Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 2 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

(1, 2) - Domination in Some Harmonius Graphs

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Abstract - In this paper we discuss (1, 2) - domination in some harmonious graphs namely ladder graph, wheel graph and tetrahedral graph.

Keywords : dominating set, domination number, (1,2) - dominating set, (1,2) - domination number.

GJSFR-F Classification : MSC 2010: 05C10, AMS: 05C69

1, 2 - DOMINATION IN SOME HARMONIUS GRAPHS

Strictly as per the compliance and regulations of :

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(1, 2) - Domination in Some Harmonius Graphs

N. Murugesan [α] & Deepa.S.Nair [σ]

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I. Introduction

Let G = (V,E) be a simple graph. A subset D of V is a *dominating set* of G if every vertex of V- D is adjacent to a vertex of D. The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G.

A (1,2) - dominating set in a graph G = (V,E) is a set S having the property that for every vertex v in V - S there is atleast one vertex in S at distance 1 from v and a second vertex in S at distance almost 2 from v. The order of the smallest (1,2) - dominating set of G is called the *(1,2) - domination number* of G and we denote it by $\gamma_{(1,2)}$.

A harmonius graph is a connected labeled graph with n graph edges in which all graph vertices can be labeled with distinct integers (mod n) so that the sums of the pairs of numbers at the ends of each graph edge are also distinct (mod n). The ladder graph and wheel graph are harmonius. The n-ladder graph can be defined as $P_2 \square P_n$, where P_n is a path graph. It is therefore equal to the $2\times n$ grid graph. This graph looks like a ladder, having two rails and n rungs between them. A wheel graph Wn of order n, contains a cycle of order n-1, and for which every graph vertex in the cycle is connected to one other graph vertex. *The tetrahedral graph* is the platonic graph that is the unique polyledral graph on four nodes which is also the complete graph K_4 and therefore the wheel graph W_4.

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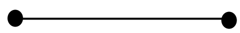
II. (1,2) - Domination in Ladder Graphs

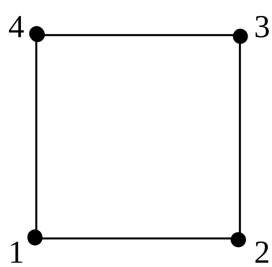
In this section we consider ladder graphs of order upto 10 and find out their domination number and (1,2) - domination number.

i) For n = 1,



This is a graph of order 2. (1,2) - domination number is defined for graphs of order atleast 3.

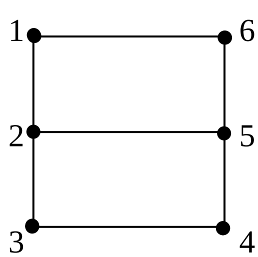
For n = 2,



{1,2} is a dominating set and also a (1,2) - dominating set.{1,2} is a dominating set.

$\therefore \quad \gamma_{(1,2)} = 2$ and $\gamma = 2$.

For n = 3,



{1,2,3} is a (1,2) - dominating set.{2,5} is a dominating set.

$\therefore \quad \gamma_{(1,2)} = 3$ and $\gamma = 2$.

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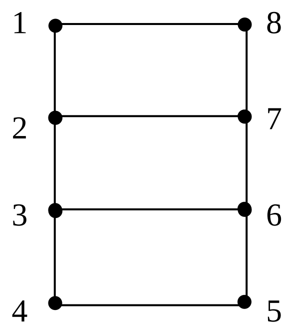
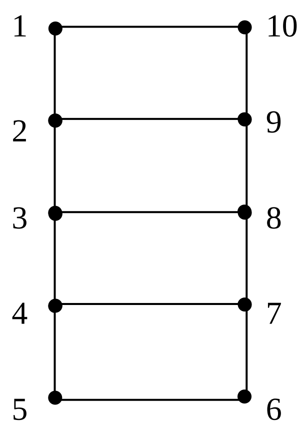
For n = 4,



{1,2,3,4} is a (1,2) - dominating set.{1,3,5,7} is a dominating set.

$\gamma_{(1,2)} = 4$ and $\gamma = 4$.

