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Semirelib Cutvertex Graph

By Venkanagouda M Goudar & K B Manjunatha Prasad
CMJ University, Meghalaya

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Ref.

[2] Harary F., Graph theory, Addition - Weseley Reading .Mass.(1969), pp 34 and 107.

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I. Introduction

The concept of block edge cut vertex graph was introduced by Venkanagouda M Goudar [6]. For the graph G(p,q), if $B = u_1, u_2, ...u_r : r \geq 2$ is a block of G, then we say that the vertex u_i and the block B are incident with each other. If two blocks B_1 and B_2 are incident with a common cutvertex, then they are adjacent blocks. The lict graph n(G) was defined in [5]. For the graph G the lict graph n(G) is the graph whose vertex set is the union of the set of edges and the set of cutvertices of G in which two vertices are adjacent if and only if the corresponding edges of G are adjacent or the corresponding members of G are incident.

All undefined terminology will conform with that in Harary[2]. All graphs considered here are finite, undirected, planar and without loops or multiple edges.

The semirelib cutvertex graph of a planar graph G denoted by $R_c(G)$ is the graph whose vertex set is the union of set of edges, set of blocks, set of cutvertices and set of regions of G in which two vertices are adjacent if and only if the corresponding edges of G are adjacent, the edges lies on the blocks, the edges lies on the region and edges are incident to the cutvertex.

The edge degree of an edge uv is the sum of the degree of the vertices of u and v. For the planar graph G, the inner vertex number i(G) of a graph G is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of G in the plane. A graph G is said to be minimally nonouterplanar if i(G)=1 as was given by Kulli [4].

Author α : Sri Siddhartha Institute of Technology, Tumkur, Karnataka, India. E-mail : vmgouda@gmail.com
Author σ : Sri Siddaganga college for women, Tumkur, India Research Scholar, CMJ University, Meghalaya.
E-mail : meenakshisutha.43@gmail.com

II. Preliminary Notes

We need the following results to prove further results.

Theorem 2.1 (2) . *If G is a (p,q) graph whose vertices have degree* d_i *then the line graph L(G) has q vertices and* q_L *edges, where* $q_L = -q + \frac{1}{2}\sum d_i^2$ *edges.*

Theorem 2.2 (2) . *The line graph L(G) of a graph is planar if and only if G is planar,* $\Delta(G) \leq 4$ *and if* $degv = 4$*, for a vertex v of G, then v is a cutvertex.*

Theorem 2.3 (4) . *A graph is outerplanar if and only if it has no subgraph homeomorphic to* K_4 *or* $K_{2,3}$*.*

III. Main Results

We start with few preliminary results.

Lemma 3.1 *For any planar graph G,* $L(G) \subseteq R_c(G)$*.*

Lemma 3.2 *If a graph G contains a block as* K_2*, then it becomes a pendent vertex in* $R_c(G)$*.*

In the following theorem we obtain the number of vertices and edges of a semirelib cutvertex graph of a plane graph.

Theorem 3.3 *For any planar graph G with r regions and b blocks, the semirelib cutvertex graph* $R_c(G)$ *whose vertices have degree* d_i*,has (q +r+b+ c)vertices and* $\frac{1}{2}\sum d_i^2 + \sum q_j + \sum c_k$ *edges where* c_k *be the number of edges incident to the cutvertex* c_k*.*

Proof. By the definition of $R_c(G)$, the number of vertices is the union of edges, regions, blocks and the number of cutvertices of G. Hence $R_c(G)$ has (q + r + b + c) vertices. Further by the Theorem 2.1, number of edges in L(G) is $q_L = -q+\frac{1}{2}\sum d_i^2$. Thus the number of edges in $R_c(G)$ is the sum of the number of edges in L(G), the number of edges bounded by the regions which is q, the number of edges lies on the blocks is $\sum q_j$ and the number of edges incident to the cut vertices which is $\sum c_k$. Hence $E[R_c(G)] = -q+\frac{1}{2}\sum d_i^2+q+\sum q_j+\sum c_k$ $= \frac{1}{2}\sum d_i^2 + \sum q_j + \sum c_k$.

Theorem 3.4 *For any edge* $e_i \in G$ *which is either an internal edge not incident to a cutvertex or a pendent edge with edge degree n, then degree of the corresponding vertex in* $R_c(G)$ *is n+1.*

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Proof. Suppose a pendent edge $e_i \in E(G)$ have edge degree n. Let $v_i^{'}$ be the corresponding vertex in$R_c(G)$. Clearly there are n - 1 edges adjacent to e_i. Also, by the definition of semirelib cutvertex graph, e_i lies on only one region and also on one block. Hence in $R_c(G)$, deg $(v_i^{'})$= n-1+1+1=n+1. Further, if e_i is an internal edge which is not incident to a cutvertex and $v_i^{'}$ be the corresponding vertex in$R_c(G)$ then there are n-2 edges adjacent to $v_i^{'}$. Also e_i lies on only one block and lies in two regions. Hence deg $v_i^{'}$= n-2+1+2=n+1.

