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Using of Semi-Empirical Models and Fick's Second Low for Mathematical Modeling of Mass Transfer in Thin Layer Drying of Carrot Slice

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USING OF SEMI-EMPIRICAL MODELS AND FICKS SECOND LOW FOR MATHEMATICAL MODELING OF MASS TRANSFER IN THIN LAYER DRYING OF CARROT SLICE

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Isa Hazbavi [α], Seyed Hashem Samadi [σ] & Hamid Khafajeh [ρ]

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I. Introduction

Drying of moist materials is a complicated process involving simultaneous heat and mass transfer. Mathematical modeling and simulation of drying curves under different conditions is important to obtain a better control of this unit operation and an overall improvement of the quality of the final product [1, 2, 3]. Currently, there are three types of thin-layer drying models used to describe the drying phenomenon of agriculture product, namely, theoretical model, which considers only the internal resistance to moisture transfer between product and heating air, semi-theoretical and empirical models which consider only the external resistance.

The drying process takes place in two stages. The first stage happens at the surface of the drying material at a constant drying rate and is similar to the vaporization of water into the ambient. The second stage drying process takes place with decreasing drying rate [4].

When drying process is controlled by the internal mass transfer, mainly in the falling rate period, modeling of drying is carried out through diffusion equations based on Fick's second law [4, 5, 6, 7]. Molecular diffusion is the main water transport mechanism and to predict the water transfer in food materials diffusion models based on Fick's second law are used.

Effective moisture diffusivity describes all possible mechanisms of moisture movement within the foods, such as liquid diffusion, vapour diffusion, surface diffusion, capillary flow and hydrodynamic flow. A knowledge of effective moisture diffusivity is necessary for designing and modeling mass-transfer processes such as dehydration, adsorption and desorption of moisture during storage. Researchers reported that effective moisture diffusivity increased with moisture content up to a limit value and then decreased and eventually became constant at high moisture contents [8, 9].

The aim of this research was the study and modelling of the drying kinetics of mass transfer during the microwave drying process of carrot slices. The effect of drying powers and moisture content on the effective moisture diffusivity of the dried slices was also studied.

II. Mateials and Methods

Carrot samples were procured from local vegetable market in Tehran, Iran. The samples were stored at 4±0.5 °C before they were used in experiments. The samples were removed from the refrigerator before experimentation and were allowed to attain room temperature. Carrots were washed under running water to remove the adhering impurities, and thinly sliced in thicknesses of 5 mm using a sharp stainless steel knife. The average initial moisture content of the samples were found to be 78.2 ± 0.7 % wet basis, as determined by using convective oven at 105 ^{o}C for 24h.

A domestic microwave oven (M945, Samsung Electronics Ins) with maximum output of 1000 W at 2450MHz was used for the drying experiments. The

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dimensions of the microwave cavity were 327×370×207 mm. The oven has a fan for air flow in drying chamber and cooling of magnetron. The moisture from drying chamber was removed with this fan by passing it through the openings on the right side of the oven wall to the outer atmosphere. The microwave dryer was operated by a control terminal which could control both microwave power level and emission time. Experiments were performed at four initial mass of 30 g at four microwave powers of 200, 300, 400 and 500 W. The moisture losses of samples were recorded at 15 s intervals during the drying process by a digital balance (GF-600, A & D, Japan) and an accuracy of ± 0.001 g.

For measuring the weight of the sample during experimentation, the tray with sample was taken out of the drying chamber, weighed on the digital top pan balance and placed back into the chamber. Drying was carried out until the final moisture content reaches to a level less than 10 % (w.b.).

The moisture ratio of samples during the thin layer drying experiments was calculated using the following equation:

$$MR = \frac{X_t - X_e}{X_0 - X_e} \qquad (1)$$

where MR is the moisture ratio (dimensionless), Xt is the moisture content at drying time t (d.b.) and X0 is the initial moisture content (d.b.). The values of Xe are relatively small compared to Xt or X0. Thus, Eq. (1) can be reduced to MR=Xt/X0 [5].

