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Sensitivity Analysis for the Physiographic Parameters of Overland as well as Cannel Flow Elements Based on Kinematic Wave (Kw) Theory

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Abstract - In the recent past, in many developing countries a good deal of research has been carried to solve the problems of 'large basins' whereas not much has been done with regard to the hydrologic problems of small watersheds. Small watersheds to play important roles e.g. a village pond catered by its own small watershed; in hilly watersheds, the generated runoff causes flash flood, resulting into disruptions of communication lines etc. Therefore, it is necessary to look into these aspects of the hydrologic problems with greater attention. The hydrologic responses of a small watershed depend upon the mechanics of surface runoff, which is primarily a nonlinear process. We have discussed the sensitivity of physiographic parameters through lumped kinematics wave models and found that the overland roughness and overland slope are more sensitive than other physiographic parameters of overland and channel flows. Appropriate discussions of results and conclusions as arrived at in different parts have been summarized.

Keywords : kinematic wave, overland flow, channel flow, lumped physiographic models, watershed, sensitivity analysis.

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SENSITIVITY ANALYSIS FOR THE PHYSIOGRAPHIC PARAMETERS OF OVERLAND AS WELL AS CANNEL FLOW ELEMENTS BASED ON KINEMATIC WAVE KWTHEORY

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Sensitivity Analysis for the Physiographic Parameters of Overland as well as Cannel Flow Elements Based on Kinematic Wave (Kw) Theory

Dr. M. M. Hossain[α], J. Ferdous [σ] & M. Motateb Hossain [ρ]

Abstract - In the recent past, in many developing countries a good deal of research has been carried to solve the problems of 'large basins' whereas not much has been done with regard to the hydrologic problems of small watersheds. Small watersheds to play important roles e.g. a village pond catered by its own small watershed; in hilly watersheds, the generated runoff causes flash flood, resulting into disruptions of communication lines etc. Therefore, it is necessary to look into these aspects of the hydrologic problems with greater attention. The hydrologic responses of a small watershed depend upon the mechanics of surface runoff, which is primarily a nonlinear process. We have discussed the sensitivity of physiographic parameters through lumped kinematics wave models and found that the overland roughness and overland slope are more sensitive than other physiographic parameters of overland and channel flows. Appropriate discussions of results and conclusions as arrived at in different parts have been summarized.

Keywords : *kinematic wave, overland flow, channel flow, lumped physiographic models,watershed, sensitivity analysis.*

I. Introduction

Water is one of the most important natural resources vitally needed for sustenance of life. The use of water is continuously increasing with the increase in population. This increased demand for water has forced the planners, scientists and hydrologist not only to concentrate on major 'basin planning' but also to pay sufficient attention to small watersheds. The watershed planning may help in micro-level water budgeting to meet the localized demands of water for small population concentrations scattered over the entire basin. In Bangladesh, in majority cases, each village has its own pond (or tank) from which the water is drawn for domestic and other needs. These ponds have their own watersheds from which the rainwater is fed to them. Prior to onset of monsoon, after the sustained dry period most of the ponds either get dries up or are left with insignificant quantities of water. The fine silt or clay, which is accumulated at the bottom of the tank, is removed by the local people for plastering of huts and other general repairs of the houses. Thus, the lost capacity of the pond due to accumulation of silt is recouped. Though is may appear quite insignificant, yet the role of small watersheds is very important is most of the developing tropical counties is general and in the Indian subcontinent is particular. The micro-climatic influences onto the small watersheds are of

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particular significance. Response of a watershed to small, medium heavy and very heavy rainfall leasing to near cloud burst conditions is of importance not only for meeting the water demands but also for the protection of life and property. In hilly and mountainous catchments, the situations may get worse. Heavy intensity rains and resulting runoffs may cause flash floods and landslides and thus disrupt the communication lines of vital importance. These disruptions may result into major economic problems as one region gets completely cutoff from rest of the country.

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To find suitable solutions to these problems, it is necessary that micro-level in-depth studies be carried out for computations of watershed responses to vastly varying meteorological conditions. There are different types of watershed models, which have been developed in the past to estimate the peak flows and runoff hydrographs for small watersheds. With the advent of powerful high-speed digital computers, it is now possible to study through mathematical models the basic physical processes viz. transformation of rain into the runoff land also the movement of water in the surface as well as in channels. These models rely on mathematical statements to represent the system.

In a small watershed, the generation of runoff can better be studied through the 'overland phase' and 'channel phase'. The conversion of rainfall excess into surface runoff and then the movement of flood flows in the channel is recognized as a nonlinear process. These days this process is generally modeled through two approaches, viz. the hydrodynamic approaches and the system's approach. For small watersheds the hydrodynamic approaches have been found more useful for the transformation of rainfall excess into the surface runoff (viz. in the overland phase) and the movement of flood flows in the channel (viz. in the channel phase).

a) Objective of the Study

The objectives of the present work may thus be summarized as below:

(i) To develop suitable mathematical models based on KW theory, best suited to the typical problems of some small watersheds.

