Online ISSN: 2249-4626
Print ISSN: 0975-5896

Global Journal of Science Frontier Research: A

Physics and Space Science

Highlights

Higher Order Statistics

Hawking-Penrose Singularity

Spectra in Communication

Negative Cosmological Constant

Discovering Thoughts, Inventing Future

VOLUME 13 ISSUE 4 VERSION 1.0

Global Journal of Science Frontier Research: A
Physics & Space Science

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Physics & Space Science
Volume 13 Issue 4 (Ver. 1.0)

Open Association of Research Society

Incorporation No.: 0423089
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USA Tax ID: 98-0673427

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200 USD (B/W) & 250 USD (Color)

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Contents of the Volume

Global Journal of Science Frontier Research
Physics and Space Science
Volume 13 Issue 4 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Higher Order DNS on an Airfoil Fx77-W500 using Spectral/Hp Method

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Indexterms : *reynolds number (RE), direct numerical simulation (DNS), spectral/hp elements method, polynomial orders (P), forces, contour plots.*

GJSFR-A Classification : *FOR Code: 090101*

HIGHER ORDER DNS ON AN AIRFOIL FX77-W500 USING SPECTRALHP METHOD

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M. M. Hossain [α], B. Stoevesandt [σ], & J. Peinke [ρ]

Abstract - The prediction of aerodynamic properties of thick airfoils is a tricky business for the blade manufactures. Computational fluid dynamics (CFD) offers a number of significant advantages to investigate the flow properties around airfoils, however, most turbulence models are working properly except having some problems in case of turbulent separation. The spectral/hp elements method offers fast convergence due to the better resolution in a short time. Therefore, we perform direct numerical simulation (DNS) at the Reynolds number (Re) $= 3900$, polynomial orders (P)$=9$ and an angle of attack of $\alpha = 12^0$ round an airfoil (Fx77-W500) using spectral/hp elements method to solve the unsteady, viscous and two dimensional (2D) incompressible Navier-Stocks equations. A frequent fluctuation of the lift and drag forces has been found over the entire time. The contour plots of pressure and velocity magnitude around the airfoil are investigated for certain cases, which show more vortices at the upper portion of the airfoil and around the trailing edge The presented results show that some interesting and peculiar behaviors due to unsymmetrical airfoil shape, which remind more of a bluff body, that the lift forces show negative value very frequently.

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I. Introduction

Wind turbines (WT) are used for extracting kinetic energy from wind and generate a significant growing factor of the world's energy demand, and offer a promise for inexpensive and clean energy. The foremost aims of the most wind turbines are to extract as much energy as possible from wind, and the each component of WT has to be optimized for that goal. Therefore, suitable blade geometry is needed to achieve maximum power coefficient. Due to the turbulent nature of wind, the prediction of aerodynamic and aero acoustic properties of WT is a major concerning threats for engineers (e.g. involving wind gusts, turbulence, icing), which affect turbine performance (energy output, noise emission etc), life expectancy and safety, see Tony et al. [1].

Considering the factors, conducting experiment on such type problem is much more expansive, computational fluid dynamics (CFD) modeling and experiments are interesting alternative. This method has both advantages and disadvantages. Nevertheless, CFD provide more detailed and comprehensive information and is increasingly cost effective than wind tunnel test. On the other hand, CFD consume less energy, by Flecther [2]. There are several methods to simulate turbulence flow such as: direct numerical simulation(DNS), large eddy simulation(LES), Reynolds averaged Navier-Stocks model (RANS model), Reynolds stress models, very large eddy simulation, detach eddy simulation, vortex method, Vorticity conformed method, two phase method, see Ferziger and Peric [3]. Bernhard St. et al. [4], performed two dimensional simulation using the spectral element code on fx79-w151a airfoil at moderate Reynolds number Re=33000 and at angle of attack 120. Their result suggested that vortex shedding appear in front of the airfoil and was confirmed by HPIV measurement. In another work, Bernhard St. et al. [5] investigated that the effects of dynamic stall using spectral/hp code on the same airfoil [4] at Re=11111 and initial angle of attack of 120. They found that the spectral element code is promising in simulating turbulent flows using DNS and LES. Dong et al. [6] inspected the effects of the Reynolds number on the statistical characteristics of the cylinder wake and on the share-layer instability in the transition range. Both PIV measurements and DNS were performed at Reynolds number Re=3900/4000 and 10000 and found all the quantitative patterns move upstream with increasing Reynolds number, and high frequency fluctuation caused by share layer vortices for both cases. Another study made by Monty and Chong in [7] on a comparison between stream wise velocity data from experiment and DNS. The results showed that there were excellent agreements between the stream wise velocity statistics of the two data sets. The spectra are also very similar, however, throughout the logarithmic region the secondary peak in energy is clearly reduced in the DNS results. Therefore, we perform DNS on the airfoil (FX77-W500) using spectral/hp method to provide a prospect for aerodynamic control of modern wind turbine.

In section II, the analytical descriptions of the numerical methods are introduced including the spectral/hp element approach. The numerical setup is

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discussed in section III. Simulating results are presented in section IV. Discussion and conclusions are given in section V.

II. The Numerical Method

The solver which is used for the numerical simulation is spectral element based solver called *Nektar.* It is a DNS solver. It provides fast convergence, small diffusion and dispersion errors, easier implementation of the in-sup condition for income-pressible Navier-Stokes, and better data volume over-surface-ratio for efficient parallel processing and also better input/output handing due to the smaller volume of data. Since wind turbines are operating for low wind speed, it is enough to consider incompressible flows for DNS. The governing equations are the Navier-Stokes equations for incompressible, transient (or unsteady) and viscous flow can be written in dimensional form as:

$$\nabla \cdot u = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\nabla p + v \nabla u + F \tag{2}$$

where u is stand for the velocity vector, P is the pressure, F is the external forces and v is the kinematics viscosity. The procedures used in $N\varepsilon\kappa\tau\alpha r$ consider a solution domain Ω (where Ω is a domain of $\Re^2$) and which is decomposed into N_{el} elemental domains Ω^e such that

$$\Omega = \bigcup_{e=1}^{e=N_{el}} \Omega^e, \quad \text{Where} \quad \bigcap_{e=1}^{N_{el}} = \phi, \tag{3}$$

For the solution domain $\Omega = \{x \,|0 < x < l\}$ a specific mesh can be denoted by points

$$0 = x_0 < x_1 < -----< x_{N_{el}-1} < x_{N_{el}} = l \tag{4}$$

Therefore, the domain is defined as:

$$\Omega^e = \{x \,|x_{e-1} < x < x_e\} \tag{5}$$

and grid is obtained by partitioning the physical domain. One of the main reasons of using unstructured expansion for time dependent computations is rotationally symmetric considerations which increase the numerical efficiency, which was described by Sherwin et al [8]. More efficient numerical operation can be achieved by using expansions. The spectral expansion is being employed into each sub domains. A function $\Phi(x, y)$ over the e^{th} sub-domains can be represented a polynomial basis, $\psi_{i,j}(r,s)$ where $\Phi(x,y)$ can be written as:

$$\Phi(x,y) = \sum_e \sum_i \sum_j \Phi^e_{i,j} \, \psi_{i,j}(r,s). \tag{6}$$

Where the expansion coefficients for the polynomial are defined by $\Phi^e_{i,j,}$ and (r, s) are the local coordinate within the sub-domains. e^{th} The spatial/global coordinates can be defined as. *(x, y)* This extends the expansion bases within a standard region into multi domain expansion. Sherwin et al. [8] showed that to develop a suitable tensorial type basis within unstructured regions, such as the triangle, it is needed to develop a new coordinate system where the local coordinates have independent bounds. The advantage of such a system is that it can then define one-dimensional functions upon which we can construct our multi-domain tensorial basis. It also defines an appropriate system upon which we can perform important numerical operations such as integration and differentiation. Richard et al. [9] reported that the main drawback of spectral method is their inability to handle complex geometries and these complex geometries can easily be handled in triangular based spectral method.

A good assumption is standard triangular domain (Tri_{st}) system which is described by Sherwin [8, 10] such that,

$$Tri_{st} = \{ (r,s) | \; -1 \le r, s; r + s \le 0 \} \tag{7}$$

Now the local coordinate system*(r, s)* can be mapped for defining the global coordinate system*(x, y)* as

$$x = -\frac{(r+s)}{2} V_1 + \frac{(1+r)}{2} V_2 + \frac{(1+s)}{2} V_3 \tag{8}$$

Where the V_1 (-1-1),V_2 (1-1), *and* V_3 (-1,1) are the physical coordinates of the vertices of the elementary (standard) triangle.

As the standard triangular domain is not independently bounded by the coordinates*(r, s),* Sherwin et al. [8] introduced the so-called collapsed coordinate system *(x, y).*

The collapsed Cartesian system is suitable for Gauss integration in the unstructured regions. The transformation to collapsed coordinate system can be interpreted as a mapping to a standard quadrilateral region (see Fig.1).

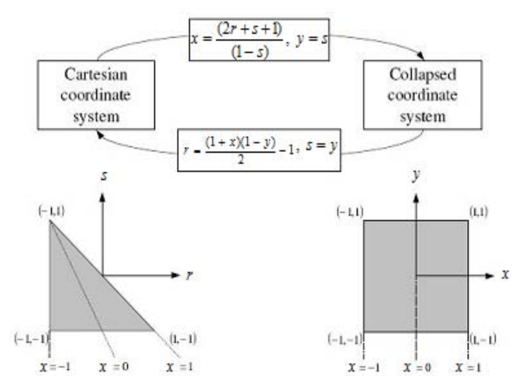

Figure1 : The standard triangular and collapsed coordinates

The transformation from $(r, s)\rightarrow(x, y)$ is given by:

$$x = \frac{(2r+s+1)}{(1-s)} \text{ and } y = s \quad . \tag{9}$$

The reverse transformation can be written as:

$$r = \frac{(1+x)(1-y)}{2} - 1 \text{ and } s = y \quad . \tag{10}$$

The standard triangle can now be defined in terms of collapsed coordinates as:

$$Tri_{st}^{\ 2} = \{ (x, y) \mid -1 \le x, y \le 1 \}, \tag{11}$$

The Spectral/hp method allows both h-type and p-type extensions. So, to construct p-type expansions, orthogonal polynomials have been considered. The Jacobi polynomials $P_n^{\alpha,\beta}(x)$ are an orthogonal set of polynomials in the interval [-1, 1] with respect to the function $(1-x)^{\alpha}(1+x)^{\beta}$ where, $(\alpha, \beta > -1)$ can be defined as

$$P_n^{\alpha,\beta}(x) = \frac{(-1)^n}{2^n n!}(1-x)^{-\alpha}(1+x)^{-\beta}\frac{d^n}{dx^n}[(1-x)^{\alpha+n}(1+x)^{\beta+n}] \tag{12}$$

n stands for the polynomial order, and the coefficients α and β define the characterizing coefficient of the polynomial.

$P_n^{\alpha,\beta}(x)$ is Legendre polynomial $P_n^{0,0}(x)$ when and $(\alpha = \beta = 0)$ and Chebychev polynomial can be obtained by imposing . $(\alpha = \beta = -\frac{1}{2})$ The orthogonal affiliation is important for the Jacobi polynomials and this can be written as

$$\int_{1}^{1}(1-x)^{\alpha}(1+x)^{\beta}P_i^{\alpha,\beta}(x)P_j^{\alpha,\beta}(x)dx = C\delta_{ij} \tag{13}$$

Where C depends on α, β, and i, and δ_{ij} is the Kronecker-Delta in which triangular sub-domains are mapped into a rectangular domain using mapping functions.

Finally full expansion basis which were already described by Dubiner [11], who considered a triangular basis as below:

$$\psi_{i,j}(r,s) = P_i^{0,0}\frac{(2r+s+1)}{(1-s)}(1-s)^i P_j^{2i+1,0}(s) \tag{14}$$

Where i and j are the expansion orders which could be differing for different directions, and $P_i^{\ a,\beta}$ denotes Jacobi Polynomials.

It is well known that the splitting method is a very popular approach to constructing efficient numerical methods for unsteady problems because it permits high-order time accuracy, Karniadakis et al. [12]. This approach is in common use in CFD as well. The splitting scheme used in the *Nektar* solver is a higher order time accurate scheme for the solution of the incompressible Navier Stokes equations which was developed by Karniadakis et al. [12]. In their studies, incompressible flows with constant properties were considered, which are governed by the Navier-Stokes equations (1) and (2) written in the form,

$$\frac{\partial u}{\partial t} = -\nabla p + vL(u) + N(u) \quad \text{in } \Omega \tag{15}$$

subject to incompressible constraint

$$\Omega \equiv \nabla.u = 0 \quad \text{in } \Omega \tag{16}$$

Where is the velocity vector P, is the pressure v, is the kinematics viscosity. The linear and nonlinear operators are $N(u)$ and $L(u)$, respectively and can be defined as follows:

$$L(u) \equiv \nabla^2.u \tag{17}$$

$$N(u) \equiv -\frac{1}{2}[u\nabla.u + \nabla(u.u)] \tag{18}$$

The nonlinear terms are written here in a skew symmetry form in order to minimize aliasing effects. Finally, equation (17) is integrated, using high-order time-stepping schemes; such schemes are routinely used for the numerical solution of ordinary differential equations. The linear and nonlinear terms are approximated via implicit schemes for stability regions and explicit schemes for efficiency reasons, respectively and can be written in the integration form as:

$$\int_{t_l}^{t_{l+1}} L(u)dt = \Delta t \sum_{q=0}^{J_i-1} \gamma_q L(u^{l+1-q}), \tag{19}$$

$$\int_{tl}^{t_{l+1}} N(u) \equiv \Delta t \sum_{q=0}^{J_e-1} \beta_q N(u^{l-q}) \tag{20}$$

Where Y_q and β_q are appropriately chosen weights for implicit and explicit schemes, respectively J_e. and j_i are denote the explicit and implicit order of the scheme respectively. Karniadakis et al. [12] reported that the splitting scheme involves these steps as:

$$\frac{\hat{u} - u^l}{\Delta t} = \sum_{q=0}^{J_e-1} \beta_q N(u^{l-q}) \quad \text{in } \Omega \tag{21}$$

$$\frac{\hat{\hat{u}} - \hat{u}}{\Delta t} = -\nabla p^{-l+1} \quad \text{in } \Omega \tag{22}$$

$$\frac{u^{l+1} - \hat{\hat{u}}}{\Delta t} = v\sum_{q=0}^{J_i-1} \gamma_q L(u^{l+1-q}) \quad \text{in } \Omega, \tag{23}$$

Where,

$$\Delta t \nabla p^{-l+1} = \int_{t^l}^{t^{l+1}} \nabla p dt \tag{24}$$

Global Journal of Science Frontier Research (A) Volume XIII Issue IV Version I Year 2013

III. Numerical Setup

The simulation of a flow over the profile (FX77-W-500) was undertaken and evaluated for several aspects such as force coefficients, the flow dynamics, vortices contours, and pressure contours. During the 2D simulation, the Reynolds number of 3900 and an angle of attack of α=120, with polynomial orders P = 9 were chosen.

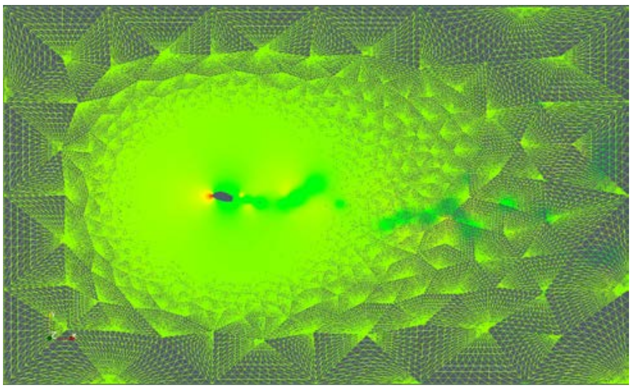

Figure 2 : Full mesh view with the polynomial nodes around the airfoil (FX77-W500)

Fig. 2 illustrates the used mesh with the polynomial nodes around an airfoil (FX77-W500). The unstructured gird was used throughout this work, the spectral/hp method employ permits the use of both structured and unstructured meshes, based on triangular and/or quadrilateral elements. The unstructured grid used was composed of 3814 triangular elements and 200 quadrilateral elements. The resolution of the grid was refined around the airfoil as well as in the wake region (Fig. 3 - 4) since the most interesting phenomenon was expected there; it already studied by Folkerts L. et al. [13] and Bernhard St. et al. [14].

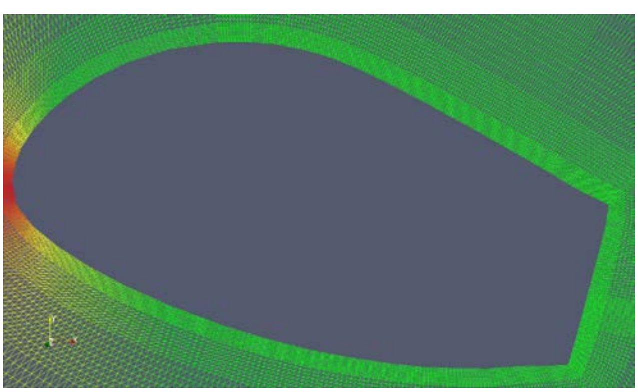

Figure 3 : Close-up view of the selected airfoil (FX77-W500)

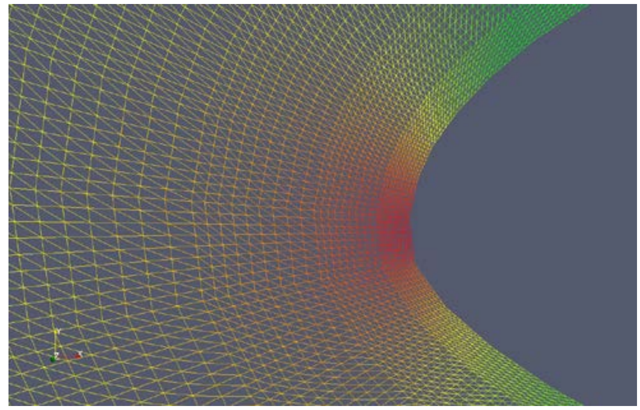

Figure 4 : Close-up view of the mesh around the leading edge of the airfoil with the polynomial nodes

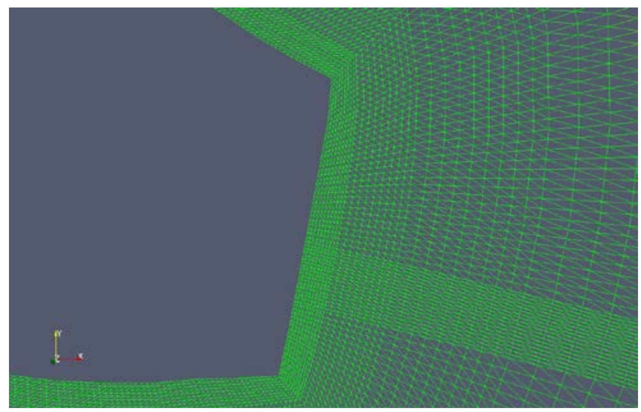

Figure 5 : Close-up view of the mesh around the tailing edge of the airfoil with the polynomial nodes

The computational domain extended from -10C (C is the cord length of the profile) in the inflow direction to 20C in the out flow direction, and from -10C to 10C in the cross flow direction. Hubscher [15] reported that specification of the time step is an essential parameter for the simulation because a too short time step can increase the computational cost. On the other hand, simulation could be crashed due to high velocity which is created around the leading edge of the airfoil for a too high time step. To achieve higher resolution, lower time step (DT = 4*10-5) considered during this work. The product of the time step and the number of time steps gives the required time units.

$$\text{Time units} = \text{time step} \times \text{number of steps} \qquad (25)$$

This time units denotes that one time unit is needed for the flow to cover the distance of the chord length of the airfoil. The dimensionless time unit forms as:

$$t \rightarrow \frac{tU_0}{c} \qquad (26)$$

Where U_0 is the free stream velocity and c is stand for chord length of the airfoil.

IV. Simulation and Results

The simulations were performed with an open source solver *Nektar* [16] which is developed by Sherwin et al. [8]. Paraview for visualization technique [17] was used to analyze the data. The force in the code is derived from equation (2) in section II as:

$$F = -\int_s Pnds + \int_s \tau .nds \qquad (27)$$

The lift and drag forces and the flow dynamics, which are one of the most significant matters for the wind turbine blade designers. In Fig. 6 illustrates, the lift force (left side) and drag force (right side) over 80 dimensionless time units at Re = 3900, $\alpha=12^0$ and P=9 which confirm the response of the lift and drag force to the turbulent separation at the blade surface. It has been seen that the lift forces (F_l) are not constant but highly unsteady. The average lift and drag forces coefficient are F_{lmean} = 0.08528 and F_{dmean}= 0.2446 respectively. The maximum drag F_{dmax} = 0.6568 is observed due to the high inertia at the time $(t = 0)$ of simulation started and decreases to F_{lmin} = 0.1124 over the whole simulation period. The fluctuation of the lift is F_{lmax} = 0.88610 to F_{lmin} = -0.76800. At the time unit of t =2.156, the lift forces has reached the maximum value F_{lmax} = 0.88610 due to the fact of a strong fast separation of vortices which occurs at the time unit of t = 1.04.

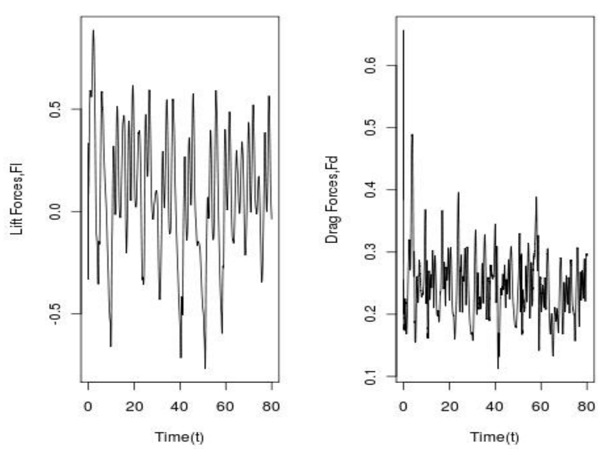

Figure 6 : Lift (left) and Drag (right) forces showing fluctuation over dimensionless time unit at Re=3900, $\alpha=12^0$ and P=9 obtained using 2D *Nektar* solver

The standard deviation for the lift and drag forces are F_{lstd} = 0.3052 and F_{dsts} = 0.0519. The frequency of the vortex shedding could be described by the Strouhal Number (St), which is 0.15.

A screen shot of velocity magnitude contours is depicted in Fig. 7 that is modeled for the stationary airfoil (FX77-W500) when lift forces $F_l = 0$ at different time units $t = 32.00$ (in Fig. 7a) and $t = 46.00$ (in Fig.7b). It can be observed that the vortices are developed just behind the upper portion of the trailing edge of the airfoil for both cases due to the fact of lift forces.

