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Study of Nonlinear Evolution Equations in Mathematical Physics

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Study of Nonlinear Evolution Equations in Mathematical Physics

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Abstract - In the present paper, we construct the traveling wave solutions involving parameters for some nonlinear evolution equations in the mathematical physics via the Konopelchenko-Dubrovsky Coupled System equation and the (1+1)-dimensional nonlinear Ostrovsky equation by using the Bernoulli Sub-ODE method. By using this method exact solutions involving parameters have been obtained. When the parameters are taken as special values, solitary wave solutions have been originated from the hyperbolic function solutions. It has been shown that the method is effective and can be used for many other NLEEs in mathematical physics.

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I. Introduction

NLEEs are encountered in various fields of mathematics, physics, chemistry, biology, engineering and numerous applications. Exact solutions of NLEEs play an important role in the proper understanding of qualitative features of many phenomena and processes in various areas of natural science. Exact solutions of nonlinear equations graphically demonstrate and allow unscrambling the mechanisms of many complex nonlinear phenomena such as spatial localization of transfer processes, multiplicity or absence steady states under various conditions, existence of peaking regimes and many others. Even those special exact solutions that do not have a clear physical meaning can be used as test problems to verify the consistency and estimate errors of various numerical, asymptotic, and approximate analytical methods. Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving NLEEs. It is significant that many equations of physics, chemistry, and biology contain empirical parameters or empirical functions. Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore, investigation exact traveling wave solutions are becoming successively attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the (G'/G)-expansion method [1-7], the Hirota's bilinear transformation method [8,9], the modified simple equation method [10-13], the tanh-function method [14], the first integral method[15], the Exp-function method [16-18], the Jacobi elliptic function method [19], the

Ref

1. Wang M., Li X., Zhang J., The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A. 372(2008) 417-423.

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homotopy perturbation method [20-22], the Bernoulli Sub-ODE method [23-24], the enhanced (G'/G)-expansion Method [25-27], the $\exp(-\Phi(\xi))$-expansion method [28] and so on.

The objective of this paper is to find the exact solutions then the solitary wave solutions for the Konopelchenko-Dubrovsky Coupled System equation and the (1+1)-dimensional nonlinear Ostrovsky equation through Bernoulli Sub-ODE method.

The article is arranged as follows: In section II, the Bernoulli Sub-ODE method is discussed. In section III, we apply this method to the nonlinear evolution equations pointed out above; in section IV, graphical representation and in section V, conclusions are given.

II. Methodology

In this section, we describe the Bernoulli Sub-ODE method for finding traveling wave solutions of NLEEs. Suppose that a nonlinear partial differential equation, say in two independent variables x and t is given by

$$\Re(u,u_t,u_x,u_{tt},u_{xx},u_{xt},\cdots\cdots\cdots\cdots)=0, \tag{1}$$

where $u(\xi)=u(x,t)$ is an unknown function, $\Re$ is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [23, 24]:

Step 1. Combining the independent variables x and t into one variable ξ, we suppose that

$$u(\xi)=u(x,t), \qquad \xi = x \pm \omega t. \tag{2}$$

The traveling wave transformation Eq. (2) permits us to reduce Eq. (1) to the following ODE:

$$\Re(u,u',u'',\cdots\cdots\cdots)=0, \tag{3}$$

where $\Re$ is a polynomial in $u(\xi)$ and its derivatives, while $u'(\xi)=\frac{du}{d\xi}$, $u''(\xi)=\frac{d^2u}{d\xi^2}$ and so on.

Step 2. We suppose that Eq.(3) has the formal solution

$$u(\xi)=\sum_{i=0}^{n} a_i G^i, \tag{4}$$

where $G=G(\xi)$ satisfy the equation $G'+\lambda G=\mu G^2$, (5)

in which $a_i\ (-n\le i\le n; n\in \mathrm{N})$ are constants to be determined later, and $\mu\neq 0, \lambda\neq 0$.

then the Eq. (5) is the type of Bernoulli equation, and we can obtain the solution as

$$G=-\frac{\lambda}{2\mu}\left(\tanh\left(\frac{\lambda}{2}\xi\right)-1\right). \tag{6}$$

Ref

20. Mohiud-Din S.T., Homotopy perturbation method for solving fourth-order boundary value problems, *Math. Prob. Engr.* Vol. 2007, 1-15, Article ID 98602, doi:10.1155/2007/98602.

$$G=-\frac{\lambda}{2\mu}\left(\coth\left(\frac{\lambda}{2}\xi\right)-1\right). \tag{7}$$

Step 3. The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq.(1) or Eq.(3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi))=n$ which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right)=n+q,\ D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right)=np+s(n+q). \tag{8}$$

Therefore we can find the value of n in Eq.(4), using Eq.(1).

