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Computation of a Summation Formula Clung To Recurrence Relation

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Computation of a Summation Formula Clung To Recurrence Relation

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I. Basic Introduction

The Pochhammer symbol or **generalized factorial function** or **shifted factorial** or **falling factorial** is defined by

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{bmatrix} 1 & ; & n=0 \\ a(a+1)(a+2).....(a+n-1); & & n=1,2,3.... \end{bmatrix} \tag{1}$$

where $a \neq 0, -1, -2, ...$ and the notation Γ stands for Gamma function.

$$(b)_{-n} = \frac{\Gamma(b-n)}{\Gamma(b)} = \frac{(-1)^n}{(1-b)_n}\ ; \tag{2}$$

where $b \neq .....-3, -2, -1, 0, 1, 2, 3.....$ and $n = 1, 2, 3, ...$

If $m = 1, 2, , 3, 4 \ldots$ and $n = 0, 1, 2, 3, \ldots,$ then

$$(b)_{mn} = m^{mn}\left(\frac{b}{m}\right)_n\left(\frac{b+1}{m}\right)_n \cdots \left(\frac{b+m-2}{m}\right)_n\left(\frac{b+m-1}{m}\right)_n \tag{3}$$

Generalized Gaussian hypergeometric function of one variable is followed as

$${}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!}$$

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or

$$ {}_AF_B\left[\begin{array}{ccc}(a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ccc}(a_j)_{j=1}^{A} & ; & \\ & & z \\ (b_j)_{j=1}^{B} & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{4} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

Contiguous Relation[E. D. p.51(10), Andrews p.363(9.16)] is defined as follows

$$ (a-b)\ {}_2F_1\left[\begin{array}{cc}a,\ b\ ; & \\ c\ ; & z\end{array}\right] = a\ {}_2F_1\left[\begin{array}{cc}a+1,\ b\ ; & \\ c\ ; & z\end{array}\right] - b\ {}_2F_1\left[\begin{array}{cc}a,\ b+1\ ; & \\ c\ ; & z\end{array}\right] \tag{5} $$

Recurrence relation of gamma function is defined as follows

$$ \Gamma(z+1) = z\ \Gamma(z) \tag{6} $$

Legendre duplication formula[Bells & Wong p.26(2.3.1)] is defined as follows

$$ \sqrt{\pi}\ \Gamma(2z) = 2^{(2z-1)}\ \Gamma(z)\ \Gamma\left(z+\frac{1}{2}\right) \tag{7} $$

$$ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{8} $$

$$ = \frac{2^{(a-1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{9} $$

Bailey summation theorem [Prudnikov, p.491(7.3.7.8)]is defined as follows

$$ {}_2F_1\left[\begin{array}{cc}a, 1-a & ;\ \frac{1}{2}\\ c & ;\end{array}\right] = \frac{\Gamma(\frac{c}{2})\ \Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-1}\ \Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} \tag{10} $$

II. Main Result of Summation Formula

$$ {}_2F_1\left[\begin{array}{cc}a\ , & -a-51\ ;\ \frac{1}{2}\\ c & ;\end{array}\right] $$

$$ = \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+51}}\left[\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+51}{2})}\left\{+39215629363216388314063441609359360000000\right.\right. $$