The leading edge structure is shown in light color and is a less effective area on the airfoil. The pressure contour plots in Fig. 8 explain the distribution of lift forces, when $F_l = 0$ as well. There is a low pressure contours just behind the trailing edge of airfoil that is shown in Fig. 8(b). A large area of low pressure contours is revealed in Fig. 8(a). These regions of low pressure are vortices that form, and detach from the airfoil. The leading edge structures are shown in lighter red colour with same graphical view for both cases.

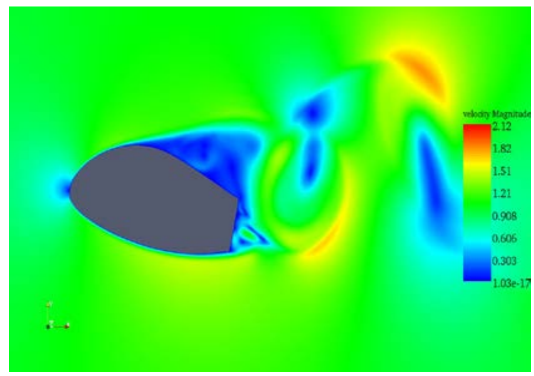

Figure 7(a) : Velocity magnitude contours and flow structure when the lift forces F_l = 0 and t = 32.00

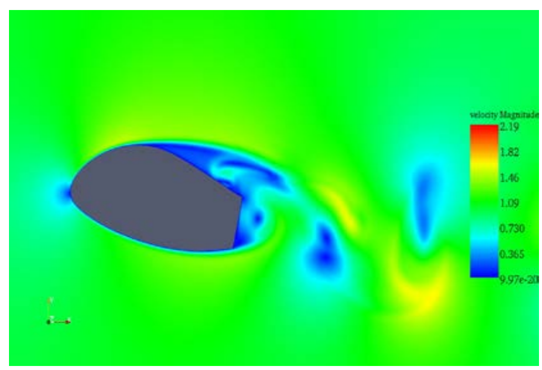

Figure 7(b) : Velocity magnitude contours and flow structure when the lift forces F_l = 0 and t = 46.00

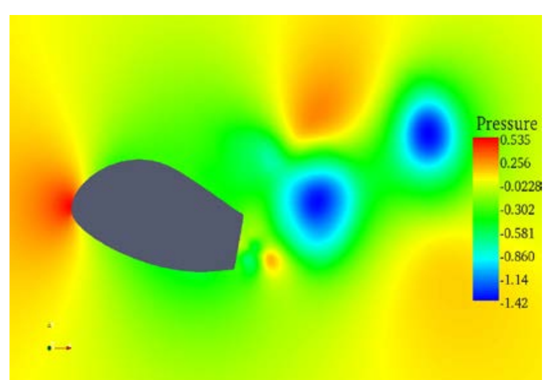

Figure 8(a) : Pressure contours and flow structure when the lift forces F_l = 0 and t =32.00

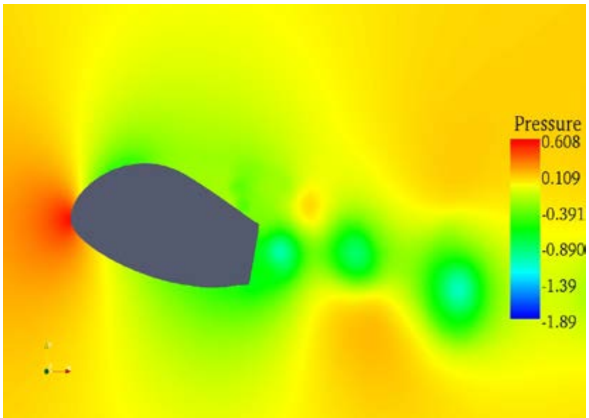

Figure 8 (b) : Pressure contours and flow structure when the lift forces $F_l = 0$ and $t = 46.00$

In the flowing Fig. 9 depict the velocity magnitude contour of the flow over the airfoil for Re = 3900 and P = 9 at F_l = + 0.54 and +0.57 when time unit t =39.90 (in Fig. 9a) and t = 45.70 (in Fig. 9b), respectively. While the leading edge flow structure shows the same structure compared to the $F_l = 0$ case, parabolic shaped vortices have been developed just behind the trailing edge, and are responsible for the maximum positive lift forces. In Fig. 10(a) and 10(b), the pressure distributions are shown. Two circles, one small and another one big size, develop just above the trailing edge. These two circles of low pressure are also responsible for the positive lift forces. The pressure at the leading edge of the airfoil show maximum values for both cases.

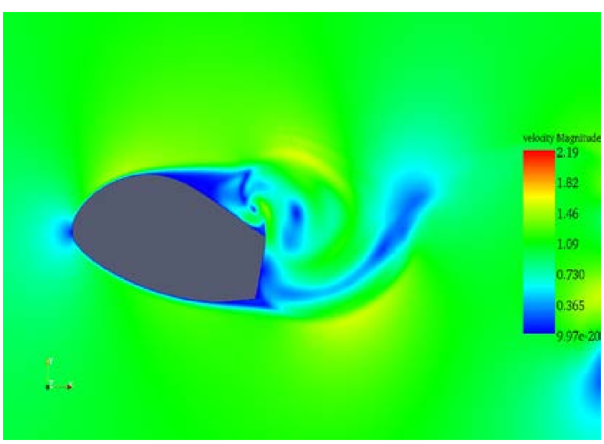

Figure 9 (a) : Velocity magnitude contours and flow structure when the lift forces $F_l = +0.54$ and t =39.90

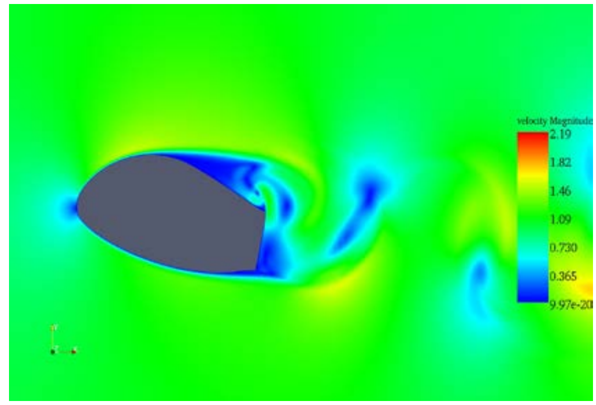

Figure 9 (b) : Velocity magnitude contours and flow structure when the lift forces $F_l = +0.57$ and $t = 45.70$

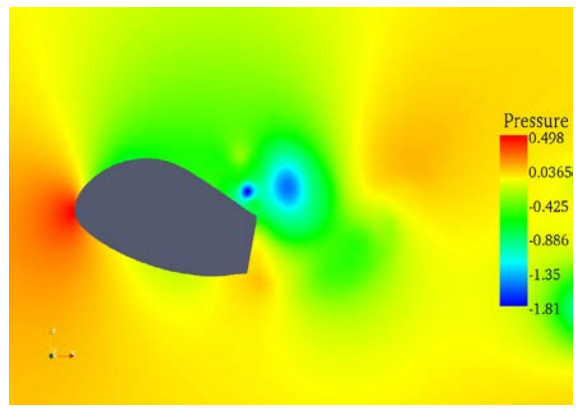

Figure 10 (a) : Pressure contours and flow structure when the lift forces $F_l = +0.54$ and $t = 39.90$

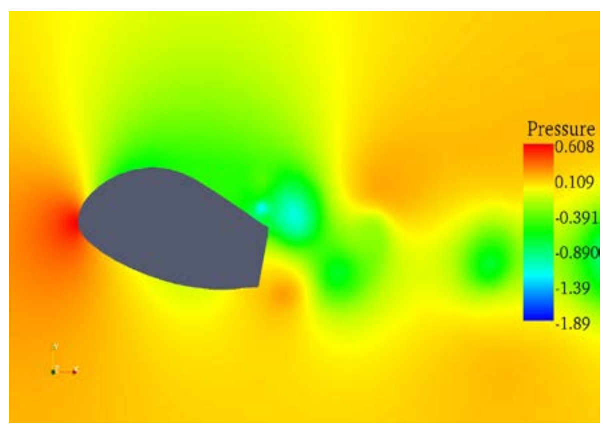

Figure 10 (b) : Pressure contours and flow structure when the lift forces $F_l = +0.57$ and $t = 45.70$

At the maximum negative lift forces F_l = - 0.71 (in Fig. 11a) *and* - 0.74 (in Fig. 11b), *and* the contours of velocity magnitude indicate at the time unit t = 40.40 and t = 50.90, respectively. The vortices just behind the trailing are completely opposite in structure when compared to the cases for positive lift forces. The separation has started at half chord length of the airfoil. Parabolic shaped vortices only at the upper part of the airfoil are responsible for the negative forces. One small region of lower pressure is shown at the lower part of the trailing edge in Fig. 12(a). Another bigger region of the lower pressure is shown at the upper part of the trailing edge in Fig. 12(b). Combinations of both create vortices at the upper surface of the trailing edge and are responsible for lift forces.

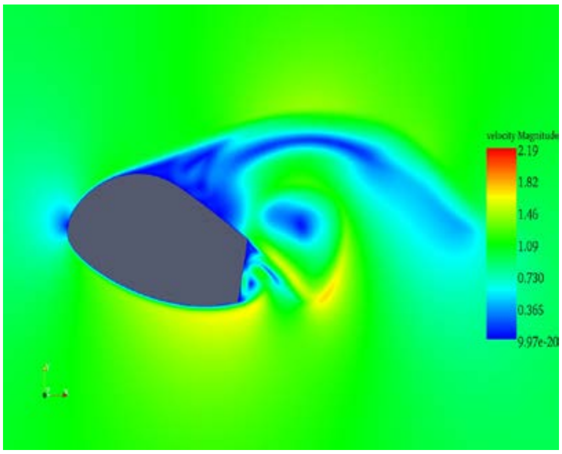

Figure 11(a) : Velocity Magnitude and flow structure when the lift forces F_l = -0.71 and t = 40.40

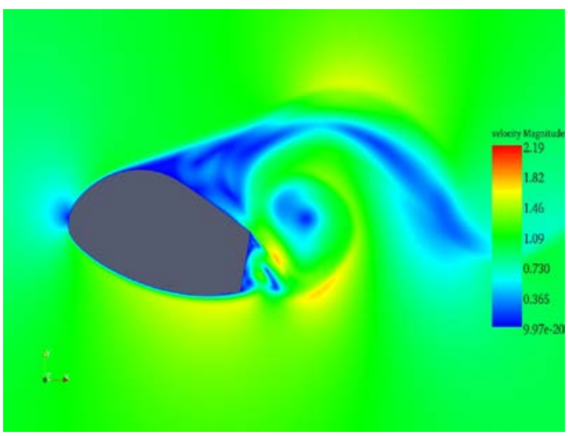

Figure 11 (b) : Velocity Magnitude and flow structure when the lift forces F_l = -0.74 and t = 50.90

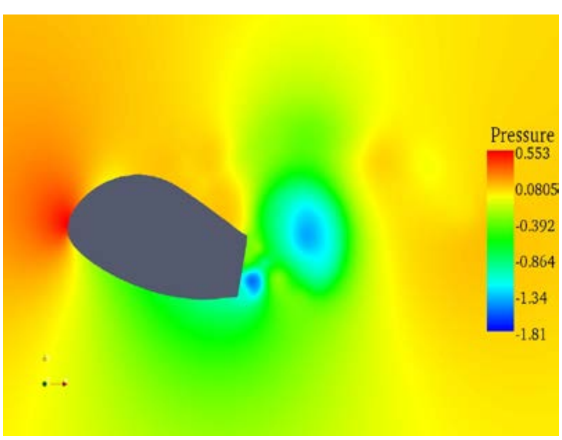

Figure 12 (a) : Pressure contours and flow structure when the lift forces F_l = -0.71 and t = 40.40

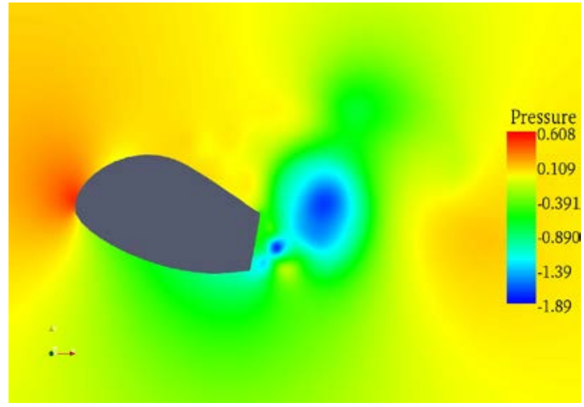

Figure 12 (b) : Pressure contours and flow structure when the lift forces F_l = -0.74 and t = 50.90

V. Summery and Discussions

The direct numerical simulations(DNS) were used on a stationary airfoil (fx77-w500) using an open source CFD solver, Nektar, based on spectral/hp method. The simulations were done at Reynolds number Re= 3900 and polynomial order P = 9. The contour plots of velocity magnitude and pressure were analyzed and the flow structure investigated around the airfoil and the wake. More vortices in the wake region were identified. Further studies should be simulated for longer time units (at least up to 100 time units) for 2D simulations with the objective to maximize realistic results. A comparison between *Nektar* and with any other CFD solver should be included. In this paper, 3D simulations were not carried out, so the 3D could be done for further analysis. Finally, it is also suggested that since in this work stationary airfoil (fx77-w500) was used for angle of an attack of 120. So, therefore it could be another option to do simulation for a different angle of attacks and for a moving airfoil.

Acknowledgement

The authors like to take this opportunity to thank all of the colleagues in the Turbulence, Wind Energy and Stochastic (TWIST) group well known as ForWind research group at the Carl von Ossietzky University Oldenburg, D-26111 Oldenburg, Germany.

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Global Journals Inc.

Global Journal of Science Frontier Research
Physics and Space Science
Volume 13 Issue 4 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Magneto Resistance of $In_xGa_{1-x}As$ Thin Film Prepared by MBE

By Alam. M. K

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Abstract - Magnetoresistance measurements were made on Thin Films of $In_xGa_{1-x}As$ from Electrical resistivity and Hall Effect measurements. Films used in this study were grown on single crystal GaAs substrate by Molecular Beam Epitaxial method. Measurements were made at room temperature and up to field strength of 5.5KG.

Resistivity of GaAs and $In_{0.14}Ga_{0.86}As$ at room temperature are 65.3 Ohm-cm and 7.77×10^{-1} Ohm-cm respectively and for samples with $x \approx 0.185$ and ≈ 0.205 resistivity at room temperature are 1.06×10^3 Ohm-cm and 2.54×103 Ohm-cm respectively. Hall coefficients were calculated for all samples. Temperature variations of these values are presented in a temperature region 273K to 323K. From the measurements it is found that GaAs and $In_{0.14}Ga_{0.86}As$ samples $\Delta R/R_0$ show a linear variation with magnetic field. But for $In_{0.185}Ga_{0.815}As$ and $In_{0.205}Ga_{0.795}As$ films $\Delta R/R_0$ initially decreases and then increases with increasing fields. Results obtained are explained in the light of the existing theories and they are found to be in good agreement with theories and also with the published results of other workers.

Keywords : resistivity, thin film, hall coefficient.

GJSFR-A Classification : FOR Code: 040406

MAGNETO RESISTANCE OF INXGA1-XAS THIN FILM PREPARED BY MBE

Strictly as per the compliance and regulations of :

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Keywords : resistivity, thin film, hall coefficient.

I. Introduction

The studies of thin film phenomena have attracted considerable attention in past three decades because of their potential applications such as magnetic memory devices (such as ROM, RAM etc.), hetero junction solar cells, liquid crystal displays, a variety of active and passive microminiaturized components and devices, radiation detections, and interference filters to the modern civilization. At present, for magnetic memory device the low cost production of semiconducting thin film has become a necessary precondition. The most modern development in the field of thin film physics is that of optoelectronic devices, photovoltaic, photoconductive and solid state laser devices are now under consideration of the experimental physicists of this respect.

The development of high speed high frequency optoelectronic devices as well as their integrated modulus and circuits shows the necessity of obtaining defect free III-V semiconductor compounds, substrates and active layers. The epitaxial growth technique is a method to obtain this improvement. Epitaxial film of III-V compounds are grown by three techniques: a) Liquid Phase Epitaxy (LPE[1]); b) Vapor Phase Epitaxy (VPE[2-4]); c) Molecular Beam Epitaxy (MBE[5]). The films used are deposited on a single crystal GaAs substrate by MBE. The thicknesses of all samples were 2.5 μ and shape of the samples were square and the surface area of each film was 5.64 mm^2. All measurements were made at atmospheric pressure.

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II. Experimental Measurements

a) Electrical Resistivity

Electrical resistivity of various samples of $In_xGa_{1-x}As$ was measured at 77K and from 273 K to 323 K. Vander Pauw's method was used to measure the electrical resistivity of the specimens. Four electrical contacts were made at the four corners; say A, B, C & D (Figure. 2.1) of the sample with silver paste. When a dc was passed through any two terminals of the sample, say A & B; an electrometer (Keithley 614) was used to measure the dc passing through AB, it produced a potential difference between the contact CD and a digital voltmeter was used to measure the voltage developed between C & D. A copper-constantan thermocouple was used to measure the temperature. If $I_{AB,}$ the dc entering a specimen through AB and $V_{CD,}$ the potential difference between CD, then the resistance $R_{AB,CD}$ is defined as V_{CD}/I_{AB}. Similarly $R_{BC,DA}$ and $R_{AC,BD}$ are defined as V_{DA}/I_{BC} and V_{AC}/I_{BD} respectively.

For our square shaped and constant thickness samples, the resistivity ρ is given by the expression

$$\rho = 2.266t\,(R_{AB,CD}+R_{BC,DA})\ \text{ohm-cm} \qquad \text{(i)}$$

where t is the thickness of the film.

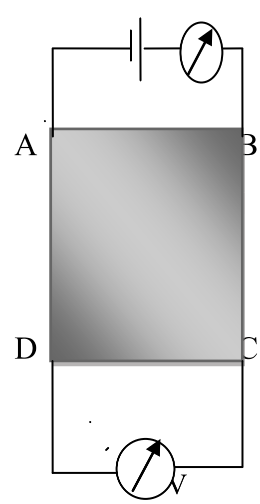

Figure 2.1 : Electrical contact of a film

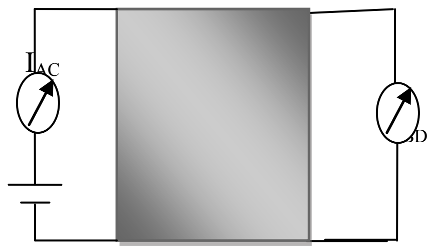

Figure 2.2 : Circuit for Hall Effect Measurement

b) Hall Effect

Van-der Pouw's method was also used for the measurement of Hall Effect. The (Van-der Pouw's) specimen provided with four electrical contacts was used for this measurement. The electrical circuit used in Hall Effect measurements is shown in Figure. 2. The specimen with the heating arrangement was placed in between parallel pole-pieces. The sample was placed between the pole faces in such a way that magnetic lines of forces are perpendicular to the faces of the sample. In our work the gap between pole-pieces was kept at 4 cm and fields from 0 to 5.5 KGs were used.

Hall effects were measured using conventional dc method. The current drawn from a voltage regulated power supply unit. Currents and voltages were measured by a digital electrometer (Keithley 614) and a digital multimeter. A dc from the power supply unit was passed between the contacts A and C. Variable magnetic field was then applied normal to the faces of the specimen and the corresponding Hall voltage was measured between the points B and D. That voltage was measured carefully with the help of a high precession digital multimeter.

c) Hall Constant

For a constant current between A and C terminals, voltages induced between B and D in zero field, say V_1 and at any other applied field, say V_2 were measured. Then the change in resistance $\Delta R_{AC,BD}$ was calculated by using the relation

$$\Delta R_{AC,\,BD} = (V_2 - V_1)/I \qquad \text{(ii)}$$

Nernst and Righi-Ledue effects were eliminated by reversing the current and the magnetic field and taking the average of the readings. Four sets of readings were taken first with I in positive direction and a pair of readings with the magnetic field normal and reversed and then reversing the current and another pair of readings with the magnetic field normal and reversed were taken. Average of these four readings eliminates all errors and Hall voltage was calculated by using the relation

$$R_H = \Delta R_{AC,\,BD}\,(10^8 t/H)\ \text{cm}^3/\text{Coul.} \qquad \text{(iii)}$$

where the magnetic field **H** is in Gauss and the film thickness **t** is in cm.

d) Magnetoresistance

The magnetoresistance of a crystal refers to the change of electrical resistance of a crystal when placed in the magnetic field. This effect is due to the fact that when the magnetic field is applied, the path of the electron becomes curved and do not go exactly in the direction of the superimposed electric field. If R_0 is the resistance of the film in zero fields at a given temperature and ΔR be the increase of resistance caused by the application of a magnetic field, then the ratio $\Delta R/R_0$ increases proportionally to H^2 for comparatively low values of the field, but proportionally to H for higher values.

Considering the material of the film have two types of carriers with same relaxation time τ. Then $\Delta R/R_0$ comes out as a function of τH. But τ itself will then be inversely proportional to the resistance R_0 ; we can write the Kohler's rule[20]

$$\frac{\Delta R}{R_0} = F\left(\frac{H}{R_0}\right) \qquad \text{(iv)}$$

where F is a function depending on the nature of the material itself.

III. Results

a) Electrical Resistivity

In the present work resistivity and Hall Effect of $In_xGa_{1-x}As$ films of 2.5 μ thicknesses were studied at liquid nitrogen temperature (77K) and also within 273K to 313K temperature range. The temperature variation of resistivity for different concentration of In are observed. For pure GaAs i.e. for x=0 the curve shows that with the increase in temperature, resistivity decreases gradually goes through minima around room temperature and then increases with temperature. For x=0.14 resistivity does not show such minima but there is a small but continuous increase in its value with the increase in temperature. For films with x=0.185 and x=0.205 Indium concentrations resistivity first increases then pass approximately through a flat region and then again increase with the decrease in temperature. In both the samples, the observed flat region appears slightly below the room temperature. A comparison of the resistivity of all films shows that the resistivity of the sample with x=0.14 Indium concentration is less than the value for GaAs by almost two order of magnitude but films with x =0.185 and x=0.205 Indium concentration the resistivity is higher than the value obtained for GaAs and this value increases with increasing concentration of Indium.

The variation of conductivity corresponding to resistivity curves for all these samples are also depicted in graph (Figure 3.1).