The experimental sets of (MR, t) were fitted to six empirical models from the literature, using IBM SPSS Statistics 19. (SPSS, Inc.).

Newton model:

$$MR = \exp(-kt) \qquad (2)$$

Henderson and Pabis model:

$$MR = a\exp(-kt) \qquad (3)$$

Page's model:

$$MR = \exp(-kt^n) \qquad (4)$$

Midilli model:

$$MR = a\exp(-kt^n) + bt \qquad (5)$$

Wand and Singh model:

$$MR = 1 + bt + at^2 \qquad (6)$$

Logarithmic model:

$$MR = a\exp(-kt) + b \qquad (7)$$

Where k is the drying rate constant (1/min), and a, b and n are equation constants model.

The terms used to evaluate goodness of fit of the tested models to the experimental data were the coefficient of determination (R^2); root mean square error (RMSE) and the reduced chi-square (χ^2) between the experimental and predicted moisture ratio values. Statistical values are defined as follows:

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{N}(MR_{pre,i} - MR_{exp,i})^2}{\sum_{i=1}^{N}(MR_{pre,i} - \overline{MR}_{exp})^2}\right) \qquad (8)$$

$$\chi^2 = \frac{\sum_{i=1}^{n}(MR_{exp,i} - MR_{pre,i})^2}{N - z} \qquad (9)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(MR_{exp,i} - MR_{pre,i})^2}{N}} \qquad (10)$$

In these equations, N is the number of observations, z is the number of constants, MRexp and MRpre are the experimental and predicted moisture ratios, respectively.

The drying rate of samples was calculated by using Eq. (11):

$$DR = \frac{X_{t+\Delta t} - X_t}{\Delta t} \qquad (11)$$

where $X_{t+\Delta t}$ is moisture content at time $t+\Delta t$ (d.b.), t is the time (min) and DR is the drying rate (d.b./min).

Fick's second equation of diffusion was used to calculate the effective diffusivity, considering a constant moisture diffusivity, infinite slab geometry and uniform initial moisture distribution:

$$MR = \frac{8}{\pi^2}\sum_{n=o}^{\infty}\frac{1}{(2n+1)}\exp\left(-\frac{(2n+1)\pi^2}{L^2}D_{eff}\,t\right) \qquad (12)$$

where D_{eff} is the effective diffusivity (m^2/s), and L is the thickness (here half) of slab (m).

The Eq. (12) can be simplified by taking the first term of Eq. (13):

$$MR = \frac{8}{\pi^2}\exp\left(-\frac{\pi^2 D_{eff}\,t}{L^2}\right) \qquad (13)$$

Eq. (13) is evaluated numerically for Fourier number, $F_0 = D_{eff} \times t/L^2$, for diffusion and can be rewritten as Eq. (14) can be rewritten as:

$$MR = \frac{8}{\pi^2}\exp(-\pi^2 F_0) \qquad (14)$$

Thus:

$$F_0 = -0.101\ln(MR) - 0.0213 \qquad (15)$$

The effective moisture diffusivity was calculated using Eq. (16) as:

$$D_{eff} = \frac{F_0}{\left(\frac{t}{L^2}\right)} \qquad (16)$$

III. Results and Discussion

Fig. 1 shows the change in moisture ratio of carrot slices with time by microwave drying. A reduction in drying time occurred with increasing the microwave power level. On the other hand, mass transfer within the sample was more rapid during higher microwave power heating because more heat was generated within the sample creating a large vapor pressure difference between the centre and the surface of the product due to characteristic microwave volumetric heating. The time required for the lowering of moisture content of carrot slices to 0.1 from 3.59 on dry basis varied between 9.5 and 17.5 min depending on the microwave power level.