$$ -74021568132398532674161337237569536000000a $$

$$ \left.\left.+46295981506391174974374821359954329600000a^2\right.\right. $$

Notes

$$-1320609504346225933240581617829869568000a^3$$
$$+1811930446458959824391495038134193152 00a^4$$
$$-9311856861379155575957484232594867200a^5$$
$$-319305701534575139018589169352966400a^6+432906651408504871126269609925056 00a^7$$
$$+1459526532650544719250939889840 00a^8-8771917410859920237540675575520 0a^9$$
$$-4575088744242843615633002444 00a^{10}+1005808770049491450042585156 00a^{11}$$
$$+1508122770646685337604700500a^{12}-5142016025722708393563120 0a^{13}$$
$$-1509631951413201653983400a^{14}+4302016118331891960 0a^{15}+4606163067186269875 00a^{16}$$
$$+585472610368039680 0a^{17}-662319052263440 0a^{18}-74418821316840 0a^{19}$$
$$-6243725988500a^{20}-9352543200a^{21}+138868600a^{22}+795600a^{23}+1300a^{24}$$
$$+10161656819477326305691970258274877440000c$$
$$-1446367255959290974968755905793910374400 0ac$$
$$+7213434913697898840014034554316432998400a^2c$$
$$-1652471481542170230203835091179109662720a^3c$$
$$+1746582112809621535161394811697886126 08a^4c$$
$$-4969371876573656445853216824494834688a^5c$$
$$-4756041029223851855577462614864386 56a^6c$$
$$+2690942552956382033480148191746022 4a^7c+7668477053102870426726336078793 60a^8c$$
$$-4686074174974908708180859439020 8a^9c-1208658606098948851658543849776a^{10}c$$
$$+3578734555798042658974146062 4a^{11}c+1310981764885248713045228020a^{12}c$$
$$-4121598466147286396065248a^{13}c-6208563979723478843193 36a^{14}c$$
$$-6671743900559708043216a^{15}c+6558127104990267950 0a^{16}c+1896520433242735872a^{17}c$$
$$+1153021136901462 4a^{18}c-53516453726736a^{19}c-1025323839540a^{20}c-4458181728a^{21}c$$
$$-1725256a^{22}c+31824a^{23}c+52a^{24}c+1076616397008140759739536209336074240 0000c^2$$
$$-1222065339153341113055562146980665753600 0ac^2$$
$$+4964847384594138558862158629137140940800a^2c^2$$
$$-9173553475857202963004609719592828928 00a^3c^2$$
$$+7284818328107943034599645981002711040 0a^4c^2$$
$$-4613333979910927893660172636314624 00a^5c^2$$
$$-2344905737318877803070406580027392 00a^6c^2$$
$$+5607883301081174731722705232665600a^7c^2+4045327426057874772699839668992 00a^8c^2$$
$$-7562908655286410131260619449600a^9c^2-4823146433791448180907007104 00a^{10}c^2$$

$$+16497683403078933715422672 00a^{11}c^2+3158754417227862826724624 00a^{12}c^2$$
$$+3057010359220073337681600a^{13}c^2-7134279093442306908960 0a^{14}c^2$$
$$-1619095636189622448000a^{15}c^2-4986468790418544000a^{16}c^2+157146003545731200a^{17}c^2$$
$$+1822144519395200a^{18}c^2+4701184488000a^{19}c^2-34522488000a^{20}c^2-245044800a^{21}c^2$$
$$-436800a^{22}c^2+6529011725984880559625498384381509632000c^3$$
$$-607527203112514220836692567188777730048 0ac^3$$
$$+2032304372461996734448911257402664812544a^2c^3$$
$$-3020678620572310923886949723004789719 04a^3c^3$$
$$+1718072055568936330350721831353316147 2a^4c^3$$
$$+3045162780146217013937529575302471 68a^5c^3$$
$$-5928200045143129512117233626136985 6a^6c^3$$
$$+1515324587191551134306980047022 08a^7c^3+935587100140229553968545979586 56a^8c^3$$
$$+1127331108146289426687197406 72a^9c^3-8250765809298635820087200947 2a^{10}c^3$$
$$-8798814973420162230349031 04a^{11}c^3+3139362688575582221563283 2a^{12}c^3$$
$$+6860746929250194791690 88a^{13}c^3-6672368856121662545 28a^{14}c^3$$
$$-1399368420511416326 40a^{15}c^3-1285493802358113920a^{16}c^3+1721586727276416a^{17}c^3$$
$$+89439005591936a^{18}c^3+471090459840a^{19}c^3+340500160a^{20}c^3-3267264a^{21}c^3$$
$$-5824a^{22}c^3+2612866415715646813807239647295700992000c^4$$
$$-2023055775046023696675566334528454656000ac^4$$
$$+5597606675261636184600124256007290880 00a^2c^4$$
$$-6632956167167984828404965598970265600 0a^3c^4$$
$$+2455512005609343344145848292777984000a^4c^4$$
$$+1237535851513536362173688224473600 00a^5c^4$$
$$-8848826342631603836821350432640000a^6c^4$$
$$-1424035278140995663311034379520 00a^7c^4+1180714737565094411440833323520 00a^8c^4$$
$$+1958535340174562865381338400 00a^9c^4-7044583957774727005157880000a^{10}c^4$$
$$-1752821145922705335116640 00a^{11}c^4+9269200774452512894000 00a^{12}c^4$$
$$+6064650048474121104000 0a^{13}c^4+4374362730688922400 00a^{14}c^4$$
$$-4560723705938400000a^{15}c^4-81537114234576000a^{16}c^4-305264559600000a^{17}c^4$$
$$+1313672360000a^{18}c^4+12252240000a^{19}c^4+24024000a^{20}c^4$$
$$+7464487195938416781946417894200296079 36c^5$$