(ii) To confirm the applicability of finite difference schemes which are most suitable for small watersheds under study.

(iii) To carry out the sensitivity analysis for the physiographic parameters of overland as well as cannel flow elements and to identify the parameters which are most sensitive. Further, to find out the role of effective overland roughness in the context of natural high slope watersheds.

Computer can solve only a finite number of digits to represent each number at each step of a calculation. Hence, round-off errors are incorporated. The computation is stable if the growth of these errors is within reasonable limits or controlled.

A numerical model with consistent equations, convergent solutions and stable error propagation forms a computationally stable scheme and gives results, which are quite close to the exact solution.

b) Nature of Hydrologic System

The nature of the hydrologic system is completely defined, through one property each, mentioned in the following three sets of possible system behaviors.

(1) Linear or nonlinear

(2) Lumped or distributed

(3) Time-invariant or time-variant.

A brief description of these three characteristics is given in the following subsections.

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i. *The Linear and Nonlinear Systems*

A hydrologic system is said to be linear, if a step input fed to the system produces an output, which is directly proportional to their input. A linear system may be represented mathematically by a linear equation. The principle of superposition applies to it may be defined mathematically as under:

$$f(\Psi_1)_t + f(\Psi_2)_t + --- + f(\Psi_n)_t = f(\Psi_1 + \Psi_2 + --- + \Psi_n)_t$$

where Ψ_i is any response function and t is the time of its applicability.

The principle of homogeneity, which is a particular case of principle of superposition, thus may be described as below $f(k\Psi)_t = kf(\Psi)_t$ where k is a constant function.

A nonlinear system is represented by a nonlinear function. The extent of nonlinearity depends upon the system itself.

The hydrologic system may be defined by a general type of differential equation and can be written as follows:

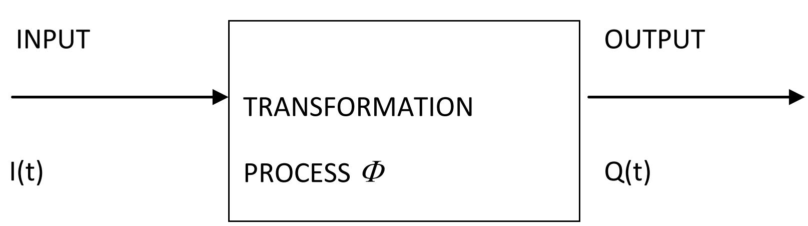
$$f(\Psi) = a_n \frac{d^n\Psi}{dt^n} + a_{n-1}\frac{d^{n-1}\Psi}{dt^{n-1}} + --- + a_1\frac{d^1\Psi}{dt^1} + a_0\Psi\ .$$

The system would be nonlinear if any on these coefficients a_0, a_1, a_2, - - -, a_n etc. are the function of Ψ or the function Ψ carries an exponent other than unity.

ii. *The Lumped and Distributed System*

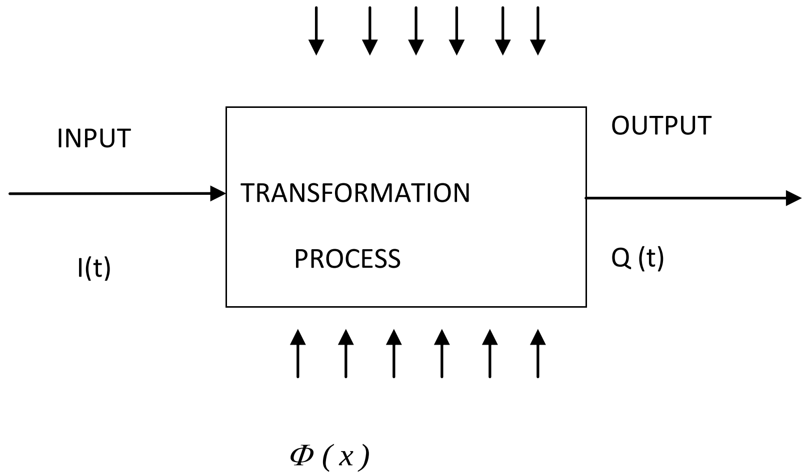
The hydrologic system is said to be lumped if its input functions or parameters do not vary with respect to spatial coordinates (Fig.2.3 (a)). For the lumped systems, average conditions or values of input and parameters are applicable. Thus, lumped systems are represented by ordinary differential equations.

