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Classes of Fuzzy Real-Valued Double Sequences Related to the Space ℓ^p

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Abstract - In this article, corresponding to certain general sequence $\phi = \{\phi_{nk}\}$, we introduce the fuzzy real valued double sequence spaces ${}_2m(\phi, p)$ space where $p = (p_{nk})$ is a double sequence of bounded strictly positive numbers, closely related to the space ℓ^p. We study their different properties. We study some algebraic and topological properties of the space ${}_2m(\phi, p)$. Also we obtained the necessary and sufficient conditions for inclusion and equality of ${}_2m(\phi, p)$ and ${}_2m(\psi, p)$.

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CLASSES OF FUZZY REAL-VALUED DOUBLE SEQUENCES RELATED TO THE SPACE LP

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Bijan Nath [α] & Santanu Roy [σ]

Abstract - In this article, corresponding to certain general sequence $\phi = \{\phi_{nk}\}$, we introduce the fuzzy real valued double sequence spaces ${}_2m(\phi, p)$ space where $p = (p_{nk})$ is a double sequence of bounded strictly positive numbers, closely related to the space ℓ^p. We study their different properties. We study some algebraic and topological properties of the space ${}_2m(\phi, p)$. Also we obtained the necessary and sufficient conditions for inclusion and equality of ${}_2m(\phi, p)$ and ${}_2m(\psi, p)$.

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I. Introduction

To overcome limitations induced by vagueness and uncertainty of real life data, neoclassical analysis [6] has been developed. It extends the scope and results of classical mathematical analysis by applying fuzzy logic to conventional mathematical objects, such as functions, sequences and series etc. Since the introduction of the concept of fuzzy sets by Zadeh [28] in 1965, fuzzy set theory has become an active area of research in science and engineering. The ideas of fuzzy set theory have been used widely not only in many engineering applications, such as, in the computer programming [10], in quantum physics [15], in population dynamics [2], in the control of chaos [9], in bifurcation of non-linear dynamical system [12], but also in various branches of mathematics, such as, theory of metric and topological spaces [8], in the theory of linear systems [17], studies of convergence of sequences of functions [5,13,27] and in approximation theory [1].

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Ref

[2] L. C. Barros, R. C. Bassanezi, and P. A.Tonelli, Fuzzy modelling in population dynamics, *Ecol. Model.*, 128 (2000), 27-33.

Using the notion of fuzzy real numbers, different types of fuzzy real-valued sequence spaces have been introduced and studied by several mathematicians. The initial works on double sequences of real or complex terms are found in Bromwich [4]. Hardy [11] introduced the notion of regular convergence for double sequences of real or complex terms. Moričz [16], Basarir and Solancan [3], Choudhary and Tripathy [7], Tripathy and Dutta [22,23], Tripathy and Sarma [24,25] are a few to be named those who studied different aspect of double sequences of fuzzy real numbers

The space $m(\phi)$ was introduced by Sargent [19]. He studied some properties of the space $m(\phi)$. Later on it was studied from sequence space point of view and some matrix classes with one member as $m(\phi)$ were characterized by Rath and Tripathy [18], Tripathy and Sen [26] and many others.

II. Definitions and Background

Throughout the thesis *N*, *R* and *C* denote the sets of **natural**, **real** and **complex** numbers respectively and *w*, ℓ_∞ denote the spaces of all and bounded sequences of complex terms respectively.

A fuzzy real number *X* is a fuzzy set on *R*, ie. a mapping $X : R \to L(=[0,1])$

associating each real number *t* with its grade of membership $X(t)$.

A fuzzy real number *X* is said to be **convex** if

$$X(t) \geq X(s) \wedge X(r) = \min[X(s) \wedge X(r)], \text{ where } s < t < r.$$

If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number *X* is said to be **normal**.

A fuzzy real number *X* is said to be **upper semi-continuous** if for each $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$, for all $a \in L$ is open in the usual topology of *R*. The set of all upper semi continuous, normal, convex fuzzy number is denoted by $R(L)$. Throughout the thesis, by a fuzzy real number we mean that the number belongs to $R(L)$.

The **α-level** set of a fuzzy real number *X*, $0 < \alpha \leq 1$, is denoted and defined as

$$[X]^\alpha = \{t \in R : X(t) \geq \alpha\}.$$

Ref.