Notes

$$-48484234224919834080743341409765739724 8ac^5$$
$$+1109848016894475735519120702247379927 04a^2c^5$$
$$-1031723368727946254444011135645650124 8a^3c^5$$
$$+1950939528571816894404651335140638 72a^4c^5$$
$$+2364932148757800197825670685221888 0a^5c^5$$
$$-7967507845695813932055290677811 20a^6c^5 - 3126389268620074980188047934361 6a^7c^5$$
$$+8231557848426858833582541788 16a^8c^5 + 3005939207175143872612779072 0a^9c^5$$
$$-2326353187477596442959830 40a^{10}c^5 - 1522791942814168592697331 2a^{11}c^5$$
$$-8185281180894762424480 0a^{12}c^5 + 2562162110375475928320a^{13}c^5$$
$$+364163020462906579 20a^{14}c^5 + 3736483199729280 0a^{15}c^5 - 2170921221076608a^{16}c^5$$
$$-14204266876800a^{17}c^5 - 15116221120a^{18}c^5 + 98017920a^{19}c^5 + 192192a^{20}c^5$$
$$+1599367617240372884217170181803212 80000c^6$$
$$-8749212737111082488373415636390379 5200ac^6$$
$$+1651600970029225647429459878551224 3200a^2c^6$$
$$-1173671805520416747746112210311577 600a^3c^6$$
$$+1775391309788925186503726533836800a^4c^6$$
$$+2862264047025912796435298741452800a^5c^6 - 3631355003658491460239375728 6400a^6c^6$$
$$-3479560389180212846538284236800a^7c^6 + 2098728159618006248183255040 0a^8c^6$$
$$+2413106726268002114404454400a^9c^6 + 1081923974698429393090560 0a^{10}c^6$$
$$-7153662492242325835776 00a^{11}c^6 - 9335180202255799091200a^{12}c^6$$
$$+347359470239029248 00a^{13}c^6 + 131913011269079040 0a^{14}c^6 + 6809187280896000a^{15}c^6$$
$$-17303430144000a^{16}c^6 - 235243008000a^{17}c^6 - 512512000a^{18}c^6$$
$$+2657638401472618758221576819347292 1600c^7$$
$$-1225677240375840964909193396983588 4544ac^7$$
$$+1895875818441715846981287505091887 104a^2c^7$$
$$-9876339246552438748626259617723187 2a^3c^7$$
$$-1653746068979970114211115631509504a^4c^7$$
$$+2383166266278887193826963246940 16a^5c^7 + 5908658433080967378318473297 92a^6c^7$$
$$-2436036550682117691745516584 96a^7c^7 - 1564667020221574346476019712a^8c^7$$
$$+1158420607582797015031111 68a^9c^7 + 1563137249722283122348032a^{10}c^7$$
$$-1629693584699384763187 2a^{11}c^7 - 4194859863705361776 64a^{12}c^7$$