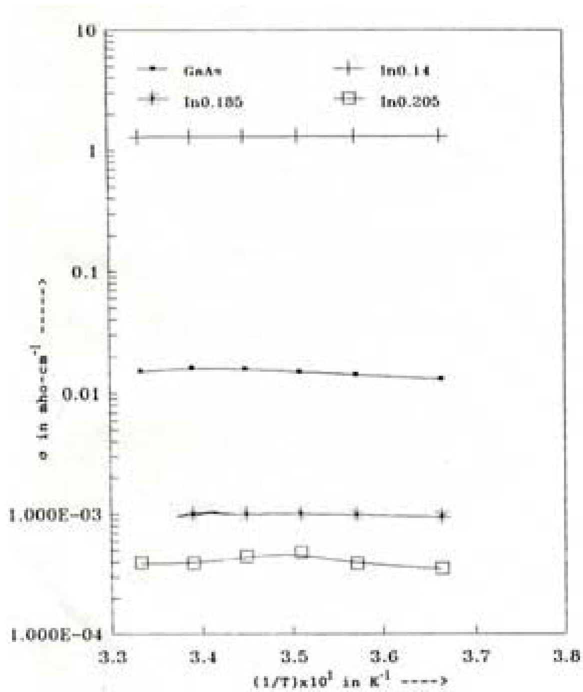

Figure 3.1 : Variation of conductivity with temperature of $In_xGa_{1-x}As$

b) Hall Effect

Measurements of Hall voltage are a natural compliment to resistivity measurements in studies of transport phenomena of crystalline semiconductors. Van-der Pouw's method was used for Hall Effect measurements. Hall Effect measurements were made on pure GaAs and on $In_xGa_{1-x}As$ at 77K and also within temperature range of 273 to 313K for magnetic field up to 5.5 KG. At 77K Hall coefficient remains almost constant with field for x=0.14 In composition but for pure GaAs and x=0.205 composition the value decreases with increasing field. At room temperature R value for GaAs and for x=0.14 remains almost invariant with field but for x=0.185 the value decreases first and then remain almost constant with field and for x=0.205 the value increases with field.

Hall coefficients were also measured at various temperatures. Temperature variation of Hall coefficient for these films is also observed. For GaAs, R_H decreases gradually with the increase in temperature, for x=0.l4 the value remain almost constant up to room temperature and then decreases with temperature. For x=0.185 concentration, R_H decreases with temperature similar to GaAs but for x=0.205 the value increases with increasing temperature.

Magnitude of R_H for GaAs, x=0.14, x=0.185, and x=0.205 at 300K are 1.20×10^5, 1.8×10^3, 1.7×10^5 and 2×10^6 cm^3/Coul respectively.

One interesting fact that was observed from Hall Effect measurement is that for GaAs and $In_xGa_{1-x}As$ with x=0.14 films, Hall coefficients are negative, so these two films are n-type and the carriers are electrons. But samples with x=0.185 and x=0.205 shows positive Hall coefficients, hence they are p-type and the carriers are holes.

c) Magnetoresistance

The magnetoresistance of a crystal refers to the change of electrical resistance of a crystal when placed in the magnetic field. This effect is due to the fact that when the magnetic field is applied, the path of the electron becomes curved and do not go exactly in the direction of the superimposed electric field. Transverse magnetoresistance measurements were made on $In_xGa_{1-x}As$ (with x=0, 0.14, 0.185 & 0.205) at room temperature up to a field strength of 5.5 KG. Fig 3.2 shows the plot of $\Delta R/R_0$ versus H for different samples at 300 K. For films with x=0 and x=0.14 composition $\Delta R/R_0$ shows a liner variation with field. But for films with x=0.185 and x=0.205 composition $\Delta R/R_0$ initially decreases and then increases with increasing field.

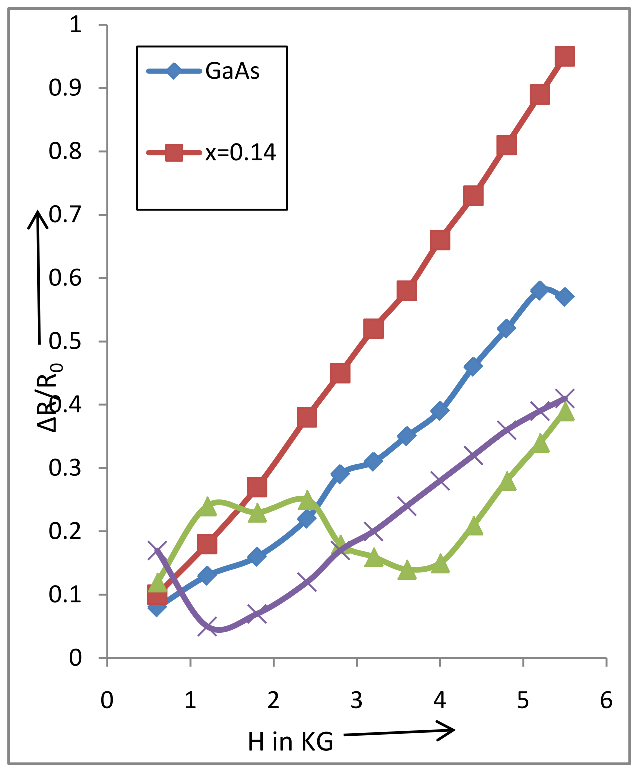

Figure 3.2 : $\Delta R/R_0$ versus H graph for $In_xGa_{1-x}As$

IV. DISCUSSIONS

Compounds formed by the elements of the third and fifth column of the periodic table crystallize with spherelite structure. In this compound since each group III atom is tetrahedrally surrounded by group V atoms and vice-versa it is reasonable to assume that, on the average each atom has four valence electrons. This suggests that the bonding has a covalent character and that the semiconducting properties of these types of compounds are similar to those of the corresponding group IV elements.

Single phase solid solutions of ternary III-V mixed crystal system such as $In_xGa_{1-x}As$ alloy can be prepared over the complete compositional range[9] $0 \leq x \leq 1$. Optical-absorption measurements show that the band gap varies monotonically from GaAs to InAs[9-11] and is believed to remain direct in the alloy system.

In the preparation of our samples a crystalline GaAs was used as a substrate for the epitaxial growth of $In_xGa_{1-x}As$. Since lattice constant of $In_xGa_{1-x}As$ is larger than that of GaAs, a difference of the lattice constants between GaAs and $In_xGa_{1-x}As$ gives rise to lattice mismatching at the interface. Crystal imperfections due to this lattice mismatching cause a degradation of electrical characteristics of the epitaxial layers. The surface of $In_xGa_{1-x}As$ layer in the case of small composition 'x' usually is smooth as that of GaAs epitaxial layers. The surface degrades with increasing In composition. The degradation of the crystal is attributed to accumulation of the strain due to the lattice mismatching. These lattice mismatches induces defects into the epitaxial layer and thus greatly influence the electrical properties of the layers.

Therefore the cause of resistivity must be sought in the deviations from the perfect regularity of the potential in which the electrons move. Deviations from the periodicity of the potential causing resistivity are due to (i) Lattice vibration, (ii) Lattice defects such as vacancies, interstitial and dislocations, (iii) The order in the lattice and (iv) Grain boundaries.

According to electronic theory of solids the electrical conduction or electrical resistivity results from the scattering of these electrons by lattice. Electron can pass through a perfect lattice without any attenuation. Actually no lattice is perfect. Even a lattice which has no structural defects or foreign atoms cannot be completely regular at any temperature.

From our Hall Effect measurements we observed that Hall coefficients are negative for GaAs and $In_xGa_{1-x}As$ with x=0.14. For pure GaAs and with x=0.14 when the temperature is increased conductivity increases.

As mentioned in the theory of magnetoresistance when a magnetic field is applied perpendicular to the direction of current flow in a metal or semiconductor, a change of resistance is observed. The magnetoresistance is usually measured as a change of resistance ΔR relative to the resistance in zero magnetic field R_0. Magnetoresistance arises because groups of conduction electrons have different mean velocities. Hence although the transverse electric field on the average balances the effect of magnetic field, individual electron will travel in a curved path. In crystal line semiconductors, measurements of magnetoresistance in a magnetic field, like Hall Effect, can be used to determine carrier motilities and also gives information concerning the energy surface near the edge of the conduction and valance bands.

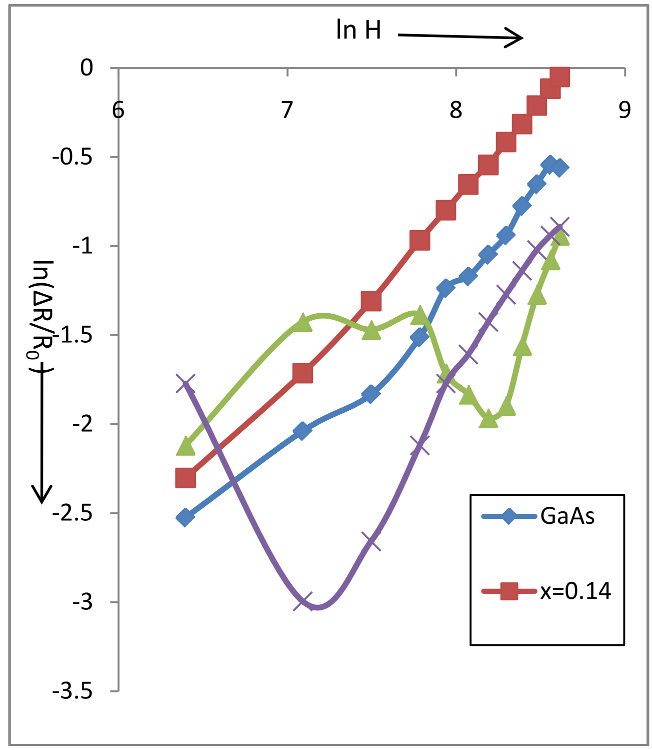

Figure 3.3 : ln ($\Delta R/R_0$) vs lnH graph for $In_xGa_{1-x}As$

Normally $\Delta R/R_0$ is positive and proportional to B^n, with n=2 for low field and n=1 for high field. Magnetoresistarice measured at 300 K on GaAs and $In_xGa_{1-x}As$ as presented in Figure 3.2 shows that for GaAs and for x=0.14 composition magnetoresistance is positive. From $\ln(\Delta R/R_0)$ vs. lnH graph (Figure3.3) we obtained the value of n=1, that is magnetoresistance for these two samples are found to be proportional to H up to 5.5 KG field. So the behavior follows the Kohler's rule approximately[20]. For sample with x=0.185, n=2 at low field up to 2.0 KG, within a field of 2.3 KG n=2 and above this field n=1.2. But for sample with x=0.205 of In concentration, value of n was more than 2. These two samples also do not follow the Kohler's rule. This is the evidence that in our films particularly films with x=0.185 and x=0.205 In composition different types of scattering mechanism are in operation and they are having different effect on the different group of carriers.

Acknowledgements

Author is grateful to his supervisor Professor Dr. M. G. M Chowdhury for invaluable guidance and Professor Dr. M. Obaidul Hakim for his constant help to carry out this research work.

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Global Journal of Science Frontier Research
Physics and Space Science
Volume 13 Issue 4 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

On Contributions of S. T. Yau in Mathematics and Physics Related to General Relativity

By C. Y. Lo
Applied and Pure Research Institute, Nashua

Abstract - Due to inadequacy in pure mathematics among physicists, the non-existence of dynamic solutions for the Einstein equation was not recognized. Instead, his field equation was perpetuated. The positive energy theorem of Schoen and Yau also continues such an error. Although Yau may not have made errors in mathematics, their positive energy theorem produced not only just useless but also misleading results in physics. Since the Einstein equation does not have a bounded dynamic solution, the condition of asymptotically flat implies that the case can only be a static solution. Thus, their positive energy theorem actually does not have a meaningful application in physics. Yau failed to see this problem of misleading since he has not attempted to find explicit examples to illustrate their theorem. Moreover, E. Witten made the same mistake in his alternative proof, but that was also cited as an achievement for his Fields Medal in 1990. Concurrently, outstanding mathematical contributions of Yau to the completion of the Poincare conjecture are pointed out. Related issues are also discussed. 04.20.-q, 04.20.Cv

Keywords : the positive energy theorem; asymptotically flat; dynamic solution.

GJSFR-A Classification : FOR Code: 010504

ON CONTRIBUTIONS OF S. T. YAU IN MATHEMATICS AND PHYSICS RELATED TOGENERAL RELATIVITY

On Contributions of S. T. Yau in Mathematics and Physics Related to General Relativity

C. Y. Lo

Abstract - Due to inadequacy in pure mathematics among physicists, the non-existence of dynamic solutions for the Einstein equation was not recognized. Instead, his field equation was perpetuated. The positive energy theorem of Schoen and Yau also continues such an error. Although Yau may not have made errors in mathematics, their positive energy theorem produced not only just useless but also misleading results in physics. Since the Einstein equation does not have a bounded dynamic solution, the condition of asymptotically flat implies that the case can only be a stable solution. Thus, their positive energy theorem actually does not have a meaningful application in physics. Yau failed to see this problem of misleading since he has not attempted to find explicit examples to illustrate their theorem. Moreover, E. Witten made the same mistake in his alternative proof, but that was also cited as an achievement for his Fields Medal in 1990. Concurrently, outstanding mathematical contributions of Yau to the completion of the Poincare conjecture are pointed out. Related issues are also discussed. 04.20.-q, 04.20.Cv

Keywords : the positive energy theorem; asymptotically flat; dynamic solution.

I. Introduction

In my recent paper [1], "On the Nobel Prize, Controversies and Influences" and related papers [2-4], the problem of inadequate background in pure mathematics among physicists is identified as one of the main reasons that the errors in general relativity have a long life. Here, it is pointed out further that an inadequate background in physics among mathematicians also contributes to such a long life for the errors. Understandably, mathematicians usually rely on physicists for the validity of the physical conditions being used. However, for the case of general relativity, this turns out to be problematic.

For instance, Einstein [5] claimed that his field equation provides a solution for the perihelion of Mercury, and a renounced mathematician D. Hilbert also approved the related calculation of Einstein [6] which is based on a perturbation approach. However, Gullstrand [7], Chairman of the Nobel Prize for Physics (1922-1929) still suspected that Einstein's calculation is invalid. This controversy was not solved until 1995 when Gullstrand was proven correct [8]. It is interesting to note that Hilbert did not make any mistake in mathematics, but he was not aware Einstein's calculation on the perihelion Mercury requires an implicit perturbation approach of calculation for a many-body problem. The problem is, however, that the Einstein field equation cannot have a dynamic solution even for a two body problem [8]. Thus, the claim that general relativity is already an over all improvement from Newtonian theory is actually exaggerated.

Since the non-existence of dynamic solutions for the Einstein equation was not recognized at the beginning, subsequently based on such a false result, mathematicians naturally obtain further incorrect conclusions. For instance, if physics requires a bounded solution for a two-body problem such as the binary pulsars of Hulse and Taylor, the coupling constants of the Einstein equation cannot be unique [4]. However, the physical assumption of unique sign for all the coupling constants is the crucial implicit assumption of the space-time singularity theorems of Penrose and Hawking [9].

II. Yau's Work and Errors on General Relativity

Another example to illustrate this problem is the work of mathematician Yau. Now, it is clear that the contribution of Prof. Yau to physics is at least over estimated and even incorrect while his contribution to mathematics in other areas is underestimated. From the free encyclopedia Wikipedia, the contributions of Professor Yau were summarized as follows:

"Yau's contributions have had a significant impact on both physics and mathematics. Calabi–Yau manifolds are among the 'standard tool kit' for string theorists today. He has been active at the interface between geometry and theoretical physics. His proof of the positive energy theorem in general relativity demonstrated—sixty years after its discovery—that Einstein's theory is consistent and stable. His proof of the Calabi conjecture allowed physicists—using Calabi-Yau compactification—to show that string theory is a viable candidate for a unified theory of nature."

Thus, it was claimed that Yau's "proof" of the positive energy theorem [10, 11] in general relativity would have profound influence that leads to even the large research efforts on string theory. Based on his proof, it was claimed that Einstein's theory is consistent and stable. This would be in a direct conflict with the fact that there is no dynamic solution for the Einstein equation. Moreover, Yau's proof would also be

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inconsistent with the space-time singularity theorems of Penrose and Hawking that use the same implicit assumption of unique sign for all the coupling constants.

Thus, it is clear that, in terms of physics, there should be problems in Yau's proof since in 1995 Einstein's theory has been proven inconsistent without the necessary rectifications [12]. The controversy of whether the Einstein equation has dynamic solutions was raised in 1921 by Gullstrand [7]. Now, this conclusion of non-existence of dynamic solution is no longer in doubt because there are explicit examples at the undergraduate level to illustrate such mathematical errors [2, 4].

Moreover, there are some well-known problems related to this issue. To mention a few, they are as follows:

1. It is known that Einstein's covariance principle is not valid as pointed out by Zhou Pei-Yuen of Peking University [13,14] and its invalidity is proven recently with counter examples [15]. Being an outstanding physicist, Zhou actually identified this problem and questioned Einstein directly when they met in 1937 although Zhou wrote papers on this subject only after 1982. On the other hand, Yau probably still believed Einstein's covariance principle since C. N. Yang still incorrectly advocates his notion of unconditional gauge invariance [16,17].
2. A related problem is Einstein's theory of measurement since the covariance principle was created to remedy the short comings of his theory of measurement. Note that such a theory was criticized by Whitehead as clearly invalid in physics [18] and is inconsistent with the observed light bending [19]. However, these criticisms were rejected although the light bending is calculated alternatively to avoid the obvious conflict [20]. On the other hand, Yau probably was unaware that Einstein's theory of measurement would lead to inconsistency in the Schwarzschild solution [1, 4].
3. A major reason that theorists held on to Einstein's covariance principle is due to that his theory of measurement was justified with special relativity. Recently, it is proven that Einstein's justifications are based on invalid applications of special relativity. Thus, the invalidity of Einstein's covariance principle is finally settled [12]. On the other hand, Yau probably was unaware that Einstein failed to justify his theory of measurement.
4. In fact, Einstein's covariance principle and Einstein's equivalence principle [5, 21] actually are directly in conflict [22]. However, many misidentified Einstein's equivalence principle of 1916 wrongly as the equivalent assumption of 1911 [23], which have been proven invalid by observations [19]. Moreover, due to an inadequate background in mathematics, the Wheeler School and her associates [1, 18] misinterpreted Einstein's equivalence principle, and created the invalid notion of local Lorentz invariance [24] (as a distortion of the Einstein-Minkowski condition [5, 21]). On the other hand, Yau probably was unaware of this because it was likely that Yau learned general relativity through second hand information [9, 20, 25] instead of from the work of Einstein directly [5, 21].
5. The famous formula E = mc^2 is only conditionally valid [26]. In fact, Einstein failed to prove the formula for other cases beyond the photons in spite of his efforts of several years (1905-1909) [27]. Moreover, Einstein or others actually have not completed the proof even for the case of photons [28]. It is safe to assume that Yau was unaware of this common error among physicists.
6. It has been proved in 1993 [29] and published in 1995 [8, 30] that there is no dynamic solutions for the Einstein equation. In fact, Yau and I have discussed this issue in the Chinese University of Hong Kong in 1993. It was expected that Yau would improve general relativity, but he only avoided this issue by claiming a loss of earlier interest.
7. It has been further proved that the book of Christodoulou and Klainerman [31] that claimed the existence of a dynamic solution (published by Princeton University as a mathematical classic) is actually invalid [32-34], due to elementary errors at undergraduate level in mathematics. I was informed that Yau also avoided the participation in the selection of Christodoulou for the 2011 Shaw Prize for mathematics [2].

Nevertheless, after the necessary rectifications, Einstein's general relativity can prove the need of unification, a dream of Einstein that he himself failed to complete [1]. Thus, one may wonder what went wrong in Yau's proof. Many believed that Yau and Richard Schoen proved the long-standing conjecture that the total mass in general relativity is positive.

Now, let us examine their theorem that would imply that flat space-time is stable, a fundamental issue for the theory of general relativity. Briefly, the positive mass conjecture says that if a three-dimensional manifold has positive scalar curvature and is asymptotically flat, then a constant that appears in the asymptotic expansion of the metric is positive. A crucial assumption in the theorem of Schoen and Yau is that the solution is asymptotically flat. However, since the Einstein equation has no dynamic solution, which is bounded, the assumption of asymptotically flat implies that the solution is a stable solution such as the Schwarzschild solution, the harmonic solution, the Kerr solution, and etc.

Therefore, Schoen and Yau actually prove a trivial result that the total mass of a stable solution is positive. Note that since the dynamic case is actually excluded from the consideration in the positive energy theorem, this explains why it was found from such a theorem that Einstein's theory is consistent and stable. This is, of course, misleading.

In fact, Yau [10] and Christodoulou [31] make essentially the same error of defining a set of solutions that actually includes no dynamic solutions [32-34]. Their fatal error is that they neglected to find explicit examples to support their claims. Had they tried, they should have discovered their errors.

The problem started around 1981 because Yau did not understand the physics of the non-linear equation of Einstein. Note that Yau has wisely avoided committing himself to the errors of Christodoulou & Klainerman, by claiming that his earlier interest has changed [31]. However, he was unable to see that the binary pulsars experiment of Hulse & Taylor not only confirms that there is no dynamic solution but also that the signs of coupling constants are not unique [32]. In fact, Yau has made the same errors of Penrose and Hawking [9], and implicitly uses the invalid assumption of unique sign in his positive energy theorem of 1981. Nevertheless, Prof. Yau is a good mathematician as shown by his other works although he does not understand physics well. Since the Einstein equation must be modified for a dynamic case, their positive energy theorem is also irrelevant to physics just as the space-time singularity theorem of Penrose and Hawking.

Unfortunately, Yau also made the crucial mistake of assuming that famous theorists would understand physics correctly. Moreover, even Einstein and Princeton University can make a crucial error in spite of the warning of Gullstrand [7], a member of the 1921 Nobel Committee. Who could have discovered that the Wheeler School actually does not understand Einstein's equivalence principle if you do not read carefully and question them [3]?

Due to the influences of the Wheeler School, general relativity is incorrectly believed as effective only for large scale problems. Thus, the study for the applications of general relativity on earth and understanding material structure is neglected [1-4]. Moreover, due to the invalid speculation of unconditional E = mc^2, the study of unification between electromagnetism and gravitation failed. The facts that Hilbert and Yau were unable (or neglected) to identify their errors, they misleadingly created a false impression that the Wheeler School did not make errors in mathematics.

In China, it seems that there is little theoretical progress in the field of gravitation after Zhou Pei-Yuan. Currently, the incorrect views of L. Z. Fang and C. N. Yang dominate the opinions of the Chinese academy. C. N. Yang incorrectly disagrees with Zhou Pei-Yuan because of the invalid notion of gauge invariance [16, 17]. However, experiments of gravitational physics have very interesting developments although this is not matched by their theoretical developments [35].