The system is defined as distributed if the input or the transfer function and other parameters do vary with the spatial coordinates (Fig.2.3 (b)). Such systems are mathematically represented by the partial differential equations. The theoretical solution of such systems (a differential equation) this requires completes knowledge of the boundary conditions.



(a) LUMPED SYSTEM

$\Phi(x)$



(b) DISTRIBUTED SYSTEM

Figure : The Lumped and Distributed Systems

iii. *Time-Invariant and Time-Variant Systems*

A time-variant system is one in which the input-output relationship is not dependent upon the time at which the input is applied to the system. The concept of time-invariant makes the analysis simpler.

A time-variant system is one in which the input-output relationship is a dependent function of time. It may be concluded that a lumped, linear and time-invariant system is easiest to work with. But in surface hydrology, the hydrologic system happens to be distributed, nonlinear and time-variant in its behavior. This behavior of the system is the most complex one and quite difficult not only to formulate mathematically but also to solve. Therefore, a compromise has to be reached so that the complicated natural hydrologic system may be solved satisfactorily by making suitable assumptions with respect to the aspects stated above.

c) *Numerical Technique (Finite Difference Methods) for Solving the Kinematic Wave Equations*

The computational schemed will require the initial values for the entire domain $(X_i's)$ and the upstream boundary conditions for all $(t_j$'s). Solutions of the governing equations through the finite difference scheme will be obtained at each of this grid point. Computations advance along the downstream direction for a time step Δt, until all the discharges as well as water areas are computed at all the grid points in the entire longitudinal length L.

Next, the computations are advance ahead in time by another time step Δt and the computations proceed likewise.

The rainfall excess intensity (i_e) is assumed constant within a time step Δt. But it may change from one time step to the next time step, to account for the variation in rates of rainfall excess intensities occurring within a storm event.

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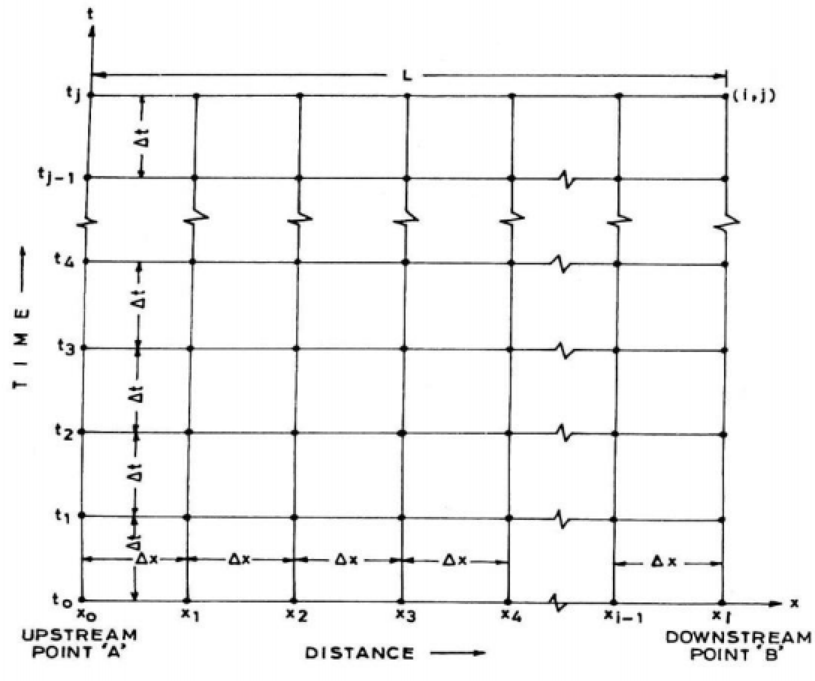