$$-1367035409521926144a^{13}c^7 + 22577009199833088a^{14}c^7 + 191257494405120a^{15}c^7$$
$$+274524856320a^{16}c^7 - 1344245760a^{17}c^7 - 2928640a^{18}c^7$$
$$+3507093480716382082057454435172352000c^8$$
$$-136190227379755457345546565797806080 0ac^8$$
$$+17094042739830324058091479337533440 0a^2c^8$$
$$-6067926395459215226369951249203200a^3c^8$$
$$-2382423862432601243329431773184 00a^4c^8 + 139581712425222863160597872640 00a^5c^8$$
$$+2239138355792106881873080320 00a^6c^8 - 1114032374517839103003156480 0a^7c^8$$
$$-1926068902089500539058688 00a^8c^8 + 3143176542139018125312000a^9c^8$$
$$+8056355222033739878400 0a^{10}c^8 + 4549225380767907840 0a^{11}c^8$$
$$-1004782518133954560 0a^{12}c^8 - 71472471662592000a^{13}c^8 + 91162266624000a^{14}c^8$$
$$+2285217792000a^{15}c^8 + 5601024000a^{16}c^8 + 373936129332143623419537730431877120c^9$$
$$-12188199359647766088654846316825 8048ac^9$$
$$+12244622122591681114343641738379264a^2c^9$$
$$-2546410607640370619633091721297 92a^3c^9$$
$$-1966367023335741556657324861030 4a^4c^9 + 56008479467022568667676278784 0a^5c^9$$
$$+1809574612500475422876065792 0a^6c^9 - 3106058308727262631265402 88a^7c^9$$
$$-1020747663135337611727052 8a^8c^9 + 2322307780121925353472 0a^9c^9$$
$$+2355636749511444439040a^{10}c^9 + 1396666394732799590 4a^{11}c^9$$
$$-117516116735553536a^{12}c^9 - 1384090399211520a^{13}c^9 - 2582047170560a^{14}c^9$$
$$+10156523520a^{15}c^9 + 24893440a^{16}c^9 + 3262443149751044008415240126464 0000c^{10}$$
$$-8881346851313890235603771562393600ac^{10}$$
$$+7007702800107856248737834729472 00a^2c^{10} - 5276222909482495066628397465600a^3c^{10}$$
$$-1131639476780264378939159347200a^4c^{10} + 1328631196452522577931796480 0a^5c^{10}$$
$$+8813451963415592102658048 00a^6c^{10} - 3076458866038702861516800a^7c^{10}$$
$$-3313785127571059654656 00a^8c^{10} - 1597994314386127257600a^9c^{10}$$
$$+4000308020175667200 0a^{10}c^{10} + 408320886900326400a^{11}c^{10} - 79710786355200a^{12}c^{10}$$
$$-12797219635200a^{13}c^{10} - 35846553600a^{14}c^{10} + 2350559102017734118725983508889600c^{11}$$
$$-5308492413293677107107666633687 04ac^{11} + 3204082190205603045447691979980 8a^2c^{11}$$
$$+1548691193163573591934406492 16a^3c^{11} - 4794024376510400642713701580 8a^4c^{11}$$
$$+1650831059349741270151987 2a^5c^{11} + 2918564816248263138685747 2a^6c^{11}$$
$$+1394154197632331673108 48a^7c^{11} - 6872246298970496630784a^8c^{11}$$

Notes

Notes

$$-68461810010010353664a^9c^{11}+31312605180100608 0a^{10}c^{11}+5905660917252096a^{11}c^{11}$$
$$+14157389955072a^{12}c^{11}-46535344128a^{13}c^{11}-130351104a^{14}c^{11}$$
$$+14075367280203107352188236595200 0c^{12}-26142919734994299344681277849600ac^{12}$$
$$+11618463325817878010549239808 00a^2c^{12}+19816175844567736562968166400a^3c^{12}$$
$$-15146808165260705389281280 00a^4c^{12}-13007910157465601064960000a^5c^{12}$$
$$+66749147310775606476800 0a^6c^{12}+74520790250798481408 00a^7c^{12}$$
$$-83890949279322931200a^8c^{12}-1363318831300608000a^9c^{12}-1212846415872000a^{10}c^{12}$$
$$+44208576921600a^{11}c^{12}+144472473600a^{12}c^{12}+70332555573175654177310349721 60c^{13}$$
$$-10627144706902857190454518087 68ac^{13}+32779414532156325397071396864a^2c^{13}$$
$$+10022692374039650387094405 12a^3c^{13}-35211188622667072624394240a^4c^{13}$$
$$-59359172675018752327680 0a^5c^{13}+10030581135956355645440a^6c^{13}$$
$$+18242041681740260966 4a^7c^{13}-343475889878532096a^8c^{13}-15530359717232640a^9c^{13}$$
$$-48184903925760a^{10}c^{13}+136026390528a^{11}c^{13}+444530688a^{12}c^{13}$$
$$+29381483038307735107010560000 0c^{14}-35630582416423246085750784000ac^{14}$$
$$+68794386734409115710259200 0a^2c^{14}+33783509590480862576640000a^3c^{14}$$
$$-57100745582772879360000 0a^4c^{14}-15474236362054041600000a^5c^{14}$$
$$+79181320218083328000a^6c^{14}+272732913008640000 0a^7c^{14}+5936389816320000a^8c^{14}$$
$$-97161707520000a^9c^{14}-381026304000a^{10}c^{14}+10259229421653434996201881600c^{15}$$
$$-98183872198774294537725542 4ac^{15}+94378071966287098145669 12a^2c^{15}$$
$$+83110564558287723823104 0a^3c^{15}-53750719883571101696 00a^4c^{15}$$
$$-26982524008999157760 0a^5c^{15}-203567309276577792a^6c^{15}+25409729750630400a^7c^{15}$$
$$+1047365104435 20a^8c^{15}-259097886720a^9c^{15}-1016070144a^{10}c^{15}$$
$$+29869828988705671689011200 0c^{16}-22087743903606947866214400ac^{16}$$
$$+34749165208324354867200a^2c^{16}+15256504595570570035200a^3c^{16}$$
$$+2601854431140249600a^4c^{16}-32104948692418560 00a^5c^{16}-12890056359936000a^6c^{16}$$
$$+136026390528000a^7c^{16}+666796032000a^8c^{16}+72163098146267657037414 40c^{17}$$
$$-40133249027490628553932 8ac^{17}-19946948191516738191 36a^2c^{17}$$
$$+20784195017805555302 4a^3c^{17}+943705218464612352a^4c^{17}-25157520819486720a^5c^{17}$$
$$-145384545976320a^6c^{17}+320062095360a^7c^{17}+1568931840a^8c^{17}$$
$$+14353770849318731776000 0c^{18}-57954785787541192704 00ac^{18}$$
$$-58868619943503462400a^2c^{18}+20460253945528320 00a^3c^{18}+150714381762560 00a^4c^{18}$$
$$-117356101632000a^5c^{18}-767033344000a^6c^{18}+23234440656339337216 00c^{19}$$