III. Important Omissions On Yau's Contribution In Mathematics

While some physicists, due to their own errors, keep attributing credits to Yau's earlier work in physics, his interest has shifted to mainly mathematics since 1993. It is known that Yau has an ambition on solving the Poincare conjecture, and he does make important contribution as pointed out by Hamilton. However, after Yau's contribution at the early stage and the subsequent important progress of Hamilton, unexpectedly Perelman claimed a complete proof by pointing out several sub-conjectures without the necessary explicit proofs that should have been there.

Being a seasoned mathematician in this field, Yau probably has already understood the extent of difficulty in obtaining such a proof. Naturally, Yau would doubt the claim of Perelman as genuine. And thus, in my view and probably in the view of Yau, Perelman's claim is a way to gain extra time needed for his efforts. In any case, there is no harm done to complete the proof himself with a team including former students Cao and Zhu. After more than three years, Yau's team succeeded as the first in completing the proof, and Yau wisely just acts as a referee. This step makes a speedy publication of the paper and thus denies the competitors the opportunity to play a delay tactic. This further infuriated the losers and they become irrational and decided to deny the due credit of the team.

However, Yau's team did nothing wrong although their action is rather unusual. Nevertheless, in Wikipedia Yau's crucial contribution in solving the Poincare conjecture is omitted. It is well known that he leads Cao and Chu to finish the final stage of solving the Poincare conjecture. Although Yau did the pioneer work that lays the foundation before even Hamilton, who laid the subsequent basis for solving the conjecture, some throw mud at Yau and give such a credit unfairly to Perelman, who has a very considerable contribution, but with questionable honesty.

In fact, Perelman only created several sub-conjectures of which most were proven correct years later. However, he claimed having solved them all when published in the NET (and thus avoided questioning). Nevertheless, one of his sub-conjectures is still not yet proved not only after Cao and Zhu have completed the proof of the Poincare conjecture but also unclear even now. It is well known that conjectures are much easier to make than to prove them. This is especially true for mathematical analysis. Also, Perelman's several unexplained acts of disappearance would strengthen the suspicion of avoiding questions from colleagues after having taken such a deceptive short cut [36]. The fact remains that there is no evidence that Perelman has actually proved the conjecture. Nevertheless, some theorists still unfairly claimed that Perelman has solved

Global Journal of Science Frontier Research (A) Volume XIII Issue IV Version I Year 2013

the conjecture, but they make such an approval months after the publication of the work of Cao and Zhu.

It is amazing that not only nobody from the mathematical community of China, except Yau's students, stands out for the team of Yau et al. for a fair credit. On the contrary, Tian (Yau's student) of Peking University made shameless statements of belittling their Chinese colleagues. Professor Tian also distinguished himself by telling Perelman privately that he is correct soon after he made the sub-conjectures – an act ignored by Perelman as probably fishing for information [36]; but failed to claim Perelman's being essentially correct publicly immediately after the publication of Cao and Zhu.

IV. Discussions And Conclusions

It has been shown that two out-standing mathematicians Hilbert and Yau produced useless misleading results in physics because they do not understand the related physics although they may still be mathematically correct. Nevertheless, their reputation may have misleadingly covered up the errors in physics with seeming validity in mathematics. Hilbert later recognized his mistake of neglecting the crucial many-body problem. Yau is not as lucky because he did not try to find explicit examples to illustrate his conclusion. Thus, for the benefit of those working on gravitation, the misleading nature of the positive energy theorem of Schoen and Yau must be exposed.

A lesson to be learned is that, to avoid misleading understanding in physics, it is necessary to provide examples to illustrate crucial conclusions. The facts that S. T. Yau was awarded a Fields Medal in 1982 1), for which the positive energy theorem is cited as an achievement and that E. Witten was awarded another Fields Medal in 1990, for which an alternative proof of the positive energy theorem is cited as an achievement,2) testifies that many pure mathematicians and applied mathematicians just do not understand physics. Thus, an error of Einstein in physics [1] becomes a formidable problem in mathematics with a misleading result in physics. Since Witten is leading the research in the string theory, understandably there is little useful result in physics.3) Moreover, from the experience of general relativity, it is clear that the development of theoretical physics requires improvements in the communication between physicists and mathematicians.

Currently, it is generally recognized that applied mathematics is a necessary skill in physics, but many incorrectly regarded pure mathematics as not useful in physics. Moreover, errors were made because theorists often confuse mathematic and physics [1]. These must be resolved. If the Nobel Prize Committee also establishes a prize in mathematics, the communication between physicists and mathematicians would be greatly improved [1]. Moreover, from the experience of Yau, it is clear that the Chinese academy is still not strong enough in some areas of physics and mathematics. This is manifested by the fact that she seldom has a voice that is independent to their colleagues outside China.

Acknowledgments

The author gratefully acknowledges stimulating discussions with C. Wong. Special thanks are to S. Holcombe for valuable suggestions. This work is supported in part by Innotec Design, Inc., USA. and the Chan Foundation, Hong Kong.

Endnote

1) S. T. Yau, 1982, Fields Medal, for "his contributions to partial differential equations, to the Calabi conjecture in algebraic geometry, to the positive mass conjecture of general relativity theory, and to real and complex Monge–Ampère equations".
2) Ludwig D. Faddeev, the Chairman of the Fields Medal Committee, wrote ("On the work of Edward Witten"):
   "Now I turn to another beautiful result of Witten proof of positivity of energy in Einstein's theory of gravitation.
   Hamiltonian approach to this theory proposed by Dirac in the beginning of the fifties and developed further by many people has led to the natural definition of energy. In this approach a metric γ and external curvature h on a space-like initial surface $S^{(3)}$ embedded in space-time $M^{(4)}$ are used as parameter in the corresponding phase space. These data are not independent. They satisfies Gauss-Codazzi constraints highly non-linear PDE, The energy H in the asymptotically flat case is given as an integral of indefinite quadratic form of $\nabla\gamma$ and h. Thus, it is not manifestly positive. The important statement that it is nevertheless positive may be proved only by taking into the account the constraints – a formidable problem solved by Yau and Schoen in the late seventy as Atiyah mentions, 'leading in part to Yau's Fields Medal at the Warsaw Congress'.
   Witten proposed an alternative expression for energy in terms of solutions of a linear PDE with the coefficients expressed through γ and h ….."
3) *Time* magazine stated that E. Witten was widely thought to be the world's greatest living theoretical physicist. This is, of course, based on the assumption that if his string theory is right. However, Witten once called string theory "a bit of 21st century physics that somehow dropped into the 20th century." So, Witten's physics is essentially based on imagination rather than observation. In fact, Witten didn't even set out to be a scientist. He majored in history at Brandeis and originally planned to be a journalist but ended up getting a Ph.D. in physics

instead. Thus, his work is often elegant in mathematics, but inadequate in physics. For instance, he gives an elegant alternative proof for the positive energy theorem, but failed to see that such a theorem has little meaning in physics since it would be valid only for the static case because of the constraints used.

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Global Journal of Science Frontier Research
Physics and Space Science
Volume 13 Issue 4 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

On the Invalidity of the Hawking-Penrose Singularity 'Theorems' and Acceleration of the Universe from Negative Cosmological Constant

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Abstract - Hawking and Penrose proposed "A new theorem on spacetime singularities … which largely incorporates and generalizes the previously know results" which they claimed "implies that space-time singularities are to be expected if either the universe is spatially closed or there is an 'object' undergoing relativistic gravitational collapse (existence of a trapped surface)" and that their 'Theorem' applies if four certain physical conditions are satisfied. Hartle, Hawking and Hertog have proposed a quantum state with wave function for the Universe which they assert "raises the possibility that even fundamental theories with a negative cosmological constant can be consistent with our low-energy observations of a classical, accelerating universe." They also relate this concept to string cosmology. It is however proven in this paper that the Hawking-Penrose Singularity 'Theorem' and accelerated expansion of the Universe with negative Λ are invalid because they are based upon demonstrably false foundations relating to Einstein's field equations, trapped surfaces, and the cosmological constant.

Keywords : hawking-penrose singularity theorem, trapped surface, negative cosmological constant, einstein's field equations, conservation of energy and momentum.

GJSFR-A Classification : FOR Code: 010599

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I. Introduction

Hawking and (1970) proposed in their paper '*The singularities of gravitational collapse and cosmology*' what they claim is a theorem which, "*implies that space-time singularities are to be expected if either the universe is spatially closed or there is an 'object' undergoing relativistic gravitational collapse (existence of a trapped surface)*".

They also assert that their Theorem applies if the following conditions are also satisfied,

i. *"Einstein's equations hold (with zero or negative cosmological constant),*
ii. *the energy density is nowhere less than minus each principal pressure no less than minus the sum of the three principal pressures (the 'energy condition'),*
iii. *there are no closed timelike curves,*
iv. *every timelike or null geodesic enters a region where the curvature is not specifically aligned with the geodesic."*

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To disprove the Hawking-Penrose Singularity Theorem requires only disproof of one of the conditions the Theorem must satisfy. Nonetheless, all of the required conditions are proven invalid herein.

In their paper '*Accelerated Expansion from Negative Λ*', Hartle, Hawking and Hertog (2012) have proposed a quantum state of the Universe with a wave function in a theory of gravitation which they claim,

"… raises the possibility that even fundamental theories with a negative cosmological constant can be consistent with our low-energy observations of a classical, accelerating universe."

This they maintain has implications for String theoretic cosmological models:

"Treating the classical behavior of our universe as an emergent property of a fully quantum mechanical treatment of cosmology opens up new possibilities for building models of inflation in string theory."

Hertog (Grossman 2012) states further,

"We have a new route towards constructing string theory models of our world."

In addition they imbue Big Bang creation with quantum cosmology:

"… quantum cosmology enables us not just to calculate quantum processes near the big bang but also gives us a deeper understanding of our universe at the classical level today."

However, the arguments adduced by all these authors have no valid basis in science bearing in mind that their concepts and arguments are based on the General Theory of Relativity which is easily proven to violate the usual conservation of energy and momentum and to not predict the black hole (Crothers 2005, 2008, 2010, 2012, 2012b), with or without cosmological constant, as proven herein. It therefore matters not whether the cosmological constant is positive, zero or negative since the General Theory of Relativity is in conflict with experiment on a deep level, rendering it invalid and hence the Big Bang creation cosmology with its alleged expansion of the Universe fallacious, along with the conditions stipulated by Hawking and Penrose for their Singularity Theorem.

Global Journal of Science Frontier Research (A) Volume XIII Issue IV Version I Year 2013

II. Invalidity of Singularity Theorem

By way of preamble it is curious that Hawking and (1970) and Penrose (2002) repeatedly refer to the "*pull of gravity*" and "*gravitational collapse*". This implies the Newtonian concept of a force of gravitational attraction. However, in General Relativity gravity is *not* a force that produces a pull or attraction on anything because Einstein's gravitation is due to a curvature of spacetime induced by the presence of matter.

"Mass acts on spacetime, telling it how to curve. Spacetime in turn acts on mass, telling it how to move." (Carroll and Ostlie 1996)

Matter according to Einstein (1916) consists of mass and electromagnetic radiation, and by implication from the latter, also charge. Furthermore, 'gravitational collapse' is not explained by Hawking and Penrose in terms of either Newtonian theory or General Relativity. They merely alleged the existence, without qualification, of the unobserved process of irresistible "*gravitational collapse*", which is the case in the literature generally to the present day. For example, in the 'Introduction' of their paper, Hawking and Penrose (1970) remark,

"An important feature of gravitation, for very large concentrations of mass, is that it is essentially unstable. This is due, in the first instance, to its r^2 attractive character. … The pull of gravity is readily counteracted by other forces. … a star of mass greater than 1.3 times that of the Sun, … cannot sustain itself against its own gravitational pull, so a gravitational collapse ensures. … some form of gravitational collapse may be taking place in quasars, or perhaps in the centres of (some?) galaxies. Finally, on the scale of the universe as a whole, this instability shows up again in those models for which the expansion eventually reverses, and the entire universe becomes involved in a gravitational collapse."

Hawking and Penrose (1970) also remark,

"Finally, on the scale of the universe as a whole, this instability shows up again in those models for which the expansion eventually reverses, and the entire universe becomes involved in a gravitational collapse. In the reverse direction in time there is also the 'big bang' initial phase which is common to most relativistic expanding models."

In Section '3. THE THEOREM', of their paper, Hawking and (1970) say,

"*We expect trapped surfaces to arise when a gravitational collapse of a localized body (e.g. a star) to within its Schwarzschild radius takes place, which does not deviate too much from spherical symmetry.*"

In the same section of their paper they write Einstein's field equations as,

$$R_{ab} - \frac{1}{2} R g_{ab} = -K T_{ab} \tag{1}$$

which they number as equation (3.5) therein and in which the constant $K > 0$. It is this form of Einstein's field equations upon which Hawking and Penrose (1970) rely for their Singularity Theorem, although they also remark in relation to eq. (1) above,

"To incorporate a cosmological constant λ, we would have to replace T_{ab} with $T_{ab} + \lambda K^{-1} g_{ab}$."

In this case eq. (1) becomes,

$$R_{ab} - \frac{1}{2} R g_{ab} = -K T_{ab} - \lambda g_{ab} \tag{2}$$

Consider the notion of a trapped surface. The reference to "*gravitational collapse of a localized body (e.g. a star) to within its Schwarzschild radius*" necessarily refers to the formation of a black hole related to Einstein's so-called *static* field equations for *empty space*, i.e. when $T_{\mu\nu} = 0$ (Dirac 1996, Eddington 1960, Foster & nightingale 1995, Tolman 1997), given by,

$$R_{\mu\nu} = 0 \tag{3}$$

Now the black hole allegedly associated with eq. (3) is alone in its universe. Indeed,

"Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat space-time." (Matzner 2001)

In fact, *all* alleged black hole solutions to Einstein's field equations pertain to a spatially infinite Universe which is eternal, contains only *one* mass, is not expanding, and is asymptotically flat (Crothers 2008, 2010). Hilbert's solution, Schwarzschild's (1916) actual solution, and the Reissner-Nordström (charged) black hole, are both very easily seen to be asymptotically flat. The Kerr solution (rotating black hole), and the Kerr-Newman (charged and rotating) black hole are also asymptotically flat (Penrose 2002) (note that Penrose's metric (1) is metric (4) given below):

"The Kerr-Newman solutions … are explicit asymptotically flat stationary solutions of the Einstein-Maxwell equation ($\lambda = 0$) involving just three free parameters m, a and e. As with the metric (1), the mass, as measured asymptotically, is the parameter m (in gravitational units). The solution also possesses angular momentum, of magnitude am. Finally, the total charge is given by e. When a = e = 0 we get the Schwarzschild solution." (Penrose 2002)

However, a localised body such as a star is *not* the only mass present in the actual Universe and since it is present amidst very many stars and galaxies its associated spacetime is not asymptotically flat. So the notion that a localised body such as a star can form a 'Schwarzschild' black hole, and hence a trapped surface at its 'Schwarzschild radius', or indeed any type of black hole, by means of irresistible gravitational collapse, is in conflict with the defining characteristics of a black hole because the said star and its associated black hole with trapped surface by means of gravitational collapse is not alone in a Universe that is spatially infinite, eternal, asymptotically flat, and not expanding. Furthermore, the notion of a black hole with its trapped surface is also in conflict with Big Bang cosmology because Hot Big Bang pertains to a Universe that is said to be finite both spatially and temporally, is expanding, is not asymptotically flat, and contains radiation and many masses, including many black holes some of which are claimed to be primordial.

Since a star is not the only mass in the Universe and since there are no known solutions to Einstein's field equations for two or more masses and no existence theorem by which it can even be asserted that the field equations contain latent solutions for two or more masses (McVittie 1978), on what theoretical basis are the many stars and radiation in the Relativistic Universe postulated to begin with? It clearly cannot be from General Relativity. It is in fact achieved by means of invalidly applying Newton's theory and its associated Principle of Superposition to General Relativity. But then "*gravitational collapse*" cannot be due to Newtonian gravitation, given the resulting black hole, which does not occur in Newton's theory of gravitation. And a Newtonian universe cannot "collapse" into a non-Newtonian universe such as that of the lonesome black hole or a cosmological singularity. Oppenheimer and Snyder (1939) began with a Newtonian Universe owing to the *a priori* presence of many stars, and allowed a star therein to collapse into a black hole to form a non-Newtonian universe, which is impossible, and also maintained that the black hole so formed is present in a universe that contains other masses in contradiction to the defining characteristics of the black hole itself. Hawking and Penrose (1970), Hawking (2002) Penrose (2002), Misner, Thorne and Wheeler (1970), Chandrasekhar (1972), do the very same. According to Oppenheimer and Snyder (1939),

"When all thermonuclear sources of energy are exhausted, a sufficiently heavy star will collapse. … this contraction will continue indefinitely. … the radius of the star approaches asymptotically its gravitational radius."

It is also routinely claimed by the proponents of the black hole that not only does it exist in multitudes, they can collide and merge, be components of binary systems, and consume other non black hole matter from the Universe, and that primordial black holes existed in the early Universe, shortly after the Big Bang; but all this is in conflict with the fact that all black hole models pertain to a spatially infinite, eternal, asymptotically flat, non-expanding universe that contains only one mass. In addition, it has been shown elsewhere (Crothers 2008, 2010, 2012) that Einstein's field equations (3) for static empty spacetime actually contain *no matter* by mathematical construction and that Einstein's field equations (1) cannot in fact reduce to equations (3).

Owing to the foregoing, the notion of a trapped surface enunciated by Hawking and Penrose (1970) and Penrose (2002) has no valid basis in General Relativity whatsoever and so their Singularity Theorem is invalid.

Consider next the so-called *"Schwarzschild radius"*, also called the "*gravitational radius*". Hilbert's metric (Abrams 1989, Antoci 2001) is usually given as,

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \left(1-\frac{2m}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$$

$$d\Omega^2 = \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$0 \le r \tag{4}$$

where *m* is claimed to represent the mass of a body that can undergo irresistible gravitational collapse to form a black hole and trapped surface. Schwarzschild's (1916) actual metric is different to Hilbert's metric and contains no black hole. In expressions (4) according to Penrose (2002),

"The quantity m is the mass of the body …"

In expression (4) the speed of light c and Newton's gravitational constant G are both set equal to unity. This practice is rather misleading and so with c and G written explicitly so that nothing is hidden, expression (4) becomes,

$$ds^2 = c^2\left(1-\frac{2Gm}{c^2 r}\right)dt^2 - \left(1-\frac{2Gm}{c^2 r}\right)^{-1}dr^2 - r^2 d\Omega^2$$

$$0 \le r \tag{5}$$

where $d\Omega^2$ is as that given in expressions (4). Consider the two terms g_{00} and g_{11} extracted from expressions (5), thus,

$$g_{00} = \left(1-\frac{2Gm}{c^2 r}\right) \qquad g_{11} = -\left(1-\frac{2Gm}{c^2 r}\right)^{-1} \tag{6}$$

It is asserted routinely that when $r = 2\,Gm/c^2$ there is a 'coordinate singularity' and when $r = 0$ there is 'physical singularity'. In the case of $r = 2Gm/c^2$ it is also routinely asserted that (Dirac 1996),

$$g_{00} = (1-1) = 0 \qquad g_{11} = \frac{-1}{(1-1)} = \frac{-1}{0} = -\infty \tag{7}$$

and that a trapped surface is produced in the course of gravitational collapse (Hawking and Penrose 1970, Penrose 2002), so that the quantity $r = 2Gm/c^2$ is the 'Schwarzschild radius' of the black hole, that is, the 'radius' of the *event horizon* of the black hole. Penrose (2002) says in relation to expression (4), *"The radius r = 2m is referred to as the Schwarzschild radius of the body."*

However, we note that in expressions (7) there is division by zero in the case of g_{11}; but division by zero is *undefined* in mathematics. It is also noted that not only is division by zero permitted to generate the trapped surface and the 'Schwarzschild radius' for the event horizon of the black hole, division by zero is also alleged to produce -∞. This too is false. Since g_{11} is undefined at $r = 2Gm/c^2$ expression (5) is also undefined at this value, and so no physical entity can be assigned to this value of r.

In the case of $r = 0$ we obtain from expressions (6),

$$g_{00} = \left(1 - \frac{2Gm}{0}\right) \quad g_{11} = \frac{-1}{\left(1 - \frac{2Gm}{0}\right)} \tag{8}$$

We note once again that division by zero results; and not once but twice! In this case both g_{00} and g_{11} are undefined and so expression (5) is undefined. Nonetheless, the proponents of the black hole again permit division by zero and assign to this value of *r* an infinitely dense point-mass singularity for the 'Schwarzschild' black hole. Indeed, according to Hawking (2002),

"The work that Roger Penrose and did between 1965 and 1970 showed that, according to general relativity, there must be a singularity of infinite density, within the black hole."

Furthermore, by virtue of expressions (7) it is also incorrectly claimed that $1/\infty = 0$ (Misner, Thorne and Wheeler 1970). Owing to expressions (7) and (8), the trapped surface is again invalidated and hence also the Hawking-Penrose Singularity Theorem.

Consider further the expression for the 'Schwarzschild radius' of the trapped surface (i.e. event horizon),

$$r = \frac{2Gm}{c^2} \tag{9}$$

Solving equation (9) for *c* we obtain,

$$c = \sqrt{\frac{2Gm}{r}} \tag{10}$$

We immediately recognise that this is Newton's expression for escape velocity. It is from this expression that it is so often claimed that the escape velocity of a black hole is the speed of light *c* in vacuum or even greater than c. Chandrasekhar (1972) says,

"Let me be more precise as to what one means by a black hole. One says that a black hole is formed when the gravitational forces on the surface become so strong that light cannot escape from it. ... A trapped surface is one from which light cannot escape to infinity."

In the Dictionary of Geophysics, Astrophysics and Astronomy (Matzner 2001), one finds the following assertions:

*"**black hole** A region of spacetime from which the escape velocity exceeds the velocity of light. In Newtonian gravity the escape velocity from the gravitational pull of a spherical star of mass M and radius R is*

$$v_{esc} = \sqrt{\frac{2GM}{R}},$$

where G is Newton's constant. Adding mass to the star (increasing M), or compressing the star (reducing R) increases v_{esc}. When the escape velocity exceeds the speed of light c, even light cannot escape, and the star becomes a black hole. The required radius R_{BH} follows from setting v_{esc} equal to c:

$$R_{BH} = \frac{2GM}{c^2}$$

"In General Relativity for spherical black holes (Schwarzschild black holes), exactly the same expression R_{BH} holds for the surface of a black hole. The surface of a black hole at R_{BH} is a null surface, consisting of those photon trajectories (null rays) which just do not escape to infinity. This surface is also called the black hole horizon."

According to Hawking (2002),

"Eventually when a star has shrunk to a certain critical radius, the gravitational field at the surface becomes so strong that the light cones are bent inward so much that the light can no longer escape. According to the theory of relativity, nothing can travel faster than light. Thus, if light cannot escape, neither can anything else. Everything is dragged back by the gravitational field. So one has a set of events, a region of space-time from which it is not possible to escape to reach a distant observer. Its boundary is called the event horizon. It coincides with the paths of the light rays that just fail to escape from the black hole."

