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The Errors in the Fields Medals, 1982 to S. T. Yau and 1990 to E. Witten

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Abstract- Due to inadequate understanding in physics, mathematicians have made misleading erroneous claims in general relativity that result in awarding three times the Fields Medal in 1982 to S. T. Yau, and 1990 to E. Witten, and the 2011 Shaw Prize to D. Christodoulou. It is pointed out that they failed to discover that there are no bounded dynamic solutions for the Einstein equation as Gullstrand suspected. Thus, the existence of bounded dynamic solutions was implicitly, but incorrectly assumed. This error currently prevents the necessary rectification of general relativity that leads to the unification of gravitation and electromagnetism. Also, similar errors have been made by D. Hilbert and M. Atiyah.

Keywords: *stable solution; dynamic solution.*

GJSFR-F Classification : MSC 2010: 00A69

Ref

1. R. Schoen and S.-T. Yau, "Proof of the Positive Mass Theorem. II," Commun. Math. Phys. **79**, 231-260 (1981).

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I. Introduction

In mathematics, it is commonly known that an assertion can be either right or wrong. However, in logic, there is actually a third case that the conditions in a theorem are valid in mathematics but some implicit assumption is not generally valid. Thus, the theorem is not simply right or wrong, but misleading. In fact, such an error can be made by top mathematicians such as M. Atiyah [1)] and consequently such misleading errors in mathematics were cited as a main reason to award the 1982 and the 1990 Fields Medal to Yau and Witten [2)] and to award the 2011 Shaw Prize in mathematics to Christodoulou. [3)] To this end, the Positive Energy Theorem of Yau and Schoen [1, 2] for general relativity is an example. Briefly, the positive mass conjecture says that if a three-dimensional manifold has positive scalar curvature and is asymptotically flat, then the mass in the asymptotic expansion of the metric is positive (Wikipedia). As in the space-time singularity theorems, the unique coupling signs are also implicitly used in the positive energy theorem of Schoen and Yau [1, 2]. A crucial assumption in the theorem of Schoen and Yau is that the solution is asymptotically flat. To be more specific, they [1] requires the metric,

$$g_{ij} = \delta_{ij} + O(r^{-1}). \qquad (1)$$

The motivation of (1) is clearly the linearized equation of Einstein (see eq. (C3) in Appendix C). Moreover, such an assumption can be considered as common in physics since this condition is satisfied in stable solutions such as the Schwarzschild solution, the harmonic solution, the Kerr solution, etc. [4)]

Thus, it could be "natural" to assert (as in Wikipedia) that their proof of the positive energy theorem in general relativity demonstrated—sixty years after its

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discovery—that Einstein's theory is consistent and stable. However, if one understands the physics in general relativity as well as Gullstrand, the Chairman (1922-1929) of the Nobel Prize Committee for Physics does, the above statement is clearly incorrect [3-5]. Note that the condition of asymptotically flat does not necessarily imply the inclusion of a dynamic solution. Apparently, Schoen and Yau assumed it did because they failed to see that, for a dynamic case the linearized equation and the non-linear Einstein equation are not compatible [6].

However, it has been proven that the Einstein equation has no dynamic solution, which is bounded [3-5]. Thus, the assumption of asymptotically flat implies the exclusion of the most important class of solutions, the dynamic solutions. However, the notion of a dynamic solution was not critical to mathematicians Schoen and Yau. So, they have not considered such a problem in their theorem. Therefore, they actually prove a trivial result that the total mass of a static (or stable) solution is positive. In other words, the conclusions drawn from the positive theorem are grossly misleading. This illustrates that an inadequate understanding in physics can lead to beautiful, but actually completely invalid statements in physics.

The problem rises from the Einstein equation that does not have a bounded dynamic solution as Gullstrand suspected [7]. Thus, Yau and Schoen used an implicit assumption, the existence of bounded dynamic solutions, which is actually false but was not stated in their theorem. Similarly, Witten is also essentially a mathematician because his major concern is self-consistency instead of agreement with observation. Thus he also overlooked the problem of the dynamic solutions. Moreover, Atiyah, being a pure mathematician, was not aware of the problem of non-existence of bounded dynamic solutions. Thus, one should not be surprised that such an error was made twice over eight years (1982-1990) by the Fields medal. Note that the proof for the nonexistence of a bounded dynamic solution was published in 2000 [4].[5)]

It should be noted that D. Hilbert also made a mistake on approving Einstein's calculation of the perihelion of Mercury because he was not aware that this calculation requires a bounded solution of the many-body problem [7]. However, Hilbert was lucky because he understood that the related Einstein's calculation is not valid, but Atiyah was not as lucky. Nevertheless, because of Atiyah's reputation as an outstanding mathematician, some journals such as Nature would not dare to criticize him.

In fact theorists such as Yau [1], Christodoulou [8], Wald, and Penrose & Hawking [9] make essentially the same error of defining a set of solutions that actually includes no dynamic solutions [10-13]. The fatal error is that they neglected to find explicit examples to support their claims. Had they tried, they should have discovered their errors. Moreover, the same error [5] was cited in awarding to Christodoulou the 2011 Shaw Prize.[3), 6)] Subsequently, Christodoulou was elected to the Member of U.S. National Academy of Sciences (2012). It would be interesting to see how this special case would end up. The problem of Christodoulou represents an accumulation of long standing errors committed by the top mathematicians and physicists.