$$-649251921274223984 64ac^{19} - 916234973881565184a^2c^{19} + 13778265172869120a^3c^{19}$$
$$+124580748328960a^4c^{19} - 247065477120a^5c^{19} - 1614807040a^6c^{19}$$
$$+300929768772075520 00c^{20} - 5436453465096192 00ac^{20} - 92106736533504 00a^2c^{20}$$
$$+568250597376 00a^3c^{20} + 557108428800a^4c^{20} + 304216262815252480c^{21}$$
$$-32002791262126 08ac^{21} - 59990496772096a^2c^{21} + 108238209024a^3c^{21} + 1061158912a^4c^{21}$$
$$+23113970483200 00c^{22} - 11807804620800ac^{22} - 231525580800a^2c^{22} + 12408428953600c^{23}$$
$$-20535312384ac^{23} - 402653184a^2c^{23} + 41943040000c^{24} + 67108864c^{25}\Big\}+$$
$$+\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+52}{2})}\Big\{-17464069942802715386614602906796032000000a$$
$$+20686287355803117929950207741475389440000a^2$$
$$-89378277350952189664003768283812331520 00a^3$$
$$+17781875231429143853642282718796858368 00a^4$$
$$-15295834999455270891171432131298479616 0a^5$$
$$+84580791261276992924025160195502361 6a^6$$
$$+62507388470410138827268293141233145 6a^7 - 16351780395261526276780909546342272a^8$$
$$-13877159130594298593242575586078 88a^9 + 29422980952667711908287591614240a^{10}$$
$$+21972722714969371470994180557 96a^{11} - 91546213857026072673125431 12a^{12}$$
$$-20090558959466593944440490 63a^{13} - 22907465907142301442089065a^{14}$$
$$+67806894845325092251212 6a^{15} + 18820430104162303836518a^{16}$$
$$+69913513840255423767a^{17} - 31060328718393810 55a^{18} - 48230575964749464a^{19}$$
$$-173954956930612a^{20} + 2022903890007a^{21} + 24154252265a^{22} + 87862086a^{23} + 2678a^{24}$$
$$-663a^{25} - a^{26} + 17464069942802730897824646237782016000000c$$
$$-54007367613351708706039996361205350400000ac$$
$$+41767493571718136116945131402825154560000a^2c$$
$$-13573255377452899961200986142034724864000a^3c$$
$$+20546717179456108606506652537709976576 00a^4c$$
$$-11835766148166118034864208368537495040 0a^5c$$
$$-31878105386048508220317844756794432 00a^6c$$
$$+53767945589634820729486018826559360 0a^7c+21561567877933197657113200287840 0a^8c$$
$$-11064344422506559947750729879696 00a^9c - 39005211881643018383038309708 00a^{10}c$$
$$+12993527595769132331897223012 00a^{11}c + 18242603793449626590118783300a^{12}c$$
$$-68134188058418541366674640 0a^{13}c - 19313434798930849743184200a^{14}c$$

