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Abstract - LRS Bianchi Type-II space – time is considered in the presence of a perfect fluid source in the framework of f(R,T) gravity proposed by Harko et al. [Phy.Rev-D 84, 024020(2011)] . An exact cosmological model with an appropriate choice of the function f(RT) is obtained using a special law of variation for Hubble's parameter proposed by Bermann (Nuovo Cimento B 74. 182,1983). The physical behavior of the cosmological model is, also, studied.

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I. Introduction

Recent observational data suggests that our universe is accelerating. Supernova 1A data gave the first indication of the accelerated expansion of the universe [1-4]. Another major development in cosmology is that our universe is making a transition from a decelerating phase to an accelerating one. This was confirmed by anisotropies in the cosmic microwave background (CMB) radiation as seen in the data from satellite such as WMAP and large scale structure. Astrophysical observations indicate that the accelerated expansion of the universe is driven by exotic energy with large negative pressure which is known as dark energy (For a general complete review see [5]. In spite of all the observational evidences, the nature of dark energy is, still, a challenging problem in modern cosmology.

There are two major approaches to address the problem of cosmic acceleration. One approach is to introduce a dark energy component in the universe and study its dynamics. Another alternative approach is to modify general relativity itself. This is termed as 'modified gravity' approach [5-8]. Both the approaches have novel features with some deep theoretical problems. However, here, we are interested in modified gravity approaches, several modifications of general gravity have been proposed in the last few decades. Noteworthy among them are Brans-Dicke[9]and Saez-Ballester[10] scalar-tensor theories of gravitation. In Brans-Dicke gravity besides a gravitational part, a dynamical scalar field was introduced to account for variable gravitational constant This modification was introduced because of the fact that Einstein's theory does not fully incorporate Mach's Principle. Later Saez and Ballester formulated a scalar-tensor theory of gravity in which metric is coupled with a scalar field. Here the strength of the coupling between the gravity and the field is governed by a parameter ω. This theory also enables us to solve the 'missing mass" problem. Other than these approaches some authors [11-15] considered modified gravitational action by adding a function f(R) (R being Ricci scalar curvature) to Einstein-Hilbert Lagrangian where f(R) provides a gravitational alternative for dark energy causing late time acceleration of the universe. A comprehensive review on f(R) gravity is given by Copeland et al. [16].

Very recently, Harko et al. [17] developed another modified gravity known as f(R, T) gravity. In this the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor. Adhav[18] has presented spatially homogeneous and anisotropic Bianchi type–I model in f(R,T) gravity while Reddy et al,[19-20]] have studied Bianchi type-III and Kaluza-Klein cosmological models in this theory,

Motivated by the above investigations, in this paper, we obtain LRS Bianchi type – II cosmological model in f(R, T) gravity. Bianchi type – II space time are of vital importance in describing cosmological models at the early stages of evaluation of the universe. This paper is organized as follows: Sect. 2, presents a brief description of f(R, T) gravity. In Sect. 3, we derive f(R, T) gravity field equations for LRS Bianchi type – II metric. In Sect. 4, the solutions of field equations and the model are obtained. Sect. 5 is devoted to the discussion of physical and Kinematical properties of the model. The last section contains some useful conclusions.

II. F (R,T) Theory of Gravity

In f(R,T) gravity, the field equations are derived from a variational, Hilbert-Einstein type, principle.

The action for the modified f(R,T) gravity is

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \qquad (1)$$

Where f(R,T) is an arbitrary function of the Ricci scalar, R, T is the trace of stress-energy tensor of the matter, T_{ij} and L_m is the matter Lagrangian density. We define the stress-energy tensor of matter as

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$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \quad (2)$$

and its trace by $T = g^{ij} T_{ij}$ respectively. By assuming that L_m of matter depends only on the metric tensor components g_{ij}, and not on its derivatives, we obtain

$$T_{ij} = g_{ij} L_m - 2\frac{\partial L_m}{\partial g^{ij}} \quad (3)$$

Now by varying the action S of the gravitational field with respect to the metric tensor components g^{ij}, we obtain the field equations of f(R,T) gravity as

$$f(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)\,T_{ij} - f_T(R,T)\,\theta_{ij} \quad (4)$$

Where $\theta_{ij} = -2\,T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}$ (5)

Here $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$ $\Box = \nabla^i\nabla_i$, ∇_i is the covariant derivative and T_{ij} is the standard matter energy-momentum tensor derived from the Lagrangian L_m. It may be noted that when f(R,T) ≡f(R) the equations (4) yield the field equations of f(R) gravity.

