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# Semi-Invariant Submanifolds of Nearly Hyperbolic Cosymplectic Manifold

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SEMI-INVARIANT SUBMANIFOLDS OF NEARLY HYPERBOLIC COSYMPLECTIC MANIFOLD

# Semi-Invariant Submanifolds of Nearly Hyperbolic Cosymplectic Manifold

Mobin Ahmad [α] & Kashif Ali [σ]

*Abstract -* We consider a nearly hyperbolic cosymplectic manifold and study semi-invariant submanifolds of a nearly hyperbolic cosymplectic manifold. We also study parallel distributions on nearly hyperbolic cosymplectic manifold and find the integrability conditions of some distributions on nearly hyperbolic cosymplectic manifold.

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## I. Introduction

The notion of CR-submanifolds of a Kaehler manifold as generalization of invariant and anti-invariant submanifolds was initiated by A. Bejancu in [7]. A semi-invariant submanifold is the extension of the concept of a CR-submanifold of a Kaehler manifold to submanifolds of almost contact manifolds. The study of Semi-invariant submanifolds of Sasakian manifolds was initiated by Bejancu-Papaghuic in [9]. The same concept was studied under the name contact CR-submanifold by Yano-Kon in [16] and K. Matsumoto in [14]. The study of semi-invariant submanifolds in almost contact manifold was enriched by several geometers (see, [1], [2], [3], [4], [5], [6], [11], [14]). On the other hand, almost hyperbolic $(f,g,\eta,\xi)$-structure was defined and studied by Upadhyay and Dube in [15]. Joshi and Dube studied semi-invariant submanifolds of an almost r-contact hyperbolic metric manifolds in [12]. In this paper, we study semi-invariant submanifolds of a nearly hyperbolic cosymplectic manifold.

## II. Preliminaries

Let $\bar{M}$ be an $n$-dimensional almost hyperbolic contact metric manifold with almost hyperbolic contact metric structure-$(\phi,\xi,\eta,g)$, where a tensor $\phi$ of type $(1,1)$, a vector field $\xi$, called structure vector field and $\eta$, the dual 1-form of $\xi$ satisfying the followings

$$\phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$g(X,\xi) = \eta(X), \eta(\xi) = -1, \tag{2.2}$$

$$\phi(\xi) = 0, \ \ \eta o\phi = 0, \tag{2.3}$$

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1. Ahmad, M., Semi-invariant submanifolds of nearly Kenmotsu manifold with semi-symmetric semi-metric connection, MathematickiVesnik 62 (2010), 189-198.

$$g(\phi X, \phi Y) = -g(X,Y) - \eta(X)\eta(Y) \tag{2.4}$$

for any $X, Y$ tangent to $\bar{M}$ [15]. In this case

$$g(\phi X, Y) = -g(\phi Y, X). \tag{2.5}$$

An almost hyperbolic contact metric manifold with almost hyperbolic contact metric structure-$(\phi, \xi, \eta, g)$ is said to be nearly hyperbolic cosymplectic manifold [10] if

$$(\bar{\nabla}_X \phi)Y + \phi(\bar{\nabla}_Y X) = 0, \tag{2.6}$$

$$\bar{\nabla}_X \xi = 0 \tag{2.7}$$

for all $X, Y$ tangent to $\bar{M}$.

The Nijenhuis tensor $N(X,Y)$ of a nearly hyperbolic cosymplectic manifold $\bar{M}$ is defined as

$$N(X,Y) = (\bar{\nabla}_{\phi X}\phi)Y - (\bar{\nabla}_{\phi Y}\phi)X - \phi(\bar{\nabla}_X \phi)Y + \phi(\bar{\nabla}_Y \phi)X \tag{2.8}$$

$for\ any\ \ X, Y \in T\bar{M}.$

From (2.6), we have

$$(\bar{\nabla}_{\phi X}\phi)Y = -(\bar{\nabla}_Y \phi)\phi X. \tag{2.9}$$

Also, we have

$$(\bar{\nabla}_Y \phi)\phi X = (\bar{\nabla}_Y \eta)(X)\xi + \phi(\bar{\nabla}_Y \phi)X. \tag{2.10}$$

From (2.9) and (2.10), we get

$$(\bar{\nabla}_{\phi X}\phi)Y = (\bar{\nabla}_Y \eta)(X)\xi + \phi(\bar{\nabla}_Y \phi)X. \tag{2.11}$$