The non-existence of a bounded dynamic solution for the Einstein equation was not recognized because they did not try to obtain such a solution. Thus the need of modifying the Einstein equation with an additional gravitational energy-stress tensor with the anti-gravitational coupling as the source was overlooked [3]. Then, the energy-mass formula $E = mc^2$ was still incorrectly considered as unconditionally valid [12]. Consequently, the charge-mass interaction was not only overlooked, but also explicitly denied by Einstein and his colleagues. Hence, the need of unification between gravitation and

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3. C. Y. Lo, Astrophys. J. 455, 421-428 (1995); Editor S. Chandrasekhar, a Nobel Laureate, suggests and approves the Appendix therein.

electromagnetism is missed [14]. Thus, the positive mass theorem is actually an obstacle for the progress in physics.

An urgent problem is that Misner, Thorne. & Wheeler [15] used the errors of Yau [1] and Witten [2] to strengthen their incorrect claim on the existence of bounded dynamic solutions. For instance, they incorrectly claim that for their eq. (35. 31), $L'' + (\beta')^2 L = 0$, there are dynamic solutions without a proof (see Appendix A). If such an error was overlooked, one could easily fall into agreeing with the other errors [16].

After P. Morrison passed away, general relativity at MIT is dominated by the Wheeler School whose errors are in the open courses Phys 8.033 and Phys 8.962. Although E. Bertschinger and Scott A. Hughes studied the linearized equation of the Einstein equation, they failed to understand that for the dynamic case, the non-linear Einstein equation and its linearized equation do not have any compatible solutions [3-6]. Apparently, they failed to see that this process of linearization is not valid in mathematics [6]. Moreover, Max Tegmark even failed to tell the different between mathematics and physics [16]. Thus, in the Physics Department of MIT, currently nobody understands the basic essence of general relativity, and has up-to-dated knowledge.

Moreover, MIT is not the only victim among universities because of the influences of the Wheeler School [5, 14]. Thus, it is necessary to point out their errors with a paper, [7] such that it is clearly understood that Fields Medals, 1982 to S. T. Yau and 1990 to E. Witten [8] were misleading. Different from a mathematician, a physicist usually understands the problem of dynamics and the principle of causality [3, 4].

Currently, mathematicians are often being considered in terms of a hierarchy system.[9] However, such a practice would result in errors in mathematics not being corrected. This article attempts to break such a practice by showing that the current top mathematicians can also make an elementary mistake just as Hilbert did because of inadequate consideration in physics (see also Appendices A, B, C).

II. Acknowledgments

This paper is dedicated to Professor S. Weinberg, Nobel Laureate, University of Texas at Austin, who taught us that general relativity must be understood in terms of physics. The author gratefully acknowledges stimulating discussions with Professors L. Ford, J. E. Hogarth, P. Morrison, A. Napier, H. C. Ohanian, R. M. Wald, and J. A. Wheeler, H. Yilmaz. Special thanks are to S. Holcombe for valuable suggestions. This publication is supported by Innotec Design, Inc., U.S.A. and the Chan Foundation.

Appendix A: Invalidity of Linearization for the Dynamic case & the Principle of Causality

The earliest reference of the definite non-existence of dynamics solution for the Einstein equation is probably the 1953 thesis of J. E. Hogarth [13], who conjectured that, for an exact solution of the two-particle problem, the energy-momentum tensor did not vanish in the surrounding space and would represent the energy of gravitational radiation. In 1995 and subsequently, it is proven that this is indeed the case [3].

Historically, Einstein and Rosen were the first that questioned the existence of a wave solution [17] because they found a singularity in such a solution. However, the Physical Review shows that such a singularity is removable, and thus claimed a wave solution does exist because they failed to see that a wave solution (or a dynamic solution) must be bounded in amplitude according to the principle of causality [9]. Thus, it is clear that this boundedness is needed for a dynamic solution.

Ref

14. C. Y. Lo, Phys. Essays, **23** (2), 258-267 (2010).

A1. Errors of Misner, Thorne, & Wheeler

An example is that Misner et al. [15] claimed that there is a bounded wave solution of the form,

$$ds^2 = c^2dt^2 - dx^2 - L^2\left(e^{2\beta}dy^2 + e^{-2\beta}dz^2\right) \tag{A1}$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - x$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0 \tag{A2}$$

They claimed that Eq. (A2) has a bounded solution, compatible with a linearization of metric (A1). It has been shown with mathematics at the undergraduate level that Misner et al. are incorrect [12, 16] and Eq. (A2) does not have a physical solution that satisfies Einstein's requirement on weak gravity.

Misner et al. [15] claimed that Eq. (A2) has a bounded solution, compatible with a linearization of metric (A1). Such a claim is in conflict with the non-existence of dynamic solutions [3, 4]. They further claimed,

"The linearized version of L″ = 0 since $(\beta')^2$ is a second–order quantity.