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$$+7547544375035398047600a^{15}c+60091256929518250007 00a^{16}c$$
$$+7542862590857742720 0a^{17}c-939894569306264 00a^{18}c-9694744240381200a^{19}c$$
$$-81018694656900a^{20}c-120521200800a^{21}c+1807184600a^{22}c+10342800a^{23}c$$
$$+16900a^{24}c+3332108025754864996621860032093356032 0000c^2$$
$$-61112319783077679498758852113706385408000ac^2$$
$$+3523916859244368212159736492025518981120 0a^2c^2$$
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$$-27182086039814644390155980616825488 64a^6c^2$$
$$+17346834336799238866980948672915187 2a^7c^2$$
$$+43889630745475193234758592147775 68a^8c^2-30487945465896598352737520311939 2a^9c^2$$
$$-73236132900824524921962747794 16a^{10}c^2+23810712057746792854377072602 4a^{11}c^2$$
$$+83161095798756193264712156 66a^{12}c^2-31210924941366650068334928a^{13}c^2$$
$$-40306787508500310428236 84a^{14}c^2-42456192021451570839048a^{15}c^2$$
$$+43460102747458570001 4a^{16}c^2+12315977161258028544a^{17}c^2+74364791296267472a^{18}c^2$$
$$-350917550607624a^{19}c^2-6666815293138a^{20}c^2-28956944016a^{21}c^2-11176308a^{22}c^2$$
$$+206856a^{23}c^2+338a^{24}c^2+28282653946454862828385645890424012800000c^3$$
$$-37847109234486369007419794151897563136000ac^3$$
$$+17134579371035444556951735602324491468800a^2c^3$$
$$-34348739862985467569978326427685814272 00a^3c^3$$
$$+29448904720092988909106397581109411840 0a^4c^3$$
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$$-94057616732278352210820058186275840 0a^6c^3$$
$$+25431434993231939895777252470553600a^7c^3$$
$$+16513171743660878272187645669888 00a^8c^3-34397369483011709151574125734400a^9c^3$$
$$-20286632404921551187011159264 00a^{10}c^3+86464008832314800874790512 00a^{11}c^3$$
$$+13604789058704108240147536 00a^{12}c^3+12722508323197889416200000a^{13}c^3$$
$$-31290321118970293791840 0a^{14}c^3-69763486819972751520 00a^{15}c^3$$
$$-20901902309034032000a^{16}c^3+68360450584345920 0a^{17}c^3+7884558877395200a^{18}c^3$$
$$+20265613368000a^{19}c^3-149805656000a^{20}c^3-1061860800a^{21}c^3-1892800a^{22}c^3$$

$$+1440300849635277480118978884647308492800 0c^4$$
$$-150960224819365534135896082504816892313 60ac^4$$
$$+549850181306804818472775808469427211468 8a^2c^4$$
$$-875089957529533052576034038524658614272a^3c^4$$
$$+536437469536812592432516366462773411 84a^4c^4$$
$$+745901964781113133197915990274824192a^5c^4$$
$$-184010830303513208010308817367427584a^6c^4$$
$$+818367161622811724817415382305536a^7c^4$$
$$+295069311238367211284859400869888a^8c^4 + 468788101453269526684999426 56a^9c^4$$
$$-265547644209905563443206559264a^{10}c^4 - 269551643666341209691120948 8a^{11}c^4$$
$$+102893805882101328651547536a^{12}c^4 + 220191398659140564856200 0a^{13}c^4$$
$$-256183031419614817918 4a^{14}c^4 - 455189373853931951520a^{15}c^4$$
$$-41543219817731403 20a^{16}c^4 + 5748965064434592a^{17}c^4 + 290840371773952a^{18}c^4$$
$$+1529982133680a^{19}c^4 + 1104543440a^{20}c^4 - 10618608a^{21}c^4 - 18928a^{22}c^4$$
$$+498732460082223946428109653534069227520 0c^5$$
$$-42313376901729947265139758466165899264 00ac^5$$
$$+125463585064926798763757239214389985280 0a^2c^5$$
$$-157710034258206885829138153197028147200a^3c^5$$
$$+634191211917035513660628923286159360 0a^4c^5$$
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$$-225165576987905063844560590803712 00a^6c^5$$
$$-325958511808137888230326702617600a^7c^5 + 304096559464959260798029765248 00a^8c^5$$
$$+478117034612337247952376988800a^9c^5 - 184497581315116051311406896 00a^{10}c^5$$
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$$+157223273868315560505600a^{13}c^5 + 112019339012164034880 0a^{14}c^5$$
$$-119462069789270208 00a^{15}c^5 - 211771076704387200a^{16}c^5 - 790629695856000a^{17}c^5$$
$$+3422210792000a^{18}c^5 + 31855824000a^{19}c^5 + 62462400a^{20}c^5$$
$$+125855750279546574894574649931157261516 8c^6$$
$$-879365308715679527902577076638293426176ac^6$$
$$+213226342068718901576910141848419172352a^2c^6$$
$$-208887417390440831904152609630035476 48a^3c^6$$
$$+445350239425183456899743815003930624a^4c^6$$

