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Estimation of Population Ratio in Simple Random Sampling using Variable Transformation

By Onyeka, A.C., Nlebedim, V.U. & Izunobi, C.H.
Federal University of Technology

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GJSFR-F Classification : *MSC 2010: 62D05*

ESTIMATION OF POPULATION RATIO IN SIMPLE RANDOM SAMPLING USINGVARIABLE TRANSFORMATION

Strictly as per the compliance and regulations of :

Estimation of Population Ratio in Simple Random Sampling using Variable Transformation

Onyeka, A.C. [α], Nlebedim, V.U. [σ] & Izunobi, C.H. [ρ]

Abstract - This paper proposes six new estimators of the population ratio (R) of the population means of two variables (y and x) in Simple Random Sampling (SRS) scheme, using a variable transformation of the auxiliary variable, x. Properties of the proposed estimators, including optimality conditions, are derived up to first order approximation, and conditions under which the proposed estimators perform better than the customary ratio estimator ($\hat{R} = \bar{y}/\bar{x}$) are also obtained. The results are supported with empirical illustrations, which show that some of the proposed estimators have relatively large gains in efficiency over the customary ratio estimator, $\hat{R}$ for the data set considered.

Keywords : variable transformation, ratio, product and regression-type estimators, mean squared error.

I. Introduction

It is a common practice, in sample surveys, to use information obtained on an auxiliary variable to improve the efficiency of estimates of the population mean and total of the study variable. In some studies, however, the ratio of the population means (or totals) of the study and auxiliary variables might be of great significance, hence the need to estimate such ratios. For instance, one might be interested in the unemployment rate, which is usually obtained as the ratio of the number of unemployed people to the number of employed individuals in a country's labour force. Other parameters that could be obtained as a ratio of two parameters include income per capita, which is the ratio of the total income of a household to the total number of members of the household; the average salary of workers in a given establishment or company, which is usually obtained as the ratio of the total salary funds of the establishment to the company's total number of employees; and the employment sex ratio of a country, which is the ratio of the number of employed men and women in the country's labour force.

The usual or customary estimator of the population ratio, $R = \bar{Y}/\bar{X}$, of the population means of two variables, y and x, under the simple random sampling scheme, is given as $\hat{R} = \bar{y}/\bar{x}$, which is the ratio of the sample means of the two variables [Cochran(1977)]. However, several authors have contributed to the problem of estimating the population ratio of two means, often utilizing information on single or more auxiliary variables. These include Singh (1965), Srivastava (1967), Srivastava et al. (1988), Upadhyaya et al. (2000), Khare and Sinha (2007), and Khare et al. (2012). In using information on one or more auxiliary characters to

Author α σ ρ : Department of Statistics Federal University of Technology PMB 1526, Owerri, Nigeria. E-mail : aloyonyeka@yahoo.com

estimate the population ratio, $R = \overline{Y}/\overline{X}$, the two variables, y and x, are considered as the study variables, while other variables, say $z_i \, (i = 1, 2, \cdots, k)$, are considered as auxiliary variables, known to have some strong correlation with the variables, y and x. This implies that after observing the variables, y and x, more funds would be required to obtain information on the auxiliary variables, z_i's. If the variable y is the study variable, as it is often the case in most practical surveys, then the variable x, together with the variables, z_i's, would all be considered as auxiliary variables, which require extra funds in order to be observed. In the present study, we restrict observations, and consequently, funding costs, to only two variables, y and x, taking the variable x as an auxiliary variable having strong correlation with the variable, y. The parameter of interest still remains the population ratio, $R = \overline{Y}/\overline{X}$, and the objective of the study is to estimate R using variable transformation of the (auxiliary) variable, x, on the assumption that the population mean ($\overline{X}$) of x is known.