Therefore the solution corresponding to linearized theory is

$$L = 1, \quad \beta(u) \text{ arbitrary but small.} \tag{A3}$$

The corresponding metric is

$$ds^2 = (1 + 2\beta)dx^2 + (1 - 2\beta)dy^2 + dz^2 - dt^2, \qquad \beta = \beta(t-z)." \tag{A4}$$

However, these claims are actually incorrect. In fact, $L(u)$ is unbounded even for a very small $\beta(u)$. *It should be noted that their book* [15] *includes also factual errors, in addition to a misrepresentation of Einstein* [16].

Linearization of (A2) yields $L'' = 0$, and in turn this leads to $\beta'(u) = 0$. Thus, this leads to a solution $L = C_1u + C_2$ where C_1 & C_2 are constants. Therefore, the requirement $L \approx 1$ implies $C_1 = 0$. *However,* $\beta'(u) = 0$ *implies* $\beta(u)$*=constant, i.e. no waves. Thus, metric (6) is not derived, but only claimed.*

To prove Eq. (A2) having no wave solution, it is sufficient to consider the case of weak gravity. According to Einstein, for weak gravity of metric (A1), one would have

$$L^2e^{2\beta} \cong 1 \quad \text{and} \quad L^2e^{-2\beta} \cong 1 \tag{A5a}$$

It follows that

$$L^4 \cong 1, \quad e^{\pm 2\beta} \cong 1 \quad \text{and} \quad L(u) >> |\beta(u)| \tag{A5b}$$

Since $L(u)$ is bounded, $L'(u)$ cannot be a monotonic function of u, unless L′–¿ 0. Thus, there is an interval of u such that the average,

$$\langle L'' \rangle = 0 \tag{A6}$$

Ref

15. C. W. Misner, K. S. Thorne, and J. A. Wheeler, ***Gravitation*** (W. H. Freeman, San Francisco, 1973).

Notes

On the other hand, the average of the second term of equation (A2) is always larger than zero unless $\beta'(u)=0$ in the whole interval of u.

Also, from eq. (A2), one would obtain L ($\cong 1$) > 0, and one has $0 > L''(u)$ *if* $\beta'(u) \neq 0$. Thus, $-L'(u)$ is a monotonic increasing function in any finite interval of u since $\beta'(u) = 0$ means $L'' = 0$, i.e., no wave. In turn, *since* $\beta'(u)$ *is a "wave factor",* this implies that $L(u)$ is an unbounded function of u. Therefore, this would contradict the requirement that L is bounded. In other words, eq. (A2) does not have a bounded wave solution. Moreover, the second order term L'' would give a very large term to L, after integration.

Now, let us investigate the errors of Misner et al. [15; p. 958]. Their assumption is that the signal $\beta(u)$ has duration of $2T$. For simplicity, it is assumed that definitely $/\beta'(u)/ = \delta$ in the period $2T$. Before the arrival of the signal at $u = x$, one has

$$L(u)=1, \quad \text{and} \quad \beta(u)=0 \tag{A7}$$

If the assumption of weak gravity is compatible with Eq. (A2), one would have L(u) ≅ 1. It thus follows one has

$$L'(u)=0-\int_x^u \beta'^2 dy \approx -\int_x^u \delta^2 dy = \delta^2(u-x) \quad \text{for} \quad x+2T>u>x,$$

$$\text{or} \quad \approx -\delta^2 2T \qquad \text{for} \quad u>x+2T \tag{A8}$$

Hence

$$\left.\begin{aligned} L(u) &= 1+\int_x^u L'dy \\ &\approx 1-\int_x^u \delta^2(y-x)dy = 1-\frac{\delta^2(u-x)^2}{2} \end{aligned}\right) \text{for} \quad x+2T>u>x$$

$$\text{or} \quad \left.\begin{aligned} &\approx 1-\int_x^{x+2T} \delta^2(y-x)dy - \delta^2 2T\int_{x+2T}^u dy \\ &= 1-\delta^2 2T(u-T-x) \end{aligned}\right) \quad \text{for} \quad u>x+2T \tag{A9}$$

Thus, independent of the smallness of $2\delta^2 T$ (or details of $|\beta'(u)|^2$), L could be approximately zero and violates the condition for weak gravity. Thus, eq. (A2) has no weak wave solution. Moreover, —$L(u)$— is not bounded since it would become very large as u increases. Thus, restriction of $2\delta^2 T$ being small [15] does not help.

Thus, one can get a no wave solution through linearization of Eq. (A2), which has no bounded solution. The assumption of metric form (A1) is bounded [15], and has a weak form (A4), is not valid. Thus, there is no bounded wave solution for the non-linear Einstein equation, which violates the principle of causality.

The root of their errors was that they incorrectly assumed that a linearization of the Einstein non-linear equation would produce a valid approximation. Thus, they implicitly and incorrectly assume the existence of a bounded wave solution without the necessary verification, and thus obtain incorrect conclusions.