The problem of the perfect fluids described by an energy density ρ, pressure p and four velocity u^i is complicated since there is no unique definition of the matter Lagrangian. However, here, we assume that the stress energy tensor of the matter is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \quad (6)$$

And the matter Lagrangian can be taken as $L_m = -p$ and we have

$$u^i\nabla_j u_i = 0, \quad u^i u_i = 1 \quad (7)$$

Then with the use of Equations (5) we obtain for the variation of stress-energy of perfect fluid the expression

$$\theta_{ij} = -2T_{ij} - p g_{ij} \quad (8)$$

Generally, the field equations also depend through the tensor θ_{ij}, on the physicsl nature of the matter field. Hence in the case of f(R,T) gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of f(R,T). Assuming

$$f(R,T) = R + 2f(T) \quad (9)$$

as a first choice where f(T) is an arbitrary function of the trace of stress-energy tensor of matter, we get the gravitational field equations of f(R,T) gravity from Eq. (4) as

$$R_{ij} - \frac{1}{2}R g_{ij} = 8\pi T_{ij} - 2f'(T)\,T_{ij} - 2f'(T)\theta_{ij} + f(T)\,g_{ij} \quad (10)$$

Where the prime denotes differentiation with respect to the argument.

If the matter source is a perfect fluid,

$$\theta_{ij} = -2T_{ij} - p g_{ij}$$

then the field equations become

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} + 2f'(T)\,T_{ij} + [2pf'(T) + f(T)]\,g_{ij} \quad (11)$$

III. Metric and Field Equations

We consider a homogeneous LRS Bianchi type–II space–time given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + 2B^2xdydz$$
$$(Bx^2 + A^2)dz^2 \quad (12)$$

where A and B are functions of cosmic time t.

Using co moving coordinates and equations (6)-(8), the f(R,T) gravity field equation(11) with the particular choice of the function (Harko et al.2011)

$$f(T) = \lambda T, \ \lambda, a\ constant \quad (13)$$

for the metric (12) take the form

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{1}{4}\frac{B^2}{A^4} = (8\pi + 3\lambda)p - \lambda\rho \quad (14)$$

$$]2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = (8\pi + 3\lambda)p - \lambda\rho \quad (15)$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}^2}{A^2} - \frac{1}{4}\frac{B^2}{A^4} = -(8\pi + 3\lambda)\rho + \lambda p \quad (16)$$

Where an overhead dot denotes differentiation with respect to cosmic time t.

IV. Solutions and the Model

The field equations (14)-(16) are a system of three independent equations in four unknowns A, B, ρ and p. Hence to obtain a determinate solution of the system we take the help of special law of variation proposed by Bermann [21] for Hubble's parameter that yields constant deceleration parameter models of the universe.

We consider constant deceleration parameter model defied by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = constant \quad (17)$$

where the scale factor a is given by

$$a = (A^2B)^{1/3} \quad (18)$$

Here the constant is taken as negative because our theory corresponds to the accelerating model of the universe.

