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Enhanced (G'/G)-Expansion Method to Find the Exact Complexiton Soliton Solutions of (3+1)-Dimensional Zakhrov-Kuznetsov Equation

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Abstract - In this article, an enhanced (G'/G)-expansion method has been applied to find the traveling wave solutions of the (3+1)-dimensional Zakhrov-Kuznetsov (ZK) equation. The efficiency of this method for finding these exact solutions has been demonstrated. As a result, a set of complexiton soliton solutions are derived, which are expressed by the combinations of rational, hyperbolic and trigonometric functions involving several parameters. It is shown that the method is effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics.

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Enhanced (G'/G)-Expansion Method to Find the Exact Complexiton Soliton Solutions of (3+1)-Dimensional Zakhrov-Kuznetsov Equation

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Abstract - In this article, an enhanced (G'/G)-expansion method has been applied to find the traveling wave solutions of the (3+1)-dimensional Zakhrov-Kuznetsov (ZK) equation. The efficiency of this method for finding these exact solutions has been demonstrated. As a result, a set of complexiton soliton solutions are derived, which are expressed by the combinations of rational, hyperbolic and trigonometric functions involving several parameters. It is shown that the method is effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics.

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I. Introduction

Nowadays NLEEs have been the subject of all-embracing studies in various branches of nonlinear sciences. A special class of analytical solutions named traveling wave solutions for NLEEs have a lot of importance, because most of the phenomena that arise in mathematical physics and engineering fields can be described by NLEEs. NLEEs are frequently used to describe many problems of protein chemistry, chemically reactive materials, in ecology most population models, in physics the heat flow and the wave propagation phenomena, quantum mechanics, fluid mechanics, plasma physics, propagation of shallow water waves, optical fibers, biology, solid state physics, chemical kinematics, geochemistry, meteorology, electricity etc. Therefore investigation traveling wave solutions is becoming more and more attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the Hirota's bilinear transformation method [1, 2], the tanh-function method [3, 4], the extended tanh-method [5, 6], the Exp-function method [7-14], the Adomian decomposition method [15], the F-expansion method [16], the auxiliary equation method [17], the Jacobi elliptic function method [18], Modified Exp-function method [19], the (G'/G)-expansion method [20-29], Weierstrass elliptic function method [30], the

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Ref
1. R. Hirota, Exact envelope soliton solutions of a nonlinear wave equation. J. Math. Phy. 14(1973) 805-810.

homotopy perturbation method [31-35], the homogeneous balance method [36, 37], the Modified simple equation method [38-42], He's polynomial [43], asymptotic methods and nanomechanics [44], the variational iteration method [45, 46], the casoration formulation [47], the frobenius integrable decomposition [48], the extended multiple Riccati equations expansion method [49, 50], the enhanced (G'/G)-expansion method [51] and so on.

The objective of this article is to apply the enhanced (G'/G)-expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the ZK equation. The ZK equation is completely integrable and has N-soliton solutions.

The article is prepared as follows: In section II, an enhanced $G'\ G$ -expansion method is discussed. In section III, we apply this method to the nonlinear evolution equations pointed out above; in section IV, physical explanations and in section V conclusions are given.

II. Enhanced $G'\ G$ -Expansion Method

In this section we describe enhanced (G'/G)-expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear evolution equation, say in two independent variables x and t, is given by

$$\mathcal{R}(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots\dots\dots\dots) = 0, \tag{2.1}$$

where $u(\xi) = u(x,t)$ is an unknown function, $\mathcal{R}$ is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method:

Step 1. Combining the independent variables x and t into one variable $\xi = x \pm \omega t$, we suppose that

$$u(\xi) = u(x,t), \qquad \xi = x \pm \omega t \ . \tag{2.2}$$

The traveling wave transformation Eq. (2.2) permits us to reduce Eq. (2.1) to the following ODE:

$$F(u\ u'\ u''\ \dots\dots\dots\dots) \quad , \tag{2.3}$$

where F is a polynomial in $u(\xi)$ and its derivatives, while $u'(\xi) = \frac{du}{d\xi}$, $u''(\xi) = \frac{d^2u}{d\xi^2}$, and so on.

Step 2. We suppose that Eq. (2.3) has the formal solution

$$u(\xi) = \sum_{i=-n}^{n}\left(\frac{a_i (G'/G)^i}{\left(1+\lambda(G'/G)\right)^i} + b_i (G'/G)^{i-1}\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}\right), \tag{2.4}$$

where $G = G(\xi)$ satisfy the equation $G'' + \mu G = 0$, (2.5)

in which $a_i, b_i (-n \le i \le n; n \in \mathbb{N})$ and λ are constants to be determined later, and $\sigma = \pm 1, \mu \neq 0$.

Step 3. We determine the positive integer n in Eq. (2.4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (2.3).