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$$+476970442495828897042416866157066 24a^5c^6$$
$$-173299232354517895275640971397580 8a^6c^6$$
$$-640835516165831680141567823907 84a^7c^6+1805944697836994302445981656832a^8c^6$$
$$+633748043179436102539476865 92a^9c^6-52745112795207530584067126 4a^{10}c^6$$
$$-32744884604360817430301760a^{11}c^6-17100054725544699526217 6a^{12}c^6$$
$$+55719083451714810167 04a^{13}c^6+78559241033904908992a^{14}c^6+78057691270086528a^{15}c^6$$
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$$+416416a^{20}c^6+241830426344143445812620039165050880000c^7$$
$$-140430911900394092446841334069303705600ac^7$$
$$+278342515450299241729726387709804544 00a^2c^7$$
$$-207550771145550821978660619233525760 0a^3c^7$$
$$+722454153797750835287655407288320 0a^4c^7$$
$$+507614192801360525343360477265920 0a^5c^7$$
$$-711619054898602252120168546304 00a^6c^7-626952044191716405270660464640 0a^7c^7$$
$$+422548014354573964422279168 00a^8c^7+44267148074098957188639744 00a^9c^7$$
$$+187037225255938857845760 00a^{10}c^7-13292516499724186103808 00a^{11}c^7$$
$$-171626158029500821504 00a^{12}c^7+65746142983045324800a^{13}c^7$$
$$+2448226396398182400a^{14}c^7+12606023079936000a^{15}c^7-32232026112000a^{16}c^7$$
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$$-176691127417865776195061927021673185 28ac^8$$
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$$-231574837489625776642035778643558 4a^4c^8$$
$$+376349415587111659860654974263296a^5c^8+614552347269208953427966926848a^6c^8$$
$$-389764985733751875918189023232a^7c^8-234525647004655720732486041 6a^8c^8$$
$$+187773430052693688565519872a^9c^8+24942416722340159185228 80a^{10}c^8$$
$$-267533933666265602519 04a^{11}c^8-679369622518788810752a^{12}c^8$$
$$-219445442315837337 6a^{13}c^8+36737957249190912a^{14}c^8+310595376199680a^{15}c^8$$
$$+445617469440a^{16}c^8-2184399360a^{17}c^8-4759040a^{18}c^8$$
$$+440406877603934096777026673311744 0000c^9$$
$$-178326131008147415788221034030694 4000ac^9$$
$$+232113864710755881803031756996608 000a^2c^9$$

$$-864406634285854196587884783206400 0a^3c^9$$
$$-3201107777856395066129853194240 00a^4c^9+19869691087509808561877532672000a^5c^9$$
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$$+11550243977838018252800 0a^{10}c^9+5687883165675110400 0a^{11}c^9$$
$$-14511749989248512000a^{12}c^9-102960741531648000a^{13}c^9+132455504896000a^{14}c^9$$
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$$+15168496614286038219626167337746432a^2c^{10}$$
$$-33429760249731875864504419260825 6a^3c^10$$
$$-24435417055526058339828463517696a^4c^{10}+72746188959478259574173093068 8a^5c^{10}$$
$$+22833231529388666525619544064a^6c^{10}-40703143038001266074284032 0a^7c^{10}$$
$$-13106081077916016150737920a^8c^{10}+31814940216382283710464a^9c^{10}$$
$$+30548440455509786501 12a^{10}c^{10}+18015970550045884416a^{11}c^{10}$$
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$$-18531460853097873408 00a^9c^{11}+47307691166058086400a^{10}c^{11}$$
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$$-42364108800a^{14}c^{11}+23552741637156584469560740020224 00c^{12}$$
$$-54582320621870124520454660869324 8ac^{12}+33794155269834290973635734667264a^2c^{12}$$
$$+14011372388038828834651688140 8a^3c^{12}-50947210104854667731378241536a^4c^{12}$$
$$+34417799286102523775877120a^5c^{12}+31335119871918556766765056a^6c^{12}$$
$$+14591272483288663882137 6a^7c^{12}-74351866737848277893 12a^8c^{12}$$
$$-73729701177942491136a^9c^{12}+340576396405260288a^{10}c^{12}+6393967917023232a^{11}c^{12}$$
$$+15324651503616a^{12}c^{12}-50413289472a^{13}c^{12}-141213696a^{14}c^{12}$$
$$+13209108250053065305305697484800 0c^{13}-25080727018788218156955048345600ac^{13}$$