The solution of (17) is

$$a = (ct + d)^{1/(1+q)} \quad (19)$$

Where c≠0 and d are constants of integration. Also this equation implies that the condition for cosmic acceleration is $(1 + q) > 0$

The scalar expansion θ , shear scalar σ^2 in the model (12) are defined by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \tag{20}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{21}$$

We also observe that the expansion θ in the model is proportional to the Shear scalar σ which leads to

$$A = B^m \tag{22}$$

Where m is constant [22]

Solving the field Equations (14)-(16) with the help of (19) and (22), we obtain the expressions for metric coefficients as follows

$$A = (ct + d)^{\frac{3m}{(2m+1)(1+q)}} \quad , \quad B = (ct + d)^{\frac{3}{(2m+1)(1+q)}} \tag{23}$$

After a suitable choice of constants (c=1,d=0) the metric (12) can be written as

$$\mathrm{ds}^2 = -\mathrm{dt}^2 + \mathrm{t}^{\frac{6\mathrm{m}}{(2\mathrm{m}+1)(1+\mathrm{q})}}\mathrm{dx}^2 + \mathrm{t}^{\frac{6}{(2\mathrm{m}+1)(1+\mathrm{q})}}\mathrm{dy}^2 + 2\mathrm{t}^{\frac{6}{(2\mathrm{m}+1)(1+\mathrm{q})}}\mathrm{xdydz} + \left(\mathrm{t}^{\frac{6}{(2\mathrm{m}+1)(1+\mathrm{q})}}\mathrm{x}^2 + \mathrm{t}^{\frac{6\mathrm{m}}{(2\mathrm{m}+1)(1+\mathrm{q})}}\right)\mathrm{dz}^2 \tag{24}$$

This represents LRS Bianchi type – II universe in f(R,T) gravity.

V. Some Physical Properties of the Model

Equation (24) represents a perfect fluid Bianchi type – II cosmological model in f(R,T) gravity with the following physical and Kinematical parameters:

The Spatial Volume is

$$V^3 = A^2 B = t^{\frac{3}{(1+q)}} \tag{25}$$

The Scalar Expansion is

$$\theta = \frac{3}{(1+q)} \tag{26}$$

The Shear Scalar is

$$\sigma^2 = \frac{3(m-1)^2}{(2m+1)^2\,(1+q)^2}\frac{1}{t^2} \tag{27}$$

The Generalized Hubble's parameter is

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{1}{(1+q)}\frac{1}{t} \tag{28}$$

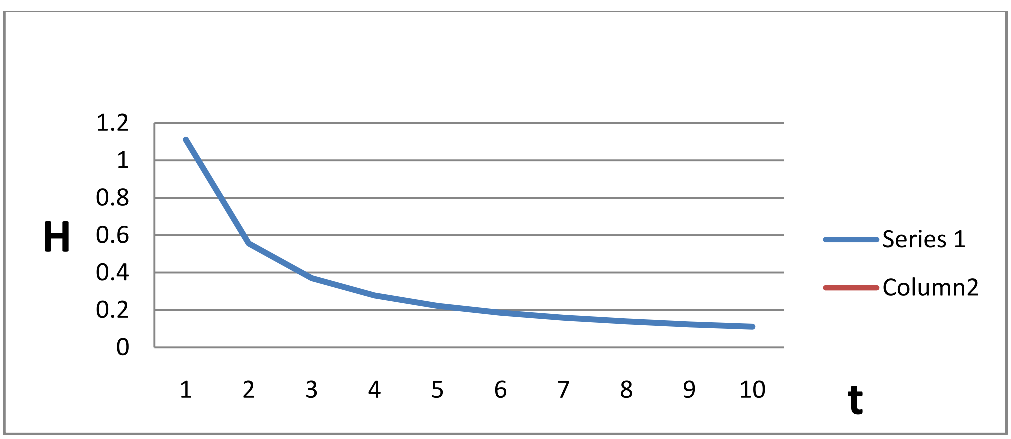

Figure 1 : The plot of Hubble's parameter H Vs. t. Here q= -0.1

The Average anisotropic parameter is

$$A_\alpha = \frac{1}{3}\Sigma\left(\frac{\Delta \mathrm{H}_1}{\mathrm{H}}\right)^2 \tag{29}$$