Theorem 3.5 *For any edge $e_i \in G$ which is an internal edge incident to a cutvertex with edge degree n, then degree of the corresponding vertex in $R_c(G)$ is n+2.*

Proof. Suppose a pendent edge $e_i \in E(G)$ have edge degree n. Let $v_i^{'}$ be the corresponding vertex in$R_c(G)$. Clearly there are n - 1 edges adjacent to e_i. Since e_i is an internal edge which is incident to a cutvertex then there are n-2 edges adjacent to $v_i^{'}$. Also e_i lies on only one block, lies in two regions and incident to a cutvertex. Hence deg $v_i^{'}$= n-2+1+2+1=n+2.

Theorem 3.6 *For any edge $e_i \in G$ which is an internal edge incident to two cutvertices with edge degree n, then degree of the corresponding vertex in $R_c(G)$ is n+3.*

Proof. Suppose a pendent edge $e_i \in E(G)$ have edge degree n. Let $v_i^{'}$ be the corresponding vertex in$R_c(G)$. Clearly there are n - 1 edges adjacent to e_i. Since e_i is an internal edge which is incident to a cutvertex then there are n-2 edges adjacent to v_i. Also e_i lies on only one block,lies in two regions and incident to two cutvertices. Hence deg $v_i^{'}$= n-2+1+2+2 = n+3.

Theorem 3.7 *For any graph G with each block contains at least three edges, then $R_c(G)$ is nonseparable.*

Proof. Let $e_1, e_2, ...e_n \in E(G)$, $r_1, r_2, ...r_t$ be the regions, $b_1, b_2, ...b_m$ be the blocks which are not K_2 and $c_1, c_2, ...c_k$. By the definition of line graph, the corresponding vertices $v_1^{'}, v_2^{'}, ...v_n^{'} \in R_c(G)$ form a subgraph without isolated vertex. Since each block contains at least three edges and each region of G has at least three edges. By the definition of $R_c(G)$, each vertex is of degree at least three. Also the cutvertices adjacent to $v_1^{\prime}, v_2^{\prime}, ...v_n^{\prime}$. Hence $R_c(G)$ is nonseparable.

In the following theorem we obtain the condition for the planarity on semirelib cutvertex graph of a graph.

Theorem 3.8 *For any planar graph G, the $R_c(G)$ is planar if and only if G is a tree such that $\Delta(G) \leq 3$.*

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Proof. Suppose $R_c(G)$ is planar. Assume that $\exists v_i \in G$ such that $degv_i \geq 4$. Suppose $degv_i = 4$ and e_1, e_2, e_3, e_4 are the edges incident to v_i. Let $v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}$ be the vertices of $R_c(G)$ corresponding to e_1, e_2, e_3, e_4. By the definition of line graph, $v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}$ form K_4 as an induced subgraph. In $R_c(G)$, the region vertex r_i is adjacent with all vertices of L(G) to form K_5 as an induced subgraph. Further the corresponding block vertices $b_1, b_2, b_3, ..., b_{n-1}$ of blocks $B_1, B_2, B_3, ..., B_n$ in G are adjacent to vertices of K_5, which forms graph homeomorphic to K_5. By the Theorem 2.3, it is non planar, a contradiction.

Conversely, Suppose $degv \leq 3$ and let e_1, e_2, e_3 be the edges of G incident to v. Let $v_1^{'}, v_2^{'}, v_3^{'}$ be the vertices of $R_c(G)$ corresponding to e_1, e_2, e_3. By the definition of line graph $v_1^{'}, v_2^{'}, v_3^{'}$ form K_3 as a subgraph. By the definition of $R_c(G)$, the region vertex r_i is adjacent to $v_1^{'}, v_2^{'}, v_3^{'}$ to form K_4 as a subgraph. Further, by the lemma 2.2, the blocks vertices b_1, b_2, b_3 of T forms a pendent vertices of $R_c(G)$. The cutvertex c_i is adjacent to $v_1^{'}, v_2^{'}, v_3^{'}$ to form a graph which is homeomorphic to K_4. Hence $R_c(G)$ is planar.

In the following theorem we obtain the condition for the outer planarity on semirelib cutvertex graph of a graph.

Theorem 3.9 *For any planar graph G, $R_c(G)$ is outer planar if and only if G is a path P_n.*

Proof. Suppose $R_c(G)$ is outer planar. Assume that G is a tree with at least one vertex v such that $degv = 3$. Let e_1, e_2, e_3 be the edges of G incident to v. Let $v_1^{'}, v_2^{'}, v_3^{'}$ be the vertices of $R_c(G)$ corresponding to e_1, e_2, e_3. By the definition of line graph $v_1^{'}, v_2^{'}, v_3^{'}$ form K_3 as a subgraph. In $R_c(G)$, the region vertex r_i is adjacent to $v_1^{'}, v_2^{'}, v_3^{'}$ to form K_4 as induced subgraph. Further by the lemma 3.2, $b_1 = e_1, b_2 = e_2, ..., b_{n-1} = e_{n-1}$ becomes n-1 pendant vertices in $R_c(G)$. Also the cutvertex c_i is adjacent to $v_1^{'}, v_2^{'}, v_3^{'}$ to form a graph homeomorphic to K_4. Clearly $i[R_c(G)] \geq 1$, which is non-outer planar ,a contradiction. Hence G must be a path.