Step 4. We substitute Eq. (4) into Eq.(3) using Eq. (5) and then collect all terms of same powers of $G(\xi)$ together, then set each coefficient of them to zero to yield a system of algebraic equations, solving this system we obtain the values of a_i and ω.

Finally, substituting a_i, ω and Eq. (6), Eq. (7) into Eq. (4) we obtain exact traveling wave solutions of Eq. (1).

III. Applications

a) *The Konopelchenko-Dubrovsky Coupled System equation*

In this section, we will consider the following the Konopelchenko-Dubrovsky Coupled System equation:

$$u_t-u_{xxx}-6buu_x+\frac{3}{2}a^2u^2u_x-3v_y+3au_xv=0 \tag{9}$$

$$u_y=v_x \tag{10}$$

This system was studied by Taghizadeh N. and Mirzazadeh M. by the first integral method [15].

Suppose that

$$u(x,t)=u(\xi),v(x,t)=v(\xi)\ \xi=kx+\alpha y+\omega t, \tag{11}$$

where k,α,ω are constants that to be determined later.

By Eq. (9), Eq. (10) and Eq. (11) are converted into the following ODEs,

$$\omega u'-k^3u'''-6bkuu'+\frac{3}{2}a^2ku^2u'-3\alpha v'+3aku'v=0, \tag{12}$$

$$\alpha u'=kv'. \tag{13}$$

Integrating Eq.(13) once with zero constant, Eq. (13) reduces to

$$v=\frac{\alpha}{k}u, \tag{14}$$

Ref

15. Taghizadeh and Mirzazade M., (2011), Exact Travelling Wave solutions for Konopelchenko-Dubrovsky equation by the First Integral Method, Applications and Applied Mathematics Vol. 6, Issue 11 pp. 1893-1901.

Substituting Eq. (14) in Eq. (12), then

$$\omega u'-k^3u'''+\frac{3}{2}a^2ku^2u'-3\frac{\alpha^2}{k}u'+3[a\alpha-2bk]uu'=0. \quad (15)$$

Integrating Eq. (15) once, Eq. (15) reduces to

$$\omega u-k^3u''+\frac{1}{2}a^2ku^3-3\frac{\alpha^2}{k}u+\frac{3}{2}[a\alpha-2bk]u^2+R=0, \quad (16)$$

where R is the integration constant.

Suppose that the solution of Eq. (16) can be expressed by a polynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{m}a_iG^i, \quad (17)$$

where a_i are constants, and $G=G(\xi)$ satisfies the following Bernoulli equation:

$$G'+\lambda G=\mu G^2 \quad (18)$$

Balancing the order of u'' and u^3 in Eq. (16), we have $3m=m+2, m=1$. So Eq. (17) can be rewritten as

$$u(\xi)=a_1G+a_0, a_1\neq 0, \quad (19)$$

where a_1, a_0 are constants to be determined later.

Substituting Eq. (19) into Eq. (16) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0: 2\omega a_0k-6bk^2a_0{}^2+3a\alpha a_0^2k+2Rk+a^2k^2a_0^3-6\alpha^2a_0$$

$$G^1: -12bk^2a_0a_1+3a^2k^2a_0^2a_1+6a\alpha a_0a_1k-6\alpha^2a_1-2k^4a_1\lambda^2+2\omega a_1k$$

$$G^2: 6k^4a_1\mu\lambda+3a^2k^2a_0a_1^2-6bk^2a_1^2+3a\alpha a_1^2k$$

$$G^3: -4k^4a_1\mu^2+a^2k^2a_1^3$$

Solving the above system of algebraic equations, we get the following two sets of solutions:

Set-1: $R=-\frac{1}{2}\frac{1}{k^2a^4}\left(\left(k^2\lambda a-2bk+a\alpha\right)\left(-4b^2k^2+4bka\alpha-\alpha^2a^2-2bk^3\lambda a+k^2\lambda a^2\alpha\right)\right)$,

$$\omega=-\frac{1}{2}\left(\frac{-12b^2k^2+12bka\alpha+k^4\lambda^2a^2-9a^2\alpha^2}{a^2k}\right),\ a_0=-\frac{k^2\lambda a-2bk+a\alpha}{ka^2},\ a_1=\frac{2k\mu}{a}.$$

Set-2: $R=-\frac{1}{2}\frac{1}{k^2a^4}\left(\left(k^2\lambda a+2bk-a\alpha\right)\left(4b^2k^2-4bka\alpha+\alpha^2a^2-2bk^3\lambda a+k^2\lambda a^2\alpha\right)\right)$,

$$\omega=-\frac{1}{2}\left(\frac{-12b^2k^2+12bka\alpha+k^4\lambda^2a^2-9a^2\alpha^2}{a^2k}\right),\ a_0=\frac{k^2\lambda ka+2b\ -a\alpha}{ka^2},\ a_1=-\frac{2k\mu}{a},$$

