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Abstract - Due to inadequacy in pure mathematics among physicists, the non-existence of dynamic solutions for the Einstein equation was not recognized. Instead, his field equation was perpetuated. The positive energy theorem of Schoen and Yau also continues such an error. Although Yau may not have made errors in mathematics, their positive energy theorem produced not only just useless but also misleading results in physics. Since the Einstein equation does not have a bounded dynamic solution, the condition of asymptotically flat implies that the case can only be a stable solution. Thus, their positive energy theorem actually does not have a meaningful application in physics. Yau failed to see this problem of misleading since he has not attempted to find explicit examples to illustrate their theorem. Moreover, E. Witten made the same mistake in his alternative proof, but that was also cited as an achievement for his Fields Medal in 1990. Concurrently, outstanding mathematical contributions of Yau to the completion of the Poincare conjecture are pointed out. Related issues are also discussed. 04.20.-q, 04.20.Cv

Keywords : the positive energy theorem; asymptotically flat; dynamic solution.

I. INTRODUCTION

In my recent paper [1], "On the Nobel Prize, Controversies and Influences" and related papers [2-4], the problem of inadequate background in pure mathematics among physicists is identified as one of the main reasons that the errors in general relativity have a long life. Here, it is pointed out further that an inadequate background in physics among mathematicians also contributes to such a long life for the errors. Understandably, mathematicians usually rely on physicists for the validity of the physical conditions being used. However, for the case of general relativity, this turns out to be problematic.

For instance, Einstein [5] claimed that his field equation provides a solution for the perihelion of Mercury, and a renounced mathematician D. Hilbert also approved the related calculation of Einstein [6] which is based on a perturbation approach. However, Gullstrand [7], Chairman of the Nobel Prize for Physics (1922-1929) still suspected that Einstein's calculation is invalid. This controversy was not solved until 1995 when Gullstrand was proven correct [8]. It is interesting to note that Hilbert did not make any mistake in mathematics, but he was not aware Einstein's calculation on the perihelion Mercury requires an implicit perturbation approach of

calculation for a many-body problem. The problem is, however, that the Einstein field equation cannot have a dynamic solution even for a two body problem [8]. Thus, the claim that general relativity is already an over all improvement from Newtonian theory is actually exaggerated.

Since the non-existence of dynamic solutions for the Einstein equation was not recognized at the beginning, subsequently based on such a false result, mathematicians naturally obtain further incorrect conclusions. For instance, if physics requires a bounded solution for a two-body problem such as the binary pulsars of Hulse and Taylor, the coupling constants of the Einstein equation cannot be unique [4]. However, the physical assumption of unique sign for all the coupling constants is the crucial implicit assumption of the space-time singularity theorems of Penrose and Hawking [9].

II. YAU'S WORK AND ERRORS ON GENERAL RELATIVITY

Another example to illustrate this problem is the work of mathematician Yau. Now, it is clear that the contribution of Prof. Yau to physics is at least over estimated and even incorrect while his contribution to mathematics in other areas is underestimated. From the free encyclopedia Wikipedia, the contributions of Professor Yau were summarized as follows:

"Yau's contributions have had a significant impact on both physics and mathematics. Calabi-Yau manifolds are among the 'standard tool kit' for string theorists today. He has been active at the interface between geometry and theoretical physics. His proof of the positive energy theorem in general relativity demonstrated—sixty years after its discovery—that Einstein's theory is consistent and stable. His proof of the Calabi conjecture allowed physicists—using Calabi-Yau compactification—to show that string theory is a viable candidate for a unified theory of nature."

Thus, it was claimed that Yau's "proof" of the positive energy theorem [10, 11] in general relativity would have profound influence that leads to even the large research efforts on string theory. Based on his proof, it was claimed that Einstein's theory is consistent and stable. This would be in a direct conflict with the fact that there is no dynamic solution for the Einstein equation. Moreover, Yau's proof would also be

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inconsistent with the space-time singularity theorems of Penrose and Hawking that use the same implicit assumption of unique sign for all the coupling constants.

Thus, it is clear that, in terms of physics, there should be problems in Yau's proof since in 1995 Einstein's theory has been proven inconsistent without the necessary rectifications [12]. The controversy of whether the Einstein equation has dynamic solutions was raised in 1921 by Gullstrand [7]. Now, this conclusion of non-existence of dynamic solution is no longer in doubt because there are explicit examples at the undergraduate level to illustrate such mathematical errors [2, 4].