Figure 3.3 : Flow Domain Representation Through a Fixed X – t Grid

d) Physiographic Models Employed

i. *Lumped Physiographic Model*

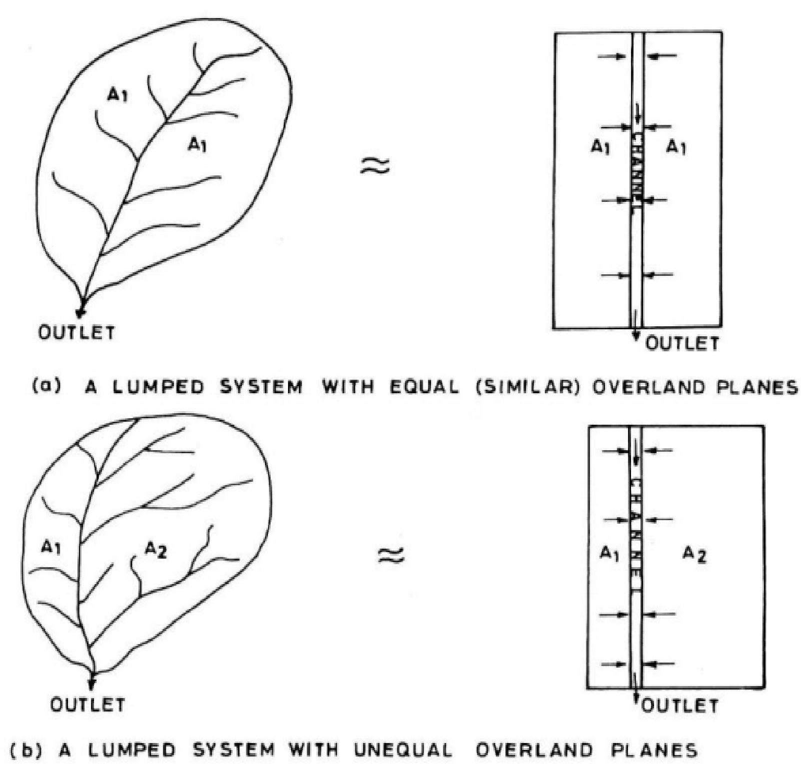

Figure 3.5 : Lumped Physiographic Models

a. Initial Conditions

$$\left.\begin{aligned} Q(x,0) &= 0 \\ A(x,0) &= 0 \end{aligned}\right\} \text{for all } x$$

For the overland flows,

$$\left.\begin{aligned} q(x,0) &= 0 \\ y_0(x,0) &= 0 \end{aligned}\right\} \text{for all } x$$

b. Boundary Conditions

For channel flow:

$$\left.\begin{aligned} Q(0,t) &= 0 \\ A(0,t) &= 0 \end{aligned}\right\} \text{for all } t$$

And for the overland flow:

$$\left.\begin{aligned} q(0,t) &= 0 \\ y_0(0,t) &= 0 \end{aligned}\right\} \text{for all } t$$

e) Physiographic Details of Watershed of Railway Bridge No. 319.

The small watershed of Railway Bridge No. 319 selected for this study is situated in Bangalore district of Karnataka of India. This bridge is located on Arsikere Bangalore Section of the Indian Railways. The index maps giving details of the watershed is given in Fig. Some important features, general information and the physiographic data as extracted from the available records and the topographic map of this bridge is given in Table.

Table 1 : General Information and Salient Features of Watershed of Railway Bridge No. 319

Sl. No.	Particulars	Units
1.	a. Name of the zone b. Name of the sub zone	Deccan Plateau 3i
2.	Geographical location	i. Longitude(app.) $77^0 10^{'}$ East ii. Latitude (app.) $13^0 18^{'}$ North
3.	Terrain	Hilly
4.	Shape of the basin	Oblong
5.	Climate	Humid
6.	Type of soil	i. Rocky ii. Red Earth

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7.	Land use	Partially Dry cultivation
8.	No. of rain gauge station	1
9.	No. of discharge location	1
10.	Altitude (average elevation from MSL)	833 meter
11.	Watershed area	82.0 hectare
12.	Length of the main channel(longest)	1650.0metre
13.	Total average annual rainfall	500-1000(mm)

