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Solution of Kinematic Wave Equation Using Finite Difference Method and Finite Element Method

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SOLUTION OF KINEMATIC WAVE EQUATION USING FINITE DIFFERENCE METHOD AND FINITE ELEMENT METHOD

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I. Introduction

Hydrology (from Greek : Yδωρ, hudōr, "water"; and λόγος, logos, "study") is the study of the movement, Distribution, and quality of water throughout the Earth and thus addresses both the hydrologic cycle and water resources. So in the broadest sense it is the study of water in all its phases and includes hydraulics, the physics and chemistry of water, meteorology, geology and biology. But the word as used by the scientists and engineers usually has a considerably narrower connotation. In this more limited sense, "Hydrology can be defined as that branch of physical geography, which is concerned with the origin. distributaries movement and properties of the waters of the Earth". The study of hydrology thus concerns itself with the occurrence and transportation of the waters through air, Over the ground and through the strata of the earth and this includes three important phases of what is known as the hydrological cycle, namely rainfall, runoff and evaporation. Hydrology is therefore, bounded above by meteorology, below by geology and at land's end by oceanology. Engineering hydrology includes those segments of hydrology pertinent to the design and operation of engineering projects for the control and use of water. Hydrology means the science of water. It is a branch of earth science. Basically it is an applied science.

Domains of hydrology include hydrometeorology, surface hydrology, hydrogeology, drainage basin management and water quality, where water plays the central role. In general sense hydrology deals with (i) Water resources estimation (ii) Acquisition of processes such as precipitation, runoff and evapo-transporation.

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II. Model Development

a) Kinemtic Wave Equations From Saint Venant Equations

The St. Venant equations characterizing the dynamic flow can be written as:

Continuity: $$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q+(i-\phi) \tag{1}$$

Momentum: $$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial y_0}{\partial x}=g(s_f-s_0)-q(\frac{u-v}{A}) \tag{2}$$

The equation (1) may be rewritten in the following form for a ready reference to the various types of wave models that are recognized.

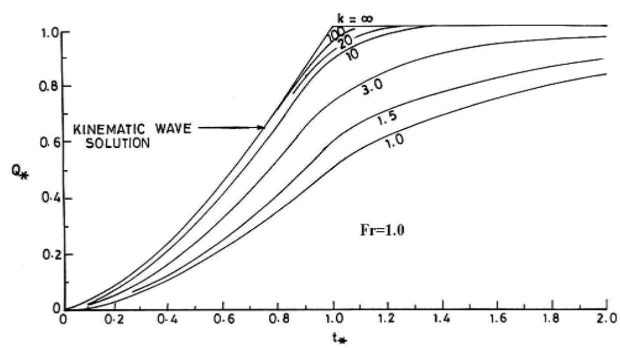
Term: I II III IV Equation of motion: $\frac{1}{g}\frac{\partial u}{\partial t}+\frac{u}{g}\frac{\partial u}{\partial x}+\frac{\partial y_0}{\partial x}+(s_f-s_0)=0$

Local Convective Depth acceleration acceleration slope

Wave model and terms used to describe it are:

Kinematics wave only term	IV= 0
Diffusion wave	III+ IV=0
Steady dynamic wave	II+III+IV=0
Dynamic wave	I+II+III+IV=0
Gravity wave	I+II+III= 0

and other terms are neglected.



b) Hydrodynamic Theory And Kinematic Wave Equations

The hydrodynamic theory for incompressible fluid flows gives the following set of equations (also known as the Navier-Stokes' equations):

$$\rho\,(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=X-\frac{\partial P}{\partial x}+\mu\,\nabla^2 u$$

$$\rho\,(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z})=Y-\frac{\partial P}{\partial y}+\mu\,\nabla^2 v$$

$$\rho\,(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z})=Z-\frac{\partial P}{\partial z}+\mu\,\nabla^2 w$$

Notes

and continuity equation: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

where $$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2};$$

ρ = the mass density ;
u, v and w are the velocity components in the x, y and z direction respectively;
X, Y, Z are the body forces per unit volume;
P = pressure and μ = viscosity.