On the other hand, from the linearization of the Einstein equation (Maxwell-Newton approximation) in vacuum, Einstein [18] obtained a solution independently as follows:

$$ds^2 = c^2dt^2 - dx^2 - (1+2\phi)dy^2 - (1-2\phi)dz^2 \qquad \text{(A10)}$$

where ϕ is a bounded function of u $(= ct - x)$. Note that metric (A10) is the linearization of metric (A1) if $\phi = \beta(u)$. Thus, the problem of waves illustrates that the linearization may not be valid for the dynamic case when gravitational waves are involved since eq. (A2) does not have a weak wave solution.

The error of Misner et al. is clearly due to the combination of a blind faith on the Einstein equation and inadequacy in mathematics at the undergraduate level. Such a blind faith is often shown in the literature.

A2. Errors of Wald

According to Einstein [19], in general relativity weak sources would produce a weak field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, \text{ where } |\gamma_{\mu\nu}| << 1 \qquad \text{(A11)}$$

and $\eta_{\mu\nu}$ is the flat metric when there is no source. However, this is true only if the equation is valid in physics. Many theorists failed to see this because they failed to see the difference between physics and mathematics clearly [14]. When the Einstein equation has a weak solution, an approximate weak solution can be derived through the approach of the field equation being linearized. However, the non-linear equation may not have a bounded solution. The linearized Einstein equation with the linearized harmonic gauge

$$\partial^\mu \bar{\gamma}_{\mu\nu} = 0 \text{ is } \quad \frac{1}{2}\partial^\alpha\partial_\alpha \bar{\gamma}_{\mu\nu} = \kappa T_{\mu\nu} \quad \text{where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\gamma \quad \text{and } \gamma = \eta^{\alpha\beta}\gamma_{\alpha\beta} \qquad \text{(A12)}$$

Note that we have

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)} \text{ and } G_{\mu\nu}^{(1)} = \frac{1}{2}\partial^\alpha\partial_\alpha \bar{\gamma}_{\mu\nu} - \partial^\alpha\partial_\mu \bar{\gamma}_{\nu\alpha} - \partial^\alpha\partial_\nu \bar{\gamma}_{\mu\alpha} + \frac{1}{2}\eta_{\mu\nu}\partial^\alpha\partial^\beta \bar{\gamma}_{\alpha\beta} \qquad \text{(A13)}$$

The linearized vacuum Einstein equation means

$$G_{\mu\nu}^{(1)}[\gamma_{\alpha\beta}^{(1)}] = 0 \qquad \text{(A14)}$$

Thus, as pointed out by Wald [9], in order to maintain a solution of the vacuum Einstein equation to second order we must correct $\gamma^{(1)}{}_{\mu\nu}$ by adding to it the term $\gamma^{(2)}{}_{\mu\nu}$, where $\gamma^{(2)}{}_{\mu\nu}$ satisfies

$$G_{\mu\nu}^{(1)}[\gamma^{(2)}{}_{\alpha\beta}] + G_{\mu\nu}^{(2)}[\gamma_{\alpha\beta}] = 0, \qquad \text{where } \gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu} \qquad \text{(A15)}$$

which is the correct form of eq. (4.4.52) in Wald's book. (In Wald's book, he did not distinguish $\gamma_{\mu\nu}$ from $\gamma^{(1)}{}_{\mu\nu}$) This equation does have a solution for the static case. However, detailed calculation shows that this equation does not have a solution for the dynamic case [3, 14]. The fact that there is no bounded solution for eq. (A15) a dynamic case means also that the Einstein equation does not have a dynamic solution.

Ref

18. A. Einstein, Sitzungsberi, Preuss, Acad. Wis. 1918, 1: 154 (1918).

For instance, a well-known example is the metric of Bondi, Pirani, & Robinson [20] as follows:

$$ds^2 = e^{2\varphi}\left(d\tau^2 - d\xi^2\right) - u^2 \begin{bmatrix} \cosh 2\beta\left(d\eta^2 + d\varsigma^2\right) \\ +\sinh 2\beta\cos 2\theta\left(d\eta^2 - d\varsigma^2\right) \\ -2\sinh 2\beta\sin 2\theta\, d\eta\, d\varsigma \end{bmatrix} \tag{A16a}$$

where ϕ, β and θ are functions of u ($= \tau - \xi$). It satisfies the equation (i.e., their Eq. [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2 \sinh^2 2\beta\right). \tag{A16b}$$

Eq. (A16b) implies ϕ *cannot be a periodic function.* The metric is irreducibly unbounded because of the factor u^2. Both eq. (A2) and eq. (A16b) are special cases of $G_{\mu\nu} = 0$. However, linearization of (A16b) does not make sense since variable u is not bounded. Thus, they incorrectly claim Einstein's notion of weak gravity invalid because they do not understand the principle of causality adequately.

Moreover, when gravity is absent, it is necessary to reduce (A16a) to

$$ds^2 = \left(d\tau^2 - d\xi^2\right) - u^2\left(d\eta^2 - d\zeta^2\right) \tag{A16c}$$

because $\phi = \sinh 2\beta = \sin 2\theta = 0$. However, this metric is not equivalent to the flat metric, and thus violates the principle of causality. Also it is impossible to adjust metric (A16a) to become equivalent to the flat metric.