Using (2.11) in (2.8), we get

$$N(X,Y) = 4\phi(\bar{\nabla}_Y \phi)X + 2g(\phi X, Y)\xi \tag{2.12}$$

$for\ X, Y \in TM.$

The paper is organized as follows. In section 2, we give a brief description of nearly hyperbolic cosymplectic manifold. In section 3, we study some properties semi-invariant submanifolds of a nearly hyperbolic cosymplectic manifold. In section 4, we discuss the integrability conditions of some distributions on nearly hyperbolic cosymplectic manifold. In section 5, we study parallel horizontal distribution on nearly hyperbolic Kenmotsu manifold.

## III. Semi-Invariant Submanifolds

Let $M$ be a submanifold immersed in $\bar{M}$. We assume that the vector field $\xi$ is tangent to $M$. Denote by $\{\xi\}$ the 1-dimensional distribution spanned by $\xi$ on $M$. Then $M$ is called a semi-invariant submanifold [8] of $\bar{M}$ if there exist two differentiable distributions $D$ and $D^{\perp}$ on $M$ satisfying.

(i) $TM = D \oplus D^{\perp} \oplus \{\xi\}$, where $D$, $D^{\perp}$ and $\{\xi\}$ are mutually orthogonal to each other.

(ii) The distribution $D$ is invariant by $\phi$, that is, $\phi D_X = D_X$ for each $X \epsilon M$,

(iii) The distribution $D^{\perp}$ is anti-invariant by $\phi$,that is, $\phi D_X^{\perp} \subset T_X M^{\perp}$ for each $X \epsilon M$,

Ref.

10. Blair, D.E., Contact manifolds in Riemannian geometry, Lecture Notes in Mathematics, Vol. 509, Springer-Verlag, Berlin, 1976.

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where $TM$ and $T^{\perp}M$ be the Lie algebra of vector fields tangential to $M$ and normal to $M$ respectively. Let the Riemannian metric induced on $M$ is denoted by the same symbol $g$ and $\nabla$ be the induced Levi-Civita connection on $M$, then the Gauss and Weingarten formulas are given respectively by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \tag{3.1}$$

$$\overline{\nabla}_X N = -A_N X + \nabla_X^{\perp} N \tag{3.2}$$

for any $X, Y \in TM$ and $N \in T^{\perp}M$, where $\nabla^{\perp}$ is a connection on the normal bundle $T^{\perp}M, h$ is the second fundamental form and $A_N$ is the Weingarten map associated with N as

$$g(A_N X, Y) = g(h(X,Y), N) \tag{3.3}$$

$for\ any\ x \in M\ and\ X \in T_x M$. We write

$$X = PX + QX, \tag{3.4}$$

where $PX \in D$ and $QX \in D^{\perp}$.
Similarly, for $N$ normal to $M$ we have

$$\phi N = BN + CN, \tag{3.5}$$

where $BN(resp.\,CN)$ is the tangential component $(resp.\, normal\ component)$ of $\emptyset N$.

**Lemma 3.1.** Let $M$ be a semi-invariant submanifold of a nearly hyperbolic cosymplectic manifold $\overline{M}$, then

$$2(\overline{\nabla}_X \phi)Y = \nabla_X \phi Y - \nabla_Y \phi X + h(X, \phi Y) - h(Y, \phi X) - \phi[X,Y]$$

$for\ all\ X, Y \in D.$

**Proof.** By Gauss formula (3.1), we have

$$\overline{\nabla}_X Y - \overline{\nabla}_Y \phi X = \nabla_X \phi Y - \nabla_Y \phi X + h(X, \phi Y) - h(Y, \phi X) - \phi[X,Y]. \tag{3.6}$$

Also, by covariant differentiation we get

$$\overline{\nabla}_X \phi Y - \overline{\nabla}_Y \phi X = (\overline{\nabla}_X \phi)Y - (\overline{\nabla}_Y \phi)X + \phi[X,Y]. \tag{3.7}$$

From (3.6) and (3.7), we obtain

$$(\overline{\nabla}_X \phi)Y - (\overline{\nabla}_Y \phi)X = \nabla_X \phi Y - \nabla_Y \phi X + h(X, \phi Y) - h(Y, \phi X) - \phi[X,Y]. \tag{3.8}$$

Adding (2.6) and (3.8), we get

$$2(\overline{\nabla}_X \phi)Y = \nabla_X \phi Y - \nabla_Y \phi X + h(X, \phi Y) - h(Y, \phi X) - \phi[X,Y]$$

$for\ all\ X, Y \in D.$

Hence Lemma is proved.

