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A Summation Formula Coupled with Contiguous Relation and Recurrence Relation

By Salahuddin, M. P. Chaudhary & Upendra Kumar Pandit

P.D.M College of Engineering, India

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A Summation Formula Coupled with Contiguous Relation and Recurrence Relation

Salahuddin^α, M. P. Chaudhary^σ & Upendra Kumar Pandit^ρ

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I. INTRODUCTION

Generalized Gaussian hypergeometric function of one variable is defined by

$${}_A F_B \left[\begin{matrix} a_1, a_2, \dots, a_A ; \\ b_1, b_2, \dots, b_B ; \end{matrix} z \right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!} \quad (1)$$

where the parameters $b_1, b_2, \dots, b_B$ are neither zero nor negative integers and A, B are positive integers.

Contiguous Relation[Andrews p.363(9.16)] is defined as follows

$$(a-b) {}_2F_1 \left[\begin{matrix} a, b ; \\ c ; \end{matrix} z \right] = a {}_2F_1 \left[\begin{matrix} a+1, b ; \\ c ; \end{matrix} z \right] - b {}_2F_1 \left[\begin{matrix} a, b+1 ; \\ c ; \end{matrix} z \right] \quad (2)$$

Recurrence relation of gamma function is defined as follows

$$\Gamma(z+1) = z \Gamma(z) \quad (3)$$

Legendre duplication formula[Bells & Wong p.26(2.3.1)] is defined as follows

$$\sqrt{\pi} \Gamma(2z) = 2^{(2z-1)} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right) \quad (4)$$

Author α : P.D.M College of Engineering, Bahadurgarh, Haryana, India. E-mails : sludn@yahoo.com, vsludn@gmail.com

Author σ : International Scientific Research and Welfare Organization, New Delhi, India. E-mail : mpchaudhary_2000@yahoo.com

Author ρ : Former Research Scholar, Department of Mathematics, University of Delhi, New Delhi, India.

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)} \Gamma\left(\frac{b}{2}\right) \Gamma\left(\frac{b+1}{2}\right)}{\Gamma(b)} \quad (5)$$

$$= \frac{2^{(a-1)} \Gamma\left(\frac{a}{2}\right) \Gamma\left(\frac{a+1}{2}\right)}{\Gamma(a)} \quad (6)$$

Bailey summation theorem [Prudnikov, p.491(7.3.7.8)] is defined as follows

$${}_2F_1 \left[\begin{matrix} a, 1-a \\ c \end{matrix} ; \frac{1}{2} \right] = \frac{\Gamma\left(\frac{c}{2}\right) \Gamma\left(\frac{c+1}{2}\right)}{\Gamma\left(\frac{c+a}{2}\right) \Gamma\left(\frac{c+1-a}{2}\right)} = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-1} \Gamma\left(\frac{c+a}{2}\right) \Gamma\left(\frac{c+1-a}{2}\right)} \quad (7)$$

II. MAIN RESULT OF SUMMATION FORMULA

$$\begin{aligned} & {}_2F_1 \left[\begin{matrix} a, -a-55 \\ c \end{matrix} ; \frac{1}{2} \right] \\ &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c+55}} \left[\frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a+55}{2}\right)} \left\{ \begin{aligned} & 42399938467509559045165393068039340032000000 \\ & -81698158291609498026693194322909777100800000a \\ & +53179477748147700050692338322143307038720000a^2 \\ & -16208201839821574661871483252462583947264000a^3 \\ & +2498820231868759868316688941501352509849600a^4 \\ & -172458010223955295273328111884661354680320a^5 \\ & -5972403528891101949572462638155844608a^6 \\ & +602428259830734229227406154866707724800a^7 \\ & -13700388831005615109859897326289532160a^8 \\ & -1086825613829916422011720567308912000a^9 \\ & +24059205457512336000383874956896320a^{10} + 1442211065939340613049446496383200a^{11} \\ & -12415780525012339706469855030960a^{12} - 1232305555998371359599497403000a^{13} \\ & -7142733122254973831799577800a^{14} + 485326427545976056159719600a^{15} \\ & +8818391979507020394687840a^{16} - 23150752912615866337800a^{17} \\ & -2129335219074860236440a^{18} - 20301435778111488000a^{19} + 35915175974517840a^{20} \\ & +1981897331614200a^{21} + 13573636844040a^{22} + 16140524400a^{23} - 223292160a^{24} \\ & -1081080a^{25} - 1512a^{26} + 111499203713698653771006621603417324257280000c \\ & -163742052796897112820242552072096619429888000ac \\ & +85944068663935914117670655771422903487692800a^2c \end{aligned} \right\} \right] \end{aligned}$$