This challenges the view that both Einstein's notion of weak gravity and his covariance principle are valid. These conflicting views are supported respectively by the editorials of the "Royal Society Proceedings A" and the "Physical Review D"; thus there is no general consensus. As the Royal Society correctly pointed out [21], Einstein's notion of weak gravity is inconsistent with his covariance principle. In fact, Einstein's covariance principle has been proven invalid by counter examples [22, 23].

Due to confusion between mathematics and physics, Wald [9] made errors in mathematics at the undergraduate level. Wald did not see that the Einstein equation can fail the principle of causality. The principle of causality requires the existence of a dynamic solution, but Wald did not see that the Einstein equation can fail this requirement. Thus, his theory does not include the dynamic solutions [3-5].

A3. The Principle of Causality

There are other theorists who also ignore the principle of causality. For example, another "plane wave", which is intrinsically non-physical, is the metric accepted by Penrose [24] as follows:

$$ds^2 = du\, dv + H du^2 - dx_i dx_i, \quad \text{where } H = h_{ij}(u) x_i x_j \text{ and } u = ct - z,\ v = ct + z. \tag{A17}$$

However, there are arbitrary non-physical parameters (the choice of origin) that are unrelated to any physical causes. Being essentially only a mathematician, Penrose [24] naturally over-looked the principle of causality.

Ref

20. H. Bondi, F. A. E. Pirani, & I. Robinson, Proc. R. Soc. London A **251**, 519-533 (1959).

Also, the plane wave solution of Liu & Zhou [25], which satisfies the harmonic gauge, is as follows:

$$ds^2 = dt^2 - dx^2 + 2\,F(dt - dx)^2 - \cosh 2\psi(e^{2\phi}\,dy^2 + e^{-2\phi}dz^2) - 2\sinh 2\psi\;dy\;dz. \tag{A18}$$

where $\phi = \phi(u)$ and $\psi = \psi(u)$. Moreover, $F = F_P + H$, where

$$F_P = \frac{1}{2}(\dot{\psi}^2 + \dot{\phi}^2 \cosh^2 2\psi)\,[\cosh 2\psi\,(e^{2\phi}\,y^2 + e^{-2\phi}\,z^2) + 2\sinh 2\phi\;yz], \tag{A19}$$

and H satisfies the equation,

$$\cosh 2\psi\,(e^{-2\phi}H_{,22} + e^{2\phi}\,H_{,33}) - 2\sinh 2\psi\;H_{,23} = 0. \tag{A20}$$

For the weak fields one has $1 >> |\phi|$, $1 >> |\psi|$, but there is no weak approximation as claimed to be

$$ds^2 = dt^2 - dx^2 - (1 + 2\phi)\,dy^2 - (1 - 2\phi)dz^2 - 4\psi dydz \tag{A21}$$

because F_p is not bounded unless $\dot{\phi}$ and $\dot{\psi}$ are zero (i.e., no wave).

A4. Other Supporting Evidence and Conclusions

Moreover, there is no bounded wave solution in the literature. The reason is later identified as the missing of a gravitational energy-momentum tensor with a coupling constant of different sign [3, 11]. An independent convincing evidence for the absence of a bounded dynamic solution is, as shown by Hu, Zhang & Ting [26], that gravitational radiation calculated would depend on the approach used. This is also a manifestation that there is no bounded solution. A similar problem in approximation schemes such as post-Newtonian approximation [8, 14, 27] is that their validity is also only assumed.

The linearized equation for a dynamic case has been illustrated as incompatible with the non-linear Einstein equation. Thus, Eq. (A2), Eq. (A16b), and Eq. (A19) serve as good simple examples that can be shown through explicit calculation that linearization of the Einstein equation is not valid. Also, metric (A17) suggests that the cause of having no physical solution would be due to inadequate source terms [3, 26, 28].

Appendix B: The So-called Space-time Singularity Theorems and the Speculation of $E = mc^2$

A surprising conclusion, from the investigation of the Einstein equation, is that the space-time singularity theorems of Penrose and Hawking are actually irrelevant to physics. This is so because their theorems have a common implicit assumption that all the couplings have the same sign. However, from the investigation of dynamic solutions, such an assumption is necessarily invalid in physics [3, 29] because it implies no dynamic solution. These theorems were accepted because Penrose won the arguments against a Russian scientist E. M. Lifshitz who claimed, with the same set of assumptions, that there is no space-time singularity [30]. However, the problem is not the mathematics in the theorems, but the earlier historical errors in mathematics and physics.

As Pauli [31] pointed out, in principle general relativity can have different signs for their coupling constants. The fact that nobody questioned the assumption of unique sign for all coupling, is probably due to the unverified speculation of formula $E = mc^2$ being generally true. This formula comes from special relativity, and the conversion of

Ref

25. Liu H. Y., & Zhou, P.-Y.: Scientia Sincia (Series A) 1985, XXVIII (6) 628-637.

some mass to various combinations of energy is verified by the fission and fusion in nuclear physics. However, the conversion of a single type of energy to mass actually has never been verified [19], but this is currently proven as the invalid main speculation.