In the Collins Encyclopaedia of the Universe (2001) there occurs the following assertion,

*"**black hole** A massive object so dense that no light or any other radiation can escape from it; its escape velocity exceeds the speed of light."*

However, it is also claimed on the other hand, that nothing at all, including light, can even *leave* the black hole. (1972) states,

"The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge."

Hawking (2002) says,

"I had already discussed with Roger Penrose the idea of defining a black hole as a set of events from which it is not possible to escape to a large distance. It means that the boundary of the black hole, the event horizon, is formed by rays of light that just fail to get away from the black hole. Instead, they stay forever hovering on the edge of the black hole."

Taylor and Wheeler (2000) assert,

"... Einstein predicts that nothing, not even light, can be successfully launched outward from the horizon ... and that light launched outward EXACTLY at the horizon will never increase its radial position by so much as a millimeter."

Equations (9) and (10) actually have nothing to do with the black hole at all; they are related only to Newton's theory of gravitation and equation (9) is the critical radius for the formation of the theoretical Michell-Laplace dark body, which is *not* a black hole since it does not possess the signatures of the black hole and is a Newtonian theoretical object, not a relativistic theoretical object (Crothers 2006, 2008, 2010, McVittie 1978). The theoretical Michell-Laplace dark body forms when,

$$r < \frac{2Gm}{c^2}$$

Nonetheless it is routinely but incorrectly claimed that the Michell-Laplace dark body is a black hole Misner, Thorne and Wheeler 1970). According to Hawking and Ellis (1973),

"Laplace essentially predicted the black hole…"

In The Cambridge Illustrated History of Astronomy (Hoskin 1997) it is asserted that,

"Eighteenth-century speculators had discussed the characteristics of stars so dense that light would be prevented from leaving them by the strength of their gravitational attraction; and according to Einstein's General Relativity, such bizarre objects (today's 'black holes') were theoretically possible as end-products of stellar evolution, provided the stars were massive enough for their inward gravitational attraction to overwhelm the repulsive forces at work."

Chandrasekhar (1972) reiterates,

"That such a contingency can arise was surmised already by Laplace in 1798. Laplace argued as follows. For a particle to escape from the surface of a spherical body of mass M and radius R, it must be projected with a velocity v such that $\frac{1}{2}v^2 > GM/R$; *and it cannot escape if* $v^2 < 2GM/R$. *On the basis of this last inequality, Laplace concluded that if* $R < 2GM/c^2 = R_s$ *(say) where c denotes the velocity of light, then light will not be able to escape from such a body and we will not be able to see it!*

"By a curious coincidence, the limit R_s *discovered by Laplace is exactly the same that general relativity gives for the occurrence of the trapped surface around a spherical mass."*

But it is not surprising that General Relativity gives the same R_s *"discovered by Laplace"* because the Newtonian expression for escape velocity is deliberately inserted *post hoc* into Hilbert's solution by the astrophysical scientists, in order to make it so.

Now if the black hole has an escape velocity c, then, by definition, light can escape, contrary to the frequent claim that it cannot. Moreover, material bodies can *leave* the black hole but not escape since in Relativity theory no material body can acquire the speed c and because 'escape velocity' does not mean bodies cannot leave, only that if they leave they cannot escape. If the escape velocity of the black hole is greater than c then light cannot escape and no material body can escape, on the basis that no material body can acquire the speed of light in vacuum, but once again that does not mean that material bodies and light cannot *leave*, only that they cannot escape. Yet black hole theory also maintains that neither light nor material bodies can even leave the event horizon of the black hole. The notion of black hole escape velocity is just a play on the words 'escape velocity' (McVittie 1978). The black hole has no escape velocity. This fact also invalidates the Hawking-Penrose concept of 'trapped surface' and hence the Hawking-Penrose Singularity Theorem.

That the Michell-Laplace dark body is not a black hole is easy to prove. The Michell-Laplace dark body possesses an escape velocity, whereas the black hole has no escape velocity; masses and light can leave the Michell-Laplace dark body, but nothing can leave the black hole; it does not require irresistible gravitational collapse, whereas the black hole does; it has no infinitely dense singularity, whereas the black hole does; it has no event horizon, whereas the black hole does; there is always a class of observers that can see the Michell-Laplace dark body, but there is no class of observers that can see the black hole; the Michell-Laplace dark body persists in a space which by consistent theory contains other Michell-Laplace dark bodies and other matter and they interact with themselves and other matter, but the spacetime of all types of black hole pertain to a Universe that contains only one mass (but actually contains no mass by mathematical construction) and so cannot interact with any other masses; the space of the Michell-Laplace dark body is 3-dimensional and Euclidean, but the black hole is in a 4-dimensional non-Euclidean spacetime; the space of the Michell-Laplace dark body is not asymptotically flat whereas the spacetime of the black hole is asymptotically flat. Thus the Michell-Laplace dark body does not possess the characteristics of the black hole and so it is not a black hole.

Furthermore, although expression (10) contains only one mass term, it is implicitly a *two-body* relation: one body escapes from another body. But all alleged black hole solutions to Einstein's field equations pertain to an infinite Universe that contains only one mass, and so Newton's expression for escape velocity *cannot* rightly appear in any mathematical expression alleging a black hole. Indeed, Newton's expression for escape velocity is inserted *post hoc* into Hilbert's solution (5) in order to satisfy the misleading words "*gravitational field outside a body*" or "*point-mass*" when in fact there is no mass present since expression (3) contains no matter by mathematical construction because the energy-momentum tensor is set to zero for no material sources present (Crothers 2008, 2010, 2012, 2012b) bearing in mind that Einstein's field equations,

"... couple the gravitational field (contained in the curvature of spacetime) with its sources." (Foster and Nightingale 1995)

"Thus the equations of the gravitational field also contain the equations for the matter (material particles and electromagnetic fields) which produces this field." (Landau and Lifshitz 1951)

It is clear that indeed Newton's expression for escape velocity is included in Hilbert's solution (5) by an invalid application of Newton's theory of gravitation, and from it the fallacious notions of black hole escape velocity and black hole trapped surface at the 'Schwarzschild radius' are obtained, along with multiple black holes and black holes with other forms of matter by application of the Principle of Superposition, which is valid in Newton's theory but which *does not* hold in General Relativity:

"The Einstein equations are nonlinear. Therefore for gravitational fields the principle of superposition is not valid." (Landau and Lifshitz 1951)

In direct violation of this fact, it is also routinely claimed that the black hole exists in multitudes, that black holes can collide and merge and interact with other matter, be components of binary systems, and can form by gravitational collapse in a Universe that contains many masses and radiation. In other words, Newtonian theory is inadmissibly employed to produce many masses, including multiple black holes, in a non-Newtonian General Relativistic Universe. According to Chandrasekhar (1970),

"From what I have said, collapse of the kind I have described must be of frequent occurrence in the Galaxy; and black-holes must be present in numbers comparable to, if not exceeding, those of the pulsars. While the black-holes will not be visible to external observers, they can nevertheless interact with one another and with the outside world through their external fields.

"In considering the energy that could be released by interactions with black holes, a theorem of Hawking is useful. Hawking's theorem states that in the interactions involving black holes, the total surface area of the boundaries of the black holes can never decrease; it can at best remain unchanged (if the conditions are stationary).

"Another example illustrating Hawking's theorem (and considered by him) is the following. Imagine two spherical (Schwarzschild) black holes, each of mass ½M, coalescing to form a single black hole; and let the black hole that is eventually left be, again, spherical and have a mass M. Then Hawking's theorem requires that

$$16\pi\overline{M}^{2} \geq 16\pi\left[2\left(\frac{1}{2}M\right)^{2}\right] = 8\pi M^{2}$$

or

$$\overline{M} \geq \frac{M}{\sqrt{2}}\,.$$

Hence the maximum amount of energy that can be released in such a coalescence is $M\left(1-1/\sqrt{2}\right)c^{2} = 0.293M^{2}$".

Hawking (2002) says,

"Also, suppose two black holes collided and merged together to form a single black hole. Then the area of the event horizon of the final black hole would be greater than the sum of the areas of the event horizons of the original black holes."

According to Schutz (1990),

"... Hawking's area theorem: in any physical process involving a horizon, the area of the horizon cannot decrease in time. ... This fundamental theorem has the result that, while two black holes can collide and coalesce, a single black hole can never bifurcate spontaneously into two smaller ones.

"Black holes produced by supernovae would be much harder to observe unless they were part of a binary system which survived the explosion and in which the other star was not so highly evolved."

Carroll and Ostlie (1996) remark,

"The best hope of astronomers has been to find a black hole in a close binary system. ... If a black hole coalesces with any other object, the result is an even larger black hole. ... If one of the stars in a close binary system explodes as a supernova, the result may be either a neutron star or a black hole orbiting the companion star. ... the procedure for detecting a black hole in a binary x-ray system is similar to that used to

measure the masses of neutron stars in these systems. … What is the fate of a binary x-ray system? As it reaches the endpoint of its evolution, the secondary star will end up as a white dwarf, neutron star, or black hole."

In the case of Hilbert's solution (4) and (5) the quantity *r* has never been correctly identified by astrophysics. It has been variously and vaguely called a *"distance"*, *"the radius*", the *"radius of a 2-sphere*", the *"coordinate radius"*, the *"radial coordinate"*, the *"Schwarzschild r-coordinate"*, the *"radial space coordinate"*, the *"areal radius"*, the *"reduced circumference"*, *"the shortest distance a ray of light must travel to get to the center"*, and even *"a gauge choice: it defines the coordinate r"*. In the particular case of $r = 2Gm/c^2$ it is invariably called the *"Schwarzschild radius"* or the "*gravitational radius*". Dirac (1996) says in relation to Hilbert's solution in the form of expression (4),

"It would seem that $r = 2m$ *gives a minimum radius for a body of mass m."*

According to Penrose (2002), in relation to expression (4),

"The radius $r = 2m$ *is referred to as the Schwarzschild radius of the body."*

However, none of the foregoing various and vague conceptions of r are correct because the *irrefutable* geometrical fact is that r, in the spatial section of Hilbert's solution (4) and (5), is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section, and as such it is not a distance at all, let alone the 'radius', in Hilbert's solution. In Newton's theory the quantity *r* in expressions (9) and (10) *is* the radius and since it is incorporated, invalidly, into Hilbert's solution, it is misinterpreted as the radius in Hilbert's solution as well. It is easy to prove that in Hilbert's solution r is not even a distance. Consider the surface in the spatial section of expressions (4) and (5). It is given by the First Fundamental Quadratic Form,

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{11}$$

The Gaussian curvature K of a surface can be calculated by,

$$K = \frac{R_{1212}}{g} \tag{12}$$

Where R_{1212} is a component of the Riemann tensor of the first kind and g is the determinant of the metric tensor. The calculation can be made with the assistance of the following relations,

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma} R^{\lambda}_{.\,\nu\rho\sigma}$$

$$R^{1}_{.212} = \frac{\partial\Gamma^1_{22}}{\partial x^1} - \frac{\partial\Gamma^1_{21}}{\partial x^2}{}_{2} + \Gamma^k_2\Gamma^1_{k1} - \Gamma^k_2\Gamma^1_{k2}$$

$$\Gamma^i_{ij} = \Gamma^i_{ji} = \frac{\partial\left(\frac{1}{2}\ln\left|g_{ii}\right|\right)}{\partial x^j}$$

$$\Gamma^i_{jj} = -\frac{1}{2g_{ii}}\frac{\partial g_{jj}}{\partial x^i} \quad (i \neq j)$$

and all other Γ^i_{jk} vanish. Making the calculation gives for expression (11),

$$K = \frac{1}{r^2}$$

and so

$$r = \frac{1}{\sqrt{K}}$$

which proves that *r* is neither a radius nor a distance in Hilbert's solution (4) and (5). Hence the 'Schwarzschild radius' is the radius of *nothing* in Hilbert's metric because it is not even a distance therein. This once again invalidates the concept of a trapped surface as proposed by Hawking and Penrose and hence also invalidates their Singularity Theorem.

Remarkably, in relation to Hilbert's metric, Celotti, Miller and Sciama (1999) make the following incorrect assertion:

"The 'mean density' $\bar{\rho}$ *of a black hole (its mass M divided by* $\frac{4}{3}\pi r_s^3$*) is proportional to* $1/M^2$*"*

where r_s is the so-called 'Schwarzschild radius'. The volume they adduce for a black hole cannot be obtained from Hilbert's solution: it is a volume associated with the Euclidean 3-space pertaining to Newton's theory: and the 'Schwarzschild radius' is not even a distance in Hilbert's solution, as proven above. Furthermore, at the 'Schwarzschild radius' Hilbert's metric is *undefined*. Misner, Thorne and Wheeler (1970) also incorrectly implicitly assert that the volume of a star according to Hilbert's solution is given, by $4\pi r^3/3$ via the expression,

"total mass-energy inside radius $r \equiv m(r)$*"*

$$= \int_0^r 4\pi r^2 \rho\, dr$$

Where $\rho = \rho(r)$ is the *"density of total mass-energy"*.

It has been shown (Crothers 2005, 2010) that the actual radius R_p for Hilbert's metric is given by,

$$R_p = \sqrt{r(r-\alpha)} + \alpha \ln\left(\frac{r + \sqrt{r-\alpha}}{\sqrt{\alpha}}\right)$$

where α is a constant given by $\alpha = 2Gm/c^2$. However, as explained above, the equality $\alpha = 2Gm/c^2$ is produced by an invalid application of Newton's theory by which Newton's expression for escape velocity is inserted into Hilbert's metric to obtain a source mass, escape velocity and 'Schwarzschild radius'. Hilbert's metric actually contains no matter in any case and is therefore physically meaningless (Crothers 2008, 2010, 2012, 2012b).

It is common practice to write Einstein's field equations compactly by means of Einstein's tensor $G_{\mu\nu}$ defined by,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

so that equations (1) become,

$$G_{\mu\nu} = -K T_{\mu\nu} \tag{13}$$

However, this form of Einstein's field equations is incorrect and so equations (1) and (13) *cannot* reduce to $R_{\mu\nu} = 0$ when $T_{\mu\nu} = 0$. When the conservation of energy and momentum is accounted for correctly, and bearing in mind that $R_{\mu\nu} = 0$ has no physical meaning because it contains no matter by mathematical construction, Einstein's field equations must take the following form (Crothers 2008, 2010, 2012, Lorentz 1916, Levi-Civita 1917),

$$\frac{G_{\mu\nu}}{K} + T_{\mu\nu} = 0 \tag{14}$$

Inclusion of the cosmological constant λ gives,

$$\frac{\left(G_{\mu\nu} + \lambda g_{\mu\nu}\right)}{K} + T_{\mu\nu} = 0 \tag{15}$$

This expression can be written in terms of mixed tensors as follows,

$$\frac{\left(G_{\nu}^{\mu} + \lambda g_{\nu}^{\mu}\right)}{K} + T_{\nu}^{\mu} = 0 \tag{16}$$

Compare this now to Einstein's (1916) expression for the total energy and momentum of his gravitational field given by,

$$\left(t_{\sigma}^{\mu} + T_{\sigma}^{\mu}\right) = E \tag{17}$$

The term t_{σ}^{μ} is Einstein's pseudo-tensor, which he calls *the energy components of the gravitational field* (Einstein 1916, Pauli 1981), and it is defined as follows (Levi-Civita 1917),

$$\sqrt{-g}\, t_{\nu}^{\mu} = \frac{1}{2}\left[\delta_{\nu}^{\mu} L - \left(\frac{\partial L}{\partial g_{,\mu}^{\sigma\beta}}\right) g_{,\nu}^{\sigma\beta}\right] \tag{18}$$

where L is given by,

$$L = -g^{\alpha\beta}\left(\Gamma_{\alpha\kappa}^{\gamma}\Gamma_{\beta\gamma}^{\kappa} - \Gamma_{\alpha\beta}^{\gamma}\Gamma_{\gamma\kappa}^{\kappa}\right)$$

Since the pseudo-tensor is not a tensor, Einstein could not take a tensor divergence of equation (17) and so he took an ordinary divergence (Einstein 1916), thus,

$$\frac{\left(t_{\sigma}^{\mu} + T_{\sigma}^{\mu}\right)}{\partial x_{\sigma}} = 0 \tag{19}$$

and since this is zero Einstein (1916) asserts,

"Thus it results from our field equations of gravitation that the laws of conservation of momentum and energy are satisfied."

However, it is noted that not only is expression (16) the correct form of Einstein's field equations with cosmological constant, it is also a total energy and momentum expression where the energy components of the gravitational field are in place of Einstein's pseudo-tensor actually given by the term,

$$\frac{\left(G_{\nu}^{\mu} + \lambda g_{\nu}^{\mu}\right)}{K} \tag{20}$$

Einstein's pseudo-tensor does not describe *the energy components of the gravitational field* for the following two reasons:

1. t_{ν}^{μ} is *not* a tensor and so it does not meet Einstein's requirement that all the equations of physics be tensorial in General Relativity;
2. Setting $\nu = \mu$ in expression (18) produces an invariant by the contraction of the pseudo-tensor, thus,

$$t_{\mu}^{\mu} = t = \frac{L}{\sqrt{-g}}$$

which is easily shown to be a first-order intrinsic differential invariant, i.e. an invariant that depends *solely* upon the components of the metric tensor and their first derivatives (Crothers 2008, 2010, 2012, Levi-Civita 1917). However, the mathematicians G. Ricci-Curbastro and T. Levi-Civita (1900), inventors of the tensor calculus, proved that such invariants *do not exist*! Hence Einstein's pseudo-tensor is a meaningless collection of mathematical symbols and so it cannot be used to make any calculations or to represent any physical entity whatsoever.

Taking the tensor divergence of the left side of expression (16) produces zero and so there is conservation of energy and momentum, but since expression (16) is also an energy equation the total energy is *always* zero and so the usual conservation of energy and momentum is violated (Crothers 2008, 2010, 2012, Pauli 1981) and gravitational energy cannot be localised i.e. there are no Einstein gravitational waves (Crothers 2008, 2010, 2012, Levi-Civita 1917). Thus, Einstein's field equations are in conflict with experiment on a deep level, rendering them invalid. Hence *all* the conditions specified for the Hawking-Penrose Singularity Theorem are invalidated. Note that by expressions (16) and (20) this result holds whether λ is negative, zero, or positive.

III. Invalidity of Acceleration with Negative Λ

In the abstract of their paper, Hartle, Hawking and (2012) say that,

"Wave functions specifying a quantum state of the universe must satisfy the constraints of general relativity, in particular the Wheeler-DeWitt equation (WDWE)."

However, it follows immediately from Section II above that the foundations of the quantum state with wave function containing negative λ proposed by Hartle, Hawking and Hertog (2012) are also invalidated, since the "*constraints of general relativity*" are meaningless. Hartle, Hawking and Hertog (2012) also say that,

"… even theories with a negative cosmological constant can predict accelerating classical histories.

"At the level of the wave function there is a close connection between asymptotic Lorentzian de Sitter (dS) spaces and Euclidean anti-deSitter (AdS) spaces."

But since the presence of λ in expressions (16) and (20) makes no difference whatsoever to the violation of the usual conservation of energy and momentum by General Relativity, the cosmological constant has no physical meaning and so its inclusion in the quantum state as a negative constant proposed by the three aforementioned authors is also meaningless. Similarly de Sitter spaces and anti-de Sitter spaces have no physical meaning.

The claim (Hartle Hawking and Hertog 2012) that,

"The observed classical expansion of our universe is accelerating at a rate consistent with a positive cosmological constant of order $\Lambda{\sim}10^{-123}$ in Planck units"

Finds no justification in either General Relativity or Newtonian gravitation and so the *"observed classical expansion"* cannot be interpreted in terms of General Relativity as due to Big Bang cosmology.

The Cosmic Microwave Background (CMB) is ubiquitously employed to justify Big Bang cosmology on empirical grounds (Hawking and Penrose 1970). However, it is now known that the CMB is not cosmic. Its true source is the oceans of the Earth (Robitaille 2007, 2009) owing to the hydrogen bonds in water (Robitaille 2009b) which cause the oceans to emit radiation in the microwave and far-infrared bands, which are scattered by the atmosphere to produce an isotropic signal from an isotropic source. It is now also known that COBE and WMAP spectra and images are meaningless (Robitaille 2007, 2009).

IV. Conclusion

The black hole is not predicted by General Relativity or by Newton's theory of gravitation, and so it has no theoretical basis whatsoever. The Hawking-Penrose 'trapped surface' and Singularity Theorem are invalid. All singularity theorems for General Relativity are invalid. General Relativity violates the usual conservation of energy and momentum and is therefore in conflict with experiment on a deep level, making it invalid. Consequently the black hole, Einstein gravitational waves, and the Big Bang Cosmology have no theoretical basis. The Cosmic Microwave Background is not the afterglow of the birth of the Universe from a Big Bang *creatio ex nihilo* or otherwise. The Hartle, Hawking, Hertog quantum wave function with negative cosmological constant is invalid.

Dedication

I dedicate this paper to my late beloved brother:

Paul Raymond Crothers

12th May 1968 – 25th December 2008

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Global Journal of Science Frontier Research
Physics and Space Science
Volume 13 Issue 4 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

A Review of Higher Order Statistics and Spectra in Communication Systems

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Abstract - There are many statistical tools to extract information from random signals. They predominantly use first and second order statistics. However, in the presence of nonlinearity in systems, many signals cannot be analyzed adequately by second order statistical methods. For this reason, higher order statistical methods have been developed. These methods are very useful in problems where non-Gaussian, non-minimum phase, phase coupling or nonlinear behavior and robustness to additive noise are important. Detection and classification using higher order statistical and spectral techniques have been proposed for use in communication and pattern recognition. They have the potential to elicit better performance from sensors, sensor networks and channels with applications in coding, filtering and detection techniques. This paper provides an introduction to higher order spectra and reviews a number of these techniques.

Keywords : higher order spectra, high order statistics, interference, bispectrum, invariant, entropy, fading.

GJSFR-A Classification : FOR Code: 010401

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Strictly as per the compliance and regulations of :

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Abstract - There are many statistical tools to extract information from random signals. They predominantly use first and second order statistics. However, in the presence of nonlinearity in systems, many signals cannot be analyzed adequately by second order statistical methods. For this reason, higher order statistical methods have been developed. These methods are very useful in problems where non-Gaussian, non-minimum phase, phase coupling or nonlinear behavior and robustness to additive noise are important. Detection and classification using higher order statistical and spectral techniques have been proposed for use in communication and pattern recognition. They have the potential to elicit better performance from sensors, sensor networks and channels with applications in coding, filtering and detection techniques. This paper provides an introduction to higher order spectra and reviews a number of these techniques.

Keywords : higher order spectra, high order statistics, interference, bispectrum, invariant, entropy, fading.