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II. The Proposed Estimators

Let $y_i(x_i)$ be observations on two variables, and let a random sample of size n be drawn from a population of N units using simple random sampling without replacement (SRSWOR) scheme. Consider the variable transformation,

$$x_i^* = \frac{N\overline{X} - nx_i}{N - n}, \quad i = 1, 2, \cdots, N, \tag{2.1}$$

The transformation (2.1) has been used by authors like Srivenkataramana (1980), Singh and Tailor (2005), Tailor and Sharma (2009), and Sharma and Tailor (2010) to improve estimates of the population mean, $\overline{Y}$, under the simple random sampling scheme, and Onyeka (2013) under the poststratified sampling scheme. The associated sample mean is obtained as:

$$\overline{x}^* = (1 + \pi)\overline{X} - \pi\overline{x}, \qquad \pi = \frac{n}{N - n} \tag{2.2}$$

Using the (sample) means, $\overline{y}$, $\overline{x}$, and $\overline{x}^*$, and assuming knowledge of the population mean, $\overline{X}$, of the (auxiliary) variable, x, we propose the following six estimators of the population ratio, $R = \overline{Y}/\overline{X}$, in simple random sampling without replacement (SRSWOR) scheme:

$$\hat{R}_1 = \frac{\overline{y}}{\overline{x} - b(\overline{x}^* - \overline{X})} \quad \text{(regression-type estimator of sample mean, } \overline{x}\text{)} \tag{2.3}$$

$$\hat{R}_2 = \frac{\overline{y}}{\left(\frac{\overline{x}}{\overline{x}^*}\overline{X}\right)} = \frac{\overline{y}\overline{x}^*}{\overline{x}\overline{X}} \quad \text{(ratio-type estimator of sample mean, } \overline{x}\text{)} \tag{2.4}$$

$$\hat{R}_3 = \frac{\overline{y}}{\left(\frac{\overline{x}\overline{x}^*}{\overline{X}}\right)} = \frac{\overline{y}\overline{X}}{\overline{x}\overline{x}^*} \quad \text{(product-type estimator of sample mean, } \overline{x}\text{)} \tag{2.5}$$

$$\hat{R}_4 = \frac{\overline{y}}{\overline{x}^*} \quad \text{(transformed mean estimator, } \overline{x}^*\text{)} \tag{2.6}$$

$$\hat{R}_5 = \frac{\overline{y}}{\overline{x}^* - b(\overline{x} - \overline{X})} \quad \text{(regression-type estimator of transformed mean, } \overline{x}^*\text{)} \tag{2.7}$$

$$\hat{R}_6 = \frac{\bar{y}}{\left(\frac{\bar{x}^*}{\bar{x}}\bar{X}\right)} = \frac{\bar{y}\bar{x}}{\bar{x}^*\bar{X}} \quad \text{(ratio-type estimator of transformed mean, } \bar{x}^*\text{)} \tag{2.8}$$

where b is a suitable constant, often chosen to be very close to the population regression coefficient of y on x.

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It is worth mentioning here, that Adewara et al. (2012) proposed some modified estimators of the population mean, $\bar{Y}$, involving the transformed (sample) means, $\bar{x}^*$ and $\bar{y}^*$, having the relationships:

$$\bar{X} = f\bar{x} + (1-f)\bar{x}^* \text{ and } \bar{Y} = f\bar{y} + (1-f)\bar{y}^* \quad , \; f = n/N \tag{2.9}$$

This is quite worrisome in view of the fact that the transformed mean, $\bar{y}^*$, is a function of the population mean, $\bar{Y}$, which is usually unknown. If the population mean ($\bar{Y}$) of the study variable, y, is already known, then there is no need constructing estimators to estimate what is already known. Consequently, there seems to be no justification for the use of the transformed mean, $\bar{y}^*$, in estimating the population mean, $\bar{Y}$, as well as the population ratio, $R = \bar{Y}/\bar{X}$. In estimating the population ratio, in particular, the much one could do is to assume that one of the population means, (say $\bar{X}$), is known, hence the justification for the use of the transformed mean, $\bar{x}^*$. To use both the transformed means, $\bar{x}^*$ and $\bar{y}^*$, in constructing estimators of the population ratio is to suggest or assume that both the population means, $\bar{X}$ and $\bar{Y}$, are already known, which implies that the population ratio is equally known and needs not to be estimated in the first place.