c) Elements Used In Kinematics Wave Models

In this work, for computational purpose, the following two types of elements have been identified:

(i) Overland flow elements and

(ii) Channel flow elements (Fig. 3.1)

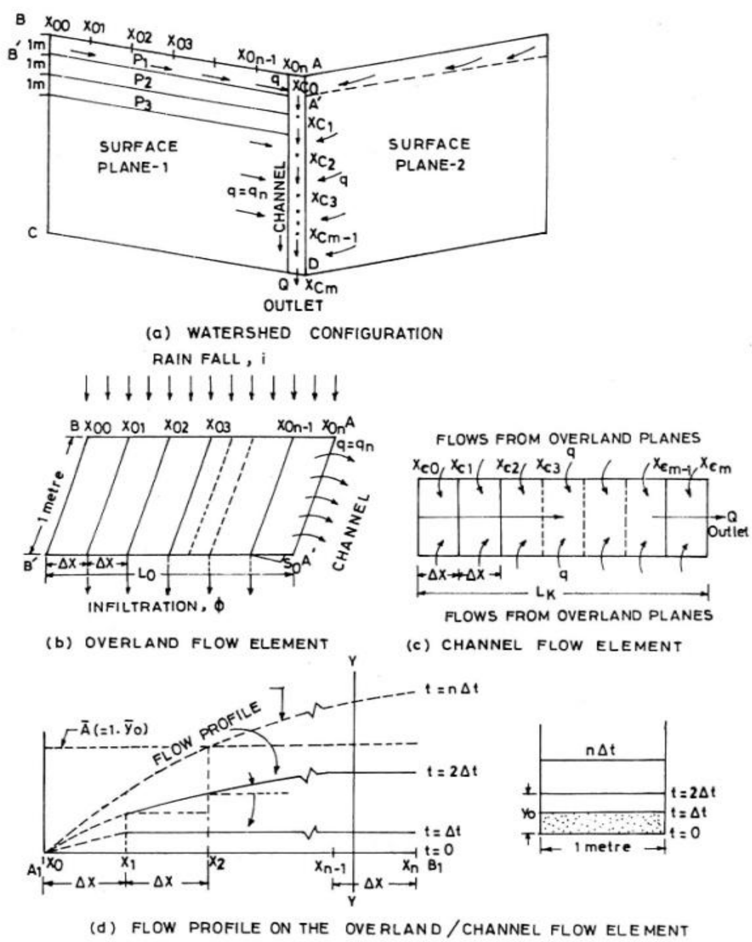


FIG. 3.1 GENERATION OF FLOW PROFILE

d) Trapezoidal Channel Cross Section

A trapezoidal cross-section is the most general type of channel cross-section. It is defined by the cannel side slope (Z), and the channel bottom width (B) (Fig.3.2).

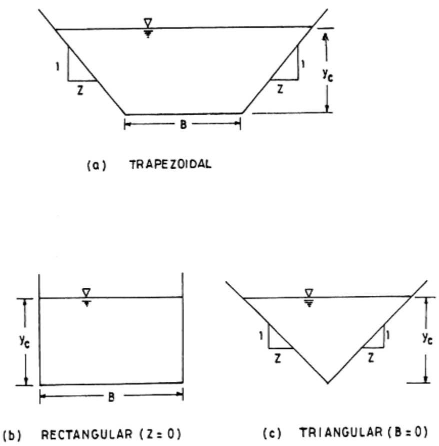


FIG. 3.2 CHANNEL SHAPES FOR KW CHANNEL ROUTING

e) The Final Form of Kinematic Wave Equations For The Channel Flows

The unknown parameters for the channel shapes under consideration i.e. α_k and m_k being the unknown functions. The KW equation for the channel flow can be written by combining equations (3.19) and (3.20) as given below:

$$\frac{\partial A}{\partial t} + \frac{\partial(\alpha_k A^{m_k})}{\partial x} = q$$

If α_k is independent of x, then the equation becomes:

$$\frac{\partial A}{\partial t} + \alpha_k m_k \frac{\partial(A^{m_k - 1})}{\partial x} = q$$