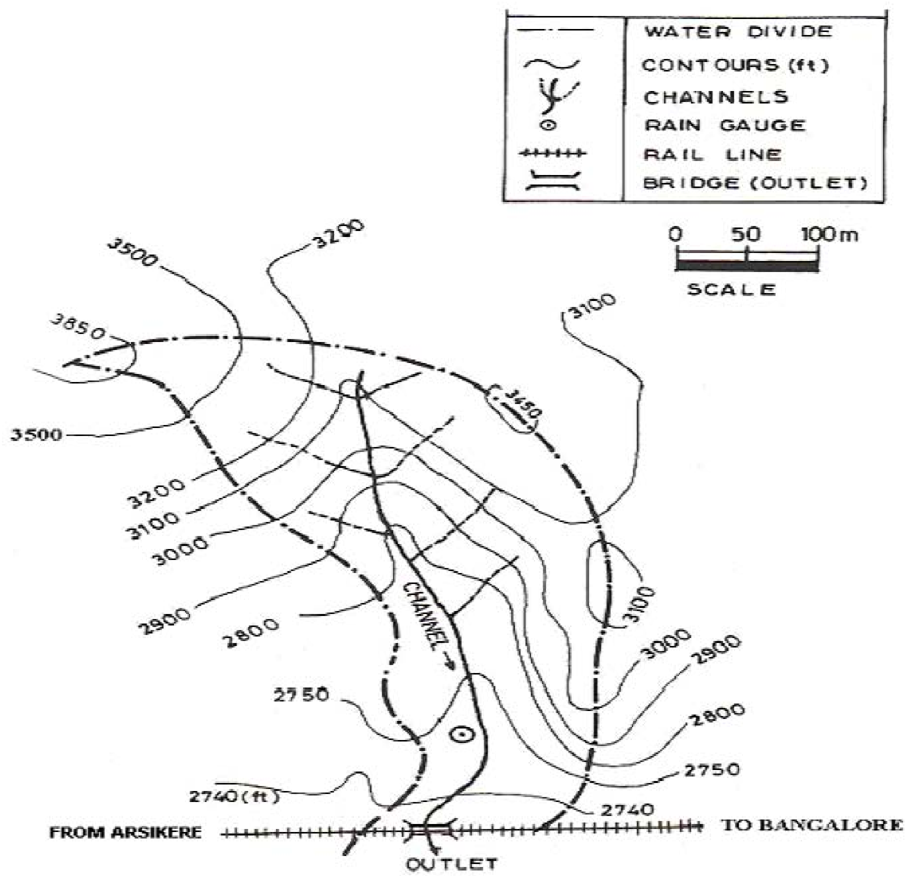

Figure 4.1 : The Index Map of Bridge no. 319

f) Estimation of Physiographic Parameters for Bridge No. 319

The topographic details of this watershed are shown in Fig. 4.1. In this natural watershed, only one main drainage channel exists that too in the central part of the watershed. As discussed the lumped physiographic model is used to compute the model parameters for the application of KW theory. For this purpose, a lumped model of the type given in Fig. (a) is adopted for the estimation of parameters. The equivalent watershed has been obtained by dividing equally the total drainage area into the two sides 1650 meters long main channel. The schematic representation of this model is shown in Fig.

As discussed in Section, the physiographic parameters were computed. The values of physiographic parameters are given in Table.

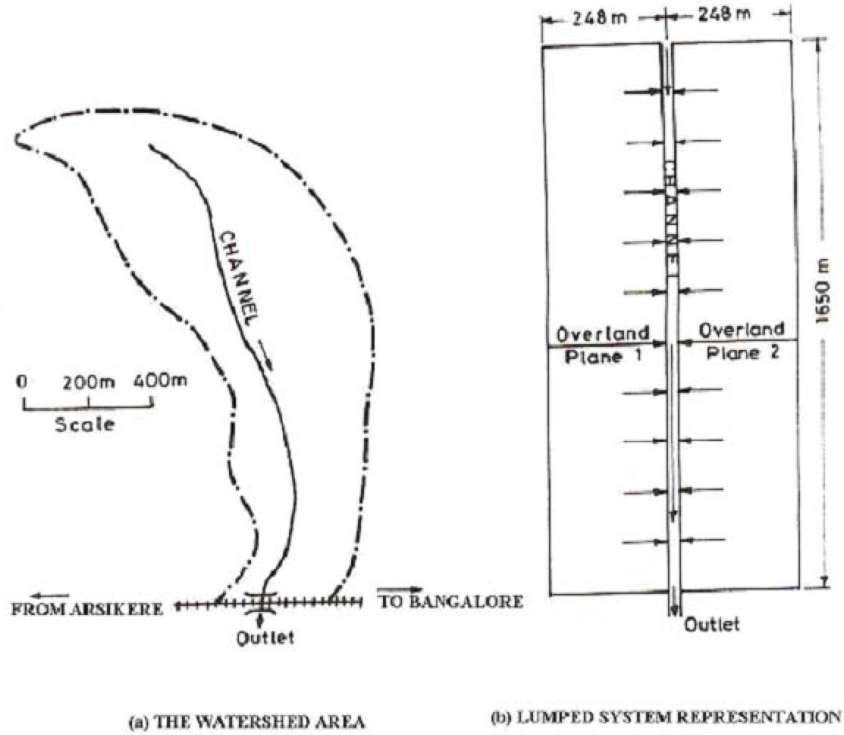

Figure 4.2 : Schmatic Representation Through a Lumped Model (Bridge No. 319)

Table 2 : The Lumped Physiographic Parameter Values of Bridge No. 319

Sl.No.	Particular	Unit
1.	Area	82.0 hectares
2.	Overland (Plane): (a). Average length (each side) (b). Average slope (each side) (c). Overland roughness	 248.0 meters 0.092 0.052
3.	Channel: (a). Average length (b). Average slope (c). Average roughness (d). Average bed width (e). Average side slope	 1650.0 meters 0.072 0.035 3.0 meters 2.5 H: IV

g) Overland Flow of Watershed of Bridge No. 319 for Different Values of Overland Roughness (An)

In overland there are two physiographic parameters land roughness and slope. The effect of overland roughness on overland flow of watershed of bridge no. 319 is shown in the following Fig.