[4] T.J.I. Bromwich, An Introduction to the Theory of Infinite Series, *Macmillan & Co. Ltd.*New York, 1965.

Notes

Every real number r can be expressed as a fuzzy real number $\bar{r}$ as follows :

$$\bar{r}(t) = \begin{cases} 1 & \text{if} \quad t = r \\ 0 & \text{otherwise} \end{cases}$$

The additive identity and multiplicative identity in $R(L)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

The **absolute value** $|X|$ of $X \in R(L)$, is defined as (one may refer to Kaleva and Seikkla [42]):

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if} \quad t \geq 0 \\ 0 & \text{if} \quad t < 0. \end{cases}$$

Let D be the set of all closed bounded intervals $X = [X^L, X^R]$ on the real line R.

If $Y = [Y^L, Y^R]$, then $X \leq Y$ if and only if $X^L \leq Y^L$ and $X^R \leq Y^R$.

Also let $d(X,Y) = \max(|X^L - X^R|, |Y^L - Y^R|)$. Then (D, d) is a complete metric space.

Let $\bar{d} : R(L) \times R(L) \to R$ be defined by

$$\bar{d}(X,Y) = \sup_{0 \leq \alpha \leq 1} d([X]^\alpha, [Y]^\alpha), \text{ for } X, Y \in R(L).$$

Then $\bar{d}$ defines a metric on $R(L)$ and $(R(L), \bar{d})$ is a complete metric space.

A fuzzy real-valued double sequence is a double infinite array of fuzzy real numbers X_{nk} for all $n, k \in N$ and is denoted by (X_{nk}), where $X_{nk} \in R(L)$.

A fuzzy real-valued double sequence (X_{nk}) is said to be **convergent** in Pringsheim's sense to the fuzzy real number X_0, if for every $\varepsilon > 0$, there exists $n_0 = n_0(\varepsilon)$, $k_0 = k_0(\varepsilon) \in N$ such that $\bar{d}(X_{nk}, X_0) < \varepsilon$, for all $n \geq n_0$ and $k \geq k_0$.

A fuzzy real-valued double sequence (X_{nk}) is said to be bounded if $\sup_{n,k} \bar{d}(X_{nk}, \bar{0}) < \infty$.

A fuzzy real valued double sequence space E^F is said to be **solid** (or **normal**) if $(Y_{nk}) \in E^F$, whenever $\bar{d}(Y_{nk}, \bar{0}) \leq \bar{d}(X_{nk}, \bar{0})$ for all $n, k \in N$ and $(X_{nk}) \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **monotone** if E^F contains the canonical pre-image of all its step spaces.

Throughout π denotes a permutation over $N \times N$. For $X = (X_{nk})$ a given sequence, $S(X)$ denotes the set of all permutation of the elements of (X_{nk}), that is $S(X) = \{(X_{\pi(n,k)})\}$.

A fuzzy real valued double sequence space E^F is said to be **symmetric** if $S(X) \subset E^F$, for all $X \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **sequence algebra** if $(X_{nk} \otimes Y_{nk}) \in E^F$, whenever $(X_{nk}), (Y_{nk}) \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **convergence free** if $(Y_{nk}) \in E^F$, whenever $(X_{nk}) \in E^F$ and $X_{nk} = \bar{0}$ implies $Y_{nk} = \bar{0}$.

Let $\wp$ denote the set of all subsets of N. For any $s \in N$, $\wp_s$ denote the class of all $\sigma \in \wp$ such that σ does not contain more than s elements. $\phi = (\phi_{nk})$ is a non-decreasing sequence such that

$$(n+1)(k+1)\phi_{nk} \geq nk\phi_{n+1,k+1} \text{ for all } n,k \in N.$$

A BK-space is a Banach space of complex double sequences $x = (x_{nk})$ in which the co-ordinate maps are continuous, that is,

$$\left|x_{nk}^{(i)} - x_{nk}\right| \to 0, \text{ whenever } \left\|x^{(i)} - x\right\| \to 0 \text{ as } n,k \to \infty,$$

where $x^{(i)} = \left(x_{nk}^{(i)}\right)$, for all $i \in N$ and $x = (x_{nk})$.