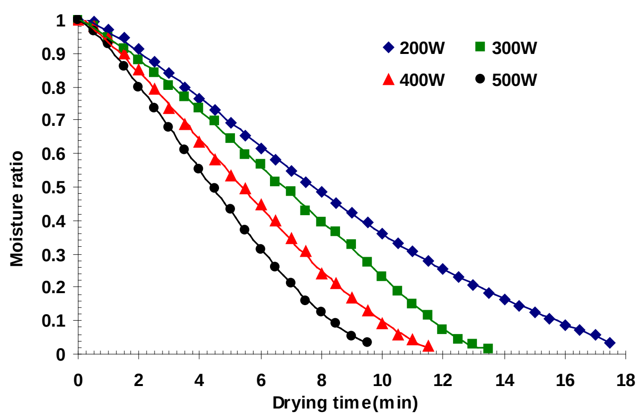

Figure 1 : Variation in moisture ratio as a function of drying time

The statistical results from models are summarized in Table 1. The values of mentioned tests were in the range of 0.7373–0.9999 for R^2, 0.00003–0.0665 for χ^2, 0.0054–0.2486 for RMSE. Based on the criteria of the highest R^2 and the lowest RMSE, and χ^2, the model of Midilli was selected as the most suitable model to represent the thin-layer drying behavior of carrot slices. Fig. 2 compares experimental data with those predicted with the Midilli model for carrot slices at 200, 300, 400 and 500W. The prediction using the model showed MR values banded along the straight line, which showed the suitability of these models in describing drying characteristics of carrot s lices.

Table 1 : Thin layer drying model constants and statistical parameters of sliced carrot

Model	P(W)	Constants	R^2	χ^2	RMSE
Newton	200	k=0.1313	0.8737	0.0125	0.1103
	300	k=0.1848	0.7373	0.0285	0.1656
	400	k=0.2153	0.8154	0.0211	0.1423
	500	k=0.2505	0.8477	0.0177	0.1266
Henderson and Pabis	200	k=0.1644; a=1.4794	0.9244	0.0182	0.1310
	300	k=0.2505; a=1.8251	0.8131	0.0665	0.2486
	400	k=0.2806; a=1.6672	0.8802	0.0428	0.1982
	500	k=0.319; a=1.5604	0.9047	0.0327	0.1674
Wang and Singh	200	a=0.0007; b=-0.0690	0.9945	0.0006	0.0228
	300	a=-0.0005; b=-0.0698	0.9969	0.0082	0.0875
	400	a=0.0005, b=-0.0951	0.9949	0.0006	0.0229
	500	a=0.0013; b=-0.1182	0.9946	0.0006	0.0228
Page	200	k=0.028; n=1.574	0.9980	0.0002	0.0145
	300	k=0.023; n=1.813	0.9903	0.0012	0.0327
	400	k=0.041; n=1.707	0.9940	0.0007	0.0248
	500	k=0.060; n=1.680	0.9961	0.0004	0.0193
Midilli	200	k=0.036; n=1.379; a=1.010; b=-0.007	0.9999	0.00003	0.0054
	300	k=0.024; n=1.519; a=0.994; b= -0.022	0.9999	0.0001	0.0094
	400	k=0.034; n=1.550; a=0.997; b=-0.020	0.9982	0.0007	0.0246
	500	k=0.063; n=1.492; a=0.996; b=-0.015	0.9999	0.0001	0.0064
Logarithmic	200	k=0.041; a=2.015; b=-0.961	0.9983	0.0002	0.0149
	300	k=0.015; a=5.931; b=-4.885	0.9964	0.0007	0.0250
	400	k=0.033; a=3.342; b=-2.298	0.9971	0.0003	0.0169
	500	k=0.047; a=2.888; b=-1.847	0.9973	0.0004	0.0179

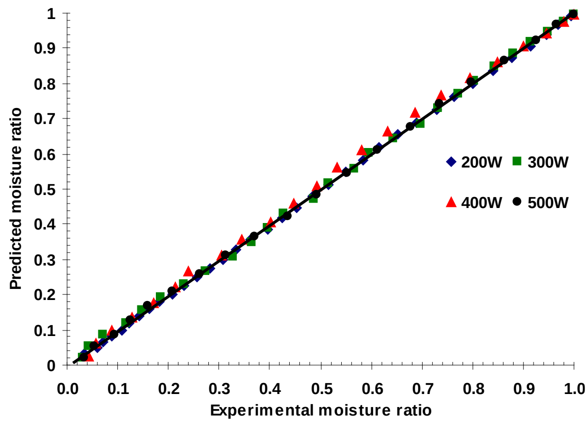

Figure 2 : Comparison of experimental moisture ratio with predicted moisture ratio from the Midilli model for carrot slices