Einstein and theorists have shown that the photons can be converted into mass thorough absorption [32]. This conversion is supported by the fact that the π_0 meson can be decayed into two photons. Thus, it was claimed that the electromagnetic energy can be converted into mass because they failed to see that the photons must have non-electromagnetic energy. When Einstein proposed the notion of photons, he had not conceived general relativity yet. Thus, understandably he neglected the gravitational component of light. However, after general relativity, a light ray consists of a gravitational component is natural because the electron has a mass. Besides, the electromagnetic energy-momentum tensor has a zero trace. In fact, Einstein failed to show the general validity of $E = mc^2$ in spite of several years effort [33]. Experimentally, in contrast of Einstein's claim, $E = mc^2$ is not always valid because a piece of heated up metal has reduced weight [34].

Physically the dynamic solution must exist for a rectified equation. A problem of the Einstein equation is that it does not include the gravitational energy-stress tensor of its gravitational waves in the source and thus the principle of causality is violated. Since a gravitational wave carried energy-momentum and the source of gravity is the energy-stress tensors, as Hogarth [13] pointed out, the presence of a non-zero energy-momentum in the source is necessary for a gravitational wave. Thus, to fit the Hulse-Taylor data of the binary pulsar, it is necessary [3] to modify the Einstein equation,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T(m)_{\mu\nu} \qquad \text{(B1a)}$$

to

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa \left[T(m)_{\mu\nu} - t(g)_{\mu\nu} \right] \qquad \text{(B1b)}$$

where $t(g)_{\mu\nu}$ is the energy-stress tensor for gravity. For radiation, the tensor $t(g)_{\mu\nu}$ is equivalent to Einstein's notion of the gravitational energy-stress. However, his notion is a pseudo-tensor and can become zero by choosing a suitable coordinate system, but the energy-momentum of a radiation cannot be zero [3].

Moreover, the geodesic equation is not the exact equation of motion for a particle because the radiation reaction force is not included. Moreover, the mass-charge interaction is only partially involved. Thus, general relativity is clearly not yet a complete theory [35].

It is crucial to note that for the existence of a dynamic solution, the new tensor necessarily has a different sign for its coupling [3]. Thus, the implicit assumption of Penrose and Hawking is proven necessarily invalid. Note that the absence of a dynamic solution and the presence of space-time singularities are related to the same invalid assumption. It is the long standing bias and errors in mathematics that some theorists accepted one but rejected the other. Other victims are the positive mass theorem of Yau [1] and Witten [2] because they used the same invalid implicit assumption as Hawking and Penrose.

Ref

35. C. Y. Lo, On the Nobel Prize in Physics, Controversies and Influences, GJSFR Vol. 13-A Issue 3 Version 1.0, 59-73 (June 2013).

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Appendix C: The Necessity of the Maxwell-Newton Approximation

A problem in general relativity [3] is that, for a dynamic case, there is no bounded solution,

$$|g_{ab}(x, y, z, t)| < \text{constant}, \tag{C1}$$

for the Einstein equation, where g_{ab} is the space-time metric. In fact, eq. (C1) is also a necessary implicit assumption in Einstein's radiation formula [27] and the light bending [28]. One might argue that requirement (C1) violates the covariance principle. However, the covariant principle is proven invalid in physics [36]. Moreover, Einstein's notion of weak gravity [19] is also in agreement with the principle of causality. It will be shown that weak gravity is also compatible with Einstein's equivalence principle.

The question of dynamic solutions was raised by Gullstrand [37]. He challenged Einstein and also D. Hilbert who approved Einstein's calculations [7]. However, Hilbert did not participate in the subsequent defense and he would probably have seen the deficiency. Nevertheless, theorists such as Christodoulou & Klainerman [8], Misner et al. [15] and Wald [9] etc. failed to see this, and tried very hard to prove otherwise.

The failure of producing a dynamic solution would cast a strong doubt to the validity of the linearized equation that produces many effects including the gravitational waves. In fact, for the case that the source is an electromagnetic plane wave, the linearized equation actually does not have a bounded solution [38].

Nevertheless, when the sources are massive, some of such results from the linearized equation have been verified by observation. Thus, there must be a way to justify the linearized equation, independently. To this end, Einstein's equivalence principle [29] is needed, although rejected by the 1993 Nobel Prize Committee for Physics implicitly [39]. As a result, it becomes even clearer that the non-existence of a bounded dynamic solution for massive sources is due to a violation of the principle of causality [12].

C1. Gravitational Waves and the Einstein Equation of 1915

Relativity requires the existence of gravitational waves because physical influence must be propagated with a finite speed [40]. To this end, let us consider the Einstein equation of 1915 [19], which is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T(m)_{\mu\nu} \ . \tag{C2}$$

Einstein believed that his equation satisfied this requirement since its linearized "approximation" gives a wave.