$$\begin{aligned}
& -21370396371652979215143578801590652211363840a^3c \\
& +2623550525769583347331377063702319807021056a^4c \\
& -124497318095462860387905918553038985236480a^5c \\
& -3612097702465706840440965975322699226112a^6c \\
& +492218605823122896902587070587500049920a^7c \\
& -275805202029035686979482288671812864a^8c \\
& -859547133037731586445840086808643200a^9c \\
& +53849592348935331827315036697152a^{10}c + 907578552960077055195680523419680a^{11}c \\
& +7651803335938703791473697804496a^{12}c - 500906035361948659636478818280a^{13}c \\
& -9640252794712151486284261528a^{14}c + 74956460074277553035389840a^{15}c \\
& +3727566043077344399337536a^{16}c + 26910119291435727816040a^{17}c \\
& -318378226807212008008a^{18}c - 6425222117022051200a^{19}c - 30386831619814384a^{20}c \\
& +140938447609960a^{21}c + 2091326625752a^{22}c + 7603195600a^{23}c + 2344160a^{24}c \\
& -40040a^{25}c - 56a^{26}c + 120646700333168033241532139957502439587840000c^2 \\
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& +3244821684646720859831682549389687040a^8c^2 \\
& -245702704791967536868878693236832000a^9c^2 \\
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& -9563925571200a^{21}c^2 + 66245830560a^{22}c^2 + 389188800a^{23}c^2 + 589680a^{24}c^2 \\
& +75111045620180303476410127043804050489344000c^3 \\
& -73507508824541531903012227142592915843317760ac^3 \\
& +26513867296308549533659020799938205718151168a^2c^3
\end{aligned}$$

$$\begin{aligned}
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& -879055550623931467328348348096780532736a^6c^3 \\
& +23987662118376858165743025588752184320a^7c^3 \\
& +1256494416477929651719763990568806656a^8c^3 \\
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& -31785915326939604073792309364524800a^9c^3
\end{aligned}$$

$$\begin{aligned}
& -1346820308106917157002616520689472a^{10}c^3 + 13926112163924055052673129895360a^{11}c^3 \\
& + 899367515248838855171703348592a^{12}c^3 + 3278792338038966450112611200a^{13}c^3 \\
& - 262042670867732864765158880a^{14}c^3 - 3635009226005737914459840a^{15}c^3 \\
& + 10612189685023190270864a^{16}c^3 + 586314123608120793600a^{17}c^3 \\
& + 4074779563518801280a^{18}c^3 - 7018160331582400a^{19}c^3 - 217313554425968a^{20}c^3 \\
& - 940039900800a^{21}c^3 - 540772960a^{22}c^3 + 4804800a^{23}c^3 + 7280a^{24}c^3 \\
& + 31009445220607609294077663842837270102016000c^4 \\
& - 25504306718499832156564208872296542864670720ac^4 \\
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\end{aligned}$$

$$\begin{aligned}
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\end{aligned}$$

$$\begin{aligned}
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& + 541697212588646400a^{14}c^9 + 4666979314483200a^{15}c^9 + 6766335713280a^{16}c^9 \\
& - 24644505600a^{17}c^9 - 49786880a^{18}c^9 + 522321157076120153860613276073637969920c^{10} \\
& - 163647792587518877982477527908614144000ac^{10} \\
& + 16307955693287241034058073891510681600a^2c^{10}
\end{aligned}$$