**Lemma 3.2.** Let $M$ be a semi-invariant submanifold of a nearly hyperbolic cosymplectic manifold $\overline{M}$, then

$$2(\overline{\nabla}_X\phi)Y = A_{\phi X}Y - A_{\phi Y}X + \nabla_X^{\perp}\phi Y - \nabla_Y^{\perp}\phi X - \phi[X,Y] \quad (3.9)$$

$for\ all\ X,Y \in D^{\perp}$.

**Proof.** By Weingarten formula (3.2), we have

$$\overline{\nabla}_X\phi Y - \overline{\nabla}_Y\phi X = A_{\phi X}Y - A_{\phi Y}X + \nabla_X^{\perp}\phi Y - \nabla_Y^{\perp}\phi X$$

$for\ any\ X,Y \in D^{\perp}$.

Also, by covariant differentiation, we have

$$\overline{\nabla}_X\phi Y - \overline{\nabla}_Y\phi X = (\overline{\nabla}_X\phi)Y - (\overline{\nabla}_Y\phi)X + \phi[X,Y]. \quad (3.10)$$

From (3.9) and (3.10), we get

$$(\overline{\nabla}_X\phi)Y - (\overline{\nabla}_Y\phi)X = A_{\phi X}Y - A_{\phi Y}X + \nabla_X^{\perp}\phi Y - \nabla_Y^{\perp}\phi X - \phi[X,Y]. \quad (3.11)$$

Adding (2.6) and (311), we get

$$2(\overline{\nabla}_X\phi)Y = A_{\phi X}Y - A_{\phi Y}X + \nabla_X^{\perp}\phi Y - \nabla_Y^{\perp}\phi X - \phi[X,Y]$$

$for\ all\ X,Y \in D^{\perp}$.

Hence Lemma is proved.

**Lemma 3.3.** Let $M$ be a semi-invariant submanifold of a nearly hyperbolic cosymplectic manifold $\overline{M}$. Then

$$2(\overline{\nabla}_X\phi)Y = -A_{\phi Y}X + \nabla_X^{\perp}\phi Y - \nabla_Y\phi X - h(Y,\phi X) - \phi[X,Y]$$

$for\ all\ X \in D\ and\ Y \in D^{\perp}$.

**Proof.** Using Gauss and Weingarten formulas, we have

$$\overline{\nabla}_X\phi Y - \overline{\nabla}_Y\phi X = -A_{\phi Y}X + \nabla_X^{\perp}\phi Y - \nabla_Y\phi X - h(Y,\phi X). \quad (3.12)$$

Also, by covariant differentiation, we have

$$\overline{\nabla}_X\phi Y - \overline{\nabla}_Y\phi X = (\overline{\nabla}_X\phi)Y - (\overline{\nabla}_Y\phi)X + \phi[X,Y]. \quad (3.13)$$

From (3.12) and (3.13), we get

$$(\overline{\nabla}_X\phi)Y - (\overline{\nabla}_Y\phi)X = -A_{\phi Y}X + \nabla_X^{\perp}\phi Y - \nabla_Y\phi X - h(Y,\phi X) - \phi[X,Y]. \quad (3.14)$$

Adding (2.6) and (3.14), we obtain

$$2(\overline{\nabla}_X\phi)Y = -A_{\phi Y}X + \nabla_X^{\perp}\phi Y - \nabla_Y\phi X - h(Y,\phi X) - \phi[X,Y]$$

$for\ all\ X \in D\ and\ Y \in D^{\perp}$.

Hence Lemma is proved.

