy vessel. In a less elastic atherosclerotic vessel, the ability of the vessels to expand in order to accommodate the volume of ejected blood at the onset of the cardiac cycle is diminished, and consequently blood flow through the diseased vessel may be substantially reduced. In order to maintain proper levels and rates of blood flow, the wall shear stress will increase. Thus there is complementary relation between the growth of stenosis and the flow of blood. Very high wall shear stress can activate the platelets which can cause thrombosis and may totally block the vessels. The wall shear stress distribution is an important diagnostic factor for examining the blood flow characteristics through the arteries. Accurate predictions of the distribution of the wall shear stress are particularly useful for the understanding of the effect of blood flow on endothelial cells.

Srivastava (2003) considered the effects of stenosis assuming blood to behave like couple stress fluid, peripheral layer plasma Newtonian fluid, which is acceptable for high shear rate flow (shear rate > 10 sec^{-1}) in layer arteries and with viscosity coefficients $\mu_{p,}$ and a core region of suspension of erythrocytes an non-Newtonian fluid, which is acceptable for low shear rate flow (shear rate <10 sec^{-1}) through small diameter arteries and with viscosity coefficients $\mu_{c.}$. Sanker and Lee (2007) have assumed the non-Newtonian fluid as Herschel-Bulkley fluid.

Kapur (1985) reported that Herschel-Bulkley fluid model is the fluid model with non-zero yield stress and it is more suitable for the studies of the blood flow through narrow arteries. It has been established by Merrill et al. (1965) that Herschel-Bulkley fluid model holds satisfactorily for blood flowing in tubes of diameter 130 – 1300 μm.

Musad and Khan (2010) have developed a mathematical model to study blood flow in stenosed region the fluid in core layer as Casson fluid. Thus in this paper, we study core layer as Herschel-Bulkley fluid. and; compare the results of these models.

II. The Model Developed

The small artery under consideration for the development of model is with an axially symmetric stenosis. The blood flow is assumed to behave like a couple stress fluid. Peripheral layer is plasma with viscosity coefficient μ_p (Newtonian fluid) and core

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suspension layer of erythrocytes is with viscosity coefficient μ_H (Herschel-Bulkley fluid) and δ_p is a uniform thickness of plasma layer

Under the above conditions Kapure 1985 have indicated the equations of velocities for the both layers as follow

$$V_P = \frac{G}{4\mu_P}[R^2(z) - r^2], R_1(z) < r \le R(z) \tag{1}$$

$$V_H = \frac{G}{4\mu_H}[R^2(z) - r^2] + \frac{G}{\mu_H}[R^2(z) - R_1^2(z)][\frac{\mu_H}{\mu_p} - 1], 0 < r \le R_1(z) \tag{2}$$

And the equation of non-Newtonian fluid flow as

$$\tau = (-\mu\frac{\partial v}{\partial r})^n + \tau_y \tag{3}$$

The boundary of the stenosis is represented by the following equations

$$R(z) = \begin{cases} 1 - \frac{\delta_P}{2R_0}(1 + \cos(\frac{2\pi}{z_0}z)), & -z_0 \le z \le z_0 \\ 1, & 2z_0 < .z < -z_0 \end{cases}$$

$$R_1(z) = \begin{cases} \beta - \frac{\delta_H}{2R_0}(1 + \cos(\frac{2\pi}{z_0}z)), & -z_0 \le z \le z_0 \\ \beta, & 2z_0 < z < -z_0 \end{cases} \tag{4}$$

Where $R(z)$, $R_1(z)$, are the radii of stenosis region and R_0, βR_0 are radii of normal region of plasma layer and core layer respectively, μ_p, μ_H are the viscosity of plasma and core layer respectively, $2z_0$ is the length of stenosis region and δ is the height of stenosis.
By taking the derivative of the above equation we get

$$(\frac{\partial v}{\partial r})_{r=R(z)} = \frac{G}{2\mu_H}[(1 + 4(\frac{\mu_H}{\mu_p} - 1))R(z) - 4(\frac{\mu_H}{\mu_p} - 1)R_1(z)] \tag{5}$$

Put $R_1(z) = \beta R(z)$ and $\beta = 0.95$ then equation (5) reduced to

$$(\frac{\partial v}{\partial r})_{r=R(z)} = \frac{G}{10\mu_H}[(4 + \frac{\mu_H}{\mu_p})R(z) \tag{6}$$

Then from equations (3) and (6), wall shear stress along the stenosed artery can be expressed as

$$\tau = [-\frac{G}{10}(4 + \frac{\mu_H}{\mu_p})\int R(z)dz]^n + \tau_y \tag{7}$$

$$\tau = [-\frac{G}{10}(4 + \frac{\mu_H}{\mu_p})\int_{-z_0}^{z_0}(1 - \frac{\delta_p}{2R_0}(1 + \cos(\frac{2\pi}{z_0}z))dz + \int_{2z_0}^{-z_0} dz]^n + \tau_y \tag{8}$$

By integrating equation (8) we get

$$\tau = [\frac{G}{10}z_0(4 + \frac{\mu_H}{\mu_p})(1 + \frac{\delta}{R_0}]^n + \tau_y \tag{9}$$

Equation (9) is the model for Herschel-Bulkley fluid flow, where τ is wall shear stress, μ_p and μ_H are the viscosities of peripheral plasma layer and erythrocytes suspension of core layer respectively, τ_y is yield stress, δ is the height of stenosis, R_0 is the radius of normal part of artery, G is pressure gradient, z_0 is the half length of stenosis region and n is the power law index

III. Results and Conclusion

The results based on the numerical solution of equation (9) using mathematical software program called (Microsoft Mathematics 3.0) for R_0 = 0.2mm, z_0 = $4R_0$ and height of stenosis ranging from 0.08 to 0.1mm, indicates that wall shear stress increases with the increase of stenosis height δ. Further wall shear stress also increases with the increase of yield stress, while the other parameters are constant. It is observed that wall shear stress increases with the increase of viscosity core layer μ_H when the other parameters are invariable.