Let $$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}} \text{ and } e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}} . \tag{2.10}$$

Then,

$$E(e_0) = E(e_1) = 0 \tag{2.11}$$

$$E(e_0^2) = \frac{V(\bar{y})}{\bar{Y}^2} = \left(\frac{1-f}{n}\right)\frac{S_y^2}{\bar{Y}^2} \tag{2.12}$$

$$E(e_1^2) = \frac{V(\bar{x})}{\bar{X}^2} = \left(\frac{1-f}{n}\right)\frac{S_x^2}{\bar{X}^2} \tag{2.13}$$

and

$$E(e_0e_1) = \frac{Cov(\bar{y},\bar{x})}{\bar{Y}\bar{X}} = \left(\frac{1-f}{n}\right)\frac{S_{yx}}{\bar{Y}\bar{X}} \tag{2.14}$$

where $S_y^2(S_x^2)$ is the variance of $y(x)$ and S_{yx} is the covariance of y and x.

To obtain the properties of the proposed estimator, $\hat{R}_1$, we first rewrite (2.3) in terms of e_0 and e_1 and expand up to first order in expected values, to obtain:

$$(\hat{R}_1 - R) = R[e_0 - (1+\pi b)e_1 - (1+\pi b)e_0e_1 + (1+\pi b)^2 e_1^2] \tag{2.15}$$

and

$$(\hat{R}_1 - R)^2 = R^2[e_0^2 + (1+\pi b)^2 e_1^2 - 2(1+\pi b)e_0e_1] \tag{2.16}$$

Taking the expectations of (2.15) and (2.16), and using (2.11) – (2.14) to make the necessary substitutions, gives the bias and mean squared error of the proposed estimators, $\hat{R}_1$, up to first order approximation, respectively as:

$$B(\hat{R}_1) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)(1+\pi b)\left[(1+\pi b)RS_x^2 - S_{yx}\right] \quad (2.17)$$

and

$$MSE(\hat{R}_1) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + (1+\pi b)^2 R^2 S_x^2 - 2(1+\pi b)RS_{yx}\right] \quad (2.18)$$

Following similar procedure, we obtain the biases and mean squared errors of the six proposed estimators, together with those of the usual or customary ratio estimator, $\hat{R} = \bar{y}/\bar{x}$, up to first order approximations as:

$$B(\hat{R}) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[RS_x^2 - S_{yx}\right] \quad (2.19)$$

$$B(\hat{R}_1) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)(1+\pi b)\left[(1+\pi b)RS_x^2 - S_{yx}\right] \quad (2.20)$$

$$B(\hat{R}_2) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)(1+\pi)\left[RS_x^2 - S_{yx}\right] \quad (2.21)$$

$$B(\hat{R}_3) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[(1-\pi+\pi^2)RS_x^2 - (1-\pi)S_{yx}\right] \quad (2.22)$$

$$B(\hat{R}_4) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\pi\left[\pi RS_x^2 + S_{yx}\right] \quad (2.23)$$

$$B(\hat{R}_5) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)(\pi+b)\left[(\pi+b)RS_x^2 + S_{yx}\right] \quad (2.24)$$

$$B(\hat{R}_6) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)(1+\pi)\left[\pi RS_x^2 + S_{yx}\right] \quad (2.25)$$

and,

$$MSE(\hat{R}) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + R^2S_x^2 - 2RS_{yx}\right] \quad (2.26)$$

$$MSE(\hat{R}_1) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + (1+\pi b)^2 R^2 S_x^2 - 2(1+\pi b)RS_{yx}\right] \quad (2.27)$$

$$MSE(\hat{R}_2) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + (1+\pi)^2 R^2 S_x^2 - 2(1+\pi)RS_{yx}\right] \quad (2.28)$$

$$MSE(\hat{R}_3) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + (1-\pi)^2 R^2 S_x^2 - 2(1-\pi)RS_{yx}\right] \quad (2.29)$$

$$MSE(\hat{R}_4) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + \pi^2 R^2 S_x^2 + 2\pi RS_{yx}\right] \quad (2.30)$$