Fig. 3 shows the changes in drying rate as a function of drying time at the different powers. It is clear that the moisture content and drying rate decrease continuously with drying time. The drying rate was rapid during the initial period but it became very slow at the last stages during the drying process. The moisture content of the material was very high during the initial phase of the drying which resulted in a higher absorption of microwave power and higher drying rates due to the higher moisture diffusion. As the drying progressed, the loss of moisture in the product caused a decrease in the absorption of microwave power and resulted in a fall in the drying rate. Similar results have been observed in the drying of different fruits and vegetables: kiwifruit [10]; hazelnut [11]; carrot pomace [12]; amelia mango [13]; pineapple, mango, guava and papaya [14] and apple [6]. Constant drying rate period was not observed during the drying of carrot slices. This shows that diffusion in dominant physical mechanism governing moisture movement in the samples.

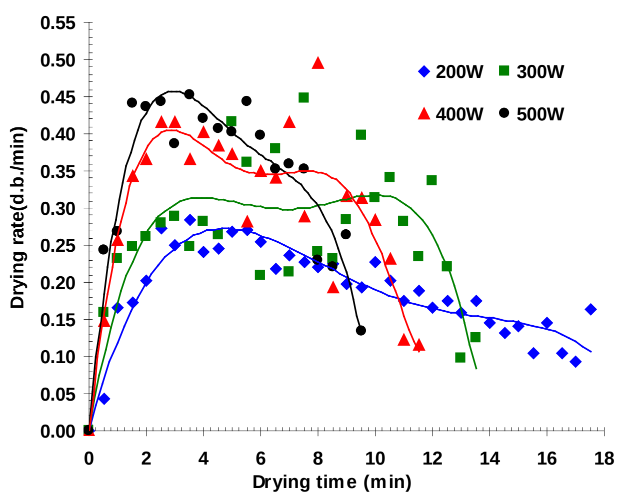

Figure 3 : Drying rates for carrot slices at different microwave powers

Variation in effective moisture diffusivity of carrot slices with moisture content at different microwave power levels is shown in Fig. 4. The effective moisture diffusivity increased with decrease in moisture content. However, the moisture diffusivity further was higher at any level of moisture content at higher microwave power level, resulting into shorter drying time. This may indicate that as moisture content decreased, the permeability to vapour increased, provided the pore structure remained open. The temperature of the product rises rapidly in the initial stages of drying, due to more absorption of microwave heat, as the product has a high loss factor at higher moisture content. This increases the water vapour pressure inside the pores and results in pressure induced opening of pores. In the first stage of drying, liquid diffusion of moisture could be the main mechanism of moisture transport. As drying progressed further, vapour diffusion could have been the dominant mode of moisture diffusion in the latter part of drying. Sharma and Prasad [8]; Sharma et al. [9] also reported similar trend in the variation in the moisture diffusivity with moisture content.

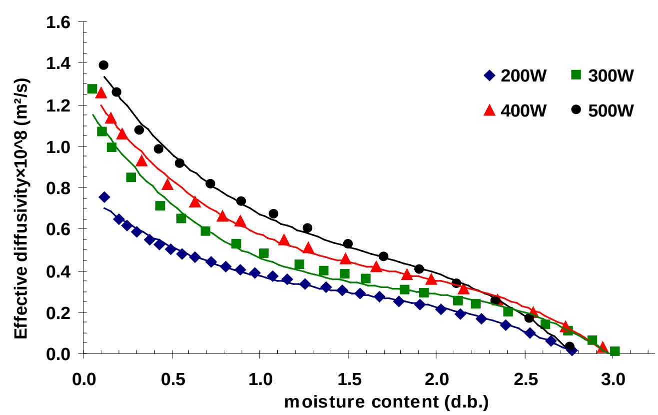

Figure 4 : Variation in effective moisture diffusivity with moisture content at different microwave powers

A third order polynomial relationship was found to correlate the effective moisture diffusivity with corresponding moisture content of carrot slices and is given by Eq. (17)

$$D_{eff} = (A + BX + CX^2 + DX^3) \times 10^{-8} \qquad (17)$$

where A, B, C, D is the constants of regression, and X is moisture content (d.b.)