$$\begin{aligned}
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\end{aligned}$$

$$\begin{aligned}
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& -1270087680a^{14}c^{13} + 6665574949154063827633621759426560c^{14} \\
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& -512337621024768a^{10}c^{15} + 1173561016320a^{11}c^{15} + 3556245504a^{12}c^{15} \\
& +8672624671198995585716296089600c^{16} - 876737959547991925898304552960ac^{16} \\
& +12566031457838852964643504128a^2c^{16} + 682183216168048743048806400a^3c^{16} \\
& -8360908484814690724085760a^4c^{16} - 240534349628722902466560a^5c^{16} \\
& +794460117519589441536a^6c^{16} + 31936837677141196800a^7c^{16} + 73279496981053440a^8c^{16} \\
& -871369054617600a^9c^{16} - 3168614744064a^{10}c^{16} + 243318147995373924999946567680c^{17} \\
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& +931301204951040a^8c^{17} - 1898407526400a^9c^{17} - 6903300096a^{10}c^{17} \\
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& -63636570480052524810240ac^{20} - 648281683007392186368a^2c^{20}
\end{aligned}$$

$$\begin{aligned}
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& -1400729763840a^5c^{21} - 8489271296a^6c^{21} + 271916697726411079680c^{22} \\
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& -64776830976000ac^{24} - 1177760563200a^2c^{24} + 67807601491968c^{25} - 95965675520ac^{25} \\
& -1744830464a^2c^{25} + 195689447424c^{26} + 268435456c^{27} \} + \\
& + \frac{1}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+56}{2})} \left\{ -196156433597560262555496976124415442944000000a \right. \\
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& +23895291859777903419490851279682873275801600a^4 \\
& -2481466472306155071649693150738696974366720a^5 \\
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& -431637831606287563218538442674021152576a^8 \\
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& -2252845991953731291870738134610483906723840a^5c
\end{aligned}$$

$$\begin{aligned}
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& +224151127200a^{23}c - 3128842080a^{24}c - 15135120a^{25}c - 21168a^{26}c \\
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\end{aligned}$$

$$\begin{aligned}
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\end{aligned}$$



$$\begin{aligned}
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\end{aligned}$$

$$\begin{aligned}
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\end{aligned}$$

$$\begin{aligned}
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\end{aligned}$$

$$\begin{aligned}
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& + 11310135236838056128290173747200a^3c^{14} - 1527131776561171967739932180480a^4c^{14} \\
& - 4106892689519208978212454400a^5c^{14} + 719013210522774123396464640a^6c^{14} \\
& + 4403814042840221568204800a^7c^{14} - 126255960168782827356160a^8c^{14} \\
& - 1293769038943086182400a^9c^{14} + 3842655127320920064a^{10}c^{14} + 80920454961561600a^{11}c^{14} \\
& + 186942935531520a^{12}c^{14} - 488983756800a^{13}c^{14} - 1270087680a^{14}c^{14} \\
& + 5904988207785551117924797482270720c^{15} - 922927548163275586115587591372800ac^{15} \\
& + 32888428808620478837542449315840a^2c^{15} + 664953901102407023723670405120a^3c^{15} \\
& - 34591391532099664608554385408a^4c^{15} - 355911683181355821013401600a^5c^{15} \\
& + 11709129775479225880412160a^6c^{15} + 137652032428798503813120a^7c^{15} \\
& - 1070145613375869026304a^8c^{15} - 18095430700892160000a^9c^{15} - 21071288048025600a^{10}c^{15} \\
& + 443606064168960a^{11}c^{15} + 1344260800512a^{12}c^{15} + 219149011598897000590157461585920c^{16} \\
& - 27917843636235046982222883061760ac^{16} + 678200311751466484764894035968a^2c^{16} \\
& + 22657947762288852899869491200a^3c^{16} - 585049837700042874668974080a^4c^{16} \\
& - 10476092935123606628597760a^5c^{16} + 125958048102699766317056a^6c^{16} \\
& + 2372329651450126663680a^7c^{16} - 2200112572171616256a^8c^{16} - 151680071948697600a^9c^{16} \\
& - 448021587492864a^{10}c^{16} + 1026865889280a^{11}c^{16} + 3111714816a^{12}c^{16} \\
& + 6904165800399098153870662041600c^{17} - 707149036748253787339515494400ac^{17} \\
& + 10352800487418100623259729920a^2c^{17} + 553607208947840288764723200a^3c^{17}
\end{aligned}$$