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**Lemma 3.4.** Let $M$ be a semi-invariant submanifold of a nearly hyperbolic cosymplectic manifold $\bar{M}$. Then

$$P(\nabla_X \phi PY) + P(\nabla_Y \phi PX) - PA_{\phi QY}X - PA_{\phi QX}Y = \phi P(\nabla_X Y) + \phi P(\nabla_Y X), \tag{3.15}$$

$$Q(\nabla_X \phi PY) + Q(\nabla_Y \phi PX) - QA_{\phi QY}X - QA_{\phi QX}Y = 2Bh(X,Y), \tag{3.16}$$

$$h(Y,\phi PX) + h(X,\phi PY) + \nabla_X^{\perp}\phi QY + \nabla_Y^{\perp}\phi QX = -2\eta(Y)\phi QX - 2\eta(X)\phi QY + \phi Q(\nabla_X Y)$$
$$+\phi Q(\nabla_Y X) + 2Ch(X,Y), \tag{3.17}$$

$$\eta(\nabla_X \phi PY) + \eta(\nabla_Y \phi PX) - \eta\big(A_{\phi QY}X\big) - \eta\big(A_{\phi QX}Y\big) = 0 \tag{3.18}$$

*for any* $X, Y \in T\bar{M}$.

**Proof.** Differentiating (3.4) covariantly and using (3.1) and (3.2), we get

$$\bar{\nabla}_Y \phi X = (\bar{\nabla}_Y \phi)X + \phi(\nabla_Y X) + \phi h(X,Y).$$

Also,

$$\bar{\nabla}_Y \phi X = P\nabla_X \phi PY + Q\nabla_Y \phi PX + \eta(\nabla_Y \phi PX)\xi + h(Y,\phi PX) + \nabla_{\phi QX}^{\perp} Y$$
$$-PA_{\phi QX}Y - QA_{\phi QX}Y - \eta\big(A_{\phi QX}Y\big)\xi.$$

Thus, we have

$$(\bar{\nabla}_Y \phi)X + \phi(\nabla_Y X) + \phi h(X,Y) = P\nabla_X \phi PY + Q\nabla_Y \phi PX + \eta(\nabla_Y \phi PX)\xi$$
$$+h(Y,\phi PX) + \nabla_{\phi QX}^{\perp} Y - PA_{\phi QX}Y - QA_{\phi QX}Y - \eta\big(A_{\phi QX}Y\big)\xi. \tag{3.19}$$

Interchanging $X$ *and* $Y$, we have

$$(\bar{\nabla}_X \phi)Y + \phi(\nabla_X Y) + \phi h(Y,X) = P\nabla_Y \phi PX + Q\nabla_X \phi PY + \eta(\nabla_X \phi PY)\xi$$
$$+h(X,\phi PY) + \nabla_{\phi QY}^{\perp} X - PA_{\phi QY}X - QA_{\phi QY}X - \eta\big(A_{\phi QY}X\big)\xi. \tag{3.20}$$

Adding (3.19) and (3.20), we have

$$(\bar{\nabla}_X \phi)Y + (\bar{\nabla}_Y \phi)X + \phi(\nabla_Y X) + \phi(\nabla_X Y) + 2\phi h(X,Y) = P\nabla_X \phi PY + P\nabla_Y \phi PX$$
$$+Q\nabla_Y \phi PX + Q\nabla_X \phi PY + \eta(\nabla_Y \phi PX)\xi + \eta(\nabla_X \phi PY)\xi + h(Y,\phi PX) + h(X,\phi PY)$$
$$+\nabla_{\phi QX}^{\perp} Y + \nabla_{\phi QY}^{\perp} X - PA_{\phi QX}Y - PA_{\phi QY}X - QA_{\phi QX}Y - QA_{\phi QY}X - \eta\big(A_{\phi QX}Y\big)\xi$$
$$-\eta\big(A_{\phi QY}X\big)\xi. \tag{3.21}$$

By virtue of (2.6) and (3.21), we obtain

$$\phi P(\nabla_Y X) + \phi P(\nabla_X Y) + \phi Q \nabla_Y X + \phi Q \nabla_X Y + 2Bh(X,Y) + 2Ch(X,Y) = P\nabla_Y \phi PX$$

$$+P\nabla_X \phi PY + Q\nabla_Y \phi PX + Q\nabla_X \phi PY + \eta(\nabla_Y \phi PX)\xi + \eta(\nabla_X \phi PY)\xi + h(Y,\phi PX)$$

$$+h(X,\phi PY) + \nabla^{\perp}_{\phi QX} Y + \nabla^{\perp}_{\phi QY} X - PA_{\phi QX}Y - PA_{\phi QY}X - QA_{\phi QX}Y - QA_{\phi QY}X$$

$$-\eta\left(A_{\phi QX}Y\right)\xi - \eta\left(A_{\phi QY}X\right)\xi.$$

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Comparing horizontal, vertical and normal components we get the desired result.
Hence the Lemma is proved.