It is of interest to note that the wall shear stress is directly proportional relation with the length of stenosis and inversely proportional relation with the value of power law index n.

It is found also that, for a given values of the parameters wall shear stress of H-B fluid for n=1.05 is lower than that of n=0.95.

Musad and Khan (2010) have developed a mathematical model to study blood flow through stenosed artery were the blood in core layer considered as Casson fluid.

Estimate of wall shear stress of the both H-B fluid and Casson fluid for different values of height stenosis, viscosity and yield stress are computed in the tables below.

In table 1, it is observed that for the range 0.08 to 0.10 mm of the height stenosis the range of wall shear stress in (case of H-B Fluid) for n=1.05 is 0.080 to 0.124 Pascal, and 0.091 to 0.148 Pascal for n=0.95. While the range of wall shears stress (in case of Casson fluid) is 0.167 to 0.240 Pascal.

In table 2, it is found that for the range of the both yield stress and viscosity 0.01 to 0.03 Pascal, and 0.004 to 0.009 Pascal*s respectively, the corresponding range of wall shear stress in (case of H-B Fluid) for n=1.05 is 0.08 to 0.12 Pascal and 0.102 to 0.144 Pascal for n=0.95. While the range of wall shears stress (in case of Casson fluid) is 0.147 to 0.240 Pascal.

It is clear that the wall shear stress in case of Casson fluid is considerably higher than these in Herschel-Bulkley fluid case, while the viscosity and yield stress are the same. And it is observed that for the range of the height stenosis 8×10^{-5} to 10×10^{-5}, the wall shear stress in case of Herschel-Bulkley fluid is considerably lower than these in case of Casson fluid.

Table 1 : Data on wall shear stress for Herschel-Bulkley fluid and Cossan fluid in stenosis artery for values of radius R_0=0.2 mm, half length $z_0=4R_0$

Height of stenosis *(10^(-5))	Gradient Pressure	Wall shear stress For H-B fluid		Wall Shear tress For Casson fluid
		n=1.05	n=0.95	
8.000	150.000	0.075	0.091	0.167
8.222	164.444	0.080	0.097	0.175
8.444	178.888	0.085	0.103	0.183
8.666	193.333	0.091	0.109	0.190
8.888	207.777	0.096	0.115	0.198
9.111	222.222	0.101	0.112	0.200
9.333	236.666	0.107	0.128	0.213
9.555	251.111	0.112	0.134	0.221
9.777	265.555	0.118	0.141	0.230
10.00	280.000	0.124	0.148	0.240

Table 2 : Data on wall shear stress, yield stress and viscosity for Herschel-Bulkley fluid and Casson fluid where radius R_0=0.2 mm, half length $z_0=4R_0$, height of stenosis 0.10 mm, at max. Pressure gradient 267pa/m

Viscosity	Yield Stress	Wall Shear Stress For H-B fluid		Wall Shear Stress For Casson fluid
		n=1.05	n=0.95	
0.0040	0.010	0.081	0.102	0.147
0.0046	0.012	0.086	0.106	0.158
0.0051	0.014	0.090	0.111	0.169
0.0057	0.017	0.094	0.116	0.179
0.0062	0.019	0.099	0.120	0.190
00.0068	0.021	0.103	0.125	0.200
0.0073	0.023	0.107	0.130	0.210
0.0080	0.026	0.112	0.134	0.220
0.0084	0.028	0.116	0.139	0.230
0.0090	0.030	0.120	0.144	0.240

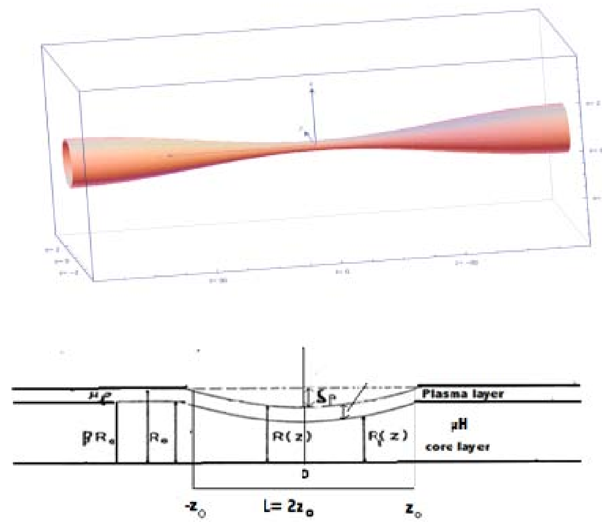

Figure 1 : physical model and coordinate system of two-layer flow in vessels of stenosis

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