Ref

31. S.T. Mohiud-Din, Homotopy perturbation method for solving fourth-order boundary value problems, Math. Prob. Engr. Vol. 2007, 1-15, Article ID 98602, doi:10.1155/2007/98602.

Step 4. We substitute Eq. (2.4) into Eq.(2.3) using Eq. (2.5) and then collect all terms of same powers of $(G'/G)^j$ and $(G'/G)^j \sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}$ together, then set each coefficient of them to zero to yield a over-determined system of algebraic equations, solve this system for $a_i, b_i(-n \le i \le n; n \in \mathbb{N})$ and λ, ω.

Notes

Step 5. The solution of Eq. (2.5) can be written as follows:
When $\mu < 0$, we get

$$\frac{G'}{G} = \sqrt{-\mu}\, tanh(\xi_0 + \sqrt{-\mu}\xi). \tag{2.6}$$

and

$$\frac{G'}{G} = \sqrt{-\mu}\, coth(\xi_0 + \sqrt{-\mu}\xi). \tag{2.7}$$

Again, when $\mu > 0$, the solutions are

$$\frac{G'}{G} = \sqrt{\mu}\, tan(\xi_0 - \sqrt{\mu}\xi). \tag{2.8}$$

and

$$\frac{G'}{G} = \sqrt{\mu}\, cot(\xi_0 + \sqrt{\mu}\xi). \tag{2.9}$$

where ξ_0 is an arbitrary constant. Finally, substituting $a_i, b_i(-n \le i \le n; n \in \mathbb{N})$, λ, ω and Eqs. (2.6)-(2.9) into Eq. (2.4) we obtain traveling wave solutions of Eq. (2.1).

III. Application

In this section, we will exert enhanced (G'/G)-expansion method to solve the ZK equation in the form,

$$u_t + auu_x + u_{xx} + u_{yy} + u_{zz} = 0, \tag{3.1}$$

where a is a positive constant.

The traveling wave transformation equation $u(\xi) = u(x,y,z,t)$, $\xi = x + y + z - \omega t$ transform Eq. (3.1) to the following ordinary differential equation:

$$-\omega u' + auu' + 3u'' = 0\,. \tag{3.2}$$

Now integrating Eq. (3.2) with respect to ξ once, we have

$$C - \omega u + \frac{1}{2}au^2 + 3u' = 0, \tag{3.3}$$

where C is a constant of integration. Balancing the highest-order derivative u' and the nonlinear term u^2, from Eq. (3.3), yields $2n = n + 1$ which gives $n = 1$.
Hence for $n = 1$ Eq. (2.4) reduces to

$$u(\xi) = a_0 + \frac{a_1(G'/G)}{1+\lambda(G'/G)} + \frac{a_{-1}[1+\lambda(G'/G)]}{(G'/G)} + b_0(G'/G)^{-1}\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}$$

$$+b_1\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]} + b_{-1}(G'/G)^{-2}\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}. \tag{3.4}$$

where $G = G(\xi)$ satisfies Eq. (2.5).

Substitute Eq. (3.4) along with Eq. (2.5) into Eq. (3.3). As a result of this substitution, we get a polynomial of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}$. From this polynomial, we equate the coefficients of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left[1+\frac{(G'/G)^2}{\mu}\right]}$ and setting them to zero, we get a over-determined system that consists of twenty-five algebraic equations. Solving these over determined system of equations, we obtain the following valid sets.

Set -1: $C = \frac{9\mu + a_0^2 a^2}{2a}, \omega = a a_0, \lambda = 0, a_{-1} = 0, a_0 = a_0, a_1 = \frac{3}{a}, b_{-1} = 0, b_0 = 0, b_1 = \pm\frac{3\sqrt{\mu}}{a\sqrt{\sigma}}$.

Set -2: $C = \frac{(a_0^2 a^2 + 12 a a_0 \lambda\mu + 36\mu + 36\mu^2\lambda^2)}{2a}, \omega = 6\mu\lambda + a a_0, \lambda = \lambda, a_{-1} = 0, a_0 = a_0,$
$a_1 = \frac{6(1+\mu\lambda^2)}{a},\ b_1 = 0, b_0 = 0, b_{-1} = 0$.

Set-3: $C = \frac{(a_0^2 a^2 - 12 a a_0 \lambda\mu + 36\mu + 36\mu^2\lambda^2)}{2a}, \omega = -6\mu\lambda + a a_0, \lambda = \lambda, a_{-1} = -\frac{6\mu}{a}, a_0 = a_0,$
$a_1 = 0, b_1 = 0, b_0 = 0, b_{-1} = 0.$

Set-4: $C = \frac{(a_0^2 a^2 + 144\mu)}{2a}, \omega = a a_0, \lambda = 0, a_{-1} = -\frac{6\mu}{a}, a_0 = a_0, a_1 = \frac{6}{b},$
$b_1 = 0, b_0 = 0, b_{-1} = 0$

Set-5: $C = \frac{(a_0^2 a^2 - 6 a a_0 \lambda\mu + 9\mu + 9\mu^2\lambda^2)}{2a}, \omega = -3\mu\lambda + a a_0, \lambda = \lambda,\ a_{-1} = -\frac{3\mu}{a}, a_0 = a_0,$
$a_1 = 0, b_1 = 0, b_0 = \pm\frac{3\mu}{a\sqrt{\sigma}}, b_{-1} = 0$.