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$$MSE(\hat{R}_5) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + (\pi+b)^2 R^2 S_x^2 + 2(\pi+b)RS_{yx}\right] \quad (2.31)$$

$$MSE(\hat{R}_6) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + (1+\pi)^2 R^2 S_x^2 + 2(1+b)RS_{yx}\right] \quad (2.32)$$

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Generally, the mean squared errors of the estimators could be written as:

$$MSE(\hat{R}_d) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\left[S_y^2 + \theta_d^2 R^2 S_x^2 - 2\theta_d RS_{yx}\right] \quad (2.33)$$

where

$$\theta_1 = (1+\pi b),\ \theta_2 = (1+\pi),\ \theta_3 = (1-\pi),\ \theta_4 = (-\pi),\ \theta_5 = [-(\pi+b)],\ \theta_6 = [-(1+\pi)] \quad (2.34)$$

III. Efficiency Comparison

The efficiencies of the six proposed estimators, $\hat{R}_d$, d = 1, 2, …, 6 are first compared to that of the customary ratio estimator, $\hat{R}$, in estimating the population ratio, R, of two population means in simple random sampling scheme. The performance of the proposed estimators among themselves is also considered. Further consideration is also given to the optimum or best estimators among the proposed set of estimators.

a) Efficiency of Proposed Estimators over the Customary Ratio Estimator, $\hat{R}$

Using (2.26) and (2.33), the proposed estimators, $\hat{R}_d$, d = 1, 2, …, 6 would perform better than the customary ratio estimator, $\hat{R}$, in terms of having a smaller mean squared error if:

$$\left.\begin{array}{ll} (1) & \theta_d < 1 \text{ and } B < R \\ \text{or} & \\ (2) & \theta_d > 1 \text{ and } B > R \end{array}\right\} \quad (3.1)$$

where θ_d is as given in (2.34), and $B = S_{yx}/S_x^2$ is the population regression coefficient of y on x. This shows that the proposed estimators are not always more efficient than the customary ratio estimator, $\hat{R}$. For instance, the proposed estimator, $\hat{R}_2$, in (2.4) would only be more efficient than the estimator, $\hat{R}$, if and only if B > R, since, from (2.2) and (2.34), the quantity, $\theta_2 = 1+\pi$ is greater than unity. This means that the customary ratio estimator, $\hat{R}$, would be more efficient than the proposed estimator, $\hat{R}_2$ for data sets in which the value of the population regression coefficient, B, is smaller than the value of the population ratio, R. However, it would be shown later in this study that the proposed estimators, under certain general optimality conditions, always perform better than the customary ratio estimator, $\hat{R}$.

b) Efficiency Comparison among the Proposed Estimators

Let $\hat{R}_j$ and $\hat{R}_k$, $j \neq k$, and $j,k = 1,2,\cdots,6$ be any two particular estimators from the six proposed estimators in (2.3) to (2.8). Then using (2.33), the estimator, $\hat{R}_j$ would be more efficient than the estimator, $\hat{R}_k$, in terms of having a smaller mean squared error, if:

$$\text{or} \qquad \left.\begin{array}{ll} (1) & \theta_j < \theta_k \text{ and } B < R\theta_k \\ (2) & \theta_j > \theta_k \text{ and } B > R\theta_k \end{array}\right\} \qquad (3.2)$$