The linearized equation with massive sources [19] is the Maxwell-Newton Approximation [3],

$$\frac{1}{2} \partial_c \partial^c \bar{\gamma}_{ab} = -\kappa T(m)_{ab} \tag{C3a}$$

where $\bar{\gamma}_{ab} = \gamma_{ab} - (1/2)\eta_{ab}$, $\gamma_{ab} = g_{ab} - \eta_{ab}$, $\gamma = \eta^{cd} \gamma_{cd}$, and η_{ab} is the flat metric. Eq. (C3a) has a mathematical structure similar to that of Maxwell's equations. A solution of eq. (C3a) is

$$\bar{\gamma}_{ab}(x_i,t) = -\frac{\kappa}{2\pi} \int \frac{1}{R} T_{ab}\,[y^i, (t-R)]\, d^3y, \quad \text{where } R^2 = \sum_{i=1}^{3} \left(x^i - y^i\right)^2 \tag{C3b}$$

Ref

40. H. A. Lorentz, Proc. K. Ak. Amsterdam 8, 603 (1900).

Note that the Schwarzschild solution, after a gauge transformation, can also be approximated by (C3b). Solution (C3b) would represent a wave if T_{ab} has a dynamical dependency on time t' (= t − R). Thus, the theoretical existence of gravitational waves seems to be assured as a certainty as believed [27, 31, 41].

However, for non-linear equations, the physical second order terms can be crucial for the mathematical existence of bounded solutions. For the Einstein equation, the Cauchy initial condition is restricted by four constraints since there is no second order time derivatives in G_{at} (a = x, y, z, t) [27]. This suggests that the Einstein equation (C2) and (C3) may not be compatible for a dynamic problem. Einstein discovered that his equation does not admit a propagating wave solution [42, 43]. Recently, it has been shown that the linearization procedure is not generally valid [3, 44]. Thus, it is necessary to justify wave solution (A18) independently since it is the basis of Einstein's radiation formula.

C2. The Weak Gravity of Massive Matter and Einstein Equation of the 1995 Update

For a massive source, the linear equation (C3), as a first order approximation, is supported by experiments [3, 27]. However, for the dynamic case, the Einstein equation is clearly invalid.

It will be shown that eq. (C3a) can be derived from Einstein's equivalence principle. Based on this, the equation of motion for a neutral particle is the geodesic equation. In comparison with Newton's second law, one obtains that the Newtonian potential of gravity is approximately $c^2 g_{tt}/2$. Then, in accord with the Poisson equation and special relativity, the most general equation for the first order approximation of g_{ab} is,

$$\frac{1}{2}\partial_c\partial^c\gamma_{ab} = -\frac{\kappa}{2}[\alpha T(m)_{ab} + \beta\hat{T}(m)\eta_{ab}], \tag{C4a}$$

where

$$\hat{T}(m) = \eta^{cd}T(m)_{cd}, \quad \kappa = 8\pi K c^{-2}, \quad \text{and} \quad \alpha + \beta = 1, \tag{C4b}$$

where α and β are constants since Newton's theory is not gauge invariant.
Then, according to Riemannian geometry [27], the exact equation would be

$$R_{ab} + X^{(2)}{}_{ab} = -\frac{\kappa}{2}[\alpha T(m)_{ab} + \beta T(m) g_{ab}], \quad \text{where } T(m) = g^{cd}T(m)_{cd} \tag{C5a}$$

and $X^{(2)}{}_{ab}$ is an unknown tensor of second order in K, if R_{ab} consists of no net sum of first order other than the term (1/2) $\partial_c\partial^c\gamma_{ab}$. This requires that the sum

$$-\frac{1}{2}\partial^c[\partial_b\gamma_{ac} + \partial_a\gamma_{bc}] + \frac{1}{2}\partial_a\partial_b\gamma, \tag{C5b}$$

must be of second order. To this end, let us consider eq. (C4a), and obtain

$$\frac{1}{2}\partial_c\partial^c(\partial^a\gamma_{ab}) = -\frac{\kappa}{2}[\alpha\partial^a T(m)_{ab} + \beta\partial_b\hat{T}(m)]. \tag{C6a}$$

Ref

27. S. Weinberg, "Gravitation and Cosmology" (John Wiley, New York, 1972), pp. 69, 251-273.

From $\nabla^c T(m)_{cb} = 0$, it is clear that K $\partial^c T(m)_{cb}$ is of second order but $K\partial_b \hat{T}(m)$ is not. However, one may obtain a second order term by a suitable linear combination of $\nabla^c \gamma_{cb}$ and $\partial_b \gamma$. From (A6a), one has

$$\frac{1}{2}\partial_c \partial^c(\partial^a \gamma_{ab} + C\,\partial_b \gamma) = -\frac{\kappa}{2}[\alpha \partial^a T(m)_{ab} + (\beta + 4C\beta + C\alpha)\partial_b \hat{T}(m)]\,. \quad \text{(C6b)}$$

Thus, the harmonic coordinates (i.e., $\partial^a \gamma_{ab} - \partial_b \gamma/2 \approx 0$), can lead to inconsistency. It follows eqs. (C5b) and (C6b) that, for the other terms to be of second order, one must have C = -1/2, α = 2, and β = -1.
Hence, eq. (C4a) becomes,

$$\frac{1}{2}\partial_c \partial^c \gamma_{ab} = -\kappa[T(m)_{ab} - \frac{1}{2}\hat{T}(m)\,\eta_{ab}]\,. \quad \text{(C7)}$$

which is equivalent to eq. (C3a), has been determined to be the field equation of massive matter. This derivation is independent of the exact form of equation (C5a). The implicit gauge condition is that the flat metric η_{ab} is the asymptotic limit. Eq. (C7) is compatible with the equivalence principle as demonstrated by Einstein in his calculation of the bending of light. Thus, the derivation is self-consistent.