Notes

Notes

Substituting Set-1 and Set-2 into Eq. (19) along with Eq. (6) and Eq. (7), we get the following exact traveling wave solutions:

Case 1: When $R=-\frac{1}{2}\frac{1}{k^2a^4}\left(\left(k^2\lambda a-2bk+a\alpha\right)\left(-4b^2k^2+4bka\alpha-\alpha^2a^2-2bk^3\lambda a+k^2\lambda a^2\alpha\right)\right)$,

$\omega=-\frac{1}{2}\left(\frac{-12b^2k^2+12bka\alpha+k^4\lambda^2a^2-9a^2\alpha^2}{a^2k}\right)$, $a_0=-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}$, $a_1=\frac{2k\mu}{a}$

$$u_1(x,t)=-\frac{k^2\lambda a-2b+a\alpha}{ka^2}-\frac{1}{a}\left(k\lambda\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)\right) \tag{20}$$

$$u_2(x,t)=-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}-\frac{1}{a}\left(k\lambda\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)\right), \tag{21}$$

where $\xi=kx+\alpha y+\left(-\frac{1}{2}\left(\frac{-12b^2k^2+12bka\alpha+k^4\lambda^2a^2-9a^2\alpha^2}{a^2k}\right)\right)t$,

Substituting Eq. (20) and Eq. (21) into Eq. (14), yields

$$v_1(x,t)=\frac{\alpha}{k}\left(-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}-\frac{1}{a}\left(k\lambda\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)\right)\right). \tag{22}$$

$$v_2(x,t)=\frac{\alpha}{k}\left(-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}-\frac{1}{a}\left(k\lambda\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)\right)\right). \tag{23}$$

Case 2: When $R=-\frac{1}{2}\frac{1}{k^2a^4}\left(\left(k^2\lambda a+2bk-a\alpha\right)\left(4b^2k^2-4bka\alpha+\alpha^2a^2-2bk^3\lambda a+k^2\lambda a^2\alpha\right)\right)$,

$\omega=-\frac{1}{2}\left(\frac{-12b^2k^2+12bka\alpha+k^4\lambda^2a^2-9a^2\alpha^2}{a^2k}\right)$, $a_0=\frac{k^2\lambda a+2b-a\alpha}{ka^2}$, $a_1=-\frac{2k\mu}{a}$,

$$u_3(x,t)=\frac{k^2\lambda a+2bk-a\alpha}{ka^2}+\frac{1}{a}\left(k\lambda\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)\right). \tag{24}$$

$$u_4(x,t)=\frac{k^2\lambda a+2bk-a\alpha}{ka^2}+\frac{1}{a}\left(k\lambda\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)\right). \tag{25}$$

Substituting Eq. (24) and Eq. (25) into Eq. (14), yields

$$v_3(x,t)=\frac{\alpha}{k}\left(\frac{k^2\lambda a+2b-a\alpha}{ka^2}+\frac{1}{a}\left(k\lambda\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)\right)\right). \tag{26}$$

$$v_4(x,t)=\frac{\alpha}{k}\left(\frac{k^2\lambda a+2bk-a\alpha}{ka^2}+\frac{1}{a}\left(k\lambda\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)\right)\right). \tag{27}$$

b) The (1+1)-dimensional nonlinear Ostrovsky equation

Consider the (1+1)-dimensional nonlinear Ostrovsky equation

$$uu_{xxt}-u_x u_{xt}+u^2u_t=0, \tag{28}$$

This equation is a model for weakly nonlinear surface and internal waves in a rotation ocean. Following the above procedure we transform Eq. (28) into ODE:

$$-(uu'')'+2u'u''-u^2u''=0, \tag{29}$$

obtained upon using $\xi=x-ct$. Integrating Eq.(29) with respect to ξ one has

$$3uu''-3(u')^2+u^3+R=0, \tag{30}$$

where R is the integration constant.

Balancing the nonlinear term u^3 with the highest order derivative uu'' that gives

$3m=m+m+2$,

so that $m=2$.

So Eq. (4) can be rewritten as

$$u(\xi)=a_2G^2+a_1G+a_0, \quad a_1,a_2\neq 0, \tag{31}$$

where a_0,a_1,a_2 are constants to be determined later.