$$\begin{aligned}
& -6894270185083927139450880a^4c^{17} - 196748052074755475374080a^5c^{17} \\
& +659552637698203189248a^6c^{17} + 26256912465788928000a^7c^{17} + 60147763406438400a^8c^{17} \\
& -717598044979200a^9c^{17} - 2609447436288a^{10}c^{17} + 184261098547060466225554391040c^{18} \\
& -14918713183406739646872289280ac^{18} + 97065957847989370543407104a^2c^{18} \\
& +10207793001459014067814400a^3c^{18} - 43886207194970676264960a^4c^{18} \\
& -2571998145530746634240a^5c^{18} - 3304798905610797056a^6c^{18} + 186064609135820800a^7c^{18} \\
& +723933740400640a^8c^{18} - 1476539187200a^9c^{18} - 5369233408a^{10}c^{18} \\
& +4149488493819913465579438080c^{19} - 259998190380059335852032000ac^{19} \\
& -66493975529545688678400a^2c^{19} + 143449614767893354905600a^3c^{19} \\
& +152929002840611880960a^4c^{19} - 23481150905253888000a^5c^{19} - 92763567371059200a^6c^{19} \\
& +772152282316800a^7c^{19} + 3509783101440a^8c^{19} + 78365298858008097828372480c^{20} \\
& -3698497425947288970199040ac^{20} - 21352983971215523708928a^2c^{20} \\
& +1519703045075202211840a^3c^{20} + 7097122848512671744a^4c^{20} - 143554548575764480a^5c^{20} \\
& -778274903949312a^6c^{20} + 1429911633920a^7c^{20} + 6499598336a^8c^{20} \\
& +1230136581121115789721600c^{21} - 42210468546500477583360ac^{21} \\
& -428783836561410097152a^2c^{21} + 11781720138527539200a^3c^{21} + 82838903555358720a^4c^{21} \\
& -529475850731520a^5c^{21} - 3208944549888a^6c^{21} + 15849918405998731591680c^{22} \\
& -376931892360107786240ac^{22} - 5091819419303149568a^2c^{22} + 63155297111244800a^3c^{22} \\
& +533284446535680a^4c^{22} - 891373486080a^5c^{22} - 5402263552a^6c^{22} \\
& +164683127651366338560c^{23} - 2535414992732160000ac^{23} - 40343282122752000a^2c^{23} \\
& +209278992384000a^3c^{23} + 1902536294400a^4c^{23} + 1345042764068290560c^{24} \\
& -12074558160896000ac^{24} - 210655982387200a^2c^{24} + 322961408000a^3c^{24} + 2936012800a^4c^{24} \\
& +8310278533939200c^{25} - 36275025346560ac^{25} - 659545915392a^2c^{25} + 36494873985024c^{26} \\
& -51673825280ac^{26} - 939524096a^2c^{26} + 101468602368c^{27} + 134217728c^{28} \} \quad (8)
\end{aligned}$$

Derivation of main result (8):

Substituting $b = -a - 55$, $z = \frac{1}{2}$ in given result (2), we get

$$\begin{aligned}
& (2a + 55) {}_2F_1 \left[\begin{matrix} a, & -a - 55 \\ c & \end{matrix} ; \frac{1}{2} \right] \\
& = a {}_2F_1 \left[\begin{matrix} a + 1, & -a - 55 \\ c & \end{matrix} ; \frac{1}{2} \right] + (a + 55) {}_2F_1 \left[\begin{matrix} a, & -a - 54 \\ c & \end{matrix} ; \frac{1}{2} \right]
\end{aligned}$$

Now involving the result established in Ref[6], we can prove the main result.

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