Einstein obtained the same values for α and β by considering eq. (C5a) after assuming $X^{(2)}{}_{ab} = 0$. An advantage of the approach of considering eq. (C4) and eq. (C5b) is that the over simplification $X^{(2)}{}_{ab} = 0$ is not needed. Then, it is possible to obtain from eq. (C5a) an equation different from (C2),

$$G_{ab} \equiv R_{ab} - \frac{1}{2}g_{ab}R = -\kappa\,[T(m)_{ab} - Y^{(1)}{}_{ab}\,], \quad \text{(C8)}$$

where

$$-\kappa Y^{(1)}{}_{ab} = X^{(2)}{}_{ab} - \frac{1}{2}g_{ab}\{X^{(2)}{}_{cd}\,g^{cd}\}.$$

The conservation law $\nabla^c T(m)_{cb} = 0$ and $\nabla^c G_{cb} \equiv 0$ implies also $\nabla^a Y^{(1)}{}_{ab} = 0$. If $Y^{(1)}{}_{ab}$ is identified as the gravitational energy tensor of $t(g)_{ab}$, Einstein equation of the 1995 update [3] is reaffirmed. Note that eq. (C3a) is the first order approximation of eq. (C8) but may not be of (C2). Note, however, that in Einstein's initial consideration, $t(g)_{ab}$ is a pseudo-tensor. It has been shown that it must be a tensor [3].

Endnotes:

1) Michael Francis Atiyah has been president of the Royal Society (1990-1995), master of *Trinity College, Cambridge* (1990-1997), chancellor of the *University of Leicester* (1995-2005), and President of the *Royal Society of Edinburgh* (2005-2008). Since 1997, he has been an honorary professor at the *University of Edinburgh* (Wikipedia).

2) Ludwig D. Faddeev, the Chairman of the Fields Medal Committee, wrote ("On the work of Edward Witten"):

"Now I turn to another beautiful result of Witten – proof of positivity of energy in Einstein's theory of gravitation.

Hamiltonian approach to this theory proposed by Dirac in the beginning of the fifties and developed further by many people has led to the natural definition of energy. In

Ref

8. D. Christodoulou & S. Klainerman, ***The Global Nonlinear Stability of the Minkowski Space*** (Princeton. Univ. Press, 1993); No. 42 of the Princeton Mathematical Series.

Notes

this approach a metric γ and external curvature h on a space-like initial surface $S^{(3)}$ embedded in space-time $M^{(4)}$ are used as parameters in the corresponding phase space. These data are not independent. They satisfy Gauss-Codazzi constraints – highly non-linear PDE, The energy H in the asymptotically flat case is given as an integral of indefinite quadratic form of $\nabla\gamma$ and h. Thus, it is not manifestly positive. The important statement that it is nevertheless positive may be proved only by taking into the account the constraints – a formidable problem solved by Yau and Schoen in the late seventy as Atiyah mentions, 'leading in part to Yau's Fields Medal at the Warsaw Congress'.

Witten proposed an alternative expression for energy in terms of solutions of a linear PDE with the coefficients expressed through γ and h ....."

3) The 2011 Shaw Prize also made a mistake by awarding a half prize to Christodoulou for his work, based on obscure errors, against the honorable Gullstrand [37] of the Nobel Committee. Although Christodoulou has misled many including the 1993 Nobel Committee [39], his errors are now well-established and they have been illustrated with mathematics at the undergraduate level [5]. Christodoulou claimed in his autobiography that his work is essentially based on two sources: 1) The claims of Christodoulou and Klainerman on general relativity as shown in their book ***The Global Nonlinear Stability of the Minkowski Space*** [8]; 2) Roger Penrose had introduced, in 1965, the concept of a trapped surface and had proved that a space-time containing such a surface cannot be complete [9]. However, this work of Penrose, which uses an implicit assumption of unique sign for all coupling constants, actually depends on the errors of Christodoulou and Klainerman [8]. However, such a relation was not clear until 1995 [3] (see Appendix B).
4) These solutions have no gravitational radiation.
5) At MIT, only P. Morrison surely read the proof for the non-existence of a dynamic solution. Apparently, Yau probably did not read such a proof since his interest is no longer in general relativity since 1993 [8].
6) M. Atiyah was in the 2011 Selection Committee for the Shaw Prize in Mathematics Sciences.
7) MIT President Reif would be able to do little without our help to counter his incompetent subordinates [45], who disobey his directive of communication because of their out-dated knowledge [45, 46].
8) E. Witten is a leader of string theorists. Thus, his error in general relativity represents a common deficiency.
9) Thus, many journals just decline to consider a critical article as this since Atiyah is a well-known mathematician and was the President of the Royal Society (1990-1995). The intention is to avoid his mistake in physics becoming an embarrassment to the scientific community. Moreover, the schools also have an informal hierarchy system. For instance, MIT would decline to think Harvard University could be wrong. In spite of an eloquent speech of the MIT President Reif on basic research, so far no MIT professor has made a single move to correct the errors of Harvard professor S. T. Yau [1, 2].

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