Now for $\mu < 0$, substituting the values of $C, \omega, a_{-1}, a_0,\ a_1, b_1,\ b_0,\ b_{-1}$ into Eq. (3.4) from the above Set-1 to Set-5, we get the following hyperbolic function solutions of ZK equation.

Family -1:
$$u_1(\xi) = a_0 + \frac{3}{a}\sqrt{-\mu}\, tanh(\xi_0 + \sqrt{-\mu}\xi) \pm \frac{3\sqrt{\mu}}{a}\mathrm{sech}(\xi_0 + \sqrt{-\mu}\xi),$$
$$u_2(\xi) = a_0 + \frac{3}{a}\sqrt{-\mu}\, coth(\xi_0 + \sqrt{-\mu}\xi) \pm \frac{3i\sqrt{\mu}}{a} cosech(\xi_0 + \sqrt{-\mu}\xi),$$
where
$$\xi = x + y + z - a a_0 t.$$

Family-2:
$$u_3(\xi) = a_0 + \frac{6(1+\mu\lambda^2)\sqrt{-\mu}\, tan\,h(\xi_0+\sqrt{-\mu}\xi)}{a(1+\lambda\sqrt{-\mu}\, tan\,h(\xi_0+\sqrt{-\mu}\xi))},$$
$$u_4(\xi) = a_0 + \frac{6(1+\mu\lambda^2)\sqrt{-\mu}\, cot\,h(\xi_0+\sqrt{-\mu}\xi)}{a(1+\lambda\sqrt{-\mu}\, cot\,h(\xi_0+\sqrt{-\mu}\xi))},$$

Notes

where $$\xi = x + y + z - (6\mu\lambda + aa_0)t$$

Family-3: $$u_5(\xi) = a_0 - \frac{6\mu\lambda}{a} + \frac{6\sqrt{-\mu}\, coth(\xi_0+\sqrt{-\mu}\xi)}{a}.$$

$$u_6(\xi) = a_0 - \frac{6\mu\lambda}{a} + \frac{6\sqrt{-\mu}\, tanh(\xi_0+\sqrt{-\mu}\xi)}{a}.$$

where $$\xi = x + y + z - (aa_0 - 6\mu\lambda)t$$

Family-4: $$u_7(\xi) = a_0 + \frac{6}{a}\sqrt{-\mu}\left(tanh(\xi_0 + \sqrt{-\mu}\xi) + coth(\xi_0 + \sqrt{-\mu}\xi)\right).$$

$$u_8(\xi) = a_0 + \frac{6}{a}\sqrt{-\mu}\left(coth(\xi_0 + \sqrt{-\mu}\xi) + tanh(\xi_0 + \sqrt{-\mu}\xi)\right).$$

where $$\xi = x + y + z - aa_0 t$$

Family-5: $$u_9(\xi) = a_0 - \frac{3\mu\lambda}{a} + \frac{3\sqrt{-\mu}}{a}(\coth(\xi_0 + \sqrt{-\mu}\xi) \mp cosech(\xi_0 + \sqrt{-\mu}\xi)),$$

$$u_{10}(\xi) = a_0 - \frac{3\mu\lambda}{a} + \frac{3\sqrt{-\mu}}{a}(\tanh(\xi_0 + \sqrt{-\mu}\xi) \mp isech(\xi_0 + \sqrt{-\mu}\xi)),$$

where $$\xi = x + y + z - (aa_0 - 3\mu\lambda)t$$

Similarly for $\mu > 0$, we get the following periodic solutions of ZK equation.

Family -6: $$u_{11}(\xi) = a_0 + \frac{3}{a}\sqrt{\mu}(tan(\xi_0 - \sqrt{\mu}\xi) \pm \sec(\xi_0 - \sqrt{\mu}\xi)),$$

$$u_{12}(\xi) = a_0 + \frac{3}{a}\sqrt{\mu}(cot(\xi_0 + \sqrt{\mu}\xi) \pm co\sec(\xi_0 + \sqrt{\mu}\xi)),$$

where $$\xi = x - aa_0 t.$$

Family-7: $$u_{13}(\xi) = a_0 + \frac{6(1+\mu\lambda^2)\sqrt{\mu}\, tan\,(\xi_0-\sqrt{\mu}\xi)}{a(1+\lambda\sqrt{\mu}\, tan\,(\xi_0-\sqrt{\mu}\xi))},$$

