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Some Results on a Lorentzian Para Sasakian Manifolds

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SOME RESULTS ON A LORENTZIAN PARA SASAKIAN MANIFOLDS

Strictly as per the compliance and regulations of :

Ref.

6. M.C.Chaki and S.Koley, On generalized Pseudo Ricci-symmetric manifolds, Periodica Mathematica Hungarica, 28 (2), (1993), 123-129.

Some Results on a Lorentzian Para Sasakian Manifolds

Mobin Ahmad [α], Archana Srivastava [σ] & Amit Prakash [ρ]

Abstract - The object of this paper is to study a type of non-flat differentiable manifold called Generalized Pseudo Symmetric, Generalized Pseudo Ricci symmetric, Generalized Ricci Recurrent, Semi pseudo symmetric and Semi pseudo Ricci symmetric manifold in a Lorentzian para-Sasakian manifold.

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I. Introduction

A non -flat differentiable manifold (M^n, g) (n> 3), is called generalized pseudo symmetric G$(PS)_n$, if there exits a vector field P and 1- form A, B, C, D on M Such that

$$(D_X S)(Y,Z) = 2A(X)S(Y,Z) + B(R(X,Y),Z) + C(Y)S(X,Z)$$

$$+D(Z)S(X,Y) + p(R(X,Z)Y). \tag{1.1}$$

In 1993, Chaki and Koley [6] introduced another type of non-flat differentiable manifold (M^n, g) (n> 2), satisfies the condition

$$(D_X S)(Y,Z) = 2A(X)S(Y,Z) + \mathrm{B(Y)}\, S(X,Z) + C(Z)S(X,Y), \tag{1.2}$$

where A,B,C are three non-zero 1-form and D denotes the operator of covariant differentiation with respect to g. Such a manifold were called by them a generalized pseudo Ricci-symmetric manifold and an n - dimensional manifold of this kind were denoted by G(PRS)n.

De, Guha and Kamilya [7] introduced and studied a type of differentiable manifold (M^n, g) (n> 2), whose Ricci tensors of type (0,2) satisfies the condition

$$(D_X S)(Y,Z) = A(X)S(Y,Z) + \mathrm{B(X)}\, g(Y,Z), \tag{1.3}$$

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where A and B are two non-zero 1-forms, P,Q are two vector fields such that

g(X,P)=A(X),

g(X,Q)=B(X),

such a manifold were called by them a generalized Ricci-recurrent manifold and an n-dimensional manifold of this kind were denoted by $(GR)_n$.

Tarafdar and Jawarneh [4] introduced a type of non-flat differentiable manifold (M^n, g) (n> 3), whose curvature tensor R satisfies the condition

$$(D_X R)(Y,Z)W = 2A(X)R(Y,Z)W + A(Y)R(X,Z)W + A(Z)R(Y,X)W + A(W)R(Y,Z)X + A(W)R(Y,Z)X, \quad (1.4)$$

where A is a non- zero 1-form and g(X,P)=A(X),

Such a manifold were called by them a Semi pseudo symmetric and an n-dimensional manifold of this kind is denoted by (SPS)n .

Tarafdar and Jawarneh [5] introduced another type of non - flat Riemannian manifold (M^n, g) (n> 3), Whose Ricci- tensor of type (0,2) satisfies the condition

$$(D_X S)(Y,Z) = A(Y)S(X,Z) + A(Z)S(X,Y), \quad (1.5)$$

where A is a non zero 1-form , such a manifold were called by them Semi pseudo Ricci -symmetric and an n – dimensional manifold of this kind is denoted by (SPRS)n.

II. Preliminaries

An *n*-dimensional differentiable manifold M^n is a Lorentzian para – Sasakian (*LP*-Sasakian) manifold if it admits a (1,1) – tenser field ϕ, a contravariant vector field ξ, a covariant vector field η and a Lorentzian metric g which satisfy ([2], [3])

$$\phi^2 X = X + \eta(X)\xi, \quad (2.1)$$

$$\eta(\xi) = -1, \quad (2.2)$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \quad (2.3)$$

$$g(X,\xi) = \eta(X), \quad (2.4)$$

$$(D_X \phi)(Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \quad (2.5)$$

and

$$D_X \xi = \phi X, \quad (2.6)$$

for any vector fields *X* and *Y*, where *D* denotes covariant differentiation with respect to *g* ([1],[2],[3]).

In an LP-Sasakian manifold M^n with structure (ϕ, ξ, η, g), it is easily seen that

$$\text{(a) } \phi\xi = 0 \qquad (b)\ \eta(\phi X) = 0 \qquad (c)\ rank(\phi) = n-1. \quad (2.7)$$

Ref.