**Definition 3.5.** The horizontal distribution $D$ is said to be parallel [10] on $M if \nabla_X Y \in D$ for all vector field $X, Y \in D$.

**Theorem 3.6.** Let $M$ be a semi-invariant submanifoldof a nearly hyperbolic cosymplectic manifold $\bar{M}$. If the horizontal distribution $D$ is parallel, then

$$h(X,\phi Y) = h(Y,\phi X)$$

$for\ all\ X, Y \in D.$

**Proof.** Let $X, Y \in D$ and $D$ is parallel then $\nabla_X \phi Y \in D\ and\ \nabla_Y \phi X \in D$. From (3.12), we have

$$h(Y,\phi X) + h(X,\phi Y) = 2\phi h(X,Y). \quad (3.22)$$

Replacing $X$ by $\phi X$ in (3.22) and using (2.1), we get

$$h(Y,X) + h(\phi X,\phi Y) = 2\phi h(\phi X,Y). \quad (3.23)$$

Again replacing $Y\ by\ \phi Y$ in (3.22) and using (2.1), we get

$$h(\phi Y,\phi X) + h(X,Y) = 2\phi h(X,\phi Y). \quad (3.24)$$

By virtue of (3.23) and (3.24), we have

$$\phi h(\phi X,Y) = \phi h(X,\phi Y). \quad (3.25)$$

Operating $\phi$ on both sides of (3.25), we get

$$h(\phi X,Y) = h(X,\phi Y).$$

Hence the theorem is proved.

**Definition 3.7.** A semi-invariant submanifold is said to be mixed totally geodesic [8] if $h(X,Y) = 0\ for\ all\ X \in D\ and\ Y \in D^{\perp}$.

**Theorem 3.8.** Let $M$ be a semi-invariant submanifold of a nearly hyperbolic cosymplectic manifold $\bar{M}$. Then $M$ is a mixed totally geodesic if and only if $A_{NX} \in D\ for\ all\ X \in D$.

**Proof.** Let $A_{NX} \in D\ for\ all\ X \in D$.

Now, $g(h(X,Y),N) = g(A_{NX},Y) = 0\ for\ Y \in D^{\perp}$, which is equivalent to $h(X,Y) = 0$.
Hence $M$ is totally mixed geodesic.

Conversely, Let $M$ is totally mixed geodesic, that is $h(X,Y) = 0\ for\ X \in D\ and\ Y \in D^{\perp}$.

Now, $g(h(X,Y),N) = g(A_{NX},Y)$ gives that $g(A_{NX},Y) = 0$. Consequently, we have $A_{NX} \in D$ for all $Y \in D^{\perp}$.
Hence the theorem is proved.

## IV. INTEGRABILITY CONDITIONS FOR DISTRIBUTIONS

Notes

**Theorem 4.1.** Let $M$ be a semi-invariant submanifold of a nearly hyperbolic cosymplectic manifold $\bar{M}$. Then the distribution $D \oplus \langle \xi \rangle$ is integrable if

$$h(\phi X, Z) + h(\phi Z, X) = 0 \tag{4.1}$$

*for any* $X, Y, Z \in D \oplus \langle \xi \rangle$.

**Proof.** The torsion tensor $S(X,Y)$ of an almost contact structure $(\phi, \xi, \eta, g)$
Using Gauss formula in (2.12), we have

$$N(X,Y) = 4\phi\big(\nabla_Y \emptyset X - \phi \nabla_Y X - \phi h(Y,X) + h(Y,\phi X)\big). \tag{4.2}$$

If $D \oplus \langle \xi \rangle$ is integrable, then $N(X,Y) = 0$. Hence

$$4\phi \nabla_Y \phi X - 4\nabla_Y X - 4\eta(\nabla_Y X)\xi - 4h(Y,X) + 4\phi h(Y,\phi X) = 0. \tag{4.3}$$