Moreover, there are some well-known problems related to this issue. To mention a few, they are as follows:

1. It is known that Einstein's covariance principle is not valid as pointed out by Zhou Pei-Yuen of Peking University [13,14] and its invalidity is proven recently with counter examples [15]. Being an outstanding physicist, Zhou actually identified this problem and questioned Einstein directly when they met in 1937 although Zhou wrote papers on this subject only after 1982. On the other hand, Yau probably still believed Einstein's covariance principle since C. N. Yang still incorrectly advocates his notion of unconditional gauge invariance [16,17].
2. A related problem is Einstein's theory of measurement since the covariance principle was created to remedy the short comings of his theory of measurement. Note that such a theory was criticized by Whitehead as clearly invalid in physics [18] and is inconsistent with the observed light bending [19]. However, these criticisms were rejected although the light bending is calculated alternatively to avoid the obvious conflict [20]. On the other hand, Yau probably was unaware that Einstein's theory of measurement would lead to inconsistency in the Schwarzschild solution [1, 4].
3. A major reason that theorists held on to Einstein's covariance principle is due to that his theory of measurement was justified with special relativity. Recently, it is proven that Einstein's justifications are based on invalid applications of special relativity. Thus, the invalidity of Einstein's covariance principle is finally settled [12]. On the other hand, Yau probably was unaware that Einstein failed to justify his theory of measurement.
4. In fact, Einstein's covariance principle and Einstein's equivalence principle [5, 21] actually are directly in conflict [22]. However, many misidentified Einstein's equivalence principle of 1916 wrongly as the equivalent assumption of 1911 [23], which have been proven invalid by observations [19]. Moreover, due to an inadequate background in mathematics, the Wheeler School and her associates [1, 18] misinterpreted Einstein's equivalence principle, and created the invalid notion of local Lorentz invariance [24] (as a distortion of the Einstein-Minkowski condition [5, 21]). On the other hand, Yau probably was unaware of this because it was likely that Yau learned general relativity through second hand information [9, 20, 25] instead of from the work of Einstein directly [5, 21].
5. The famous formula $E = mc^2$ is only conditionally valid [26]. In fact, Einstein failed to prove the formula for other cases beyond the photons in spite of his efforts of several years (1905-1909) [27]. Moreover, Einstein or others actually have not completed the proof even for the case of photons [28]. It is safe to assume that Yau was unaware of this common error among physicists.
6. It has been proved in 1993 [29] and published in 1995 [8, 30] that there is no dynamic solutions for the Einstein equation. In fact, Yau and I have discussed this issue in the Chinese University of Hong Kong in 1993. It was expected that Yau would improve general relativity, but he only avoided this issue by claiming a loss of earlier interest.
7. It has been further proved that the book of Christodoulou and Klainerman [31] that claimed the existence of a dynamic solution (published by Princeton University as a mathematical classic) is actually invalid [32-34], due to elementary errors at undergraduate level in mathematics. I was informed that Yau also avoided the participation in the selection of Christodoulou for the 2011 Shaw Prize for mathematics [2].

Nevertheless, after the necessary rectifications, Einstein's general relativity can prove the need of unification, a dream of Einstein that he himself failed to complete [1]. Thus, one may wonder what went wrong in Yau's proof. Many believed that Yau and Richard Schoen proved the long-standing conjecture that the total mass in general relativity is positive.

Now, let us examine their theorem that would imply that flat space-time is stable, a fundamental issue for the theory of general relativity. Briefly, the positive mass conjecture says that if a three-dimensional manifold has positive scalar curvature and is asymptotically flat, then a constant that appears in the asymptotic expansion of the metric is positive. A crucial assumption in the theorem of Schoen and Yau is that the solution is asymptotically flat. However, since the Einstein equation has no dynamic solution, which is bounded, the assumption of asymptotically flat implies that the solution is a stable solution such as the Schwarzschild solution, the harmonic solution, the Kerr solution, and etc.

Therefore, Schoen and Yau actually prove a trivial result that the total mass of a stable solution is positive. Note that since the dynamic case is actually excluded from the consideration in the positive energy theorem, this explains why it was found from such a

theorem that Einstein's theory is consistent and stable. This is, of course, misleading.

In fact, Yau [10] and Christodoulou [31] make essentially the same error of defining a set of solutions that actually includes no dynamic solutions [32-34]. Their fatal error is that they neglected to find explicit examples to support their claims. Had they tried, they should have discovered their errors.