1. Mihai, I and Rosca, R.: On Lorentzian *P* - Sasakian manifolds, Classical Analysis, World scientific Publi.Singapore (1992), 155-169.

Notes

Let us put

$$F(X,Y) = g(\phi X,Y). \tag{2.8}$$

Then the tensor field F is symmetric (0, 2) tensor field i.e.,

$$F(X,Y) = F(Y,X), \tag{2.9}$$

and

$$F(X,Y)=(D_X \eta)(Y) . \tag{2.10}$$

Also in an LP-Sasakian manifold, the following relation holds:

$$'R(X,Y,Z,\xi)=g(Y,Z)\eta(X) - g(X,Z)\eta(Y), \tag{2.11}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.12}$$

$$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.13}$$

$$S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{2.14}$$

and

$$S(X,\xi) = (n-1)\eta(X) , \tag{2.15}$$

for any vector fields X,Y,Z ,Where $R(X,Y)Z$ is the Riemannian curvature tensor, S is the Ricci tensor.

III. Generalized Pseudo Symmetric Lp- Sasakian Manifold

Suppose that M^n is a generalized pseudo symmetric LP-sasakian manifold Putting Z= ξ in(1.1),we get

$$(D_X S)(Y,\xi) = 2A(X)S(Y,\xi) + B(R(X,Y)\xi) + C(Y)S(X,\xi)$$
$$+D(\xi)S(X,Y) + p(R(X,\xi)\,Y). \tag{3.1}$$

Using (2.2), (2.12) and (2.15) in (3.1), we get

$$(D_X S)(Y,\xi) = 2(n-1)A(X)\eta(Y) + \eta(Y)B(X) - \eta(X)B(Y) + (n-1)C(Y)\eta(X)$$
$$+D(\xi)S(X,Y) + \eta(Y)p(X) - g(X,Y)p(\xi). \tag{3.2}$$

We know that

$$(D_X S)(Y,\xi) = (n-1)g(\phi X,Y) - S(Y,\Phi X), \tag{3.3}$$

Therefore, from (3.2) and (3.3), we get

$$2(n-1)A(X)\eta(Y) + \eta(Y)B(X) - \eta(X)B(Y) + (n-1)C(Y)\eta(X) + D(\xi)S(X,Y)$$

$$+\eta(Y)p(X) - g(X,Y)p(\xi) = (n-1)g(\phi X, Y) - S(Y, \Phi X). \quad (3.4)$$

Therefore, putting X=Y=ξ in (3.4) and using (2.1) and (2.15), we get

$(n-1)[2A(\xi)+C(\xi)+D(\xi)]=0.$

Since n≥3, we obtain $$2A(\xi)+C(\xi)+D(\xi)=0. \quad (3.5)$$

So, the vanishing of the 1-form 2A+C+D over the vector field ξ is necessary in order that M be an LP-Sasakian manifold.
Now we will show that 2A+C+D=0 holds for all vector fields on M.
Putting Y= ξ in (1.1), we get

$$(D_X S)(\xi, Z) = 2(n-1)A(X)\eta(Z) + \eta(Z)B(X) - g(X,Z)B(\xi) + C(\xi)S(X,Z)$$
$$+(n-1)D(Z)\eta(X) + \eta(Z)p(X) - \eta(X)p(Z). \quad (3.6)$$

Also, We know that

$$(D_X S)(\xi, Z) = (n-1)g(\phi X, Z) - S(\phi X, Z). \quad (3.7)$$

From (3.6) and (3.7), We get

$$2(n-1)A(X)\eta(Z) + \eta(Z)B(X) - g(X,Z)B(\xi) + C(\xi)S(X,Z) + (n-1)D(Z)\eta(X) + \eta(Z)p(X) - \eta(X)p(Z) = = (n-1)g(\phi X, Z) - S(\phi X, Z). \quad (3.8)$$

Putting Z= ξ, in (3.8), we get

$$-2(n-1)A(X) - B(X) - \eta(X)B(\xi) + (n-1)C(\xi)\eta(X) + (n-1)D(\xi)\eta(X)$$
$$-p(X) - \eta(X)p(\xi) = 0. \quad (3.9)$$

Putting X=ξ in (3.8), we get

$$2(n-1)A(\xi)\eta(X) + (n-1)C(\xi)\eta(X) - (n-1)D(X) + p(X) + \eta(X)p(\xi) = 0. \quad (3.10)$$