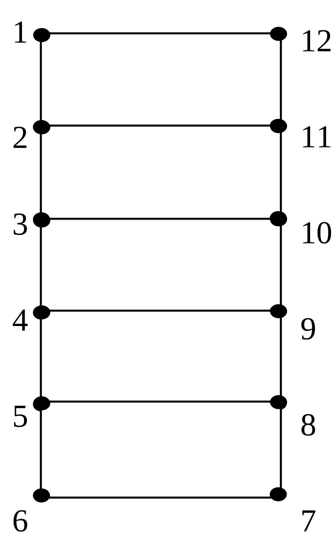
For n = 5,



{1,2,3,4,5} is a (1,2) - dominating set.{1,3,6,8} is a dominating set.

$\gamma_{(1,2)} = 5$ and $\gamma = 4$.

For n = 6,

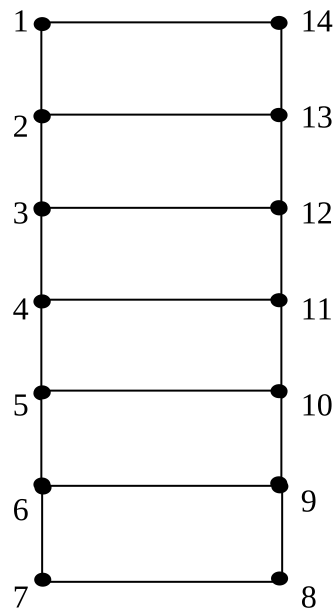


{1,2,3,4,5,6} is a (1,2) - dominating set.{1,3,5,7,9,11} is a dominating set.

$\gamma_{(1,2)} = 6$ and $\gamma = 6$.

Notes

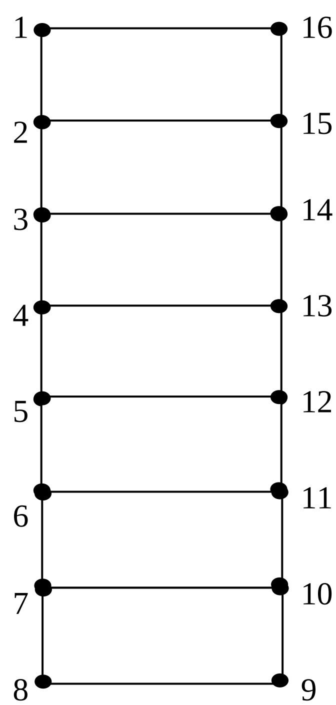
For n = 7,



{1,2,3,4,5,6,7} is a (1,2) - dominating set.{1,4,6,8,11,13} is a dominating set.

$\gamma_{(1,2)} = 7$ and $\gamma = 6$.

For n = 8,

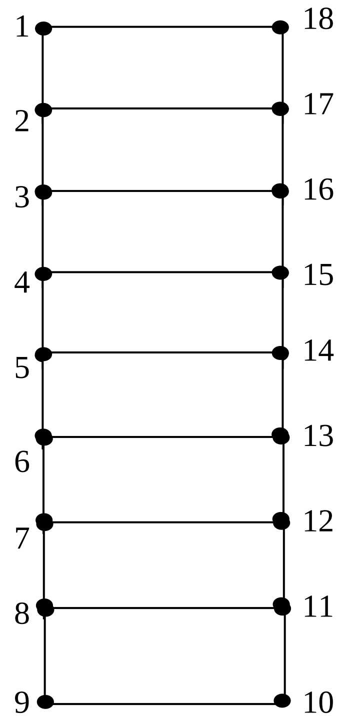


{1,2,3,4,5,6,7,8} is a (1,2) - dominating set.{1,3,5,7,9,11,13,15} is a dominating set.

$\gamma_{(1,2)} = 8$ and $\gamma = 8$.