$$u_{14}(\xi) = a_0 + \frac{6(1+\mu\lambda^2)\sqrt{\mu}\, cot\,(\xi_0+\sqrt{\mu}\xi)}{a(1+\lambda\sqrt{\mu}\, cot\,(\xi_0+\sqrt{\mu}\xi))},$$

where $$\xi = x + y + z - (6\mu\lambda + aa_0)t.$$

Family-8: $$u_{15}(\xi) = a_0 - \frac{6\mu\lambda}{a} + \frac{6\sqrt{\mu}\, cot\,(\xi_0-\sqrt{\mu}\xi)}{a},$$

$$u_{16}(\xi) = a_0 - \frac{6\mu\lambda}{a} + \frac{6\sqrt{\mu}\, tan\,(\xi_0+\sqrt{\mu}\xi)}{a},$$

where $$\xi = x + y + z - (aa_0 - 6\mu\lambda)t.$$

Family-9: $$u_{17}(\xi) = a_0 + \frac{6}{a}\sqrt{\mu}\left(tan(\xi_0 - \sqrt{\mu}\xi) - cot(\xi_0 - \sqrt{\mu}\xi)\right),$$

$$u_{18}(\xi) = a_0 + \frac{6}{a}\sqrt{\mu}\left(cot(\xi_0 + \sqrt{\mu}\xi) - tan(\xi_0 + \sqrt{\mu}\xi)\right).$$

where $$\xi = x + y + z - aa_0 t.$$

Family-10: $$u_{19}(\xi) = a_0 - \frac{3\mu\lambda}{a} - \frac{3\sqrt{\mu}}{a}(\cot(\xi_0 - \sqrt{\mu}\xi) \mp cosec(\xi_0 - \sqrt{\mu}\xi)),$$

$$u_{20}(\xi) = a_0 - \frac{3\mu\lambda}{a} - \frac{3\sqrt{\mu}}{a}(\tan(\xi_0 + \sqrt{\mu}\xi) \mp sec(\xi_0 + \sqrt{\mu}\xi)),$$

where $$\xi = x + y + z - (aa_0 - 3\mu\lambda)t.$$

Notes

IV. Physical Explanation

a) Explanation

In this section we will discuss the physical explanations of obtained solutions of ZK equation. It is interesting to point out that the delicate balance between the nonlinearity effect of uu_x and the dissipative effect of u_{xx}, u_{yy} and u_{zz} gives rise to solitons, that after a fully interaction with others, the solitons come back retaining their identities with the same speed and shape. The ZK equation has solitary wave solutions that have exponentially decaying wings. If two solitons of the ZK equation collide, the solitons just pass through each other and emerge unchanged.

The determined solutions from Family-1 to Family-10 are complexiton solution. That is the combinations of rational functions, hyperbolic functions and trigonometric functions.

For $\mu < 0$, Family-1($u_1(\xi)$) and Family-3($u_6(\xi)$) are kink solutions represented in Fig. 1 and Fig. 3 for $\mu = -1, \xi_0 = 1, a_0 = 2, a = 3, y = 0, z = 0$ and $\mu = -1, \xi_0 = 1, a = 3, a_0 = 2, \lambda = 2, y = 0, z = 0$ within the interval $-3 \le x, t \le 3$.

Fig. 2 and Fig. 5 correspond to Family-2($u_3(\xi)$) and Family-5($u_9(\xi)$) for $\mu = -1, \xi_0 = 1, a = 3, a_0 = 2, \lambda = 2, y = 0, z = 0$ and $\mu = -3, \xi_0 = 1, a_0 = 2, a = 1, \lambda = -3$, $y = 0, z = 0$ within the interval $-3 \le x, t \le 3$ are complexiton soliton solutions.

Family-4($u_7(\xi)$) provides singular kink solution for $\mu = -3, \xi_0 = 1, a_0 = 1, a = 2, y = z = 0$ within the interval $-3 \le x, t \le 3$, represented in Fig. 4.

Consequently, for $\mu > 0$, Family-6-Family-10 are combinations of rational and trigonometric functions solutions, also said to be traveling wave solutions that are periodic.

The wave speed ω plays an important role in the physical structure of the solutions obtained above. For the positive values of wave speed ω the disturbance represented by $u(\xi) = x + y + z - \omega t$ moves in the positive direction. Consequently, the negative values of wave speed ω the disturbance represented by $u(\xi) = x + y + z - \omega t$ moves in the negative direction.