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$$+1141009818986979339736134451200a^2c^{13}+188043112362196161727168512000a^3c^{13}$$
$$-14973584596144425995534336000a^4c^{13}-125412485520989620961280000a^5c^{13}$$
$$+6650787127930542571520000a^6c^{13}+736946557321491578880000a^7c^{13}$$
$$-84070453438321459200a^8c^{13}-1360371592839168000a^9c^{13}-1201288617984000a^{10}c^{13}$$
$$+44208576921600a^{11}c^{13}+144472473600a^{12}c^{13}+62048116991485759738444827852800c^{14}$$
$$-955308033798859563215320252416ac^{14}+30129275830597180093530898432a^2c^{14}$$
$$+9038814861829694073085624320a^3c^{14}-325204452383804929203240960a^4c^{14}$$
$$-54244893399483568324608000a^5c^{14}+9320577112496207134720a^6c^{14}$$
$$+1685649009185431879680a^7c^{14}-322344216242061312a^8c^{14}-14412627627540480a^9c^{14}$$
$$-44710102794240a^{10}c^{14}+126310219776a^{11}c^{14}+412778496a^{12}c^{14}$$
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$$+140283054011842560000a^4c^{17}-2450507423809536000a^5c^{17}-9827187621888000a^6c^{17}$$
$$+1040201809920000a^7c^{17}+509902848000a^8c^{17}+5116743183106645604433920000c^{18}$$
$$-287184942520039832027136000ac^{18}-1403015552680256339968000a^2c^{18}$$
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$$+969113750273523712000000000c^{19}-394179706270148198400000ac^{19}$$
$$-39879941009965056000000a^2c^{19}+1397449692807168000000a^3c^{19}+102878952816640000a^4c^{19}$$
$$-802962800640000a^5c^{19}-5248122880000a^6c^{19}+1496430120547018342400000c^{20}$$
$$-4205581734077320396800ac^{20}-592824246953050112000a^2c^{20}+8950947523854336000a^3c^{20}$$
$$+809292036833280000a^4c^{20}-1605925601280000a^5c^{20}-10496245760000a^6c^{20}$$

$$+18518937847201792000c^{21} - 335979518676172800ac^{21} - 5690809542246400a^2c^{21}$$
$$+351774179328800a^3c^{21} + 344876646400a^4c^{21} + 179146036021821440c^{22}$$
$$-1890050686058496ac^{22} - 35428864360448a^2c^{22} + 63958941696a^3c^{22} + 627048448a^4c^{22}$$
$$+1304260771840000c^{23} - 6673976524800ac^{23} - 130862284800a^2c^{23} + 6717597286400c^{24}$$
$$-11123294208ac^{24} - 218103808a^2c^{24} + 21810380800c^{25} + 33554432c^{26}\Big\}\Big] \qquad (11)$$

III. Derivation of Main Formula

Substituting $b = -a - 51, z = \frac{1}{2}$ in given result (5),we get

$$(2a+51)\ {}_2F_1\left[\begin{matrix} a \\ c \end{matrix}\ ,\ \begin{matrix} -a-51 \\ \end{matrix}\ ;\ \frac{1}{2}\right]$$
$$= a\ {}_2F_1\left[\begin{matrix} a+1 \\ c \end{matrix}\ ,\ \begin{matrix} -a-51 \\ \end{matrix}\ ;\ \frac{1}{2}\right] + (a+51)\ {}_2F_1\left[\begin{matrix} a \\ c \end{matrix}\ ,\ \begin{matrix} -a-50 \\ \end{matrix}\ ;\ \frac{1}{2}\right]$$

Now involving the result which is established in Ref[5], we can establish the main result.

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Notes