where θ_j and θ_k are obtained from θ_d, as given in (2.34). For instance, in comparing the proposed estimators, $\hat{R}_2$ and $\hat{R}_3$, we observe, from (2.34) that $\theta_2 = 1+\pi$ and $\theta_3 = 1-\pi$, indicating that the quantity, θ_2, is greater than θ_3, since $\pi = \dfrac{n}{N-n}$ is always positive. Consequently, and by using condition (2) of (3.2), the proposed estimator, $\hat{R}_2$ would be more efficient than the proposed estimator, $\hat{R}_3$ if and only if, $B > R(1-\pi)$. Comparison of the efficiencies of the remaining proposed estimators could be carry out in a similar manner, using the efficiency conditions in (3.2).

c) Optimum Estimators

The optimum estimators, among the six proposed estimators, could be obtained by minimizing the mean squared error of the proposed estimators, $\hat{R}_d$, in (2.33) with respect to the quantity, θ_d defined in (2.34). Applying the least square method, this gives the optimum value of θ_d, say θ_d^0, as

$$\theta_d^0 = \frac{B}{R} \qquad (3.4)$$

with the associated minimum mean squared error of $\hat{R}_d$ obtained as:

$$MSE_{opt}(\hat{R}_d) = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)(1-\rho^2)S_y^2 \qquad (3.5)$$

where $\rho = S_{yx}/(S_y S_x)$ is the population correlation coefficient of the variables y and x.

Notice, from (2.3) and (2.7), that the proposed estimators, $\hat{R}_1$ and $\hat{R}_5$, particularly give us the opportunity to choose suitable values of the constant, b. This means that with both estimators, it is possible and a lot easier to meet the optimality condition (3.4) by making appropriate choice of the constant, b. Using (3.4) and (2.34), it follows that the proposed estimator, $\hat{R}_1$ would be an optimum estimator if we choose the value of b, say b_1^0, as:

$$b_1^0 = \frac{B-R}{\pi R} \qquad (3.6)$$

Similarly, the proposed estimator, $\hat{R}_5$ would be an optimum estimator if we choose the value of b, say b_5^0, as:

$$b_5^0 = -\left(\frac{B+\pi R}{R}\right) \qquad (3.7)$$

The associated minimum mean squared errors of the estimators, $\hat{R}_1$ and $\hat{R}_5$ are the same as already given in (3.5). Comparing (3.5) and (2.26), we obtain the difference between the

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mean squared error of the optimum estimators and that of the customary ratio estimator, $\hat{R}$, as:

$$\Delta = MSE(\hat{R}) - MSE_{opt}(\hat{R}_d) = \frac{1}{\overline{X}^2}\left(\frac{1-f}{n}\right)(\rho S_y - RS_x)^2, \qquad (3.8)$$

which is always greater than zero. Hence the optimum estimators, using the optimality condition (3.4), are always more efficient than the customary ratio estimator, $\hat{R}$, for the purpose of estimating the population ratio, R, of two population means, under the simple random sampling scheme.

IV. Numerical Illustration

The theoretical results obtained in the present study are illustrated here numerically, using the data given on page 171 of Johnston (1982). The data set is summarized as follows:

y = Percentage of hives affected by disease

x = Date of flowering of a particular summer species (number of days from January 1)

$N = 10$, $n = 4$, $\overline{Y} = 52$, $\overline{X} = 200$, $S_y^2 = 65.97338$, $S_x^2 = 84.01556$, $S_{yx} = -69.98292$

We assume, for illustration purposes, that we are interested in the ratio of the percentage of hives affected by the disease to the number of days of flowering. That is, $R = \overline{Y}/\overline{X}$. Then, the computed percentage relative efficiencies (PRE) of the six proposed estimators, $\hat{R}_d$, $d = 1, 2, \cdots, 6$, and the optimum estimators, $\hat{R}_d^0$, over the customary ratio estimator, $\hat{R} = \overline{y}/\overline{x}$ are displayed in Table 1.