Crank-Nicolson and other methods:

First Order one-way wave equation

The first order wave equation in one-dimensional space is as follows:

$$u_t = cu_x$$

where c is a positive constant, and u(x, t) is subject to the initial condition

$$u(x, 0) = f(x), \qquad -\infty < x < \infty.$$

The solution for $t \geq 0$ and all x is a family of characteristics, which are straight lines shifted to the left in the x, t- plane, inclined to the x-axis at an angle

$$\Theta = \tan^{-1}(\tfrac{1}{c}).$$

The explicit solution is

$$u(x, t) = f(x + ct).$$

Notes

Table 1. Explicit finite difference schemes for first order 1-D wave equation

FD Scheme	Matrix Representation
Forward Euler (FEU) $u_j^{n+1} = u_j^n + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1 & \frac{1}{2}r & & \\ -\frac{1}{2}r & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -\frac{1}{2}r \\ 0 \\ \vdots \\ 0 \\ \frac{1}{2}r \end{bmatrix}$
Upwind (UPW) $u_j^{n+1} = u_j^n + r\left(u_{j+1}^n - u_j^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1-r & r & \\ & \ddots & \ddots \\ & & \ddots & \ddots \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ r \end{bmatrix}$
Lax-Friedrichs (LXF) $u_j^{n+1} = \frac{1}{2}\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 0 & \frac{1+r}{2} & & \\ \frac{1-r}{2} & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} \frac{1-r}{2} \\ 0 \\ \vdots \\ 0 \\ \frac{1+r}{2} \end{bmatrix}$
Lax-Wendroff (LXW) $u_j^{n+1} = u_j^n + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{1}{2}r^2\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1-r^2 & \frac{r(r+1)}{2} & & \\ \frac{r(r-1)}{2} & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} \frac{r(r-1)}{2} \\ 0 \\ \vdots \\ 0 \\ \frac{r(r+1)}{2} \end{bmatrix}$
Leapfrog (LFG) $u_j^{n+1} = u_j^{n-1} + r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \mathbf{u}^{n-1} + \begin{bmatrix} 0 & r & & \\ -r & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -r \\ 0 \\ \vdots \\ 0 \\ r \end{bmatrix}$
Fourth-order Leapfrog (LF4) $u_j^{n+1} = u_j^{n-1} + \frac{4}{3}r\left(u_{j+1}^n - u_{j-1}^n\right) - \frac{1}{6}r\left(u_{j+2}^n - u_{j-2}^n\right)$	$\mathbf{u}^{n+1} = \mathbf{u}^{n-1} + \begin{bmatrix} 0 & \frac{4}{3}r & -\frac{1}{6}r & & \\ -\frac{4}{3}r & \ddots & \ddots & \ddots & \\ \frac{1}{6}r & \ddots & \ddots & \ddots & \ddots \\ & \ddots & \ddots & & \\ & & \ddots & & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -\frac{4}{3}r \\ \frac{1}{6} \\ 0 \vdots \\ -\frac{1}{6}r \\ \frac{4}{3}r \end{bmatrix}$

Finite Element Formulation for Solving KW Equation: $\left.\frac{\partial h}{\partial x}\right|_{x=x_j} = \frac{h_{j+1} - h_{j-1}}{2\Delta x}$

$$h(x,t) = \sum_{j=1}^{M} \Phi_j(x)\, h_j(t)$$

Channel Discretization and Selection of Approximations Functions

The flow equations are one-dimensional. The channel is divided into small reaches called elements. Each element will be modeled with the same flow equations but with different channel geometry and hydraulic parameters. The elements equations are later assembled into global matrix equations for solution. By applying the Galerkin's principle to the continuity equation the following equation is obtained:

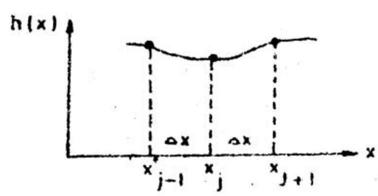


Figure (A) : Finite Difference Computational Mesh

Figure (B) : Finite Element Computational Mesh

$$\sum_{i=I}^{K-1} \int_{x_K}^{x_{K+I}} N^T \left(\frac{\partial y}{\partial t} + y\frac{\partial v}{\partial x} + v\frac{\partial y}{\partial x} - q(x,t)\right) dx = 0$$

Where $\sum_{1}^{K-1}$ is the expression for summary individual element equation from 1 to (k-1) elements; N^T transpose to the shape functions.
Using the shape functions, Equations may be written as

$$\sum_{1}^{K-1}\int_{0}^{1} N^T \frac{\partial y}{\partial t} + Y\frac{\partial v}{\partial x} + v\frac{\partial y}{\partial x} - q(x,t)\, Lds = 0$$

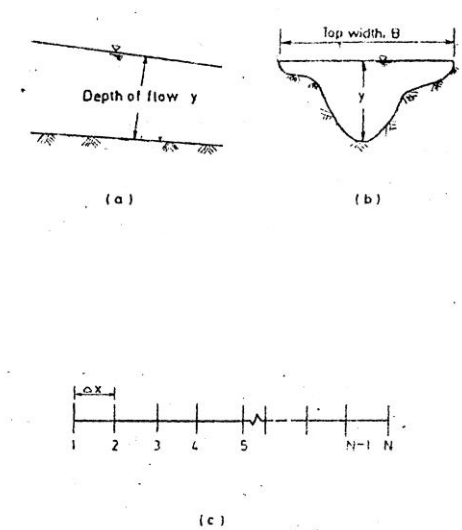


Figure : Natural Idealized Flow Sections (a) Longitudinal Profile (b) vertical Cross sectional Area Flow (c) Longitudinal Channel discretized into finite elements.

Evaluating each term of Equation (5.24) the following elements equation may be written:

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial y_1}{\partial t} \\ \frac{\partial y_2}{\partial t} \end{Bmatrix} + \frac{1}{6}\begin{bmatrix} (2y_1 + y_2)(v_2 - v_1) \\ (y_1 + 2y_2)(v_2 - v_1) \end{bmatrix} + \frac{1}{6}\begin{bmatrix} (2v_1 + v_2)(y_2 - y_1) \\ (v_1 + 2v_2)(y_2 - y_1) \end{bmatrix} - l_q\begin{Bmatrix} 3 \\ 3 \end{Bmatrix} = 0$$

Similar way the momentum Equation for an element can be derived as

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial v_1}{\partial t} \\ \frac{\partial v_2}{\partial t} \end{Bmatrix} + \frac{1}{12}\begin{bmatrix} -2v_1 - v_2 & -v_1 - 2v_2 \\ 2v_1 + v_2 & v_1 + 2v_2 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} + \frac{q}{l}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} + \frac{l_q}{2}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} (\frac{v}{y})_1 \\ (\frac{v}{y})_2 \end{Bmatrix}$$

$$+\frac{q}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} S_{f_1} \\ S_{f_2} \end{Bmatrix} + \frac{gs_0 l}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

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III. Formation of Global Matrix

The element properties originally expressed in local coordinates need to be transformed into global coordinates before a solution algorithm is initated. Based on the node to node relationship, it is possible to generate an overall element property matrix for the entire domain, a process called assembling of element equations.

Notes

The concept of discreatization employed earlier is based on the fact that a domain with varying geometric and hydraulic properties can be treated independently as subdomains but systematically from one subdomain to another. Considering N number elements of varying lengths the assembled global matrix equations for continuity and momentum equations become:

$$\begin{bmatrix} 2l_1 & l_1 & \cdots & & \circ & \\ l_1 & 2(l_1+2) & l_2 & & & \\ & l_2 & 2(l_2+l_3) & l_3 & & \\ & .. & ..\ . & .. & & \\ & \circ & & l_i & 2(l_i+l_{i+1}) & l_{i+1} \\ & & & \cdots & \cdots & .. \\ & & & & & l_{N-1}\,(l_{N-1}) \end{bmatrix} \begin{Bmatrix} \frac{\partial y_1}{\partial x} \\ \frac{\partial y_2}{\partial x} \\ \frac{\partial y_3}{\partial x} \\ \cdots \\ \frac{\partial y_N}{\partial x} \end{Bmatrix} + \begin{bmatrix} v_2\ -4v_1 & 2v_2+v_1 & & & & \\ -v_2-2v_1 & v_3-v_1 & 2v_3-v_z & & & \\ & v_3-2v_2 & v_4-v_2 & & 2v_4-v_3 & \\ & .. & ..\ . & & .. & \\ & & v_{1+1}-2v_i & v_{i+2}-v_1 & 2v_{1+2}-v_{i+1} & \\ & & & & -v_N-2v_{N-1} & v_N-v_{N-1} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_i \\ y_N \end{Bmatrix}$$

$$-3\begin{Bmatrix} l_1 q_1 \\ q_1 l_1 + q_2 l_2 \\ q_2 l_2 + l_3 q_3 \\ \ldots \\ l_i q_i + l_{i+1} q_{i+1} \\ \ldots \\ l_{N-1} q_{N-1} \end{Bmatrix} = 0$$

In matrix form the global continuity equation can be written as

$$[A]\begin{Bmatrix} \frac{dy}{dt} \end{Bmatrix} + [B]\{y\} - \{c\} = 0$$

Where A, B are the matrices and C is the column vector, Y is the dependent variable. The global momentum equation can be formed similarly.

The Solution of time dependent global matrix Equation is sought through a semi discrete approach, This approach requires the time derivative of the dependant variable at each node to be replaced by finite difference scheme (in time domain). Such as the forward, backward, and central differences and are given below with time level k as:

Forward difference, $\frac{dy}{dt} = \frac{y^{k+1} - y^k}{\Delta t}$

Backward difference, $\frac{dy}{dt} = \frac{y^k - y^{k-1}}{\Delta t}$

Central difference, $\frac{dy}{dt} = \frac{y^{k+1} - y^{k-1}}{2\Delta t}$

Substitution of Equation (5.29a) in Equation (5.28) yields

$$[A]\left\{\frac{y^{k+1}-y^{k}}{\Delta t}\right\}+[B]\left\{y^{k}\right\}-\{c\}=0$$

An implicit equation will be generated from this Equation with the aid of the time weighting factor in the next section.

Development of the Numerical models

The deterministic stream flow models are investigated with three distinct options: (1) the kinematic flow models comprises (a) the simplified version of momentum equation that neglects pressure and inertia terms are compared to friction and gravity terms and (b) the complete form of continuity equation; (2) the diffusion flow models comprises (a) the simplified momentum equation that accounts only for pressure, friction, and gravity terms and (b) the complete form of continuity equation; and (3) the complete flow model comprises (a) the complete form of momentum equation and (b) the complete continuity equation.

The kinematic flow model is investigated in both explicit and implicit sense. The explicit kinematic flow model leads to linear equations. They are solved using a direct method similar to the tridiagonal matrix algorithm set up by varga (1962). The solution proceeds by matrix reduction similar to Gaussian elimination. In contrast the explicit model, the implicit kinematic model yields a set of non-linear tridiagonal matrix equations which are solved by the functional Newton-Raphson iterative method.

The diffusion model as well as the complete flow model each results in a non-linear bitridiagonal matrix equation. The functional Newton-Raphson's method, along with the direct solution algorithm, triangular decomposition technique that yields a recursion algorithm (Douglas, et al, 1959, Von Rosenberg, 1969), is utilized to predict depth and velocity of flow for each option.

Finite Element Kinematic Wave Model

Explicit Model:

The non-linear continuity equation is easily converted to linear form by use of geometric and flow relations:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}-q(x,t)=0$$

Where, A = Area of flow, L^2 ;

Q = volumetric flow rate, $\frac{L^3}{T}$

The appropriate simplified momentum equation for coupling with the continuity equation has been obtained and is presented below

$$S_f = S_0 = \frac{n_1^2\, v^2}{R^{4/3}} = \frac{v^2 R^{4/3}}{M^2}$$

$$\text{Or}\quad Q = \frac{AR^{2/3}S_0^{1/2}}{n_1} = MAR^{2/3}\, S_0^{1/2}$$

These equations are written in matrix units. For fps units first equation to be divided by 2.216 and the second equation to be multiplied by 1.486.