Notes

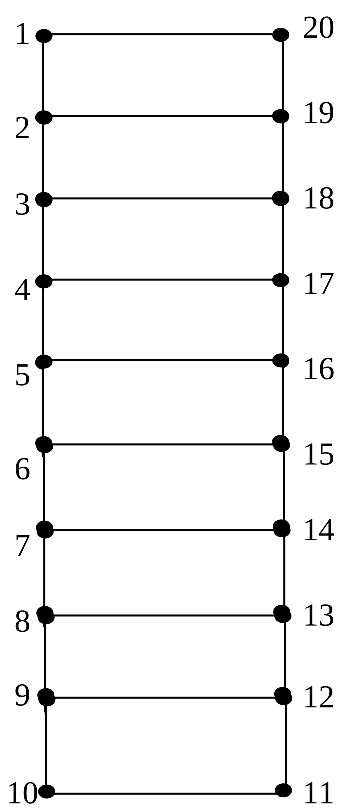
For n = 9,



{1,2,3,4,5,6,7,8,9} is a (1,2) - dominating set.{1,3,5,7,10,13,14,15} is a dominating set.

$\gamma_{(1,2)} = 9$ and $\gamma = 8$.

For n = 10,



{1,2,3,4,5,6,7,8,9,10} is a (1,2) - dominating set.{1,3,5,7,9,11,13,15,17,19} is a dominating set.

$\gamma_{(1,2)} = 10$ and $\gamma = 10$.

From the above examples we have the following theorems.

Theorem 2.1

For a ladder graph L_n, (1,2)- domination number is n. That is, $\gamma_{(1,2)}(L_n) = n$.

Proof :

For a ladder graph L_n, there are 3n-2 edges and 2n vertices. Also there are n vertices in both the rails. Suppose a vertex v_1 in the first rail is adjacent to a vertex u_1 in the second rail. Then all the remaining vertices in the first rail will be at distance greater than 1 from u_1. So to form a (1,2) - dominating set we have to include all the vertices in one rail. So the (1,2) - domination number is n.

Theorem 2.2

For a ladder graph L_n with n even, $\gamma(L_n)= n$.

Proof :

Each L_n has 3n-2 edges and 2n vertices. If n is even, the vertices in the inner rungs, that is, $\frac{n}{2}$ rungs can form a dominating set. So the number of vertices in the dominating set will be n, since each rung contains two vertices. Hence $\gamma(L_n)= n$.

Theorem 2.3

For a ladder graph L_n with n odd, $\gamma(L_n)= n-1$.

Proof :

Each L_n has 3n-2 edges and 2n vertices. Since n is odd, the vertices in the middle rung will be at equal distance from the vertices in the outer rungs. So if we take the two vertices of the middle rung and one vertex each from the alternate rungs, that set will form a dominating set. So since there are n rungs, the set will consist of n-1 vertices. So the domination number is n-1. Hence $\gamma(L_n)= n-1$.

III. Relation Between Domination Number and (1,2)-Domination Number of Ladder Graphs

Lemma 3.1([5],p.782)

In a graph G, domination number is less than or equal to (1,2)-domination number.

Ref.

5. Murugesan N. and Deepa S. Nair, (1,2) - domination in Graphs, J. Math. Comput. Sci., Vol.2, 2012, No.4, 774-783.

Notes

Proof:

Let G be a graph and D be its dominating set. Then every vertex in V-D is adjacent to a vertex in D. That is, in D, for every vertex u, there is a vertex which is at distance 1 from u. But it is not necessary that there is a second vertex at distance atmost 2 from u. So if we find a (1,2)- dominating set ,it will contain more vertices or atleast equal number of vertices than the dominating set. So the domination number is less than or equal to (1,2)- domination number.

This is true for ladder graphs also.

From the examples discussed in section 2 we have the following theorems

Theorem 3.1

For a ladder graph L_n with n even, the domination number and (1,2) - domination number are equal.

Proof :

In a ladder graph, there are 2n vertices and 3n-2 edges. The n vertices in one rail form a (1.2) - dominating set. If n is even the number of inner rungs will be $\frac{n}{2}$ even. And the vertices of these inner rungs form a dominating set. Since each rung contains 2 vertices, the dominating set will consist of n vertices. Hence the domination number and (1,2) - domination number are equal.

