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CESÁRO Mean of Product Summability of Partial Differential Equations of Sequences

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Abstract- In [4], the definition of product summability method $(\boldsymbol{D},\boldsymbol{k})(\boldsymbol{C},\boldsymbol{l})$ for functions was given and some of its properties were investigated. In [2], $(\boldsymbol{D},\boldsymbol{k})(\boldsymbol{C},\boldsymbol{\alpha},\boldsymbol{\beta})$ ($k > 0$, $\boldsymbol{\alpha} > 0$, $\boldsymbol{\beta} > -1$) summability for functions are defined and some of its properties were investigated. In [1], the Cesáro means and Cesáro summability were discussed for sequences. In this paper, we study some results of Cesáro mean of product summability $(\boldsymbol{D},\boldsymbol{k})(\boldsymbol{C},\boldsymbol{\alpha},\boldsymbol{\beta})$ ($k > 0$, $\boldsymbol{\alpha} > 0$, $\boldsymbol{\beta} > -1$) of partial differential equations of sequences.

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Ref

1. B. Kuttner, On translated quasi –Cesaro summability, Cambridge Philos.Soc.,62 (1966) ,705-712.

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I. Introduction

Kuttner [1], introduced the summability method for functions and investigated some of its properties. Pathak [4], defined the product summability method for functions and investigated some of its properties. Mishra and Srivastava [3], introduced the summability method for functions by generalizing summability method. Mishra and Mishra [2], introduced the summability method for functions and investigated some of its properties. In this paper, we study some results of Cesȧro mean of product summability $(D,k)(C,\alpha,\beta)$ $(k>0, \alpha>0, \beta>-1)$ of partial differential equations of sequences.

II. Some relations and Definitions

Let $f(x)$ be any function which is Lebesgue-measurable, and that $f : [0, +\infty) \to R$, and integrable in $(0,\ x)$, for any finite x and which is bounded in some right hand neighbourhood of origin. Integrals of the form $\int_0^\infty$ are throughout to be taken as $\lim_{x\to\infty}\int_0^x$, $\int_0^x$ being a Lebesgue integral. For any n>o, we write $a_n(x)$ for the n^{th} integral,

$$a_n(x) = \frac{1}{\Gamma(n)} \int_0^x (x-y)^{n-1}\, a(y)dy\,,$$

$$\boldsymbol{a}\_(\boldsymbol{0})(\boldsymbol{x}) = \boldsymbol{a}(\boldsymbol{x})$$

The (C,α,β) transform of $a\,(t)$, which we denote by $\partial_{\alpha,\beta}(t)$ is given by

$$a(t) \qquad (\alpha = 0)$$

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$$\frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha)\Gamma(\beta+1)}\frac{1}{t^{\alpha+\beta}}\int_0^x (t-u)^{\alpha-1}u^{\beta}a(y)dy\,,\ (\alpha>0,\beta>-1)\,, \tag{2.1}$$

If, for t > 0, the integral defining $\partial_{\alpha,\beta}(t)$ exists and if $\partial_{\alpha,\beta}(t)\to s$ as $t\to\infty$, we say that a (x) is summable (C,α,β) to s, and we write $\boldsymbol{a(x)}\to$ s (C, α, β) . We write

$g(t)=g^{(k)}(t)=kt\int_0^\infty \frac{x^{k-1}}{(x+t)^{k+1}}a(x)dx$, (k > 0) (2.2) if this exists, We also write

$U_{k,\alpha,\beta}(t)=kt\int_0^\infty \frac{x^{k-1}}{(x+t)^{k+1}}\partial_{\alpha,\beta}(x)dx$, (2.3) if this exists.

With the usual terminology, we say that the sequence a_n is summable,

(I) (D, k) to the sum s, if g (t) tends to a limit s as t→ ∞ ,

(II) (D, k) (C,α,β) to s, if $U_{k,\alpha,\beta}(t)$ tends to s as t→ ∞. When$\beta=0$,$(D,k)(C,\alpha,\beta)$ and $(D,k)(C,\alpha)$ denote the same method. The case $\beta=0$ is due to Pathak[5] . We know that for any fixed t > 0, k > 0, it is necessary and sufficient for the convergence of (2.3) that $\int_1^\infty \frac{\partial_{\alpha,\beta}(x)}{x^2}\ dx$ should converge. (2.4)

If (2.4) converges, write for x > 0, $F_{\alpha,\beta}(x)=\int_x^\infty \frac{\partial_{\alpha,\beta}(t)}{t^2}\ dt$.

We note that $F_{\alpha,\beta}(x)=o(1)$ as x→ ∞ . Further, (since f(x) is bounded in some right hand neighbourhood of the origin) we have,

$F_{\alpha,\beta}(x)=o\left(\frac{1}{x}\right)$ as x→ 0 + .

III. Main Results

In this section, we have following theorems for sequences analogous to [2].

Theorem 3.1 : If $\alpha>\gamma\geq 1,\ k>0$ then $a\,(x)\to s\,(D,k)(C,\alpha-1,\beta)$,whenever $a(x)\to s(D,k)(C,\gamma-1,\beta)$.

Theorem 3.2 : Let $\alpha>\gamma\geq 0,\ \beta>-1$, and suppose that a(x) is summable (C,γ,β) to s and that $\int_1^\infty \frac{\partial_{\gamma,\beta}(x)}{x^2}\ dx$ converges . Then a(x) is summable $(D,k)(C,\alpha,\beta)$ to s.

References Références Referencias

1. **B. Kuttner,** On translated quasi –Cesaro summability, Cambridge Philos.Soc.,62 (1966) ,705-712.
2. **B. P. Mishra and S.N. Mishra,** Some remarks on product summability methods.Prog.of Maths.Vol 35 (nos.1 & 2) 2001.
3. **B. P. Mishra and A.P.Srivastava,** Some remarks on absolute summability of functions based on (C,α,β) summability methods. To appear in Jour.Nat.Acad.Math.
4. **S.N.Pathak,** Some investigations of summability of functions, Ph.D. Thesis,Gorakhpur University,Gorakhpur (1986).

Notes