Where $\Delta H_l = H_l - H \ (l = 1,2,3)$

Which gives

$$A_\alpha = \frac{3}{4} \tag{30}$$

Also

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(2m+1)^2} \tag{31}$$

The Pressure in the model is

$$p = \frac{1}{(8\pi+3\lambda)}\left\{\left[(10m^2 - 12qm^2 - 6m - 6mq) + \frac{\lambda^3(2m^2-6m-9)(1+q) - \frac{(8\pi+3\lambda)}{\lambda}4m^2}{\lambda^3-(8\pi+3\lambda)^2}\right]\frac{t^{-2}}{(1+q)^2(2m+1)^2} + \left(\frac{\lambda^2(2\pi+\lambda)}{\lambda^3-(8\pi+3\lambda)^2} - \frac{3}{4}\right)t^{-\frac{6(2m-1)}{(2m+1)(1+q)}} - \frac{\lambda^2(16\pi+5\lambda)6m}{(\lambda^3-(8\pi+3\lambda)^2)(1+q)^2(2m+1)^2}t^{-\left(2+\frac{q}{(2m+1)(1+q)}\right)}\right\} \tag{32}$$

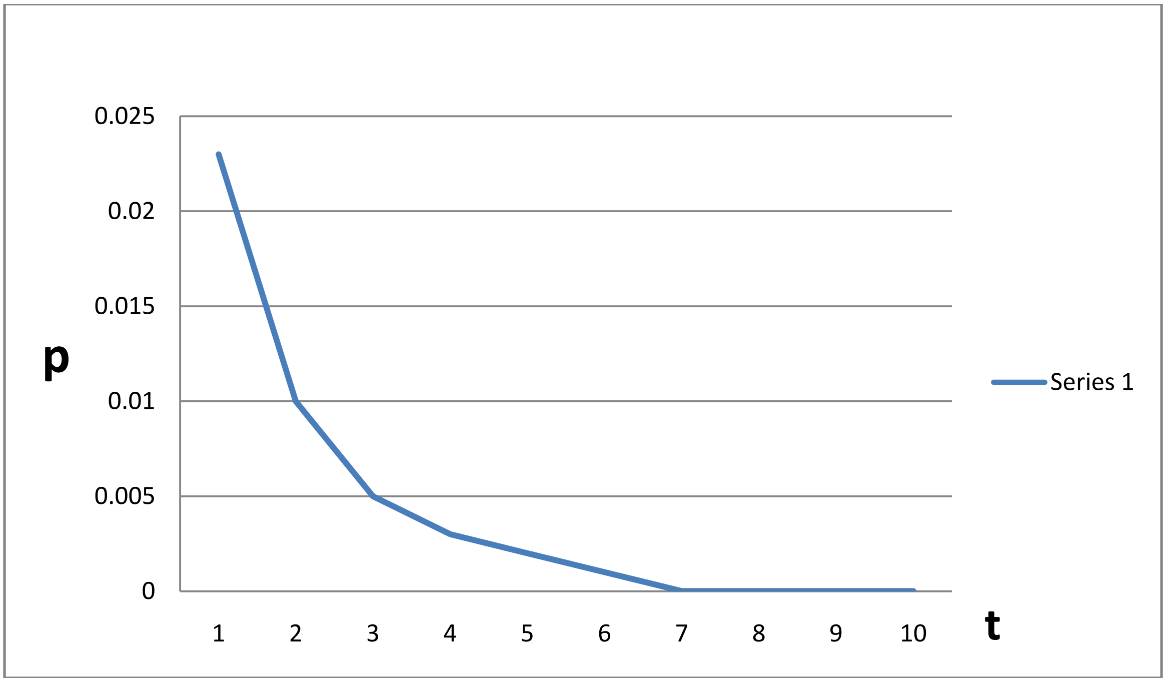

Figure 2 : The Plot of Pressure p Vs. t , Here q= -0.1, m=1.5, λ=1.5 π=3.143

The energy density in the model is

$$\rho = \frac{\lambda^2}{\lambda^3-(8\pi+3\lambda)^2}\left\{\left[(2m^2-6m-9)(1+q)-\frac{(8\pi+3\lambda)}{\lambda}4m^2\right]\frac{t^{-2}}{(1+q)^2(2m+1)^2}-\frac{(16\pi+5\lambda)6m}{\lambda(1+q)^2(2m+1)^2}t^{-\left(2+\frac{q}{(2m+1)(1+q)}\right)}+\left(\frac{2\pi+\lambda}{\lambda}\right)t^{-\frac{6(2m-1)}{(2m+1)(1+q)}}\right\} \tag{33}$$