Furthermore, the graphical demonstrations of some obtained solutions are shown in Figure-1 to Figure-10 in the following subsection.

b) Graphical representation

Some of our obtained traveling wave solutions are represented in the following figures with the aid of commercial software Maple:

Notes

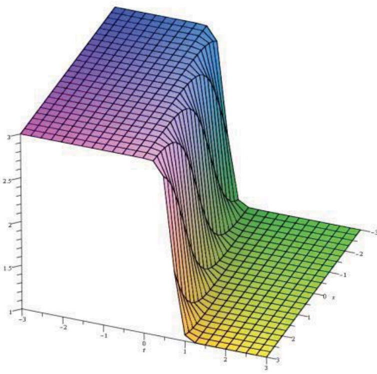

Fig. 1 : Shape of $u_1(\xi)$ for $\mu = -1,, \xi_0 = 1, a_0 = 2, a = 3, y = 0, z = 0, -3 \le x, t \le 3$.

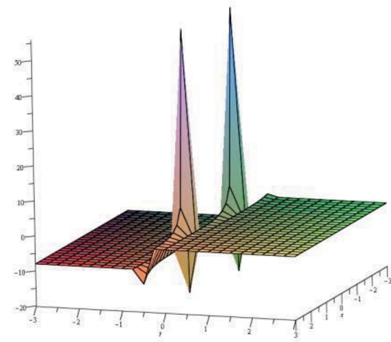

Fig. 2 : Profile of $u_3(\xi)$ for $\mu = -1, \xi_0 = 1, a = 3, a_0 = 2, \lambda = 2, y = 0, z = 0, -3 \le x, t \le 3$.

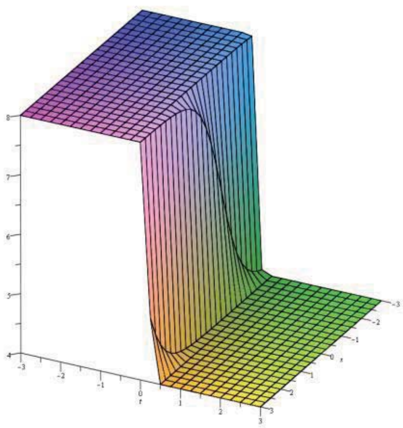

Fig. 3 : Shape of $u_6(\xi)$ for $\mu = -1, \xi_0 = 1, a_0 = 2, a = 3, \lambda = 2, y = 0, z = 0, -3 \le x, t \le 3$.

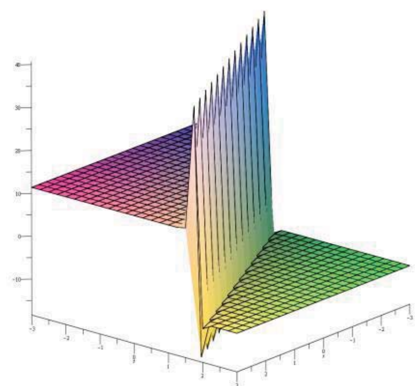

Fig. 4 : Shape of $u_7(\xi)$ for $\mu = -3, \xi_0 = 1, a_0 = 1, a = 2, y = 0, z = 0, -3 \le x, t \le 3$.

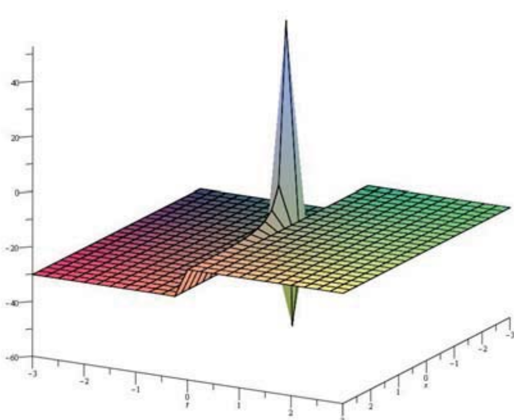

Fig. 5 : Shape of $u_9(\xi)$ for $\mu = -3, \xi_0 = 1, a_0 = 2, a = 1, \lambda = -3, y = 0, z = 0, -3 \le x, t \le 3$.

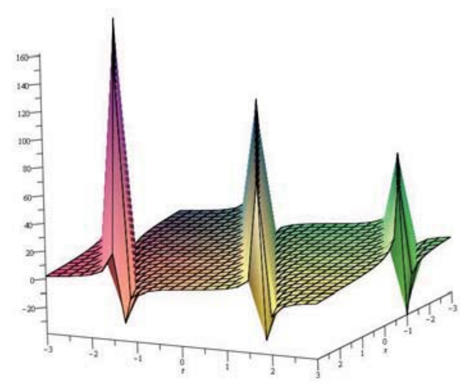

Fig. 6 : Shape of $u_{11}(\xi)$ for $\mu = 1, \xi_0 = 1, a_0 = 2, a = 1, y = 0, z = 0, -3 \le x, t \le 3$.