Adding (3.9) and (3.10) and using (3.5), we get

$$-2(n-1)A(X) - \eta(X)B(\xi) - B(X) + (n-1)C(\xi)\eta(X) - (n-1)D(X) = 0. \quad (3.11)$$

Now, Putting X= ξ in (3.4), and replacing Z with X, we get

$$2(n-1)A(\xi)\eta(X) + B(\xi)\eta(X) + B(X) - (n-1)C(X) + (n-1)D(\xi)\eta(X) = 0. \quad (3.12)$$

Adding (3.11) and (3.12) and using (3.5), we get

$$(n-1)[2A(X) + C(X) + D(X)] = 0$$

Since $n \geq 3$, Hence we get

$$2A(X) + C(X) + D(X) = 0, \text{ for all X}$$

Notes

This implies 2A+C+D=0.

Hence, we can state the following theorem:

Theorem 3.1 .There exists no generalized pseudo symmetric LP - Sasakian manifold (M^n, g), n≥ 3, if 2A+C+D is not every where zero.

Notes

IV. Generalized Pseudo Ricci Symmetric Lp - Sasakian Manifold

Suppose that M^n is a generalized pseudo Ricci symmetric LP - Sasakian manifold then

$$(D_X S)(Y,Z) = 2A(X)S(Y,Z) + B(Y)S(X,Z) + C(Z)S(X,Y). \quad (4.1)$$

Putting Z= ξ, in (4.1) and using (2.15), we get

$$(D_X S)(Y,\xi) = 2(n-1)A(X)\eta(Y) + (n-1)B(Y)\eta(X) + C(\xi)S(X,Y). \quad (4.2)$$

Also, we know that

$$(D_X S)(Y,Z) = (n-1)g(\emptyset X,Y) - S(Y,\emptyset X). \quad (4.3)$$

From (4.2) and (4.3), we get

$$2(n-1)A(X)\eta(Y) + (n-1)B(Y)\eta(X) + C(\xi)S(X,Y) = (n-1)g(\emptyset X,Y) - S(Y,\emptyset X). \quad (4.4)$$

Putting X = Y = ξ in (4.4), we get

$$(n-1)[2A(\xi) + B(\xi) + C(\xi)] = 0.$$

Which gives (since n≥ 3),

$$[2A(\xi) + B(\xi) + C(\xi)] = 0. \quad (4.5)$$

Putting X= ξ in (4.4), we have

$$(n-1)\eta(Y)[2A(\xi) + C(\xi)] - (n-1)B(Y) = 0.$$

So, By virtue of (4.5) this yields

$$(n-1)[\eta(Y)B(\xi) + B(Y)] = 0,$$

Which gives us (since $n \geq 3$ *),* $\quad B(Y) = -\eta(Y)B(\xi). \quad (4.6)$

Similarly, Taking Y=ξ in (4.4), we get

$$-2A(X) + \eta(X)[B(\xi) + C(\xi)] = 0.$$

Hence applying (4.5), we get

$$A(X) = -\eta(X)A(\xi)\ . \quad (4.7)$$

Since $(\nabla_{\xi}S)(\xi,X)=0$, then from (1.2), we obtain

$$(n-1)\eta(X)[2A(\xi)+B(\xi)]-(n-1)C(X)=0. \quad (4.8)$$

So, by making the use of (4.5), the equation (4.8) reduces to

$$C(X)= - C(\xi)\eta(X) \quad (4.9)$$

Adding equation (4.6), (4.7) and (4.9), we get

$$2A(X)+B(X)+C(X)= -[2A(\xi)+B(\xi)+C(\xi)]\eta(X) \quad (4.10)$$

And then, from (4.5), it follows that

$$2A(X)+B(X)+C(X)=0, \text{ for all } X,$$

$$\text{Thus} \quad 2A+B+C=0.$$

Hence, we can state the following theorem:

Theorem 4.1. There exists no generalized Pseudo Ricci-Symmetric LP-Sasakian manifold (M^n,g), n≥3 if 2A+B+C is not everywhere zero.