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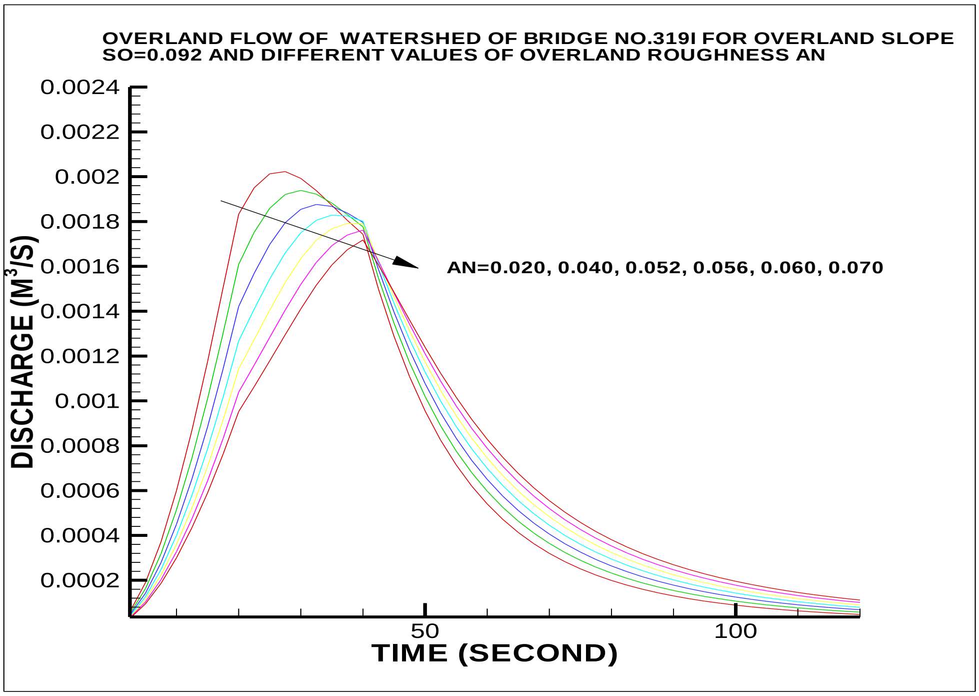

Figure : Overland Flow for Different Values of Overland Rougheness

From the above figure, we observed that if the overland roughness is increasing then peak flow is decreasing. Therefore, the overland roughness (AN) is sensitive.

h) Overland Flow of Watershed of Bridge No. 319 for Different Values of Overland Slop (So)

Different overland has different overland slope. It has an effect on overland flow. The effect of overland slope on overland flow of watershed of bridge no. 319 is shown in the following Fig. 4.4

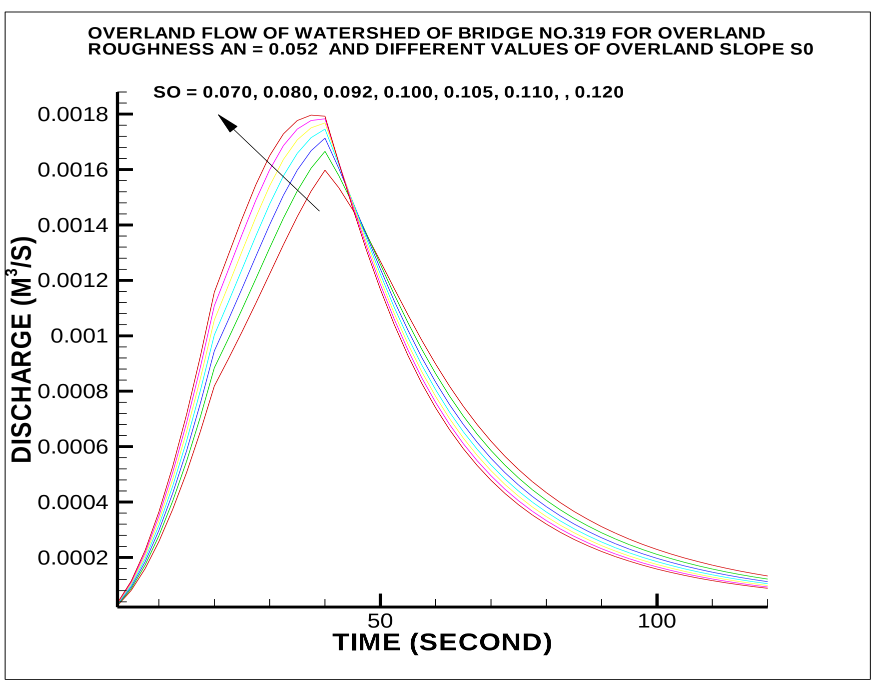

Figure : Overland Flow for Different Values of Overland Slope

From the above figure, we observe that if the overland slope is increasing then the peak flow is increasing for high slope. Therefore, the overland slope is sensitive.