I. Introduction

Higher order statistics and spectra (HOS) play an important role in digital signal processing. They are extensions of the better known concepts of correlation (in time or space) and power spectra. Just as the power spectrum is the Fourier spectral representation of the autocorrelation function which is a second order moment of the probability density function, higher order spectra are higher order Fourier spectral representations of third and higher order correlations or moments.

Higher order spectra were originally introduced as spectral representations of cumulants or moments of ergodic random processes. They were used in the identification of nonlinear systems and non-Gaussian random processes and phase coupling in wave-wave interactions. Papers by Brillinger and Rosenblatt [1, 2] laid the theoretical foundations of the area and the paper by Hasselman et al [3] in 1963 that studied bispectra of ocean waves was one of the earliest applications. Signal processing theory and applications of HOS grew with the excellent review paper in 1987 by Nikias and Raghuveer [4] and the books on HOS in 1993 by Nikias and Petropulu [5] and in 1995 by Boashash, Powers and Zoubir [6].

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Over the years, HOS have been applied in many areas in a stochastic framework and their application has been extended to deterministic signals. Although there are similarities in the forms of analytical expressions for both the stochastic and the deterministic frameworks, there are important differences that must be understood in order to apply HOS effectively to practical problems. Many of the later contributions to application of HOS to practical problems are now scattered in various journals and proceedings of conferences. Techniques for detection and classification using HOS have been proposed and many application contributions have been published in journals that are outside the IEEE/IEE research collection.

Higher order spectra have potential for application in continuing and recent research efforts in the fields of communication and pattern recognition. A communications engineer doing research on Multiple Input Multiple Output (MIMO) wireless communications could benefit from an understanding of the capabilities of HOS in order to make better use of phase information, adapt to non-Gaussian noise or adapt to non-linear channel characteristics. Similarly, a biomedical engineer can use HOS to derive features from EEG or ECG signals which represent output data from systems where the input is not known. Polyspectra play a key role in detecting and characterizing the type of nonlinearity in a system from its output data. Several signal processing methods for the detection and characterization of nonlinearities in time series using higher order spectra have been proposed. Some of the early developments are found in Rao and Gabr [14] and Nikias and Petropulu [5].

The paper is organized as follows. Section II provides definitions and properties. Section III provides a tutorial on the use of HOS. Section IV reviews contributions relevant to the chosen application areas. Research trends and potential for future applications are discussed in Section V followed by a conclusion in Section VI.

II. Higher Order Statistics: Definitions and Properties

Random variables $x \in \Re^N$ (scalars or vectors) have probability densities $p_X(x)$. If a probability density is Gaussian,

$$p_X(x)$$
$$[\ \ \pi]^{-N} \qquad (C) \qquad exp\left\{-\frac{(x-m_x)^T C^{-} \quad x-m_x}{}\right\} \tag{1}$$

it is completely characterized by the mean value, m, (a first order statistic) and the covariance , C, (a second order statistic). In an estimate of the mean, values of a random vector are considered independently.

$$m_X \qquad \iiint_{\infty}^{\infty} x\, p_X \;\; x \quad dx \tag{2}$$

In an estimate of the covariance, values of the random vector (or random vectors in the case of joint statistics) are considered in pairs.

$$\sigma_X \qquad diag\left[\iiint_{-\infty}^{\infty} \;\; x-m_x \;\; ^T \;\; x-m_x \;\; p_X \;\; x \quad dx\right] \tag{3}$$

If the probability density function of the random variable is continuous and well behaved enough for a Fourier transform to exist in the form of a Fourier-Stieltjes integral, its Fourier transform, called the characteristic function, can be represented in a power series expansion. Such an expansion has the mean and the covariance in its first two terms.

$$\Phi(f) = \sum_{k=0}^{n} \frac{1}{k!} m_X^k (jf)^k + o_n(f)$$
$$= 1 + jfm_1 - \frac{f^2}{2!} m_2 \dots\dots + \frac{(jf)^k}{k!} m_k + \cdots \tag{4}$$

where the remainder function $o_n(f)$ is such that $o_n(f)/f^n$ goes to zero in the limit as f approaches zero.

Subsequent terms are expected values of higher (than second) order products of samples of the random variable – referred to as the higher order moments of the distribution. It is often convenient to remove the mean value or first moment and expand the probability density function about the mean. This results in central moments of the distribution which have the useful property of being invariant to a shift of the origin.

It is not necessary that moments of all orders exist and are finite. Some moments can be infinite. The distribution is not always uniquely determined by the moments either even when moments of all order exist. The log-normal distribution is an example. However, in general, the higher order moments serve to determine the distribution more accurately.

Higher order moments for a Gaussian probability density are constant but not zero. If the logarithm of the characteristic function is expanded instead, the higher order terms represent expected values of combinations known as cumulants, and these are zero for a Gaussian density.

$$\Psi(f) \qquad \sum_{k}^{n} \frac{}{k} c_x(k)(jf)^k \qquad o_n(f) \tag{5}$$

where $c_x(k) = (-j)^k \left.\frac{d^k \Psi(f)}{df^k}\right|_{f=0}$ $for\ k = 1,2,3,$ and $\Psi(f) = \ln\{\Phi(f)\}$

The remainder function, $o_n(f)$, is such that $o_n(f)/f^n$ goes to zero in the limit as f approaches zero. If the probability density is non-Gaussian it is not completely represented by its mean and covariance. The higher order cumulants are then non-zero.

Random processes $\underline{x}(t): t \to \Re$ are random functions of some independent variable such as time. Each realization of the process is one of an ensemble of possible functions. If a vector of values, at different time delays from a reference variable time, is considered, there is an ensemble of realizations of random vectors.

$$x(t) = \bar{x}(t, t+\tau_1, t+\tau_2, \dots\dots, t+\tau_{n-1}) \in \Re^n \tag{6}$$

For such a process, the joint density of the random vector represents the ensemble statistics and it can in general be a function of time t. For an n-th order stationary process, the joint density is independent of time. However, it still depends on the time lags or separations between the random variables. Again assuming that the joint probability density function is well behaved and a Fourier transform exists, the characteristic function can be expressed as

$$\Phi(f) \qquad \int_{-\infty}^{\infty} \qquad (j \quad \pi f x)\, p_X \;\; x \quad dx \tag{7}$$

From the above formulation it can also be noted that the moments of this joint probability density are also the correlations in time of the underlying random process. The second moment is the auto-correlation of the process. A Fourier transform of the auto-correlation is the power spectrum of the random process. Higher order correlations will give rise to higher order spectral representations for the process. Thus higher order statistics of the joint density function of random variables separated by time lags are related to higher order spectra for the underlying random process.

If the random process is zero mean, the first moments of the density function will be zero. If the random process is white, random variables at different delays will be uncorrelated and the second moments will be described by a diagonal covariance matrix of variances. If the random process is Gaussian, the joint density function can be described by the means and covariance matrix alone, and higher order moments are functions of these parameters. Higher order moments take on non-zero values because on some sub-manifolds of the n-th order lag space, they can degenerate to products of lower order moments. For example, a fourth order moment can degenerate to a product of two variances if two of the four lags are identical and so are the other two. In this case, there are only two random variables rather than four. Such degeneracy can occur whenever the set of n random variables can be split into two or more statistically independent groups. Higher order cumulants account for these sub-manifolds and are zero for Gaussian processes. Cumulants other than the first are also invariant to a shift of the origin like the central moments.

$$\Psi(f) = \Psi_1(a_1 f) + \Psi_2(a_2 f) + \cdots + \Psi_n(a_n f) \quad (8)$$

Cumulants are not directly ascertained through summation or integration but they are related to moments and can be found using these relationships. When working with real-world signals in a random process framework, probability density functions are in most cases not known in closed form. Moments must be estimated from realizations of the signal. It is then important to understand time and ensemble statistics of the process and assumptions about stationarity and ergodicity.

If scalar random variables are considered, the expected value over the ensemble is the ensemble mean of the process. There is a corresponding expectation in time, the time mean of the process. For an ergodic process, the ensemble statistics and the time statistics are assumed to be identical. If the product of two values of the process, separated by a time lag is considered, the autocorrelation function of the process is obtained. For a second order ergodic process, the time autocorrelation and ensemble autocorrelation are equal. If the autocorrelation, $R_{xx}(\tau)$ is well behaved enough for a Fourier transform to exist, its Fourier transform yields the power spectral density of the process.

$$P_{xx}(f) \quad \int_{-\infty}^{\infty} R_{xx} \ \tau \ e^{-j \ \pi f \tau} \ d\tau \quad (9)$$

If the two values come from two different random processes, a cross-correlation is obtained and its Fourier transform represents the cross-spectral density of the two processes.

$$P_{xy}(f) \quad \int_{-\infty}^{\infty} R_{xy} \ \tau \ e^{-j \ \pi f \tau} \ d\tau \quad (10)$$

The autocorrelation and the cross-correlation and the corresponding spectra form the basis of powerful second order approaches to signal detection, estimation and tracking such as matched filtering, Wiener filtering and Kalman filtering. For a stationary and ergodic process, these concepts were extended to products of three and more values separated by lags, by Shiryaev [7] and laid on firm mathematical foundations by Brillinger [8] and Rosenblatt [1, 2]. They lead to the definition of higher order cumulant functions and corresponding higher order spectra.

For auto-correlation and auto-spectra we can omit the subscripts assuming that these are implied The Fourier transform of the third order cumulant function is referred to as the bispectrum and it is a complex-valued function of two frequencies.

$$B(f \ \ f \ ) \quad \int_{-\infty}^{\infty} R(\tau \ \ \tau \ )e^{-j \ \pi(f \ \tau \ \ \ f \ \tau \ )} d\tau \ d\tau \quad (11)$$

The Fourier transform of the fourth order cumulant function is referred to as the trispectrum. It may be noted that these Fourier spectra exist only when the cumulant functions are well behaved – they decay with increasing lag and are absolutely integrable, for example. Conditions for existence can be relaxed carefully and the definitions and estimation procedures can be extended to include cyclostationary and harmonic random processes. The power spectral density $P(f)$ of a random process $x(t)$ can be represented using

$$P(f)d \quad = E[dX(f)d \ ^{*}(f)] \quad (12)$$

This makes use of the Cramer spectral representation

$$x(t) = \int_{-\infty}^{\infty} \exp[j2\pi ft]dX(f) \quad (13)$$

Similarly the bispectrum $B(f_1, f_2)$ can be represented using

$$E[dX(f_1)dX(f_2)dX^{*}(f_3)] = \begin{cases} B(f_1, f_2)df_1 df_2 & f_1 + f_2 = f_3 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

It is estimated using the averaged biperiodogram. *E* represents expectation over an ensemble of realizations of the process. The periodogram and the biperiodogram are computed using numerical samples over a time windowed realization of the process and the Fast Fourier transform. Averaging is performed with several realizations.

The theoretical foundations have been extended and conditions for existence relaxed in a number of ways. Discrete-time processes are not integrable but the use of Z transforms, discrete Fourier transforms and absolute summability allows definition of higher order cumulants and spectra for them. Deterministic functions do not have ensemble statistics or ensemble probability density functions. However, they have time statistics and it has been shown that

$$\begin{aligned} B(f_1, f_2) &= X(f_1)X(f_2)X^{*}(f_1 + f_2) \\ &= \int_{-\infty}^{\infty} \widetilde{R}(\tau_1, \tau_2)\exp[-j2\pi(f_1\tau_1 + f_2\tau_2)]d\tau_1 d\tau_2 \end{aligned} \quad (15)$$

where* represents the complex conjugate and $\widetilde{R}(\tau_1, \tau_2)$ is the third order correlation in time (represented by $\mathscr{E}$) of the deterministic function.

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$$\widetilde{R}(\tau_1,\tau_2) = \mathscr{E}[x(t)x(t+\tau_1)x(t+\tau_2)]$$
$$= \int_{-\infty}^{\infty} x(t)x(t+\tau_1)x(t+\tau_2)d \qquad (16)$$

The third order correlation, or third order moment, is a function of two lag variables and the bispectrum is a function of two frequency variables. The above equations show that it is possible to estimate the bispectrum in two ways: either computing the third moment function and then taking its Fourier transform, or computing the Fourier transform of the function and using triple products of Fourier coefficients. For continuous-time deterministic signals, integrability of the moment and the existence of the Fourier transform are necessary for validity. For discrete-time signals or for computation via numerical methods based on sampling, summability of the moment and existence of the discrete-time Fourier transform are necessary for validity. The corresponding equations in discrete time are

$$B(f_1,f_2) = X(f_1)X(f_2)X^*(f_1+f_2)$$
$$= \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\widetilde{R}(n_1,n_2)\exp[-j2\pi(f_1n_1+f_2n_2)] \qquad (17)$$

where $B(f_1,f_2)$ is the bispectrum of x(n), $\tilde{R}(n_1,n_2)$ is the third order correlation and *X(f)* is the Fourier transform of x(n) which are shown in equation (9) and (10). In general, $B(f_1,f_2)$ is complex and a sufficient condition for its existence is that $\tilde{R}(n_1,n_2)$ is absolutely summable.

$$\widetilde{R}(n_1,n_2) = \mathscr{E}[x(n)x(n+n_1)x(n+n_2)]$$
$$= \sum_{n=-\infty}^{\infty} x(n)x(n+n_1)x(n+n_2) \qquad (18)$$

$$X(f) = \sum_{n=-\infty}^{\infty} x(n)\exp[-j2\pi fn] \qquad (19)$$

The sampling interval in the above equations is assumed to be unit, and when frequencies are normalized by the sampling frequency, the bispectrum, $B(f_1,f_2)$ is periodic in f_1 and f_2 with period 2π radians/sec or 1 cycle/sec. For a real-valued input, the spectrum is conjugate symmetric and only positive frequencies up to one-half the sampling frequency are unique in spectral values. Therefore, frequencies are often normalized by one half of the sampling frequency (or Nyquist frequency). Further, the bispectrum is symmetric in the two frequencies and knowledge of the auto-bispectrum in the triangular region $f_2 \geq 0, f_1 \geq f_2, f_1+f_2 \leq 1$ (see Fig. 1) is enough for its complete description, where frequencies are normalized by the Nyquist frequency and the input is real-valued.

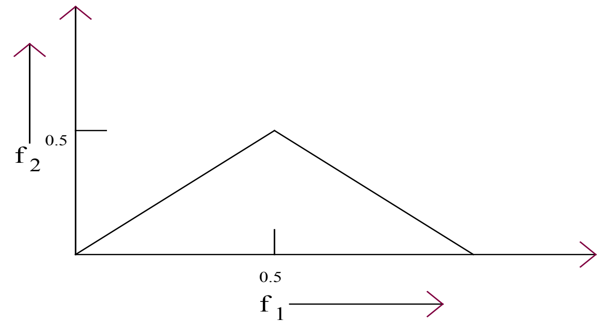


Figure 1 : Triangular bounded by $f_2 = 0, f_1 = f_2, f_1 + f_2 = 1$. Values of $B(f_1,f_2)$ on this triangle for a real-valued and band-limited random process determine its values everywhere in the $(f_1 - f_2)$ plane

For a full description of the non-redundant regions of computation of various higher order spectra, the reader is referred to [5, 9]. The discrete-time versions may be further extended to discrete versions that are also sampled in the frequency domain. This is required to be able to compute spectra numerically. It may be noted that frequency sampling inherently makes the sequence periodic in time. Equivalently, computing the spectra using finite length sequences yields the discrete Fourier transform (DFT) which is a sampled version of the discrete-time Fourier transform (DTFT).

$$X(k) = \sum_{n=0}^{N-1} x(n)\exp[-j2\pi kn] \qquad (20)$$

For deterministic signals, the assumption of periodicity outside the window of observation leads to side effects such as spectral leakage. Equation (20) is used with time samples x(n) in the frequency domain approach to obtain Fourier coefficients and their products are used in estimation via the first part of equation (15) or (17), in the frequency domain approach. Equation (20) is used with moments $\widetilde{R}(n_1,n_2)$ in the direct approach. If the observation window [0, N-1] is long enough for the moment function to have decayed sufficiently, the frequency sampling should not cause significant error.

Higher-order spectra are thus multidimensional Fourier transforms of higher-order statistical parameters [10]. For stochastic processes, a frequency domain approach is beset with some difficulty because a Fourier transform does not strictly speaking exist for a stationary random process. However, spectral representations as in equations (13) and (14) can be employed with the use of expected values as done in stochastic calculus [10]. Discrete spectral representations imply conversion to cyclostationary processes but if the moments of interest decay to nearly zero within the observation window estimation of their spectra via expectations of products of frequency domain samples as in a discrete version of equation (14) will be valid. In order to apply the frequency domain approach to estimation, each realization of a random process is windowed in time and sampled and equation (20) is used to computer Fourier coefficients. Averaging over an ensemble as in equation (14) reduces the estimation variance, taking the estimate closer to the true value.

The trispectrum is defined in terms of the fourth order cumulant by the discrete Fourier transform (DFT):

$$T(f\ \ f\ \ f\ ) \quad X(f\ )X(f\ )X(f\ )X^{*}(f \quad f \quad f\ )$$
$$\sum_{n \quad -\infty}^{\infty} \sum_{n \quad -\infty}^{\infty} \sum_{n \quad -\infty}^{\infty} c^{x}\ n\ \ n\ \ n \qquad [-j\ \ \pi(f\, n \quad f\, n$$
$$f\, n\ )] \tag{21}$$

where $c_4^x(n_1, n_2, n_3)$ is the fourth-order cumulant sequence, similar to third-order cumulant given by equation (18) for bispectrum. The expectation operation in the tri-periodogram is omitted above, when the signal is deterministic. Symmetry properties and non-redundant region of computation of the trispectrum are discussed in [9].

If a distribution is symmetric, then its third-order cumulant equals zero; hence, for such a process we must use fourth-order cumulants. For example, Laplace, Uniform, Gaussian and Bernoulli-Gaussian distributions are symmetric, whereas Exponential, Rayleigh and k-distributions are non-symmetric. Some processes have distributions with extremely small third-order cumulants and much larger fourth-order cumulants; hence, for such processes also we would also use the fourth cumulant or the trispectrum. In cubically phase coupled harmonic random processes also, the third order cumulants equal zero whereas fourth-order cumulants are non-zero.

If $\{x(k)\}$ is, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$ is a real stationary discrete-time signal and its moments up to order n exist, then

$$m_n^x(\tau_1, \tau_2, \ldots, \tau_{n-1}) \triangleq \mathrm{E}\{\mathrm{x(k)x(k+\tau_1)x(k+\tau_2)} \ldots \mathrm{x(k+\tau_{n-1}}\} \tag{22}$$

represents the nth-order moment function of the signal, which depends only on the time differences $\tau_1, \tau_2, \ldots, \tau_{n-1}, \tau_i = 0, \pm 1, \pm 2, \ldots$. for all i. The 2nd –order moment function, $m_2^x(\tau_1)$, is the autocorrelation whereas $m_3^x(\tau_1, \tau_2)$ and $m_4^x(\tau_1, \tau_2, \tau_3)$, the 3rd and 4th order moments, respectively, are the third and fourth order (auto) correlations of the signal. The nth-order cumulant function of a non-Gaussian stationary random signal $x(k)$ can be written as (for n=3, 4 only):

$$c_n^x(\tau \quad \tau \quad \ldots \ \tau_{n-}\ )$$
$$m_n^x(\tau \quad \tau \quad \ldots \ \tau_{n-}\ ) - m_n^G(\tau \quad \tau \quad \ldots \ \tau_{n-}\ ) \tag{23}$$

where $m_n^x(\tau_1, \ldots, \tau_{n-1})$ is the nth-order moment function of $x(k)$ and $m_n^G(\tau_1, \tau_2, \ldots, \tau_{n-1})$ is the nth-order moment function of an equivalent Gaussian signal that has the same mean value and autocorrelation sequence as $x(k)$.Clearly, if $x(k)$ is Gaussian, $m_n^x(\tau_1, \tau_2, \ldots, \tau_{n-1}) = m_n^G(\tau_1, \tau_2, \ldots, \tau_{n-1})$ and thus $c_n^x(\tau_1, \tau_2, \ldots, \tau_{n-1}) = 0$. Note, however, that although equation (14) is only true for orders n=3 and 4, $c_n^x(\tau_1, \tau_2, \ldots, \tau_{n-1}) = 0$ for all n if $X(k)$ is Gaussian.

The relation between moments and cumulants are shown by the following equations which exist for orders n=1,2,3,4 for the sequence of $\{x(k)\}$.

1st –order cumulants:

$$c_1^x = m_1^x = E \tag{24}$$

2nd –order cumulants:

$$c_2^x(\tau_1) = m_2^x(\tau_1) - (m_1^x)^2$$
$$= m_2^x(-\tau_1) - (m_1^x)^2 = c_2^x(-\tau_1) \tag{25}$$

where $m_2^x(-\tau_1)$ is the autocorrelation sequence. Thus, we see that the 2nd order cumulant sequence is the covariance while the 2nd order moment sequence is the autocorrelation.

3rd –order cumulants:

$$c_3^x(\tau_1, \tau_2) = m_3^x(\tau_1, \tau_2)$$
$$-m_1^x[m_2^x(\tau_1) + m_2^x(\tau_2) + m_2^x(\tau_1 - \tau_2)] + 2(m_1^x)^3 \tag{26}$$

where $m_3^x(\tau_1, \tau_2)$ is the third-order moment sequence. This follows if we combine equation (22) and (23).

4th –order cumulants:

Combining equation (22) and (23), we get

$$c^x(\tau \quad \tau \quad \tau\ ) \quad m^x(\tau \quad \tau \quad \tau\ )$$
$$-m^x(\tau\ )\ m^x(\tau \ - \tau\ ) - m^x(\tau\ )\ m^x(\tau \ - \tau\ )$$
$$-\ m^x(\tau\ )\ m^x(\tau \ - \tau\ )$$
$$-\ m^x[m^x(\tau \ - \tau \quad \tau \ - \tau\ ) \quad m^x(\tau \quad \tau\ ) \quad m^x(\tau \quad \tau\ )$$
$$m^x(\tau \quad \tau\ )]$$
$$m^x \quad [m^x(\tau\ ) \quad m^x(\tau\ ) \quad m^x(\tau\ ) \quad m^x(\tau \ - \tau\ )$$
$$m^x(\tau \ - \tau\ ) \quad m^x(\tau \ - \tau\ )]$$
$$- \quad m^x \tag{27}$$

If the signal $\{x(k)\}$ is zero mean $m_1^x = 0$, it follows from equation (25) and (26) that the second- and third-order cumulants are identical to the second- and third-order moments, respectively; however, to generate the fourth-order cumulants, we need knowledge of the fourth-order and second-order moments in equation (27), i.e.,

$$c^x(\tau \quad \tau \quad \tau\ ) \quad m^x(\tau \quad \tau \quad \tau\ ) - m^x(\tau\ )\ m^x(\tau \ - \tau\ )$$
$$-\ m^x(\tau\ )\ m^x(\tau \ - \tau\ )$$
$$-\ m^x(\tau\ )\ m^x(\tau \ - \tau\ ) \tag{28}$$

The use of cumulants rather than moments has a number of advantages as pointed out by J. Mendel [11].