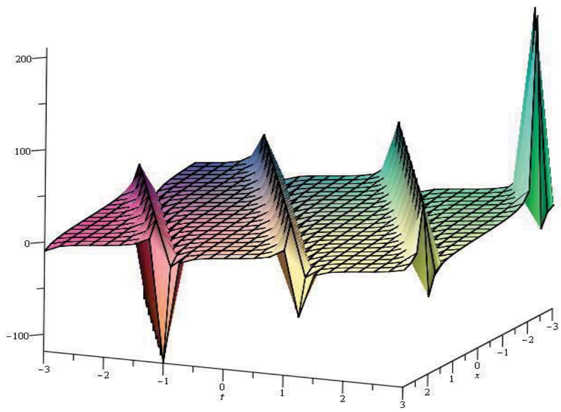

Fig. 7 : Shape of $u_{13}(\xi)$ for $\mu = 0.5, \xi_0 = 2, a_0 = 2, a = 1, \lambda = 0, y = 0, z = 0, -3 \leq x, t \leq 3$.

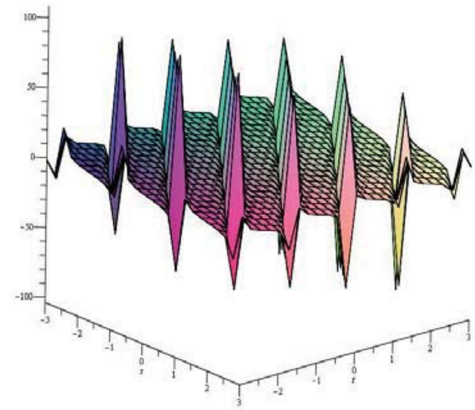

Fig. 8 : Shape of $u_{15}(\xi)$ for $\mu = 3, \xi_0 = 0, a_0 = 2, a = 3, \lambda = 0, y = 0, z = 0, -3 \leq x, t \leq 3$.

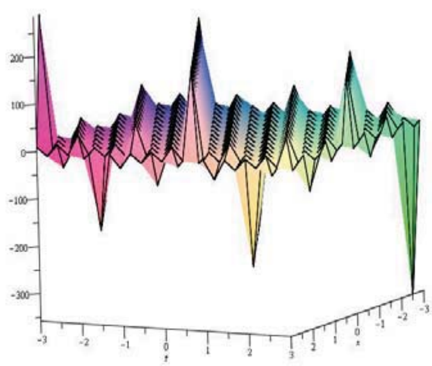

Fig. 9 : Shape of $u_{17}(\xi)$ for $\mu = 3, \xi_0 = 1, a_0 = 1, a = 2, y = 0, z = 0, -3 \leq x, t \leq 3$.

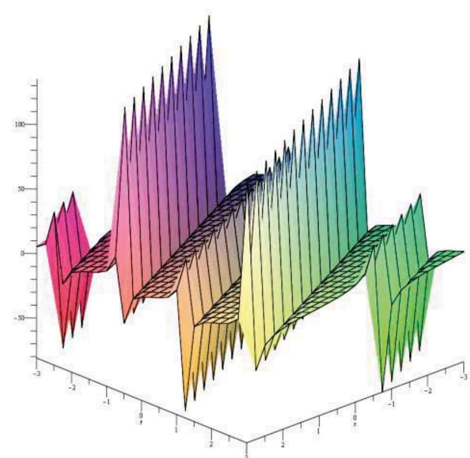

Fig. 1o : Shape of $u_{19}(\xi)$ for $\mu = 3, \xi_0 = 1, a_0 = 2, a = 1, \lambda = 0, y = 0, z = 0, -3 \leq x, t \leq 3$.

V. Conclusion

In short, we have illustrated the Enhanced (G'/G)-expansion method and utilized it to find the exact solutions of nonlinear equations with the help of commercial software Maple. We have successfully obtained some Complexiton soliton solutions of the ZK equation. When the parameters are taken as special values, the solitary wave solutions and the periodic wave solutions are obtained. Taken as a whole, it is worthwhile to mention that this method is effective for solving other nonlinear evolution equations in mathematical physics.