The space $m(\phi)$ introduced by Sargent [19] is defined as

$$m(\phi) = \left\{(x_k) \in w : \|x\|_{m(\phi)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \sum_{n \in \sigma} |x_k| < \infty\right\}.$$

Tripathy and Sen [26] introduce the sequence spaces $m(\phi, p)$ as follows :

For $1 \leq p < \infty$,

$$m(\phi, p) = \left\{(x_n) \in w : \|x\|_{m(\phi,p)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \left\{\sum_{n \in \sigma} |x_n|^p\right\}^{\frac{1}{p}} < \infty\right\},$$

For $0 < p < 1$,

$$m(\phi, p) = \left\{(x_n) \in w : \|x\|_{m(\phi,p)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \sum_{n \in \sigma} |x_n|^p < \infty\right\}.$$

Notes

Generalizing the above sequence spaces, we now introduce the spaces ${}_2m(\phi, p)$ as follows:

$$m(\phi, p)=\left\{X=(X_{nk})\in {}_2w^F : \|X\|_{{}_2m(\phi,p)} = \sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r} \frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}<\infty\right\};$$

where $p=(p_{nk})$ is a double sequence of bounded strictly positive real numbers.

Notes

III. Main Results

Theorem 1. *The class of sequences ${}_2m(\phi, p)$ is a linear space.*

Proof. With standard techniques, we can easily prove the result.

Theorem 2. *The class of sequences ${}_2m(\phi, p)$ is complete.*

Proof. Let $\left(X^{(i)}\right)$ be a Cauchy sequence in ${}_2m(\phi, p)$ where $X^{(i)}=(X_{nk}{}^{(i)})$.

$$\sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right]<\infty, \quad \text{for all } n,k\in N.$$

Then for a given $\varepsilon>0$, there exists $n_0\in N$ such that

$$\left\|X^{(i)}-X^{(j)}\right\|<\varepsilon, \text{ for all } i,j\geq n_0.$$

$$\Rightarrow \sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}{}^{(i)},X_{nk}{}^{(j)})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right]<\varepsilon, \text{ for all } n,k\in N \quad \ldots(1)$$

$$\Rightarrow \bar{d}(X_{nk}{}^{(i)},X_{nk}{}^{(j)})<\varepsilon\phi_1, \text{ for all } i,j\geq n_0, \text{ for all } n,k\in N.$$

Hence for each fixed $n,k\in N$, the sequence $\left(X_{nk}{}^{(i)}\right)$ is a Cauchy sequence in $R(L)$.

Since $R(L)$ is complete, the sequence $\left(X_{nk}{}^{(i)}\right)$ converges in $R(L)$, for each $n,k\in N$.

Let $\lim_{i\to\infty} X_{nk}{}^{(i)}=X_{nk}$, for all $n,k\in N$, where $X=(X_{nk})$.

We now show that (i) $X\in {}_2m(\phi, p)$ and

and (ii) $X^{(i)}\to X$.

From equation (1), we get for each fixed s and r

$$\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}{}^{(i)},X_{nk}{}^{(j)})]^{p_{nk}}<\varepsilon^{p_{nk}}(\phi_s\phi_r)^{p_{nk}}, \text{ for all } i,j\geq n_0,\ \sigma\in\wp_s.$$

Letting $j \to \infty$, we get

$$\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk})]^{p_{nk}} < \varepsilon^{p_{nk}}(\phi_s\phi_r)^{p_{nk}}, \text{ for all } i \ge n_0,\ \sigma \in \wp_s.$$

$$\Rightarrow \sup_{s\ge1,\sigma\in\wp_s}\ \sup_{r\ge1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \varepsilon, \text{ for all } i \ge n_0 \quad \dots(2)$$

$$\Rightarrow X^{(i)} - X \in {}_2m(\phi, p), \text{ for all } i \ge n_0.$$

Hence $X = X^{(i)} + (X - X^{(i)}) \in {}_2m(\phi, p)$, since ${}_2m(\phi, p)$ *is a linear space.*

Also (2) $\Rightarrow \left\| X^{(i)} - X \right\|_{{}_2m(\phi,p)} < \varepsilon$, for all $i \ge n_0$.

$$\Rightarrow X^{(i)} \to X \in {}_2m(\phi, p).$$

Hence ${}_2m(\phi, p)$ is complete.

Theorem 3. *The class of sequences* ${}_2m(\phi, p)$ *is a B K-space.*

Proof. By Theorem 2, ${}_2m(\phi, p)$ is a Banach space.

Let $\left\| X^{(i)} - X \right\|_{{}_2m(\phi,p)} \to 0$, as $i \to \infty$.