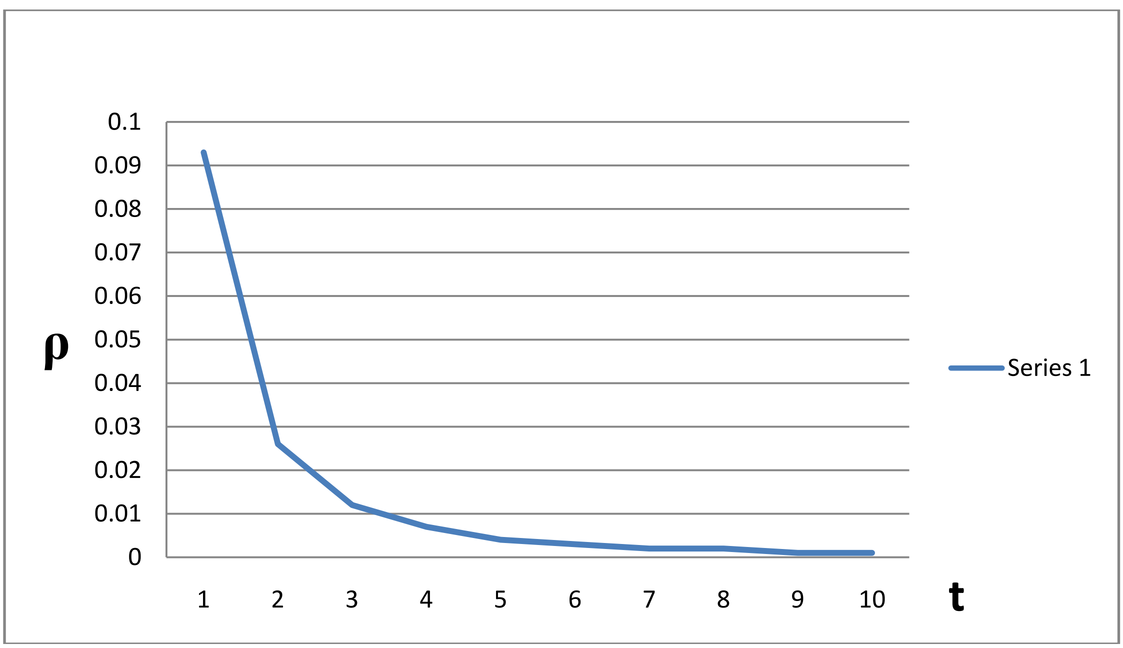

Figure 3 : The flot of energy density ρ Vs. t. Here q= -0.1, m=1.5, λ=1.5 π=3.143

From the above results it can be seen that the spatial volume is zero at t=0 and it increases with cosmic time showing the late time accelerated expansion of the universe. Also at t=0 the parameters θ, σ, H , ρ and p diverge while they vanish for infinitely large values of t. The mean anisotropic parameter is uniform through the whole evolution of the universe which shows that the dynamics of the mean anisotropic parameter does not depend on the cosmic time t. Also, since $\frac{\sigma^2}{\theta^2}$ is constant the model does not approach isotropy through the whole evolution of the universe. It may also be observed that the model (24) has no initial singularity.

VI. Conclusion

Here we have studied LRS Bianchi type-II cosmological model in the presence of perfect fluid in f(R,T) theory of gravity. It is observed that the model has no initial singularity and shows the late time accelerated expansion of the universe for large t. It is also observed that all the physical and kinematical parameters diverge at initial epoch while they approach zero for infinitely large t. The model obtained, in this paper, is of considerable interest and may be useful to study the large scale dynamics of the early universe in f(R,T) theory of gravity.

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