The problem started around 1981 because Yau did not understand the physics of the non-linear equation of Einstein. Note that Yau has wisely avoided committing himself to the errors of Christodoulou & Klainerman, by claiming that his earlier interest has changed [31]. However, he was unable to see that the binary pulsars experiment of Hulse & Taylor not only confirms that there is no dynamic solution but also that the signs of coupling constants are not unique [32]. In fact, Yau has made the same errors of Penrose and Hawking [9], and implicitly uses the invalid assumption of unique sign in his positive energy theorem of 1981. Nevertheless, Prof. Yau is a good mathematician as shown by his other works although he does not understand physics well. Since the Einstein equation must be modified for a dynamic case, their positive energy theorem is also irrelevant to physics just as the space-time singularity theorem of Penrose and Hawking.

Unfortunately, Yau also made the crucial mistake of assuming that famous theorists would understand physics correctly. Moreover, even Einstein and Princeton University can make a crucial error in spite of the warning of Gullstrand [7], a member of the 1921 Nobel Committee. Who could have discovered that the Wheeler School actually does not understand Einstein's equivalence principle if you do not read carefully and question them [3]?

Due to the influences of the Wheeler School, general relativity is incorrectly believed as effective only for large scale problems. Thus, the study for the applications of general relativity on earth and understanding material structure is neglected [1-4]. Moreover, due to the invalid speculation of unconditional $E = mc^2$, the study of unification between electromagnetism and gravitation failed. The facts that Hilbert and Yau were unable (or neglected) to identify their errors, they misleadingly created a false impression that the Wheeler School did not make errors in mathematics.

In China, it seems that there is little theoretical progress in the field of gravitation after Zhou Pei-Yuan. Currently, the incorrect views of L. Z. Fang and C. N. Yang dominate the opinions of the Chinese academy. C. N. Yang incorrectly disagrees with Zhou Pei-Yuan because of the invalid notion of gauge invariance [16, 17]. However, experiments of gravitational physics have very interesting developments although this is not matched by their theoretical developments [35].

III. IMPORTANT OMISSIONS ON YAU'S CONTRIBUTION IN MATHEMATICS

While some physicists, due to their own errors, keep attributing credits to Yau's earlier work in physics, his interest has shifted to mainly mathematics since 1993. It is known that Yau has an ambition on solving the Poincare conjecture, and he does make important contribution as pointed out by Hamilton. However, after Yau's contribution at the early stage and the subsequent important progress of Hamilton, unexpectedly Perelman claimed a complete proof by pointing out several sub-conjectures without the necessary explicit proofs that should have been there.

Being a seasoned mathematician in this field, Yau probably has already understood the extent of difficulty in obtaining such a proof. Naturally, Yau would doubt the claim of Perelman as genuine. And thus, in my view and probably in the view of Yau, Perelman's claim is a way to gain extra time needed for his efforts. In any case, there is no harm done to complete the proof himself with a team including former students Cao and Zhu. After more than three years, Yau's team succeeded as the first in completing the proof, and Yau wisely just acts as a referee. This step makes a speedy publication of the paper and thus denies the competitors the opportunity to play a delay tactic. This further infuriated the losers and they become irrational and decided to deny the due credit of the team.

However, Yau's team did nothing wrong although their action is rather unusual. Nevertheless, in Wikipedia Yau's crucial contribution in solving the Poincare conjecture is omitted. It is well known that he leads Cao and Chu to finish the final stage of solving the Poincare conjecture. Although Yau did the pioneer work that lays the foundation before even Hamilton, who laid the subsequent basis for solving the conjecture, some throw mud at Yau and give such a credit unfairly to Perelman, who has a very considerable contribution, but with questionable honesty.

In fact, Perelman only created several sub-conjectures of which most were proven correct years later. However, he claimed having solved them all when published in the NET (and thus avoided questioning). Nevertheless, one of his sub-conjectures is still not yet proved not only after Cao and Zhu have completed the proof of the Poincare conjecture but also unclear even now. It is well known that conjectures are much easier to make than to prove them. This is especially true for mathematical analysis. Also, Perelman's several unexplained acts of disappearance would strengthen the suspicion of avoiding questions from colleagues after having taken such a deceptive short cut [36]. The fact remains that there is no evidence that Perelman has actually proved the conjecture. Nevertheless, some theorists still unfairly claimed that Perelman has solved

the conjecture, but they make such an approval months after the publication of the work of Cao and Zhu.