Notes

Notes

References Références Referencias

1. R. Hirota, Exact envelope soliton solutions of a nonlinear wave equation. J. Math. Phy. 14(1973) 805-810.
2. R. Hirota, J. Satsuma, Soliton solutions of a coupled KDV equation. Phy. Lett. A. 85(1981) 404-408.
3. M. Malfliet, Solitary wave solutions of nonlinear wave equations. Am. J. Phys. 60, (1992)650–654.
4. H.A. Nassar, M.A. Abdel-Razek, A.K. Seddeek, Expanding the tanh-function method for solving nonlinear equations, Appl. Math. 2(2011) 1096-1104.
5. E.G. Fan, Extended tanh-method and its applications to nonlinear equations. Phy. Lett. A. 277(2000) 212-218.
6. M.A. Abdou, The extended tanh-method and its applications for solving nonlinear physical models. App. Math. Comput. 190(2007) 988-996.
7. J.H. He, X.H. Wu, Exp-function method for nonlinear wave equations, Chaos, Solitons and Fract. 30(2006) 700-708.
8. M.A. Akbar, N.H.M. Ali, Exp-function method for Duffing Equation and new solutions of (2+1) dimensional dispersive long wave equations. Prog. Appl. Math. 1(2) (2011) 30-42.
9. H. Naher, A.F. Abdullah, M.A. Akbar, The Exp-function method for new exact solutions of the nonlinear partial differential equations, Int. J. Phys. Sci., 6(29): (2011)6706-6716.
10. H. Naher, A.F. Abdullah, M.A. Akbar, New traveling wave solutions of the higher dimensional nonlinear partial differential equation by the Exp-function method, J. Appl. Math., Article ID 575387, 14 pages. doi: 10.1155(2012)575387.
11. A. Bekir, A. Boz, Exact solutions for nonlinear evolution equations using Exp-function method. Phy. Lett. A. 372(2008) 1619-1625.
12. M. A. Abdou, A. A. Soliman and S. T. Basyony, New application of exp-function method for improved Boussinesq equation. Phys. Lett. A, 369(2007), 469-475.
13. S. A. El-Wakil, M. A. Madkour and M. A. Abdou, Application of exp-function method for nonlinear evolution equations with variable co-efficient, Phys. Lett. A, 369(2007), 62-69.
14. S. T. Mohyud-Din, M. A. Noor and A. Waheed, Exp-function method for generalized travelling solutions of Calogero-Degasperis-Fokas equation, Zeitschrift für Naturforschung A- A Journal of Physical Sciences, 65a (2010), 78-84.
15. G. Adomian, Solving frontier problems of physics: The decomposition method. Boston (1994), M A: Kluwer Academic.
16. Y.B. Zhou, M.L. Wang, Y.M. Wang, Periodic wave solutions to coupled KdV equations with variable coefficients, Phys. Lett. A. 308(2003) 31–36.
17. Sirendaoreji, New exact travelling wave solutions for the Kawahara and modified Kawahara equations. Chaos Solitons Fract. 19(2004) 147-150.
18. A.T. Ali, New generalized Jacobi elliptic function rational expansion method. J. Comput. Appl. Math. 235(2011) 4117-4127.
19. Y. He, S. Li, Y. Long, Exact solutions of the Klein-Gordon equation by modified Exp-function method. Int. Math. Forum. 7(4) (2012) 175-182.
20. M.A. Akbar, N.H.M. Ali, E.M.E. Zayed, Abundant exact traveling wave solutions of the generalized Bretherton equation via (G'/G)-expansion method. Commun. Theor. Phys. 57(2012a) 173-178.

21. M.A. Akbar, N.H.M. Ali, E.M.E. Zayed, A generalized and improved (G'/G)-expansion method for nonlinear evolution equations, Math. Prob. Engr., Vol. 2012, 22 pages. doi: 10.1155/2012b/459879.
22. M.A. Akbar, N.H.M. Ali, S.T. Mohyud-Din, The alternative (G'/G)-expansion method with generalized Riccati equation: Application to fifth order (1+1)-dimensional Caudrey-Dodd-Gibbon equation. Int. J. Phys. Sci. 7(5) (2012c) 743-752.
23. M.A. Akbar, N.H.M. Ali, S.T. Mohyud-Din, Some new exact traveling wave solutions to the (3+1)-dimensional Kadomtsev-Petviashvili equation. World Appl. Sci. J. 16(11) (2012d) 1551-1558.
24. E. M. E. Zayed and A.J. Shorog, Applications of an Extended (G'/G)-Expansion Method to Find Exact Solutions of Nonlinear PDEs in Mathematical Physics, Hindawi Publishing Corporation, Mathematical Problems in Engineering, Article ID 768573, 19 pages, doi:10.1155/2010/768573.
25. E.M.E. Zayed, Traveling wave solutions for higher dimensional nonlinear evolution equations using the (G'/G)-expansion method. J. Appl. Math. Informatics, 28(2010) 383-395.
26. E.M.E. Zayed, K.A. Gepreel, The (G'/G)-expansion method for finding the traveling wave solutions of nonlinear partial differential equations in mathematical physics. J. Math. Phys. 50(2009) 013502-013514.
27. M. Wang, X. Li, J. Zhang, The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A. 372(2008) 417-423.
28. M.A. Akbar, N.H.M. Ali, The alternative (G'/G)-expansion method and its applications to nonlinear partial differential equations. Int. J. Phys. Sci. 6(35) (2011) 7910-7920.
29. A.R. Shehata, The traveling wave solutions of the perturbed nonlinear Schrodinger equation and the cubic-quintic Ginzburg Landau equation using the modified (G'/G)-expansion method. Appl. Math. Comput. 217(2010), 1-10.
30. M.S. Liang, et al., A method to construct Weierstrass elliptic function solution for nonlinear equations, Int. J. Modern Phy. B. 25(4) (2011) 1931-1939.
31. S.T. Mohiud-Din, Homotopy perturbation method for solving fourth-order boundary value problems, Math. Prob. Engr. Vol. 2007, 1-15, Article ID 98602, doi:10.1155/2007/98602.
32. S. T. Mohyud-Din and M. A. Noor, Homotopy perturbation method for solving partial differential equations, Zeitschrift für Naturforschung A- A Journal of Physical Sciences, 64a (2009), 157-170.
33. S. T. Mohyud-Din, A. Yildirim, S. Sariaydin, Numerical soliton solutions of the improved Boussinesq equation, International Journal of Numerical Methods for Heat and Fluid Flow 21 (7) (2011):822-827.
34. S. T. Mohyud-Din, A. Yildirim, G. Demirli, Analytical solution of wave system in R^n with coupling controllers, International Journal of Numerical Methods for Heat and Fluid Flow, Emerald 21 (2) (2011), 198-205.
35. S. T. Mohyud-Din, A. Yildirim, S. Sariaydin, Numerical soliton solution of the Kaup-Kupershmidt equation, International Journal of Numerical Methods for Heat and Fluid Flow, Emerald 21 (3) (2011), 272-281.