V. Generalized Ricci Recurrent Lp- Sasakian Manifold Admitting Codazzi Type Ricci Tensor

We know that

$$(D_X S)(Y,Z)=D_X S(Y,Z)-S(D_X Y,Z)-S(Y,D_X Z). \quad (5.1)$$

Then, from (5.1) and (1.3), we get

$$A(X)S(Y,Z)+B(X)g(Y,Z)=D_X S(Y,Z)-S(Y,D_X Z). \quad (5.2)$$

Putting Z= ξ in above relation, we get

$$(n-1)A(X)\eta(Y)+B(X)\eta(Y)=(n-1)g(\emptyset X,Y)-S(Y,\emptyset X). \quad (5.3)$$

Putting Y= ξ in (5.3), we have

$$(n-1)A(X)+B(X)=0. \quad (5.4)$$

Here we assume that a generalized Ricci recurrent manifold admits Codazzi type Ricci tensor

i.e. $$(D_X S)(Y,Z)=(D_Y S)(X,Z). \quad (5.5)$$

Then by virtue of (1.3), it follows from (5.5) that

$$A(X)S(Y,Z)+B(X)g(Y,Z)=A(Y)S(X,Z)+B(Y)g(X,Z). \quad (5.6)$$

Putting X= ξ in above, we get

$$A(\xi)S(Y,Z)+B(\xi)g(Y,Z)=[(n-1)A(Y)+B(Y)]\eta(Z). \quad (5.7)$$

In view of (5.4), (5.7) yields

$$S(Y,Z) = \lambda g(Y,Z), \text{ where } \lambda = -\frac{B(\xi)}{A(\xi)}.$$

i.e. M^n is an Einstein manifold.
Hence, we can state the following theorem:

Theorem 5.1. If a generalized Ricci recurrent LP - Sasakian manifold admits a Codazzi type Ricci tensor, then it is an Einstein manifold with constant $\lambda = -\frac{B(\xi)}{A(\xi)}$.

VI. Semi Pseudo Symmetric LP - Sasakian Manifold (SPS)n $(M^n, g), n > 3$

Suppose that M^n is a Semi pseudo symmetric LP-Sasakian manifold then from (1.4), we obtain

$$(D_X S)(Y,\xi) = (2n-1)A(X)\eta(Y) + (n-2)A(Y)\eta(X) + A(\xi)S(X,Y). \quad Since \quad g(X,P) =$$

$$A(X) \Longrightarrow A(\xi) = g(\xi,P) = \eta(P) \tag{6.1}$$

This implies $$(D_X S)(Y,\xi) = (2n-1)A(X)\eta(Y) + (n-2)\eta(X)A(Y) + \eta(P)S(X,Y). \tag{6.2}$$

We also know that

$$(D_X S)(Y,\xi) = (n-1)g(\emptyset X, Y) - S(Y, \emptyset X). \tag{6.3}$$

From equation (6.2) and (6.3), we get

$$(2n-1)A(X)\eta(Y) + (n-2)\eta(X)A(Y) + \eta(P)S(X,Y) = (n-1)g(\emptyset X, Y) - S(Y, \emptyset X). \tag{6.4}$$

Putting $X = \xi$ in (6.4), we get

$$(3n-2)\,\eta(P)\eta(Y) - (n-2)A(Y) = 0. \tag{6.5}$$

Putting $Y = \xi$ in (6.5), we get

$$\eta(P) = 0 \tag{6.6}$$

Hence, from (6.5) and (6.6), we get

$$A(Y) = 0.$$

Which is inadmissible by the definition of (SPS)n.
Hence, we can state the following theorem:

Theorem 6.1. A $(SPS)_n$ $(n > 3)$, cannot be an LP sasakian manifold.

Notes

VII. Semi Pseudo Ricci Symmetric $(SPRS)_n$ $(M^n, g), n > 3$ LP –Sasakian Manifold

In this section we assume that a $(SPRS)_n$ is an LP sasakian manifold then from (1.5), we obtain

$$(D_X S)(Y, \xi) = (n-1)A(Y)\eta(X) + A(\xi)S(X, Y). \tag{7.1}$$

Also we know that

$$(D_X S)(Y, \xi) = (n-1)g(\emptyset X, Y) - S(Y, \emptyset X). \tag{7.2}$$

From (7.1) and (7.2), we get

$$(n-1)A(Y)\eta(X) + A(\xi)S(X, Y) = (n-1)g(\emptyset X, Y) - S(\emptyset X, Y). \tag{7.3}$$

Putting $X = \xi$ in (7.3), we get

$$A(Y) = A(\xi)\eta(Y). \tag{7.4}$$

Putting $Y = \xi$ in (7.4), we get

$$A(\xi) = 0. \tag{7.5}$$

Using (7.5) in (7.4), we get

$$A(Y) = 0.$$

Which is inadmissible by the definition of $(SPRS)_n$.
Hence, we can state the following theorem .:

Theorem 7.1. A $(SPRS)_n$ $(n > 3)$ can not be an LP sasakian manifold.

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Notes