i) Channel Flow Routing

A channel is a conduit in which water flows with a free surface. Classified according to its origin a cannel may be either natural or artificial. Natural channels include all watercourse that exist naturally on the earth, varying is size from tiny hillside rivulets through brooks, streams, small and large rivers, to tidal estuaries. Underground streams carrying water with a free surface are also considered natural channels.

j) Channel Flow Routing of Watershed of Bridge No. 319 for Different Value of Overland Roughness (An)

Flow of watershed is made of overland and channel flow. Overland roughness is a important factor for channel flow. The effect of overland roughness on channel flow of watershed of bridge no. 319 is shown in the Fig. 4.5

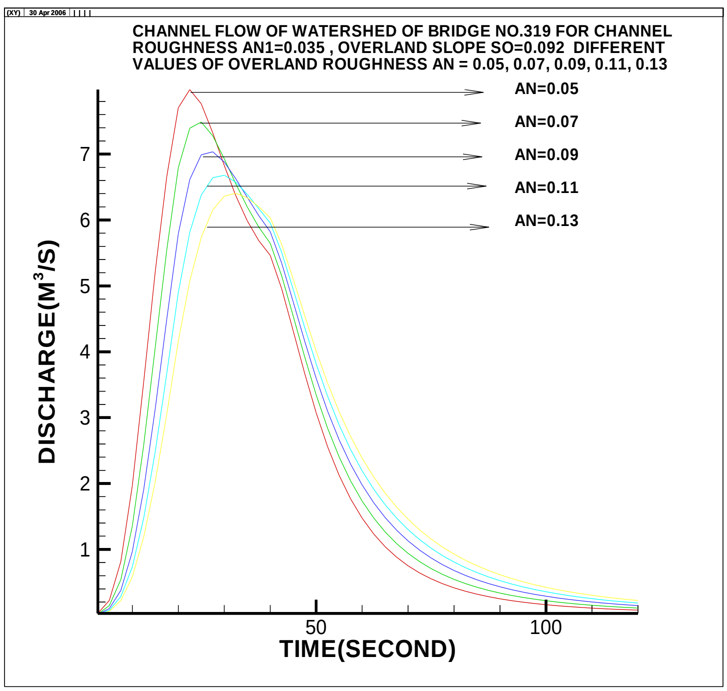

Figure : Channel Flow Routing for Different Value of Overland Roughness (An)

From above Fig, we observed that if overland roughness is increasing then the peak flow is decreasing and the flow routing changes is shapes quickly. So the overland roughness on channel flow is sensitive.

k) Channel Flow Routing of Watershed of Bridge No. 319 for Different Value of Channel Roughness (An1)

Different channel has different channel roughness. It is a factor of channel flow. The effect of channel roughness on channel flow of watershed of bridge no.319 is shown in the following Fig.4.6.

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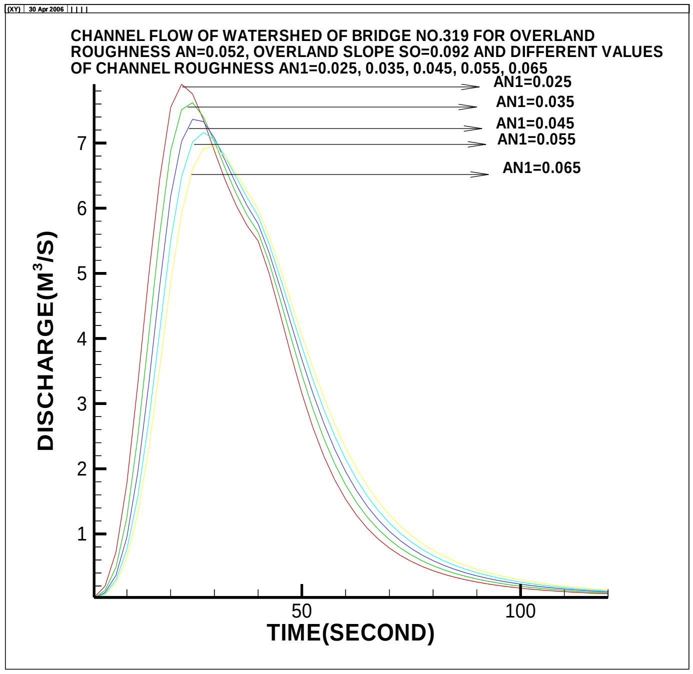

Figure : Channel Flow Routing for Different Value of Channel Roughness (An1)

From the above figure we easily observed that the effect of channel roughness on channel flow peak is sensitive but is not like overland roughness. So channel roughness is very sensitive but not like as overland roughness on channel flow.