Notes

Notes

36. M.Wang, Solitary wave solutions for variant Boussinesq equations. Phy. Lett. A. 199(1995) 169-172.
37. E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, On the solitary wave solutions for nonlinear Hirota-Sasuma coupled KDV equations, Chaos, Solitons and Fractals, 22(2004) 285-303.
38. A.J. M. Jawad, M.D. Petkovic, A. Biswas, Modified simple equation method for nonlinear evolution equations. Appl. Math. Comput. 217(2010), 869-877.
39. E.M.E. Zayed, A note on the modified simple equation method applied to Sharma-Tasso-Olver equation. Appl. Math. Comput. 218(2011) 3962-3964.
40. E.M.E. Zayed, S.A.H. Ibrahim, Exact solutions of nonlinear evolution equations in Mathematical physics using the modified simple equation method. Chinese Phys. Lett. 29(6) (2012) 060201.
41. K. Khan, M.A. Akbar and N.H.M. Ali. The Modified Simple Equation Method for Exact and Solitary Wave Solutions of Nonlinear Evolution Equation: The GZK-BBM Equation and Right-Handed Noncommutative Burgers Equations, ISRN Mathematical Physics, Hindawi Publishing Corporation, Volume 2013, doi: 10.1155/2013/146704, 5 pages.
42. K. Khan and M. Ali Akbar, Exact and solitary wave solutions for the Tzitzeica-Dodd-Bullough and the modified KdV-Zakharov-Kuznetsov equations using the modified simple equation method, Ain Shams Engineering Journal, (in press, doi:10.1016/j.asej.2013.01.010).
43. S. T. Mohyud-Din, M. A. Noor and K. I. Noor, Travelling wave solutions of seventh-order generalized KdV equations using He's polynomials, International Journal of Nonlinear Sciences and Numerical Simulation, 10 (2) (2009), 223-229.
44. J. H. He, An elementary introduction to recently developed asymptotic methods and nanomechanics in textile engineering, Int. J. Mod. Phys. B 22 (21) (2008), 3487-3578.
45. S. T. Mohyud-Din, M. A. Noor and K. I. Noor, Some relatively new techniques for nonlinear problems, Mathematical Problems in Engineering, Hindawi, (doi:10.1155/2009/234849), 25 pages.
46. S. T. Mohyud-Din, M. A. Noor, K. I. Noor and M. M. Hosseini, Solution of singular equations by He's variational iteration method, International Journal of Nonlinear Sciences and Numerical Simulation, 11 (2)(2010),81-86
47. W. X. Ma and Y. You, Rational solutions of the Toda lattice equation in Casoratian form, Chaos, Solitons & Fractals, 22 (2004), 395-406.
48. W. X. Ma, H. Y. Wu and J. S. He, Partial differential equations possessing Frobenius integrable decompositions, Phys. Lett. A, 364 (2007), 29-32.
49. K. A. Gepreel, Exact Complexiton Soliton Solutions for Nonlinear Partial Differential Equations, International Mathematical Forum, Vol. 6(2011) no. 26, 1261 - 1272.
50. K. A. Gepreel, A. R. Shehata, Exact complexiton soliton solutions for nonlinear partial differential equations in mathematical physics, Scientific Research and Essays Vol. 7(2), doi: 10.5897/2012 /SRE11.850, pp. 149-157.
51. K. Khan and M. A. Akbar. Traveling Wave Solutions of Nonlinear Evolution Equations via the Enhanced (G'/G)-expansion Method. Journal of the Egyptian Mathematical Society. doi.org/10.1016/j.joems.2013.07.009. (In Press).