Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 7 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

On Semi-Open Sets and Semi-Separability

By V. Srinivasa kumar
JNTU University, India

Abstract - In this paper, we introduce the concepts of semi-limit and semi-separability. We prove that separability and semi-separability are equivalent and also prove a few interesting results in this connection.

Keywords : semi-open, semi-closed, semi-closure, semi-neighborhood, semi-limit, semiseparability, sepa-rability, semi-topology.

GJSFR-F Classification : AMS Code: 54A10, 54A20, 54C08

ON SEMI-OPEN SETS AND SEMI-SEPARABILITY

Global Journal of Science Frontier Research (F) Volume XIII Issue VII Version I Year 2013

Ref

2. Levine, N. *Semi-open sets and semi-continuity in topological space*, Amer. Math Monthly, 70, pp. 36-41, 1963.

On Semi-Open Sets and Semi-Separability

V. Srinivasa kumar

Abstract - In this paper, we introduce the concepts of semi-limit and semi-separability. We prove that separability and semi-separability are equivalent and also prove a few interesting results in this connection.

Keywords : semi-open, semi-closed, semi-closure, semi-neighborhood, semi-limit, semiseparability, separability, semi-topology.

I. Introduction

In 1963, Norman Levin introduced the concept of semi-open sets in his paper [2]. It has drawn the attention of various authors including Crossley, Hildebrand and Dorsett and they have probed deeply into this area and developed many interesting concepts like semi-closed sets, semi-compactness etc. In this present paper, we introduce the concepts of semi-limit and semi-separability and prove that semi-separability is equivalent to separability. Also we construct a topology using semi-open sets and we call this topology a semi-topology.

In what follows (X,T) stands for a topological space. The symbols $cl(\ )$ and $Int(\ )$ denote the closure and interior in a topological space respectively.

a) *Preliminaries*

1.1 Definition: Let A be a subset of X. A is said to be

(i) semi-open in (X,T) if $A \subseteq cl(Int(A))$.

(ii) semi-closed if X - A is semi-open in (X,T).

(iii) semi-neighborhood of a point $x \in X$ if $x \in A$ and A is semi-open in (X,T).

1.2 Definition: The semi-closure of a set A in (X,T) denoted by $scl(A)$, is the intersection of all semi-closed supersets of A.

1.3 Definition: A point $x \in X$ is said to be a semi-limit point of a set A in (X,T), if every semi-neighborhood of x contains a point of A different from x in X.

Author : Assistant Professor, Department of Mathematics, JNTUH College of Engineering, JNTU, Hyderabad-500085, A.P., India.
E-mail : srinu_vajha@yahoo.co.in

b) Semi-Limit and Semi-Topology

2.1 Definition: Let $x \in X$ and let $\{ x_\lambda / \lambda \in \Delta \}$ be a net in (X, T). We say that x is a semi-limit of $\{x_\lambda / \lambda \in \Delta\}$ and we write $x = s.\lim_{\lambda \in \Delta} x_\lambda$ if for every semi-neighborhood A of x in X there exists a $\lambda_A \in \Delta$ such that $x_\lambda \in A \quad \forall \lambda \geq \lambda_A$.

2.2 Proposition: For $A \subset X$ and $x \in X$, the following are equivalent.

(i) x is a semi-limit point of A

(ii) there exists a net $\{x_\lambda / \lambda \in \Delta\}$ in A such that $x = s.\lim_{\lambda \in \Delta} x_\lambda$

(iii) $x \in scl(A)$

2.3 Remark: Let $S(T)$ be the collection of all semi-open sets in (X, T). The set $S(T)$ clearly contains T and is closed under arbitrary unions. However, being not closed under finite intersections, $S(T)$ is not a topology on X. However, if $A \in T$ and $B \in S(T)$ then $A \bigcap B \in S(T)$.

2.4 Definition: We define $S_0(T) = \{A \in S(T) / A \bigcap B \in S(T) \ \forall B \in S(T)\}$ and $S_{00}(T) = \{A \in S(S_0) / A \bigcap B \in S(S_0) \ \forall \ B \in S(S_0)\}$ where $S(S_0)$ is the collection of all semi-open sets in the topological space $(X, S_0(T))$.

2.5 Proposition:

(a) $S_0(T)$ and $S_{00}(T)$ are topologies on X.

(b) $T \subseteq S_0(T) \subseteq S(T)$.

(c) $S(S_0) \subseteq S(T)$.

(d) $S_0(T) = S_{00}(T)$.

2.6 Remark: We call the topology $S_0(T)$, a semi-topology on X.

2.7 Notation: We denote the closure of a subset A of X in the topological space $(X, S_o(T))$ by the symbol $cl_0\,(A)$ and interior of A by $Int_o(A)$.

2.8 Proposition: For $A \subseteq X$, $scl(A) \subseteq cl_0(A) \subseteq cl\,(A)$.

c) Semi-Separability

3.1 Definition: (X, T) is said to be separable if there exists a countable subset A of X such that $cl(A) = X$.

Notes

Notes

3.2 Definition: (X,T) is said to be semi-separable if there exists a countable subset A of X such that $scl(A)=X$.

3.3 Proposition: (X,T) is separable if and only if it is semi-separable.

Proof: Suppose that (X,T) is separable.

$\Rightarrow$ there exists a countable subset A of X such that $cl\,(A)=X$.

Let $x\in X$ and G be a semi-neighborhood of x in (X,T)

$\Rightarrow$ $x\in G$ and G is semi-open in (X,T)

$\Rightarrow$ there exists $O\in T$ such that $O\subseteq G\subseteq cl\,(O)$.

Assume that $G\bigcap A=\phi \Rightarrow A\subseteq X-G\subseteq X-O$

$$\Rightarrow cl\,(A)\subseteq X-O \quad \Rightarrow X=X-O$$

$$\Rightarrow O=\phi \quad \Rightarrow G=\phi$$ which is a contradiction.

Hence $G\bigcap A\neq\phi$.

Thus each semi-neighborhood of x in (X,T) intersects A

$\Rightarrow$ $x\in s\,cl(A)$. Hence $s\,cl(A)=X$ $\Rightarrow$ X is semi-separable.

The converse follows from the definitions 3.1, 3.2 and the proposition 2.8.

3.4 Proposition: $(X,S_0(T))$ is semi-separable $\Leftrightarrow$ (X,T) is semi-separable.

Proof: Suppose that $(X,S_0(T))$ is semi-separable

$\Rightarrow$ there exists a countable subset A of X such that $cl_0(A)=X$

$\Rightarrow cl(A)=X$ $\Rightarrow (X,T)$ is separable and hence it is semi-separable.

Conversely suppose that (X,T) is separable $\Rightarrow (X,T)$ is semi-separable

$\Rightarrow$ there exists a countable subset A of X such that $scl(A)=X$

$\Rightarrow$ $cl_0(A)=X$ $\Rightarrow$ $(X,S_0(T))$ is separable and hence it is semi-separable.

II. Acknowledgements

I sincerely thank my Professor, Dr. ***I. Ramabhadrasarma*** for his guidance and encouragement in making this paper.

References Références Referencias

1. **Crossley, S., and Hildebrand, S.** *Semi-Closure,* Texas J.Sci., 22, pp. 99-112, 1970.
2. **Levine, N.** *Semi-open sets and semi-continuity in topological space,* Amer. Math Monthly, 70, pp. 36-41, 1963.
3. **Ramabhadra Sarma, I., and V. Srinivasa kumar** *On Semi-Open Sets and Semi-Continuity*, Journal of Advanced Studies in Topology, Vol. 3, No. 3, pp. 06-102, 2012.

Notes