Comparing normal parts from both sides of (4.3), we get

$$\phi Q(\nabla_Y \phi X) - h(Y,X) = Ch(Y,\phi X) = 0. \tag{4.4}$$

Replacing $Y$ *by* $\phi Z, Z \in D$ in (4.4), we have

$$\phi Q\big(\nabla_{\phi z} \phi X\big) - h(\phi Z, X) + Ch(\phi Y, \phi X) = 0. \tag{4.5}$$

Interchanging $X$ *and* $Z$ in (4.5), we obtain

$$\phi Q\big(\nabla_{\phi X} \phi Z\big) - h(\phi X, Z) + Ch(\phi X, \phi Y) = 0. \tag{4.6}$$

Subtracting (4.6) from (4.5), we get

$$\phi Q[\phi X, \phi Z] - h(\phi X, Z) + h(\phi Z, X) = 0. \tag{4.7}$$

Since $D \oplus \langle \xi \rangle$ is integrable so that $[\phi X, \phi Z] \in D \oplus \langle \xi \rangle$ *for* $X, Z \in D$.
Consequently (4.7) gives

$$h(\phi X, Z) + h(\phi Z, X) = 0.$$

Hence the theorem is proved.

**Proposition 4.2.** Let $M$ be a semi-invariant submanifold of a nearly hyperbolic cosy mplectic manifold $\bar{M}$. Then

$$A_{\phi Y} Z - A_{\phi Z} Y = \phi P[Y, Z]$$

*for any* $Y, Z \in D^{\perp}$.

**Proof.** Let $X \in \chi(M)$ and $Y, Z \in D^{\perp}$. From (3.1) and (3.2), we have

$$2g(A_{\phi Z}Y, X) = g(h(Y,X), \phi Z) + g(h(X,Y), \phi Z).$$

Using (3.3) and (3.1), we get

$$2g(A_{\phi Z}Y, X) = -g(\overline{\nabla}_Y \phi X, Z) - g(\overline{\nabla}_X \phi Y, Z) + g((\overline{\nabla}_Y \phi)X + (\overline{\nabla}_X \phi)Y, Z).$$

Using (2.6) in above equation, we have

$$2g(A_{\phi Z}Y, X) = -g(\phi \overline{\nabla}_Y Z, X) + g(A_{\phi Y}Z, X). \tag{4.8}$$

Transvecting $X$ from both sides of (4.8), we have

$$2A_{\phi Z}Y = \phi \overline{\nabla}_Y Z + A_{\phi Y}Z.$$

Interchanging $Y and\ Z$, we have

$$2A_{\phi Y}Z = \phi \overline{\nabla}_Z Y + A_{\phi Z}Y.$$

Subtracting above two equations, we get

$$A_{\phi Z}Y - A_{\phi Y}Z = \frac{1}{3}\phi[Y,Z].$$

Comparing the tangential parts from both sides of above equation, we get

$$A_{\phi Z}Y - A_{\phi Y}Z = \frac{1}{3}\phi P[Y,Z], \tag{4.9}$$

where $[Y,Z]$ is Lie bracket.
Hence the proposition is proved.

**Theorem 4.3.** Let $M$ be a semi-invariant submanifold of a nearly hyperbolic cosymplectic manifold $\overline{M}$.Then the distribution $D^{\perp}$ is integrable if and only if

$$A_{\phi Y}Z - A_{\phi Z}Y = 0 \tag{4.10}$$

$for\ all\ Y, Z \in D^{\perp}$.

**Proof.** Suppose that the distribution $D^{\perp}$ is integrable. Then $[Y,Z] \in D^{\perp}\ for\ any\ Y, Z \in D^{\perp}$. Therefore, $P[Y,Z] = 0$. From (4.9), we get

$$A_{\phi Y}Z - A_{\phi Z}Y = 0.$$

Conversely, let (4.10) holds good. Then by virtue of (4.9), we get

$$\phi P[Y,Z] = 0$$

$for\ all\ Y, Z \in D^{\perp}$. Since $rank\ \phi = 2n$, therefore, either $P[Y,Z] = 0\ or\ P[Y,Z] = k\xi$. But $P[Y,Z] = k\xi$ is not possible as $P$ being a projection operator on $D$. Hence $P[Y,Z] = 0$. This implies that $[Y,Z] \in D^{\perp} for\ all\ Z \in D^{\perp}$. Thus $D^{\perp}$ is integrable.
Hence the theorem is proved.

Notes

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