1. If $\lambda_i, i = 1,2, \ldots., k$ are constants and $x_i, i = 1,2, \ldots., k$ are random variables, then $cum(\lambda_i x_i, \ldots \lambda_k x_k) = \prod_{i=1}^{k} \lambda_i \, cum\,(x_1, \ldots . x_k)$
2. Cumulants are symmetric in their arguments, i.e., $cum(x_1, \ldots \ldots, x_k) = cum(x_{i_1}, \ldots \ldots, x_{i_k})$ where $(i_1, \ldots \ldots, i_k)$ is a permutation of $(1, \ldots \ldots, k)$
3. Cumulants are additive in their arguments, i.e., $cum(x_0 + y_0, z_1, \ldots, z_k) = cum(x_0, z_1, \ldots, z_k) + cum(y_0, z_1, \ldots, z_k)$ This means that cumulants of sums equals sums of cumulants (hence, the name "cumulant").
4. If α is a constant, then $cum(\alpha + x_1, x_2, \ldots \ldots, x_k) = cum(x_1, x_2, \ldots \ldots, x_k)$
5. If the random variables $\{x_i\}$ are independent of the random variables $\{y_i\}, i = 1,2, \ldots \ldots, k$ then $cum(x_1 + y_1, \ldots, x_k + y_k) = cum(x_1, \ldots, x_k) + cum(y_1, \ldots, y_k)$
6. If a subset of the k random variables $\{x_i\}$ is independent of the rest, then $cum(x_i, \ldots, x_k) = 0$

Cumulants of an independent, identically distributed random sequence are delta functions, if $w(t)$ is an i.i.d. process, then $C_{k,w}(\pi_1, \pi_2, \ldots \ldots, \pi_{k-1}) = \gamma_{k,w} \delta(\pi_1)\delta(\pi_2) \ldots \delta(\pi_{k-1})$, where $\gamma_{k,w}$ is the kth –order cumulant of the stationary random sequence $w(n)$.

This makes the higher-order statistics more robust to additive measurement noise than second order correlation, even if that noise is colored. In essence, cumulants can draw non-Gaussian signals out of Gaussian noise; thereby boosting their signal-to-noise ratios [11].

At a glance, Figure 2 illustrates the various higher-order spectra for a given discrete-time signal. Although higher-order statistics and spectra of a signal can be defined in terms of moments and cumulants, moments and moment spectra are very useful in the analysis of deterministic signals (transient and periodic) whereas cumulant and cumulant spectra are more useful in the analysis of stochastic signals. The two spectra are identical up for order three (the bispectrum). Unlike the power spectrum which is real-valued, higher order spectra can be complex valued and have both magnitude and phase, in general. The phase of the bispectrum is referred to as biphase and that of the trispectrum as the triphase.

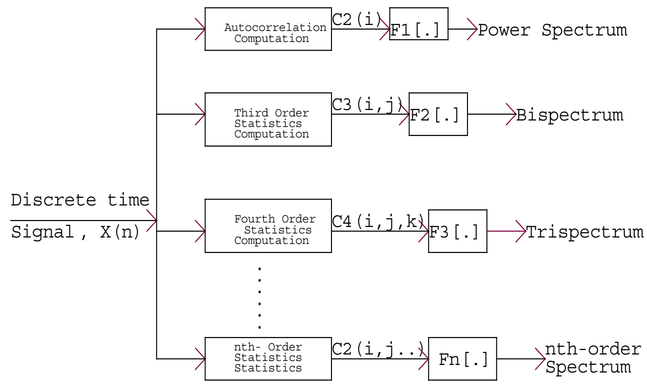


Figure 2 : The various higher order spectra for a deterministic signal. F [.] denotes n-dimensional Fourier Transform

Higher order spectra are also functions of two or more component frequencies unlike the power spectrum which is a function of a single frequency. Although numerically computed estimates of higher order spectra may have non-zero values, they may or may not be statistically significant. Statistical significance depends on the number of degrees of freedom in the estimate. Of particular interest in the analysis of phase coupling between Fourier components is the value of the magnitude of the higher order spectrum independent of the powers at the component frequencies. This can be achieved by normalizing the magnitude with powers at the component frequencies. Since non-linear interactions result in the generation of phase-coupled power at sum and difference frequencies, the normalized spectra are also useful in detection and characterization of non-linearity in systems. A normalized higher-order spectrum or nth-order coherency index is a function that combines the cumulant spectrum of order n with the power spectrum (n=2) of a signal. For a discrete- time signal, the 3rd – and 4th –order coherence are respectively defined by

$$B_{x(bico\,herence\,)}(f\;\;f\;) \qquad \frac{B_x\;f\;\;f}{\sqrt{P_x(f\;)P_x(f\;)P_x\;f\;\;\;\;f}} \tag{29}$$

$$T_{x\;\;trico\,herence} \qquad \frac{T_x\;\;f\;\;f\;\;f}{\sqrt{P_x(f\;)P_x(f\;)P_x(f\;)P_x\;f\;\;\;\;f\;\;\;\;f}} \tag{30}$$

These functions are very useful in the detection and characterization of nonlinearities in time series and in discriminating linear processes from nonlinear ones. A signal is said to be a linear non-Gaussian process of order n if the magnitude of the nth-order coherence, $|(P)_n^x(f_1, f_2, \ldots, f_{n-1})|$, is constant over all frequencies; otherwise, the signal is said to be a non-linear process[13]. They are also useful in detecting phase coupling between Fourier components.

III. Tutorial on Higher Order Statistics

A random signal, $x(n)$ is completely characterized by its Autocorrelation Function (ACF) only if it originates from a random process with Gaussian characteristics. In non-Gaussian processes, the higher order moments carry information that cannot be found in the ACF. Such signals can be found for example in speech, radar, sonar, bio-medicine, seismic data processing, plasma physics and optics. General relations for arbitrary stationary random data passing through arbitrary linear systems have been studied quite extensively for many years, but similar extensive general results are not available for arbitrary stationary random data passing through arbitrary nonlinear systems. Instead, each type of nonlinearity has been investigated as a special case. The extra information provided by HOS leads to better estimates of parameters and sheds light on non-linearity in the source of the signal. In addition, cross-polyspectra may be used for nonlinear system identification from observations of input and output data. In this section we represent some brief examples to the readers of how higher order spectral have been applied to the following tutorial topic.

a) Quadratic/Cubic Phase Coupling

Quadratic phase coupling are situations where because of interaction between two harmonic components of a process there is contribution to the power at their sum and/or difference frequencies. A special case is when have two components with one being at twice the frequency of the other. For example, suppose the signal

$$x(n) = A_1 \sin(\lambda_1 n + \theta_1) + A_2 \sin(\lambda_2 n + \theta_2)$$

is passed through the following simple nonlinear system:

$$z(n) = x(n) + cx^2(n) \qquad (31)$$

where c is a non-zero constant. The signal $z(n)$ contains sinusoidal terms in (λ_1, θ_1), (λ_2, θ_2), $(2\lambda_1, 2\theta_1)$, $(2\lambda_2, 2\theta_2)$, $(\lambda_1 + \lambda_2, \theta_1 + \theta_2)$ and $(\lambda_1 - \lambda_2, \theta_1 - \theta_2)$. Such phenomena, which give to rise to all of these phase relations, are known as quadratic phase coupling [12] and arise from the quadratic (square law) non-linearity.

In particular applications, such as EEG data analysis, oceanography and plasma physics, it is necessary to find out if peaks at harmonically related positions in the power spectrum are in fact phase-coupled. Since the power spectrum suppresses all phase relations it cannot provide an answer. The bispectrum, on the other hand, is capable of detecting and characterizing quadratic phase coupling. Consider the harmonic random process

$$x(n) \quad \sum_i \quad (\lambda_i n \quad \theta_i) \qquad (32)$$

where $\lambda_1 > \lambda_2 > 0$, $\lambda_4 > \lambda_5 > 0$, $\lambda_3 = \lambda_1 + \lambda_2$, $\lambda_6 = \lambda_4 + \lambda_5$, $\theta_1, \theta_2, \ldots\ldots \theta_5$ are all independent, uniformly distributed r.v.s over $(0, 2\pi)$ and $\theta_6 = \theta_4 + \theta_5$. In equation (32) while $(\lambda_1, \lambda_2, \lambda_3)$ and $(\lambda_4, \lambda_5, \lambda_6)$ are at harmonically related positions, only the component at λ_6 is a result of phase coupling between those at λ_4 and λ_5 while the one at λ_3 is an independent harmonic component. The power spectrum of the process consists of impulses at $\lambda_i; i = 1,2, \ldots, 6$ as illustrated in Figure 3.

Looking at the power spectrum one cannot say if the harmonically related components are, in fact, involved in quadratic phase-coupling relationships. The third-moment sequence $m_3^x(\tau_1, \tau_2)$ of $x(n)$ can be easily obtained as

$$\begin{aligned} m^x(\tau \quad \tau) \quad -\{ \quad & (\lambda\ \tau \quad \lambda\ \tau) \qquad (\lambda\ \tau \quad \lambda\ \tau) \\ & (\lambda\ \tau \quad \lambda\ \tau) \qquad (\lambda\ \tau\ - \lambda\ \tau) \\ & (\lambda\ \tau\ - \lambda\ \tau) \\ & (\lambda\ \tau\ - \lambda\ \tau)\} \end{aligned} \qquad (33)$$

It is important to observe that in equation (33) only the phase coupled components appear.

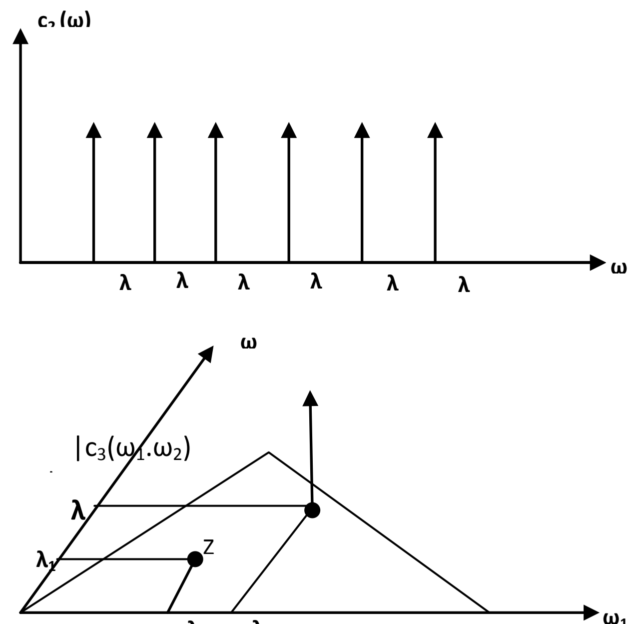


Figure 3 : Quadratic phase coupling (a) Power spectrum of the process described by equation (31) and (b) its magnitude bispectrum

Consequently, the bispectrum evaluated in the triangular region of Figure 3 shows an impulse only at (λ_4, λ_5) indicating that only this pair is phase coupled. In the total absence of phase coupling the third moment sequence and the bispectrum are both zero. Thus the fact that only phase coupled components contributes to the third moment sequence of a process is what makes the bispectrum a very useful diagnostic tool for non linear wave interactions.

In the example above, the third moment and the bispectrum were computed analytically using knowledge of the probability density functions of the phases. In practice, sampled data are available from the process and the question must be answered by numerical computation and hypothesis testing. If we simulate 100 realizations of this process, each of length 512 samples, sampled at 6 times the highest frequency component, with components of equal and unit amplitude at frequencies $\lambda_1 = 10, \lambda_2 = 20, \lambda_3 = 30, \lambda_4 = 40, \lambda_5 = 50, \lambda_6 = 90,$ such that each phase is uniform random in $[0,2\pi)$ and the phase relationship $\theta_6 = \theta_4 + \theta_5$ holds in each realization, a power spectral density of the process computed using an averaged periodogram estimate with no windowing would be as shown in Figure 4

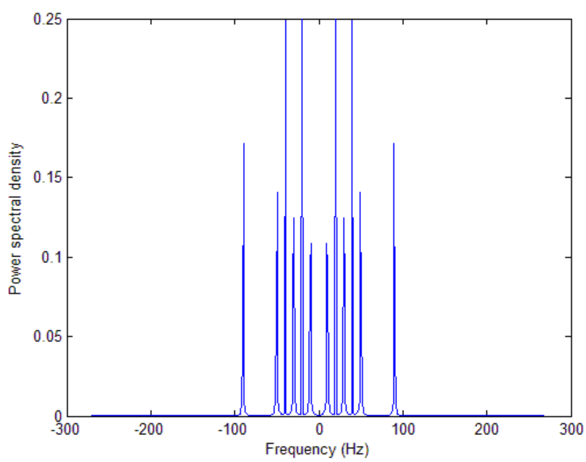


Figure 4 : Power spectral density of the process defined in equation 21, when estimated using an averaged periodogram estimate from 100 realizations

It can be observed that estimates of the power spectral density are not all equal because of the variance of the estimate and spectral leakage owing to the finite length discrete Fourier transform. The power spectrum does not reveal any phase information and does not distinguish between the peaks at the different frequencies. It does not tell us whether the peak at 90 Hz is related to those at 40 Hz and 50 Hz, or whether the peaks at 10 Hz, 20 Hz and 30 Hz are phase coupled. If we estimate the bispectrum of this process using an averaged biperiodogram method, its magnitude (Figure 5) shows a prominent peak at (50 Hz, 40 Hz) as expected. There is no such peak at (20 Hz, 10 Hz) because these components were not phase coupled to the component at 30 Hz. However, smaller peaks are also observed at other frequency combinations, again arising from the fact that statistical zero is not numerically zero and there is spectral leakage around the true peaks. The other peaks are much smaller in magnitude than the peak at (50 Hz, 40 Hz) which is where the bispectrum is statistically significantly different from zero.

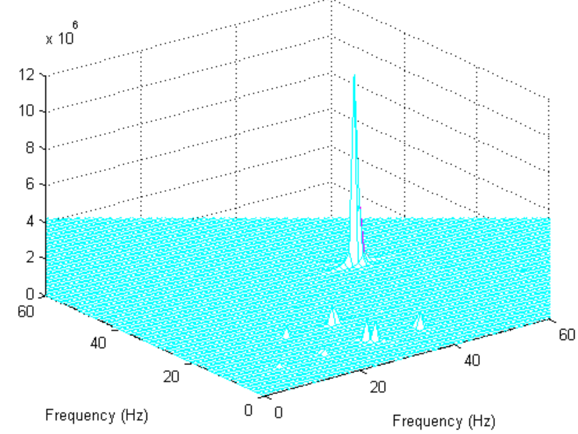


Figure 5 : Magnitude bispectrum for the process in equation (21) estimated from 100 simulated realizations

If we compute the bicoherence, normalizing as done by Kim and Powers, bicoherence values are found to be between 0 and 1. For 100 realizations averaged, the bicoherence is statistically significantly different from zero only when the value is above 6/200 or 0.03. A plot of the bicoherence is shown in Figure 6.

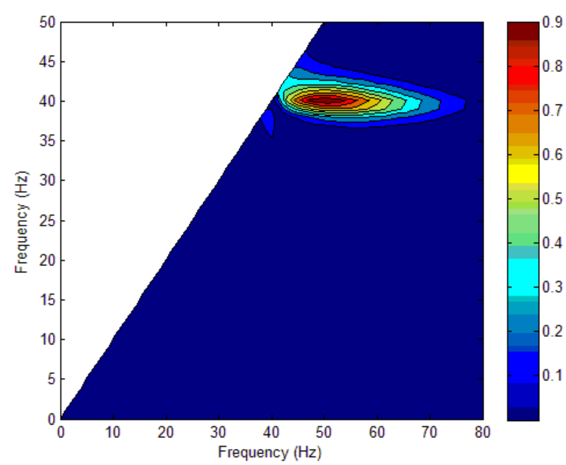


Figure 6 : Bicoherence for the processing in equation (21) estimated from 100 realizations. Only the bifrequency region of interest is shown here. A statistically significant non-zero value occurs at (50 Hz, 40 Hz) as expected

Only the region of frequencies that are of interest is shown here – not the entire triangular non-redundant region, the rest of which has statistically zero bicoherence. It can be observed here that the only value above 0.03 is at (50 Hz, 40 Hz). In reality, it is possible for an expected 5% of all the values to be above 0.03 if the process were Gaussian. In the above example, the process is harmonic and there is very little power at any other frequencies.

Let us now add a random phase component at the sum frequency of 90 Hz such that there is partial phase coupling rather than perfect phase coupling. The bicoherence at (50 Hz, 40 Hz) will now be the ratio of the power of the phase coupled component to the total power which is the sum of the powers of the phase coupled and random phase components since these are statistically independent.

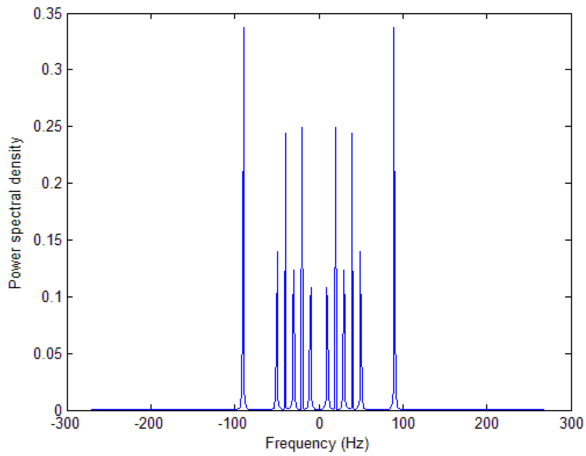


Figure 7 : Power spectral density of the process in equation (21) with an additional random phase component of equal power at 90Hz. The simulations also have additive Gaussian noise and the SNR is 20 dB

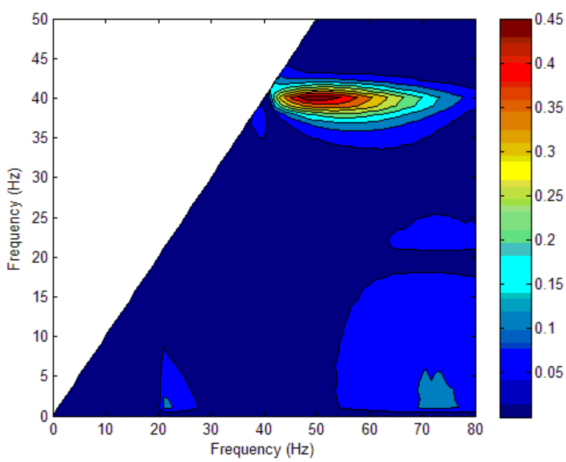


Figure 8 : Bicoherence for the same process as described for figure 7. The bicoherence at (50 Hz, 40 Hz) has now reduced to around 0.5 instead of 1.0 compared to Figure 6

Further, let us add Gaussian noise to the process such that the ratio of the total signal power to the total noise power is 20 dB. If we then compute the power spectrum and the bicoherence, they are as shown in Figure 7 and Figure 8, respectively.

We can see that the estimated bicoherence is close to the expected true value of 0.5. The estimate will in practice have a possibly non-zero bias and finite variance, which depend on a number of factors including the true value and the number of realizations averaged. Analytical expressions for these can be quite complicated. Statistics of bicoherence and tricoherence have been investigated numerically and analytically in [34],[61]. When phase coupled and random phase components are present along with additive Gaussian noise of significant variance, the estimate can be significantly different from the value expected. As the SNR falls, the statistics of the distribution will be decided by the noise rather than by the signal (the phase coupled harmonic components in the example above).

If the hypothesis to be tested is whether the signal is Gaussian or not, we can potentially use bicoherence values from all frequency triads along with the knowledge of their distribution for a Gaussian process. If we simulate 100 records consisting purely of Gaussian random noise of unit variance with the same sampling frequency and record length as in the examples above the bicoherence plot would be as shown in Figure 9 and the histogram of bicoherence values would be as shown in Figure 10. The red line indicates the bicoherence value below which 95% of all bicoherence lie. It can be observed that this is close to 6 divide by twice the number of realizations averaged or 0.03.

As we increase the number of realizations averaged, bicoherence tends towards the expected value of zero for white Gaussian noise.

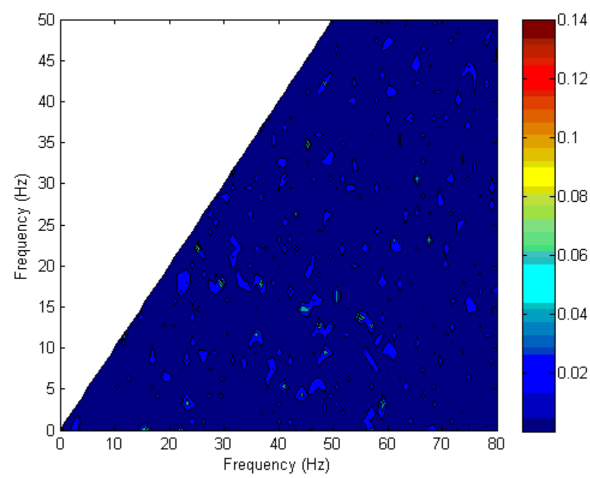


Figure 9 : Bicoherence distribution for a white Gaussian noise process over the same region as shown for the harmonic process in figures 6 and 8 and with the same number of realizations averaged. 5% of the values are now expected to be above the 95% significance level of 0.03 and some of these shows up as the lighter spots

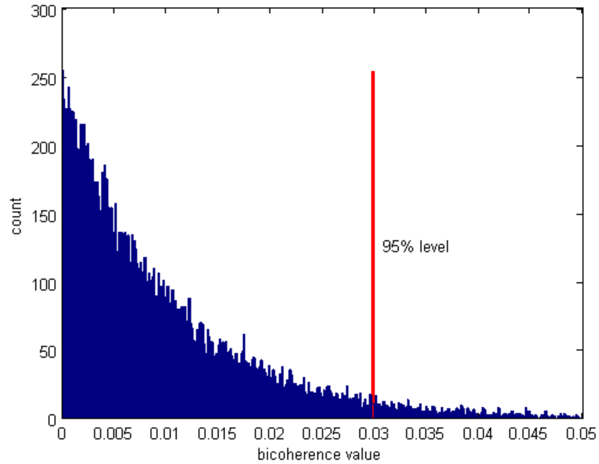


Figure 10 : Bicoherence distribution for a white Gaussian noise process when N=100 realizations are averaged in the estimate. The values are sorted and the value below which 95% lie is determined and shown using the red line above. It can be observed that this value is very nearly equal to 3/N = 0.03

If we want to test the hypothesis that a process is non-Gaussian, we can use this procedure and utilize the distribution of bicoherence over all bi-frequencies or any statistic derived from it. If we want to test whether the bicoherence at a given bi-frequency arose from phase coupled Fourier components or not, we can compare the bicoherence value to the 95% significance level for Gaussian noise. If the value is above this, we can be 95% confident that the true value was not zero. 5% of the values even from Gaussian noise can be above this level. The 99% confidence level is given by 9.2 divided by twice the number of realizations averaged. The higher the bicoherence estimate compared to these and the greater the number of realizations averaged, the greater the confidence in the estimate.