It is amazing that not only nobody from the mathematical community of China, except Yau's students, stands out for the team of Yau et al. for a fair credit. On the contrary, Tian (Yau's student) of Peking University made shameless statements of belittling their Chinese colleagues. Professor Tian also distinguished himself by telling Perelman privately that he is correct soon after he made the sub-conjectures – an act ignored by Perelman as probably fishing for information [36]; but failed to claim Perelman's being essentially correct publicly immediately after the publication of Cao and Zhu.

IV. DISCUSSIONS AND CONCLUSIONS

It has been shown that two out-standing mathematicians Hilbert and Yau produced useless misleading results in physics because they do not understand the related physics although they may still be mathematically correct. Nevertheless, their reputation may have misleadingly covered up the errors in physics with seeming validity in mathematics. Hilbert later recognized his mistake of neglecting the crucial many-body problem. Yau is not as lucky because he did not try to find explicit examples to illustrate his conclusion. Thus, for the benefit of those working on gravitation, the misleading nature of the positive energy theorem of Schoen and Yau must be exposed.

A lesson to be learned is that, to avoid misleading understanding in physics, it is necessary to provide examples to illustrate crucial conclusions. The facts that S. T. Yau was awarded a Fields Medal in 1982¹⁾, for which the positive energy theorem is cited as an achievement and that E. Witten was awarded another Fields Medal in 1990, for which an alternative proof of the positive energy theorem is cited as an achievement,²⁾ testifies that many pure mathematicians and applied mathematicians just do not understand physics. Thus, an error of Einstein in physics [1] becomes a formidable problem in mathematics with a misleading result in physics. Since Witten is leading the research in the string theory, understandably there is little useful result in physics.³⁾ Moreover, from the experience of general relativity, it is clear that the development of theoretical physics requires improvements in the communication between physicists and mathematicians.

Currently, it is generally recognized that applied mathematics is a necessary skill in physics, but many incorrectly regarded pure mathematics as not useful in physics. Moreover, errors were made because theorists often confuse mathematic and physics [1]. These must be resolved. If the Nobel Prize Committee also establishes a prize in mathematics, the communication between physicists and mathematicians would be

greatly improved [1]. Moreover, from the experience of Yau, it is clear that the Chinese academy is still not strong enough in some areas of physics and mathematics. This is manifested by the fact that she seldom has a voice that is independent to their colleagues outside China.

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Endnote

- 1) S. T. Yau, 1982, Fields Medal, for "his contributions to partial differential equations, to the Calabi conjecture in algebraic geometry, to the positive mass conjecture of general relativity theory, and to real and complex Monge–Ampère equations".
- 2) Ludwig D. Faddeev, the Chairman of the Fields Medal Committee, wrote ("On the work of Edward Witten"): "Now I turn to another beautiful result of Witten proof of positivity of energy in Einstein's theory of gravitation. Hamiltonian approach to this theory proposed by Dirac in the beginning of the fifties and developed further by many people has led to the natural definition of energy. In this approach a metric γ and external curvature h on a space-like initial surface $S^{(3)}$ embedded in space-time $M^{(4)}$ are used as parameter in the corresponding phase space. These data are not independent. They satisfies Gauss-Codazzi constraints highly non-linear PDE, The energy H in the asymptotically flat case is given as an integral of indefinite quadratic form of $\nabla \gamma$ and h . Thus, it is not manifestly positive. The important statement that it is nevertheless positive may be proved only by taking into the account the constraints – a formidable problem solved by Yau and Schoen in the late seventy as Atiyah mentions, 'leading in part to Yau's Fields Medal at the Warsaw Congress'. Witten proposed an alternative expression for energy in terms of solutions of a linear PDE with the coefficients expressed through γ and h "
- 3) *Time* magazine stated that E. Witten was widely thought to be the world's greatest living theoretical physicist. This is, of course, based on the assumption that if his string theory is right. However, Witten once called string theory "a bit of 21st century physics that somehow dropped into the 20th century." So, Witten's physics is essentially based on imagination rather than observation. In fact, Witten didn't even set out to be a scientist. He majored in history at Brandeis and originally planned to be a journalist but ended up getting a Ph.D. in physics

instead. Thus, his work is often elegant in mathematics, but inadequate in physics. For instance, he gives an elegant alternative proof for the positive energy theorem, but failed to see that such a theorem has little meaning in physics since it would be valid only for the static case because of the constraints used.

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