1) *Channel Flow Routing of Watershed of Bridge No.319 for Different Value of Side Slope (Bz)*

Channel flow depends on different physiographic parameters of overland and channel. Here the effect of side slope (BZ) on channel flow of watershed of bridge no. 319 is shown in Fig.

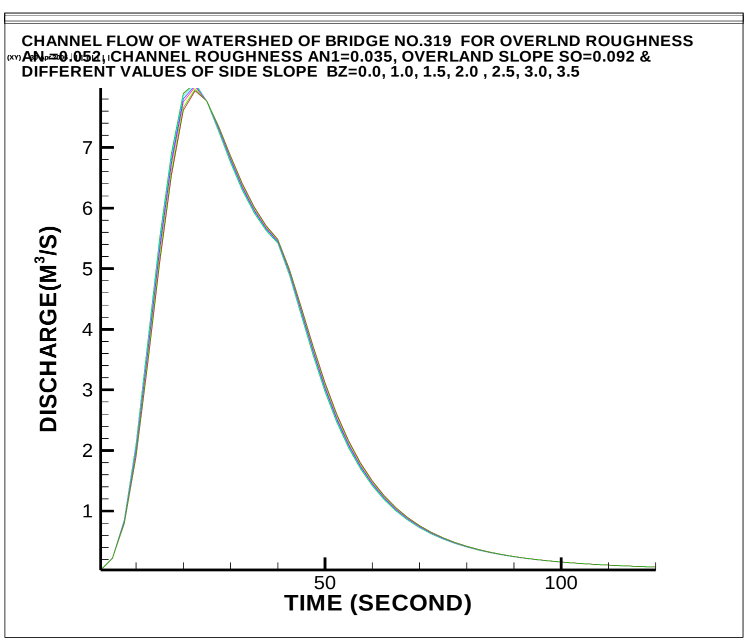

Figure : Channel Flow Routing for Different Value of Side Slope (Bz)

From the above figure we observed that if side slope is increasing then peak flow is decreasing very slowly. Therefore, the effect of side slope on the flow routing of channel flow is not important factor and hence side slope is not sensitive so much.

m) Channel Flow Routing of Watershed of Bridge No. 319 for Different Value of Overland Slope (So)

Channel flow is constituted of direct rainfall and overland flow. Channel flows depend on different physiographic parameters of overland and channel. Here the effect of overland slope (SO) on channel flow of watershed of bridge no. 319 is shown in the following Fig.

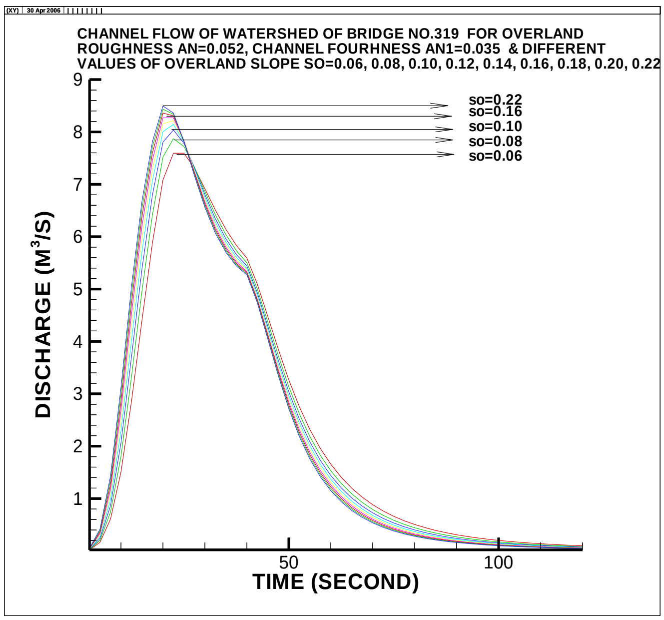

Figure : Channel Flow Routing for Different Value of Overland Slope

From the above figure, we observe that the effect of overland slope (SO) on the peak flow routing of channel flow if SO is increasing then flow routing is increasing, but after a period of flow, the flow is not increasing. So the overland slope is not so sensitive for channel flow.

II. Conclusion

From the above discussion we come in conclusion that the overland roughness and overland slope and channel roughness are more sensitive than the other parameters on overland and channel flow.

III. Scope of Furthur Study

The present study can be extended as follows:

(i) Distributed method can be used for channel flow and overland flow routing.

(ii) Three fully off-center first order explicit finite difference methods have been used for the solution of proposed KW models. A single step, second explicit method can also be used for above-mentioned models. Also, implicit finite difference methods and the finite element methods may also be used for this purpose.

(iii) In the proposed models, the contributing surface plane elements have assumed rectangular and perpendicular to the elements. The shape and the alignment of

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these planes can be oriented according to the prevailing overland slopes. An altogether, different approach will have to be developed for such cases of modeling.

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