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Applying the Galerkin's weighted residual method results in the following liner first order ordinary differential equation.

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial A_j}{\partial t} \\ \frac{\partial A_2}{\partial t} \end{Bmatrix} + \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} - \frac{ql}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 0$$

For total length of the stream reach the assembled matrix equation becomes:

$$\begin{bmatrix} 2l_1 & l_1 & \cdots & & & \text{O} \\ l_1 & 2(l_1+l_2) & l_2 & & & \\ & l_2 & 2(l_2+l_3) & l_3 & & \\ & \cdot\cdot & \cdot\cdot\ \cdot & \cdot\cdot & & \\ & \text{O} & -l_i & 2(l_i+l_{i+1}) & l_{i+1} & \\ & & & \cdots & \cdots & \cdot\cdot \\ & & & l_{N-1} & 2l_{N-1} & \end{bmatrix} \begin{Bmatrix} \frac{\partial A_1}{\partial t} \\ \frac{\partial A_2}{\partial t} \\ \frac{\partial A_3}{\partial t} \\ \cdots \\ \frac{\partial A_i}{\partial t} \\ \cdots\cdots \\ \frac{\partial AN}{\partial t} \end{Bmatrix} + \begin{Bmatrix} Q_2 - Q_1 \\ Q_3 - Q_1 \\ Q_4 - Q_2 \\ \cdots\cdots\cdots \\ Q_{i+1} - Q_{i-1} \\ \cdots \\ Q_N - Q_{N-1} \end{Bmatrix} - \frac{l}{2}\begin{Bmatrix} l_1 q_1 \\ l_1 q_1 + l_2 q_2 \\ l_2 q_2 + l_3 q_3 \\ \cdots \\ l_i q_i + l_{i+1} q_{i+1} \\ \cdots \\ l_{N-1} q_{N-1} \end{Bmatrix} = 0$$

The above Equation is expressed in a matrix form:

$$[K]\left\{\frac{dy}{dt}\right\} + [D]\{F\} = 0$$

The solution of this Equation is possible upon implementation of the forward differencing in time derivative.

$$[K]\{A\}^{N+1} = [K]\ \{A\}^N + \Delta t\ \{F\}^n - \Delta t\ \ \{D\}^N$$

The solution of the area of flow at various nodes proceeds forward in time with the right hand side evaluated at a previous time level, n. Thus, the Equation can be expressed in more compact form:

$$[K]\ \{A\}^{N+1} = \{X\}^N$$

Where X is the known column vector at previous time level. The matrix, K is a linear and tridiagonal type that easily leads to direct solution algorithm. The computer program solving Equation is facilitated by the use of the compact tridiagonal algorithms proposed by Varga (1962). The computed area of flow at current time level, n+1, is used to update cycle is repeated as new time level is reached. The coded explicit finite element scheme exhibits dynamic instability to restriction on the step. This drawback is inherent in explicit numerical schemes, is expected regardless of the finite element approach.

To solve the KW model through the above finite element method one can study the flow problem of overland flow as well as channel flow by using practical data collecting from any river in Bangladesh.

IV. Conclusion

A hydrological model is an important tool for estimating and organizing quantitative hydrologic information. The main objectives of this thesis is to develop a suitable surface hydrological model for study the movement of overland, (i.e. through its surface runoff) as well as stream flow components of the hydrologic cycle. To achieve these objectives, various techniques and available models were studied. It was concluded that the dynamic approached are the best to account for the physical processes associated with the runoff mechanics of the watersheds. Among these approaches, the kinematic wave theory is the best suited to the prevailing condition.

A further work can be done by developing computer program using these methods to solve KW equation for channel and overland flows for various practical data set collecting from any small river in Bangladesh

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