Regression constants for microwave drying of carrot slices under different powers are presented in Table 2. The high values of R^2 are indicative of good fitness of empirical relationship to represent the variation in effective moisture diffusivity with moisture content of carrot slices.

Table 2 : Regression coefficients of effective moisture diffusivity for different microwave powers

P(W)	A	B	C	D	R^2
500	1.4877	-1.3507	0.6745	-0.1372	0.9962
400	1.3081	-1.2129	0.5893	-0.1109	0.9955
300	1.2130	-1.2750	0.6410	-0.1174	0.9862
200	0.7791	-0.6854	0.3530	-0.0745	0.9944

The average effective moisture diffusivity was calculated by taking the arithmetic mean of the effective moisture diffusivities that were estimated at various levels of moisture contents during the course of drying, as shown in Fig 5. Average values of effective diffusivity for different microwave power are presented in Table 3. The D_{eff} values of the carrot slices were within the general ranges of 10^{-9} to 10^{-11} m^2/s for biological materials [6, 12, 15]. The values of D_{eff} are comparable with the reported values of 1.0465×10^{-8} to 9.1537×10^{-8} m^2/s mentioned for apple pomace microwave drying [6], 1.14×10^{-6} to 6.09×10^{-6} m^2/s for tomato pomace microwave drying at 160-800W [16], 0.55×10^{-7} to 3.5×10^{-7} m^2/s for Gundelia tournefortii microwave drying at 90-800W [7].

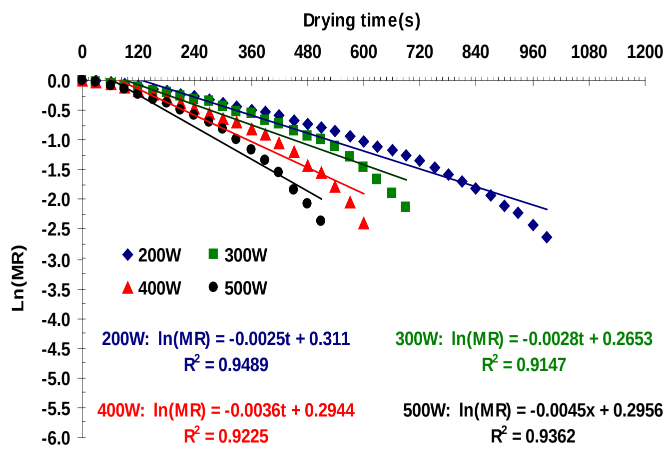

Figure 5 : Variation in ln (MR) and drying time (in seconds) for carrot slices dried at different microwave powers

Table 3 : Result of average effective diffusivity of carrot slices with different microwave power levels

P(W)	Average effective diffusivity (m^2/s)
500	1.14×10^{-8}
400	9.12×10^{-9}
300	7.09×10^{-9}
200	6.33×10^{-9}

A linear regression analysis on the average diffusion coefficient with microwave power resulted in the following relationships:

$$(D_{eff})_{average} = 4\times10^{-9}\exp(0.002P) \quad R^2 = 0.9788 \quad (18)$$

where P is the microwave power (W).

IV. Conclusion

In this study, microwave drying characteristics of carrot slices were investigated. The drying curves of carrot slices did not show a constant rate drying period. The drying took place in the falling rate period during drying process. To explain the drying characteristics of strained yoghurt, nine drying models were applied; however, the model developed by Midilli model showed good agreement with the experimental data. The effective diffusivity was computed from Fick's second law, the values of which varied between 6.33×10^{-9} to 1.14×10^{-8} m^2/s, over the microwave power level range. The effective diffusivity increases microwave power level increases. The effective moisture diffusivity increased with decrease in moisture content of carrot slices. A third order polynomial relationship existed between effective moisture diffusivity and the moisture content of carrot slices.

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