Theorem 3.2

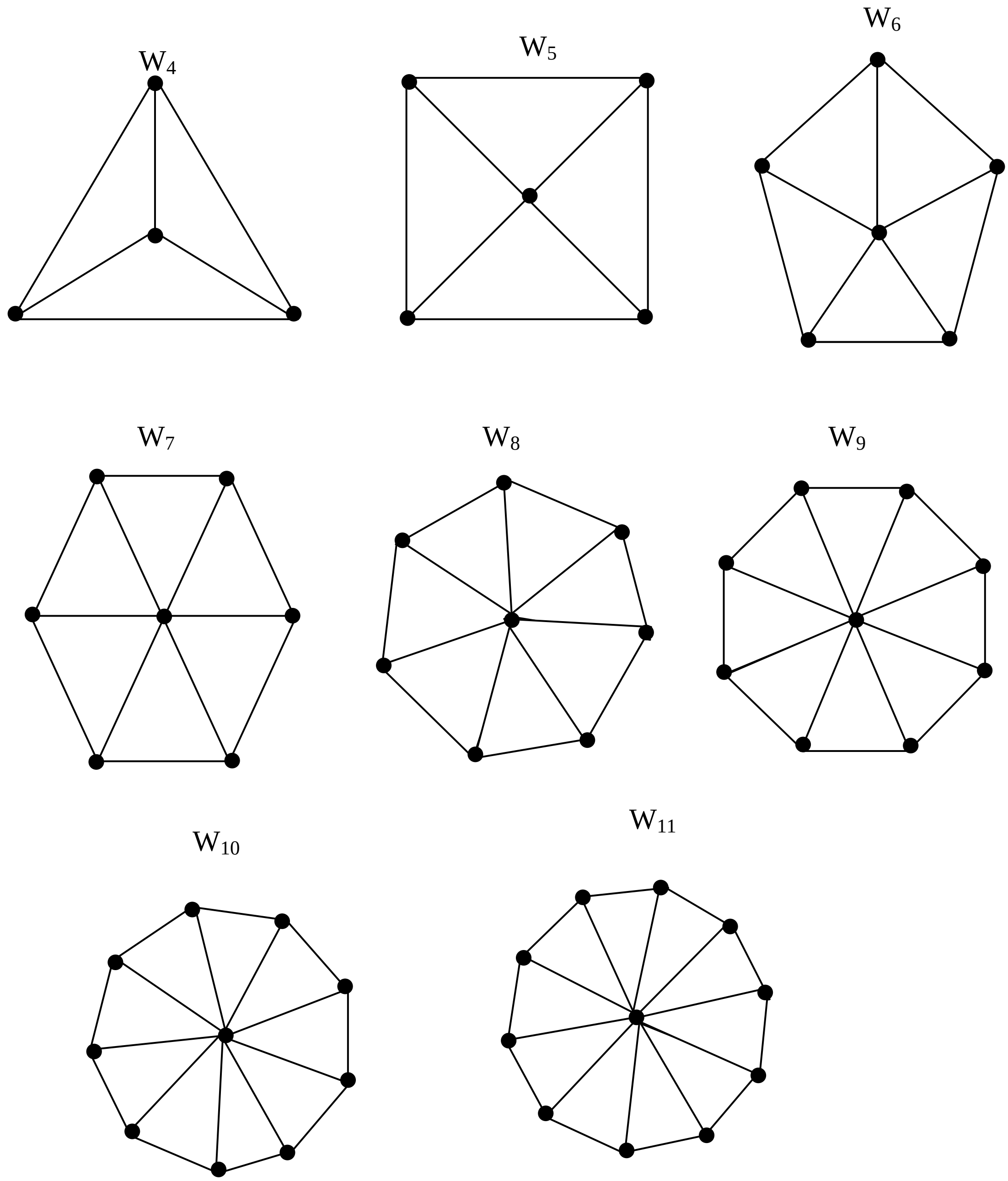
For n odd, the domination number of a ladder graph L_n is less than the (1.2) - domination number.