Then for a given $\varepsilon > 0$, there exists $n_0 \in N$ such that

$$\left\| X^{(i)} - X \right\|_{{}_2m(\phi,p)} < \varepsilon, \text{ for all } i \ge n_0.$$

$$\Rightarrow \sup_{s\ge1,\sigma\in\wp_s}\ \sup_{r\ge1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \varepsilon, \text{ for all } i \ge n_0.$$

$$\Rightarrow \bar{d}(X_{nk}^{(i)}, X_{nk}) < \varepsilon\phi_1, \text{ for all } i \ge n_0, \text{ for all } n, k \in N.$$

$$\Rightarrow \bar{d}(X_{nk}^{(i)}, X_{nk}) \to 0, \text{ as } i \to \infty.$$

Hence ${}_2m(\phi, p)$ *is a B K-space.*

This completes the proof of the theorem.

Notes

Notes

Theorem 4. (i) *The class of sequences* ${}_2m(\phi, p)$ *is a symmetric space. If* $X \in {}_2m(\phi, p)$ and $U \in S(X)$, where $S(X)$ denotes the set of all permutation of the elements of (X_{nk}), then $\|U\|_{{}_2m(\phi,p)} = \|X\|_{{}_2m(\phi,p)}$.

(ii) *The class of sequences* ${}_2m(\phi, p)$ *is a normal space.*

Proof. The proof of the result follows from definition.

Theorem 5. ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$ if and only if $\sup\limits_{s\geq 1}\left(\dfrac{\phi_s}{\psi_s}\right) < \infty$.

Proof. Let $\sup\limits_{s\geq 1,}\left(\dfrac{\phi_s}{\psi_s}\right) = K(<\infty)$.

Then $\phi_s \leq K\psi_s$.

Let $(X_{nk}) \in {}_2m(\phi, p)$. Then

$$\sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \infty$$

$$\Rightarrow \sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{K\psi_s}\frac{1}{K\psi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \infty$$

$$\Rightarrow \|X\|_{{}_2m(\psi,p)} < \infty.$$

Hence ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$.

Conversely let ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$. we have to show that $\sup\limits_{s\geq 1}\left(\dfrac{\phi_s}{\psi_s}\right) = \sup\limits_{s\geq 1}(\eta_s) < \infty$.

If possible let $\sup\limits_{s\geq 1,}(\eta_s) = \infty$. Then there exists a subsequence (η_{s_i}) of (η_s) such that $\lim\limits_{i\to\infty}(\eta_{s_i}) = \infty$.

Then for $(X_{nk}) \in {}_2m(\phi, p)$, we have

$$\sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{\psi_s}\frac{1}{\psi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] \geq \sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\eta_{s_i}\frac{1}{\phi_{s_i}}\eta_{r_i}\frac{1}{\phi_{r_i}}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] = \infty.$$

$\Rightarrow (X_{nk}) \in {}_2m(\psi, p)$, a contradiction.

This step concludes the proof of the theorem.

In view of the above theorem, we formulate the following result.

Corollary 1. ${}_2m(\phi, p) = {}_2m(\psi, p)$ if and only if $\sup_{s\geq 1,}(\eta_s) < \infty$ and $\sup_{s\geq 1,}(\eta_s^{-1}) < \infty$, where

$\eta_s = \dfrac{\phi_s}{\psi_s}$.

Theorem 7. ${}_2m(\psi, p) = \ell^p$ if and only if $\sup_{s\geq 1,}(\phi_s) < \infty$ and $\sup_{s\geq 1,}(\phi_s^{-1}) < \infty$.

Proof. Putting $\psi_{nk} = 1$, for all $n, k \in N$, in Corollary 1, we get the result easily.

Theorem 8. *If* $0 < p_{nk} < q_{nk} \leq \sup_{n,k} q_{nk}$, then ${}_2m(\phi, p) \subset {}_2m(\phi, q)$.

Proof. Using the properties of ℓ^p spaces, we get the result easily.

Theorem 9. ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$ *if* $0 < p_{nk} < q_{nk} \leq \sup_{n,k} q_{nk}$ and $\sup_{s\geq 1,}\left(\dfrac{\phi_s}{\psi_s}\right) < \infty$.

Proof. Using the properties of ℓ^p spaces and Theorem 5, we get the result.

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