Table 1 shows that apart from the estimator, $\hat{R}_2$, the remaining five proposed estimators are more efficient than the customary ratio estimator, $\hat{R}$, for the data under consideration, and the gains in efficiency of some of the estimators, like $\hat{R}_6$ and $\hat{R}_4$, are relatively large. Notice that the values of B and R are respectively obtained as $B = -0.83298$ and $R = 0.26$, showing that B is smaller than R. That is, $B < R$. Consequently, and using the efficiency condition (1) of (3.1), the proposed estimators would be more efficient than $\hat{R}$ only if the value of the associated θ_d is less than unity. Table 1 confirms this efficiency condition, since all the estimators whose values of θ_d are less than unity are found to perform better than the estimator, $\hat{R}$.

Table 1 : PRE of Proposed Estimators over the estimator, R

d	**Estimator**	π	$b = B$	θ_d	**MSE**	**PRE**
-	$\hat{R}$	0.6667		1	0.0004052	100
1	$\hat{R}_1$	0.6667	-0.83298	0.44465	0.0003123	130
2	$\hat{R}_2$	0.6667		1.66670	0.0005340	76
3	$\hat{R}_3$	0.6667		0.33330	0.0002953	137
4	$\hat{R}_4$	0.6667		-0.66670	0.0001659	244
5	$\hat{R}_5$	0.6667	-0.83298	0.16628	0.0002707	150
6	$\hat{R}_6$	0.6667		-1.66670	0.0000791	512
-	$\hat{R}_d^0$	**0.6667**		**-3.20377**	**0.0000288**	**1407**

Also to be observed from Table 1 is the fact that the first three proposed estimators, $\hat{R}_1$, $\hat{R}_2$ and $\hat{R}_3$ have smaller gains in efficiency than the last three proposed estimators, $\hat{R}_4$, $\hat{R}_5$ and $\hat{R}_6$. Notice, from (2.3) to (2.8), that while the first three proposed estimators have the sample mean, $\bar{x}$, as the lead statistic in the denominator, the transformed mean, $\bar{x}^*$, is the lead statistic in the denominator of the last three proposed estimators. Consequently, Table 1 suggests that estimators with the transformed mean, $\bar{x}^*$, as the lead statistic in the denominator are likely to be more efficient than those with the sample mean, $\bar{x}$, as the lead statistic in the denominator, when there is a strong negative correlation between the two variables, like we presently have in the data under consideration. The optimum estimators, as expected, are the most efficient estimators, in terms of having the smallest mean squared error when compared with the customary ratio estimator as well as all the proposed estimators.

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V. Concluding Remarks

Here, we have proposed and considered six new estimators of the population ratio (R) of two population means in SRSWOR scheme, using a variable transformation of the auxiliary variable, x. The biases and mean squared errors of the proposed estimators were obtained up to first order approximation. Conditions under which the proposed estimators perform better than the customary ratio estimator ($\hat{R} = \bar{y}/\bar{x}$) were derived. Also obtained were the optimality conditions under which some of the proposed estimators could become the best (optimum) estimators. The results of the study were supported and illustrated numerically. The empirical illustration confirmed, among other things, both the optimality and efficiency conditions, which we had earlier obtained theoretically in the study. The empirical study revealed that relatively large gains in efficiency over the customary ratio estimator could be obtained by using some of the new estimators proposed in the present study. Again, the direction (positive or negative) of the linear relationship between the two variables plays a role in identifying some of the proposed estimators that are likely to be more efficient than the others, for a given set of data. The first three proposed estimators make use of the sample mean, $\bar{x}$ as the lead statistic in the denominator, and are likely to perform better than the last three proposed estimators, when there is a strong positive linear relationship between the two variables. When there is a strong negative correlation between the variables, the last three proposed estimators, which incidentally make use of the transformed sample mean, $\bar{x}^*$, as the lead statistic in the denominator are likely to be more efficient than the first three proposed estimators. However, the best estimators to use for any given set of data could be obtained by using the optimality conditions given in (3.4).

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