Proof :

For a ladder graph with n odd, the number of inner rungs will be (n-2), odd. The vertices of the middle rung and one vertex each from the alternate rungs will form a dominating set. So altogether we will get (n-1) vertices. That is, the domination number is (n-1). But the (1,2) - domination number is n. Hence the domination number is less than the (1,2) - domination number.

IV. (1,2) - Domination in Wheel Graphs

Consider the following wheel graphs.



Theorem 4.1

The domination number of a wheel graph is 1. That is, $\gamma(W_n) = 1$

Proof :

In a wheel graph it contains a cycle of order n - 1 every graph vertex in the cycle is connected to one other graph vertex. In a wheel W_n, there is a vertex with degree n-1. So that vertex is adjacent to all other vertices. Hence the domination number is one.

Notes

Theorem 4.2

For a wheel graph W_n, (1,2) - domination number is 2.

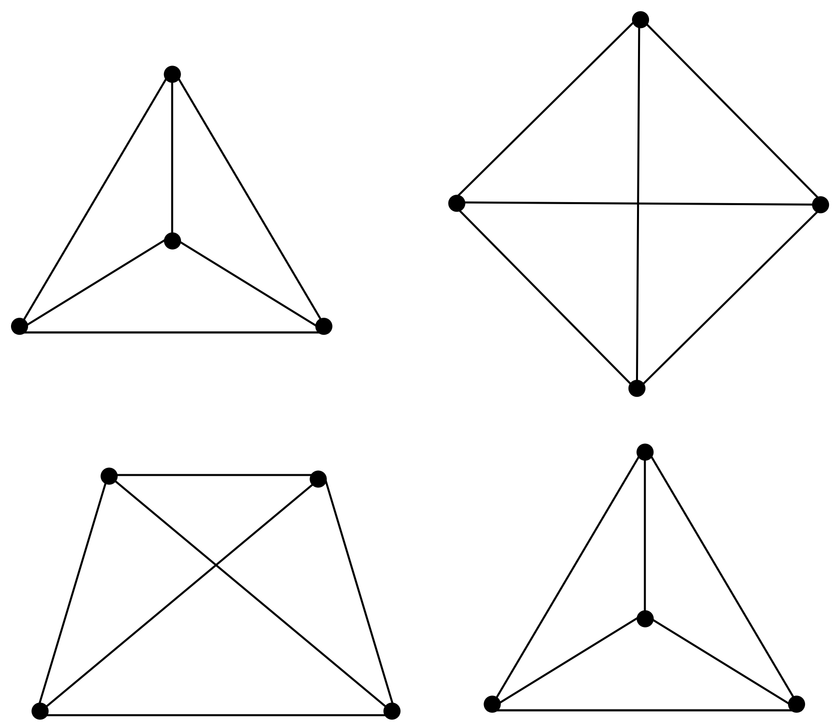
That is, $\gamma_{(1,2)}(W_n) = 2$.

Proof :

The dominating set of a wheel graph consists of only one vertex. By the definition of (1,2) - dominating set, it should contain atleast two vertices. So if we take the central vertex and any one of the vertex from the cycle, that will form a (1,2) - dominating set. The cardinality of the (1,2) - dominating is 2. Hence $\gamma_{(1,2)}(W_n) = 2$.

V. (1,2)- Domination in the Tetrahedral Graphs

Consider the following graphs



Theorem 5.1

For a tetrahedral graphs, there does not exist any (1,2) - dominating set.

Proof :

A tetrahedral graph is also a complete graph K_4. We proved in paper [5] that (1,2) - domination is not possible in complete graphs. We cannot find a (1,2) - dominating set in tetrahedral graphs.

Ref.

5. Murugesan N. and Deepa S. Nair, (1,2) - domination in Graphs, J. Math. Comput. Sci., Vol.2, 2012, No.4, 774-783.

VI. Conclusion

Here we discussed the (1,2)-domination in three types of harmonius graphs. The domination number of ladder graphs is less than or equal to (1.2) - domination number which agrees to the result of previous paper [5]. (1,2) - domination is not possible in tetrahedral graphs.

Notes

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