Substituting Eq. (31) into Eq. (30) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$G^0: a_0^3$

$G^1: 3a_0a_1\lambda^2+3a_0^2a_1$

$G^2: 3a_0^2a_2+12a_0a_2\lambda^2+3a_0a_1^2-9a_0a_1\mu\lambda$

$G^3: 6a_0a_1\mu^2-3a_1^2\mu\lambda+a_1^3-30a_0a_2\mu\lambda+6a_0a_1a_2+3a_1a_2\lambda^2$

$G^4 8:1\ a_0a_2\mu^2 +53a_0a_2^2-1\ a_1a_2\mu\lambda+3a_1^2\mu^2+3a_1^2a_2$

$G^5: 3a_1a_2^2-6a_2^2\mu\lambda+12a_1a_2\mu^2$

$G^6: 6a_2^2\mu^2+a_2^3$

Solving the above system of algebraic equations, we get the following solution:

$$R=R, c=c, a_0=0, a_1=6\mu\lambda, a_2=-6\mu^2$$

Substituting these values into Eq. (31) along with Eq. (6) and Eq. (7), we get the following exact traveling wave solutions:

Notes

$$u_1(x,t) = -3\lambda^2\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right) - \frac{3}{2}\lambda^2\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)^2, \qquad (32)$$

$$u_2(x,t) = -3\lambda^2\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right) - \frac{3}{2}\lambda^2\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)^2, \qquad (33)$$

Notes

where $\xi = x - ct$.

IV. Graphical Illustration of Some Obtained Solutions

We make graphs of obtained solutions, so that they can represent the importance of each obtained solution and physically interpret the importance of parameters. Some of our obtained traveling wave solutions are represented in Figure 1-Figure 4 with the aid of Maple:

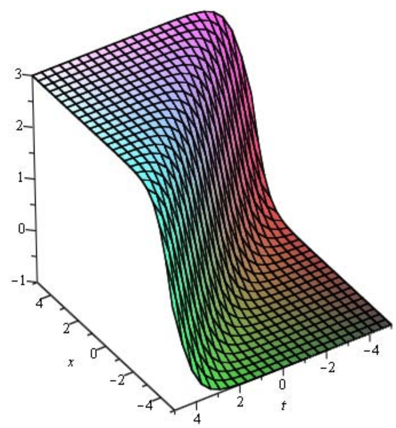

Figure (1) : Profile of Eq. (24) with $k=2, \lambda=1, a=1, b=1, \alpha=2, \omega=2$ and $y=0$.

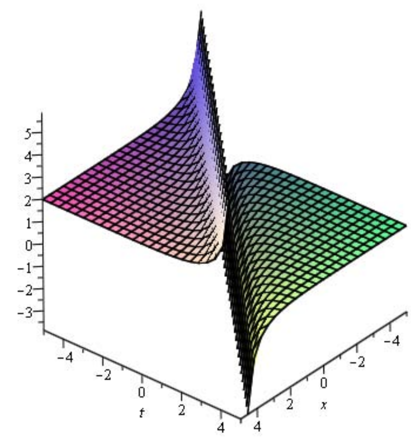

Figure (2) : Profile of Eq. (27) with $k=1, \lambda=-1, a=1, b=1, \alpha=1, \omega=-1$ and $y=0$.

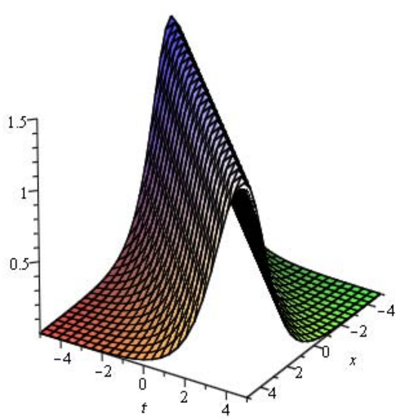

Figure (3) : Profile of Eq. (32) with $c=1, \lambda=1$.

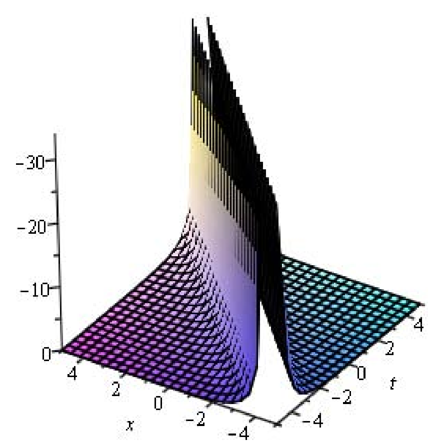

Figure (4) : Profile of Eq. (33) with $c=1, \lambda=1$.

V. Conclusion

The Bernoulli Sub-ODE method presented in this article has been applied to the NLEEs through the Konopelchenko-Dubrovsky Coupled System equation and the (1+1)-dimensional nonlinear Ostrovsky equation for finding the exact solutions and the solitary wave solutions of these equations which appeal the attention of many Mathematicians. This simple and powerful method can be more successfully applied to study nonlinear partial differential equations, which frequently arise in engineering sciences, mathematical physics and other scientific real-time application fields.

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