Conversely, Suppose G is a path. Let $e_1, e_2, e_3, ..., e_{n-1} \in E(G)$. Let $v_1^{'}, v_2^{'}, ...v_{n-1}^{'}$ be the vertices of $R_c(G)$ corresponding to $e_1, e_2, ...e_{n-2}$. By the definition of line graph, $L[P_n] = P_{n-1}$. Further by the lemma 3.2, $b_1 = e_1, b_2 = e_2, ..., b_{n-1} = e_{n-1}$ becomes n-1 pendant vertices and it becomes a caterpillar. Further the region vertex r_i is adjacent to all the vertices of $L[P_n]$. Also the cutvertices $c_1, c_2, e_3, ..., c_{n-2}$ are adjacent to $v_1^{'}, v_2^{'}, ...v_{n-1}^{'}$ to form the graph such that each block is K_3. Clearly $R_c(G)$ is outer planar.

In the following theorem we obtain the condition for the minimally non outer planar on semirelib cutvertex graph of a graph.

Theorem 3.10 *For any planar graph G, $R_c(G)$ is not minimally nonouter planar.*

Proof. Follows from the above theorem.

In the following theorem we obtain the condition for the non eulerian on semirelib cutvertex graph of a graph.

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Theorem 3.11 *For any planar graph G, $R_c(G)$ is always non Eulerian.*

Proof. We consider the following cases.

Case1. Assume that G is a tree. In a tree each edge is a block and hence $b_1 = e_1, b_2 = e_2, ..., b_{n-1} = e_{n-1} \forall e_{n-1} \in E(G)$ and $\forall b_{n-1} \in V[R_c(G)]$. In $R_c(G)$, the block vertex b_i is always a pendent vertex, which is non Eulerian.

Case2. Assume that G is a graph which contains at least one pendent vertex. Suppose $e_i \in G$ is a pendent edge. In $e_i = b_i \in V[R_c(G)]$ becomes a pendent vertex, which is non Eulerian.

Case3. Assume that G is K_2 -free graph. We have the following sub cases of case3.

Sub case1. Suppose G itself is a block with even number of edges. Clearly each edge of G is of even degree. By the definition of $R_c(G)$, both the region vertices and blocks have even degree. By the Theorem 2.3 $e_i = b_i \in V[R_c(G)]$ is of odd degree, which is non Eulerian. Further if G is a block with odd number of edges, then by the Theorem 3.3, each $e_i = b_i \in V[R_c(G)]$ is of even degree. Also the block vertex b_i and region vertex r_i are adjacent to these vertices. Clearly degree of b_i and r_i is odd, which is non Eulerian.

Sub case2. Suppose G is a graph such that it contains at least one cutvertex. If each edge is even degree then by the sub case 1, it is non Eulerian. Assume that G contains at least one edge with odd edge degree. Clearly for any $e_j \in E(G)$, degree of $e_j \in V[R_c(G)]$ is odd, which is non Eulerian. Hence for any graph G $R_c(G)$ is always non Eulerian.

In the following theorem we obtain the condition for the hamiltonian on semirelib cutvertex graph of a graph.

Theorem 3.12 *For any graph G, $R_c(G)$ is hamiltonian if and only if G is a graph such that each block contains at least three edges.*

Proof. Suppose $R_c(G)$ is hamiltonian. Assume that G has at least one block which is K_2. Let $e_i \in E(G)$ such that $e_i = b_i \in V[R_c(G)]$. By the Lemma 3.2, b_i becomes a pendent vertex. Clearly $R_c(G)$ is non hamiltonian, a contradiction.

Conversely, suppose G is K_2- free graph. Let $e_1, e_2, ...e_{n-1} \in E(G)$ and $v'_1, v'_2, ...v'_{n-1}$ be the corresponding vertices of $R_c(G)$. Let $b_1, b_2, ...b_i$ be the blocks, $r_1, r_2, ...r_k$ be the regions and $c_1, c_2, ...c_k$ be the cutvertices of G such that $e_1, e_2, ...e_l \in V(b_1)$, $e_{l+1}, e_{l+2}, ...e_m \in V(b_2)$, ... $e_{m+1}, e_{m+2}, ...e_{n-1} \in V(b_i$. By the Theorem 3.3, $V[R_c(G)] = e_1, e_2, ...e_{n-1} \cup b_1, b_2, ...b_i \cup r_1, r_2, ...r_k \cup c_1, c_2, ...c_k$. By theorem 3.7, $R_c(G)$ is non separable. By the definition, $b_1 e_1, c_1, e_2, ...e_{l-1} r_1 b_2, c_2 ...$ $r_2 e_m b_3 ... e_{k+1}, c_k, e_{k+2}, ...$ $e_{n-1} b_k r_k e_l b_1$ form a cycle which contains all the vertices of $R_c(G)$. Hence $R_c(G)$ is hamiltonian.

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