Let us now lower signal to noise power ratio to 40 dB and observe its effect on the power spectrum and the bispectrum. As shown in Figure 11, the power spectrum estimated by an averaged periodogram now shows spurious peaks at many frequencies owing to white noise. The variance at the true signal peaks is also higher. Because the signal is harmonic and comprises of narrowband components, the signal peaks are still above the wideband noise floor.

But the power spectrum does not retain any phase relationships. If we examine the bicoherence, as shown in Figure 12, we can note that the bicoherence at (50Hz, 40 Hz) is significantly above 0.03, the 95% confidence level for Gaussian noise, indicating that it is non-zero.

It is however far from the true value of 0.5 that would be expected from the harmonic components alone. This is owing to a bias introduced by the additive noise which will act as an additional random phase component or components within the bandwidth of the signal.

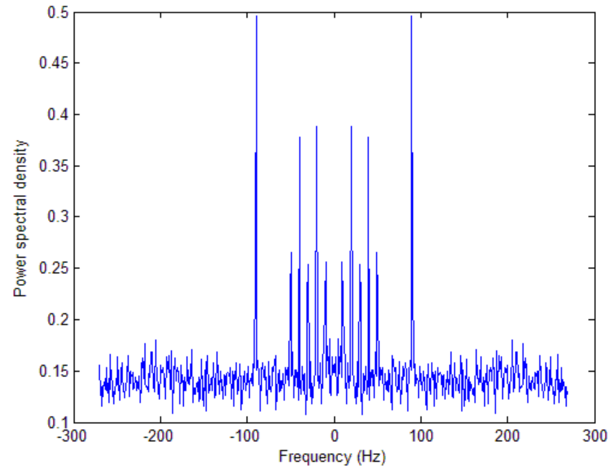


Figure 11 : Power spectral density of the same harmonic process as in figure 7 but with SNR lowered to -40 dB

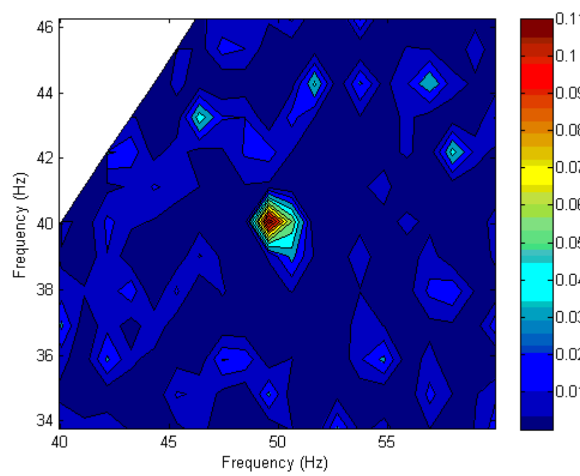


Figure 12 : Bicoherence plot for the same harmonic process as in figures 7 and 8 but with SNR lowered to -40 dB. Some values are above the 95% significance level of 0.03 as can be expected because of the additive white Gaussian noise present. The bicoherence at (50 Hz, 40 Hz) is lowered from 0.5 because of the wideband noise contribution within the signal bandwidth that acts as an additional random phase component

If we increase the number of realizations averaged to 1000, the distribution of bicoherence owing to Gaussian noise alone would tend towards zero, but the bias will depend on the SNR and will not decrease similarly. A bicoherence plot for 1000 realizations averaged is shown in Figure 13 below.

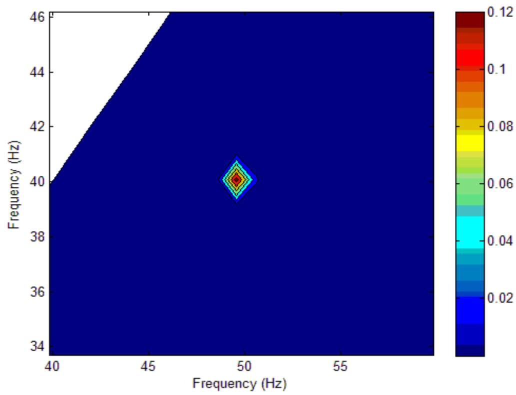


Figure 13 : Bicoherence for the same process as shown in Figure 12, but with 1000 realizations averaged in the estimate instead of 100. The 95% significance level for zero bicoherence is now 0.003. Therefore, the spuriously high bicoherence values appearing in Figure 12 have disappeared. However, the random phase components at 50 Hz, 40 Hz and 90 Hz owing to noise are present in all realizations and the bicoherence at (50 Hz, 40 Hz) remains close to 0.11, a value determined by the power ratio of phase-coupled and random phase components

Note that spurious values of high bicoherence arising at other bi-frequencies have decreased compared to Fig. 12 but the value of bicoherence at (50 Hz, 40 Hz) has not become 0.5.

In spite of a number of limitations, by contrast, the conventional methods for bispectrum estimation can serve as better quantifiers of phase coupling whereas the parametric methods such as autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA) are better as detectors rather than quantifiers [12]. Of course, the conventional bicoherence index approach to serve as a good quantifier by providing good estimates of the degree of phase coupling at harmonically related frequency pairs. For details and excellent explanation of quadratic phase coupling based on conventional and parametric methods, the readers are referred to [4, 12, 13].

Swami *et al* have shown [16] that the trispectrum can be used to resolve cubic phase coupling if a signal contains components from both quadratic and cubic phase coupling. The bispectrum of that signal is blind to the cubically-coupled components and can resolve the quadratically-coupled components, whereas the trispectrum of that signal is blind to the quadratically-coupled components and can resolve the cubically coupled components [11]. Higher-order spectra can thus resolve various types of nonlinearity. An application of higher order spectra to detect quadratic and cubic non-linear characteristics of the output of a non-linear system can be found in the work on analysis of Chua's circuit by Elgar and Chandran [40]. This circuit can exhibit various modes of operation depending on the circuit parameters. It shows periodic outputs, period doubling phenomena and can be in different chaotic regimes. Using bicoherence and tricoherence plots, Elgar and Chandran demonstrate that quadratic interactions are absent when the circuit exhibits the double scroll attractor behavior as evidenced by statistically zero bicoherence. The tricoherence remains significantly non-zero pointing to the presence of cubic phase coupling and interactions. By contrast, when the circuit exhibits Rossler attractor behavior, both the bicoherence and the tricoherence are statistically significantly non-zero and both quadratic and cubic interactions are inferred to be important to the dynamics.

b) Harmonic Retrieval

In several signal processing applications, for instance in estimating the direction of arrival (DOA) of narrow-band source signals with linear arrays and in the harmonic retrieval problem, the estimation of the number of harmonics and the frequencies and amplitudes of harmonics from noisy measurements is frequently encountered. With real signals $x(n)$ for the retrieval of harmonic in noise (RHN) problem, consider the system model is

$$y(n) \quad \sum_{i}^{p} a_i \quad (\omega_i n \quad \theta_i) \quad w(n) \quad x(n) \quad w(n) \qquad (34)$$

where the θ_i's denote random phases which are i.i.d. and uniformly distributed over $(0,2\pi)$, the ω_i's are unknown deterministic frequencies and the a_i's are unknown deterministic amplitudes. The additive noise $w(n)$is assumed to be white or colored Gaussian noise with unknown spectral density. The goal of this problem is to estimate the number of signals p, the angular frequenciesω_i's and the amplitudes a_i's.

Second-order statistics based high resolution methods such as MUSIC (Multiple Signal Classification) combined with singular value decomposition work well to estimate the number of harmonics p and their parameters if the additive noise is white. However, these methods break down in the case of colored noise to overestimate the number of sinusoids by treating the colored noise as additional sinusoids. Higher-order statistics have been successfully applied to this problem as they show robustness to additive Gaussian noise even when it is coloured.

Third-order cumulants for $y(n)$ equal zero; hence, this is an application where one must use fourth-order cumulants. According to Swami and Mendel [16], the fourth-order cumulants of $y(n)$ is a function of three lags. However, the diagonal slice of this cumulant is given as

$$c^y(\tau \quad \tau \quad \tau) \triangleq c^y(\tau) \quad -- \sum_{i}^{p} a_i \quad (\omega_i \tau) \qquad (35)$$

The autocorrelation of $y(n)$ is given by

$$c^{y}(\tau) \quad -\sum_{i}^{p} a_i \;\; sin\,(\omega_i \tau) \tag{36}$$

Except for a different scale factor, the fourth-order cumulants $c_4^y(\tau)$ from equation (35) and (36) can be easily treated as an autocorrelation function of the following signal which is directly related to $y(n)$:

$$y\;(n) \quad \sum_{i}^{p} a_i \qquad (\omega_i n \quad \theta_i) \quad w(n) \tag{37}$$

Thus, for the RHN and DOA problem, the diagonal slice of fourth-order cumulant retains all the useful signal characteristics, and consequently, signal parameters can be estimated. Additionally, we show that the 1-D slice is identical with the autocorrelation of a related noiseless signal.

c) Array Processing

One of the most fundamental problems in signal processing is that of removing noise and interference from a received sensor signal. There are two general approaches. Single-sensor methods such as the celebrated Wiener filter enhance the signal by emphasizing frequencies with a high signal- to- noise ratio (SNR) while attenuating those with a low SNR. On the other hand, multichannel techniques employ an array of sensors that perform spatial discrimination or beamforming to aid in removing the unwanted noise. Array processing problems include: directional of arrival (DOA) determination, determination of number of sources, beamforming, estimation of the source signal, source classification, sensor calibration, etc., and the readers refer to [17] for an excellent introduction to array processing and its associated models.

Although many novel and interesting array processing algorithms have appeared, higher-order statistics have been used to solve the array processing problem due to a number of reasons, namely: (1) Capon's minimum-variance distortionless response (MVDR) beamformer, that has been the starting point for both signal enhancement and high-resolution DOA estimation, requires very specific and detailed information about the so-called array steering vector(e.g., source steering angles, array geometry, receiver responses), information that is often not available, or if available is not given to a high degree of accuracy; (2)When additive noise is colored and Gaussian, a second-order statistics based high-resolution DOA algorithm, such as MUSIC, does not perform well, however a cumulant-based MUSIC algorithm does perform well; (3) most second-order statistics based beamformers assume that the received signals are not coherent, which rules out the important case of multipath propagation; cumulant-based beamformers can work in the presence of multipath[18].

Dogan et al used cumulants of received signals to estimate the steering vector of a narrowband non-Gaussian desired signal in the presence of directional Gaussian interferers with unknown covariance structure. They assume no knowledge of the DOA information about the desired signal. The desired signal could be voice speech, sonar and radar return signal.

For radar return signal, consider an array of M elements with arbitrary sensor response characteristics. Assume that the Gaussian interference signal, $i(t)$ with J number and the non-Gaussian desired signal, $d(t)$ at center frequency, f_0. The additive noise is assumed to be Gaussian with unknown covariance. Consequently, the array of M elements measurements can be collected together to give the following model.

$$\boldsymbol{r}(t) = \boldsymbol{a}(\theta_d)d(t) + A_I(\boldsymbol{\theta})\boldsymbol{i}(t) + \boldsymbol{n}(t) \tag{38}$$

where θ_d is the DOA of the desired signal, $\boldsymbol{a}(\theta_d)$ is the array steering vector of the desired signal, $A_I(\boldsymbol{\theta})$ is the array steering matrix for the J interference sources $\boldsymbol{i}(t)$, $\boldsymbol{\theta}$ is a $J \times 1$ vector of DOA's for the interferers, $\boldsymbol{r}(t)$ and $\boldsymbol{n}(t)$ are the $M \times 1$ vector of received signals and Gaussian noises respectively.

The output of an MVDR beamformer can be expressed by Capon [20] as

$$\boldsymbol{y}(t) = \boldsymbol{w}^H\boldsymbol{r}(t) = [\beta_1 R^{-1}\boldsymbol{a}(\theta_d)]^H \tag{39}$$

where constant β_1 is a constant which maintains a specified response for the desired signal, $\boldsymbol{w}$ denotes the weight vector of the processor and R is the covariance matrix of $\boldsymbol{r}(t)$.

Now, by using the properties of cumulants which are described by Mendel [11] and the receiver model in equation (28), Dogan and Mendel show that

$$\boldsymbol{c} = \beta_2\boldsymbol{a}(\theta_d) \tag{40}$$

where β_2 is a another constant. It is said that $\boldsymbol{c}$ is a replica of the steering vector of the desired signal up to a scale factor β_2.

Combining equation (39) and (40), it is shown that the cumulant-based MVDR beamformer output is

$$y(t) = \boldsymbol{w}_{cumulant}{}^{H}\boldsymbol{r}(t) = [\beta_3 R^{-1}\boldsymbol{c}]^H\boldsymbol{r}(t) \tag{41}$$

where $\beta_3 = (\boldsymbol{c}^H R^{-1}\boldsymbol{c})^{-1}$

At present, HOS has become attractive in array signal processing due to two additional reasons, namely (1) HOS can increase the effective aperture of an array, and (2) HOS can not only eliminate the effects of additive Gaussian noise, but it can also eliminate the effects of additive non-Gaussian noise.

d) Blind Deconvolution and Equilazation

The blind deconvolution or equalization problem, deals with the reconstruction of the input sequence given the output of a linear system and statistical information about the input. Blind deconvolution algorithms are essentially adaptive filtering algorithms [17] designed in such a way that do not need the external supply of a desired response to generate the error signal in the output of the adaptive equalization filter whereas Classical deconvolution is concerned with the task of recovering an excitation signal, given the response of a known time-invariant linear operator to that excitation [19]. Blind deconvolution algorithm itself generates an estimate of the desired response by applying a non-linear transformation on sequences involved in the adaptation process. Detailed discussion on blind deconvolution can be found in [5].

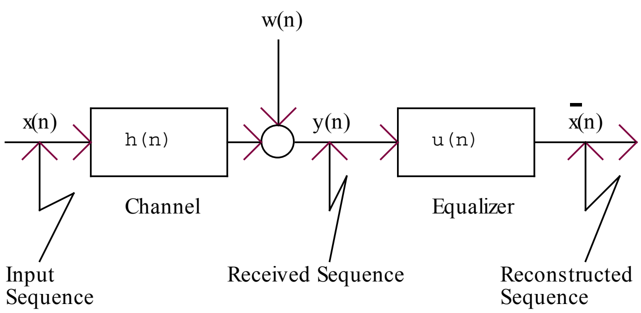


Figure 14 : Block diagram of a baseband communication system subject to additive noise

Let us consider a discrete-time linear transmission channel with impulse response, $h(n)$ which is unknown and time-varying. The input signal $x(n)$ is assumed to be independent and identically distributed (i.i.d) random variables with a non-Gaussian probability density function, with zero mean and variance. Initially the noise will not be taken into account in the output of the channel which can be written as from Figure 14.

$$y(n) \quad h(n) * x(n)$$

$$\sum_{i \quad -\infty}^{\infty} x(n)h(n-i)$$

$$\sum_{i \quad -\infty}^{\infty} h(n)x(n-i) \qquad (42)$$

Table 1 : Summary of Developed Blind Equalization Algorithm Based on Higher-order Statistics

Develop algorithm	Significant work	Related Reference
Tricepstrum Equalization Algorithm (TEA)	Estimate the equalizer impulse response by using the complex cepstrum of the fourth-order cumulants of the synchronously sampled received signal.	[Hatzinako and Nikias,1991]
Power Cepstrum and Tricoherence Equalization Algorithm (POTEA)	Recovers the Fourier magnitude of the equalizer using autocorrelations and its Fourier phase fourth-order cumulants and the cepstrum of the Tricoherence.	[Bessios and Nikias, 1991]
Cross-Trispectrum Equalization Algorithm (CTEA)	Extension of TEA to the multichannel case using the cross-cumulants of the observed signals.	[Brooks and Nikias, 1991]

The goal of this problem is to restore $x(n)$ from the received sequence $y(n)$ to identify the inverse filter, $u(n)$ of the channel.

From Figure 14, it is seen that the reconstructed signal $\tilde{x}(n)$ of the equalizer is given by

$$\tilde{x}(n) = u(n) * y(n) = u(n) * h(n) * x(n) \qquad (43)$$

It is achieved that the output reconstructed signal $\tilde{x}(n)$ is given by

$$\tilde{x}(n) = x(n-D)e^{j\theta} \qquad (44)$$

where D and θ are a constant delay and a constant phase shift respectively. The convolution sum of the equalizer function and channel response function can be written as

$$u(n) * h(n) = \delta(n-D)e^{j\theta} \qquad (45)$$

where $\delta(n)$ is the Kronecker delta function. Taking the Fourier transform of equation (45) and we obtain

$$U(\omega)H(\omega) = e^{j(\theta-\omega D)} \qquad (46)$$

Hence, the objective of the equalizer is to achieve a transfer function

$$U(\omega) = \frac{1}{H(\omega)}e^{j(\theta-\omega D)} \qquad (47)$$

In general, D and θ are unknown. However, the constant delay D does not affect the reconstructed of the original input signal, $x(n)$. The phase constant, θ can be removed by a decision device.

Blind equalization algorithms based on higher-order statistics (HOS) perform a nonlinear transformation on the input of the equalizer filter. This nonlinear transformation is memory nonlinearity and it is identical to the generation of higher-order cumulants of the received channel data. For more details, the readers are referred to the following table 1, about the various algorithms of blind equalization based higher-order statistics.

e) Narrow and Wide band Interference Cancellation

When a modulating signal is corrupted by an additive interference and an auxiliary reference signal which is highly correlated with the interference, the elimination of the interference is accomplished by an adaptive noise canceller using fourth-order statistics procedure which is shown in Fig. 4. The objective of an adaptive noise canceller is to produce a system output that best fits with the modulating signal. A conventional ANC which is called adaptive noise canceller second-order statistics (ANC-SOS) algorithm cannot be applied in practice due to two major difficulties, namely (1) the ANC-SOS filter is affected directly by uncorrelated noises at the primary and reference inputs, (2) the ANC-SOS algorithm is problem dependent, i.e., it is very sensitive to both the reference signal statistics and the choice of step size.

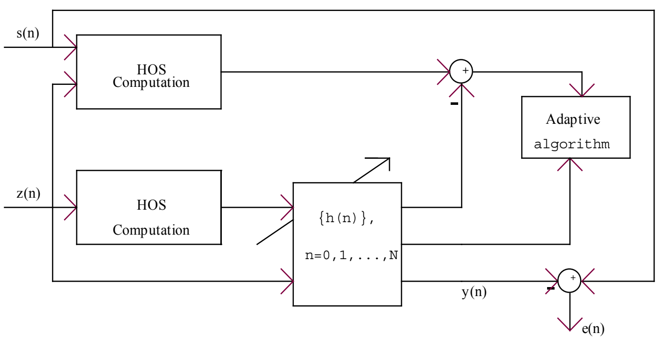


Figure 15 : Block diagram of adaptive interference canceller based on higher-order statistics

Let $x(n)$ and $z(n)$ denote the received signal and interference signal sequence respectively, they are both sampled in chip rate, satisfying

$$x(n) = s(n) + i(n) + n_p(n) \tag{48}$$

$$z(n) = w(n) + n_r(n) \tag{49}$$

where $s(n)$ is the modulating signal, $i(n)$ is the narrowband interference signal, and $w(n)$ is a zero-mean non-Gaussian reference signal highly correlated with the interference. The measurement noise $n_p(n)$ and $n_r(n)$ are stationary, zero-mean, white or colored Gaussian process. The interference can be described by the interference signal as

$$i(n) \quad \sum_{k \quad -\infty}^{\infty} g(k)w(n-k) \tag{50}$$

Let $y(n)$ be an adaptive filter output

$$y(n) \quad \sum_{k}^{N-} h(k)z(n-k) \tag{51}$$

where N denotes the number of taps and $\{h(n), n = 0,1, \dots, N-1\}$ is the adaptive filter coefficients. The system output $e(n)$ is an evaluate of modulating signal $s(n)$, which is given by

$$e(n) = x(n) - y(n) \tag{52}$$

f) Time-Delay Estimation

Time-delay estimation is one of most important method for broad-band source bearing and range calculation.

Assume that $x(n)$ and $y(n)$ are two discrete time signals which are spatially separated sensor measurements that satisfy the following equations

$$x(n) = s(n) + w_1(n) \tag{53}$$

$$y(n) = As(n-D) + w_2(n) \tag{54}$$

where $s(n)$ is an unknown broad-band source bearing signal, $s(n-D)$ is D unit delay version of $s(n)$ and $w_1(n)$ and $w_2(n)$ are unknown noise source. The basic approach to solve the time delay estimation problem is to shift measurement sequence $x(n)$ with respect to $y(n)$ by taking cross correlation technique and we obtain.

$$\begin{aligned} c_{xy}(\tau) \quad & E\{x(n)y \; n \quad \tau\} \quad E\{x(n \quad \tau)y \; n\} \\ & \sum_{n \quad -\infty}^{\infty} x(n)y(n \quad \tau) \quad \sum_{n \quad -\infty}^{\infty} x(n \quad \tau)y \; n \\ & Ac^{s}(\tau - D) \qquad - \infty \quad \tau \quad \infty \end{aligned} \tag{55}$$

Equation (53) and (54) provided that $w_1(n)$ and $w_2(n)$ are zero-mean stationary signals which are independent with each other with $s(n)$ and $c_2^s(\tau) = E\{s(n)s(n+\tau)\}$ is the covariance sequence of $s(n)$. In practical situations, due to finite length data records and noise source that are not exactly independent, the $c_{xy}(\tau)$ does not necessary to show peak at $\tau = D$ at time delay D. It is well known to [21], Carter said that various window technique such as ROTH, SCOT, PHAT, Eckart and Hannan-Thompson have been suggested to smooth the cross-correlation function in order to improve the quality of time delay estimation.

g) Classification

Signal classification can be done working directly with the signal samples, or with the attributes related to them. It is difficult to handle additive colored Gaussian noise with traditional approaches. A new approach [22] that is blind to additive colored or white Gaussian noise, works with vector of cumulants or higher order spectra, and extends correlation-based classification to HOS-based classification. It is based on the important fact that estimates of cumulants or higher order spectra are asymptotically Gaussian. Consequently, one is able to begin with an equation like 'Estimate of HOS = HOS + estimation error' In which the "estimation error" is asymptotically Gaussian and extend traditional classification or detection procedures to this information. Working with a vector of higher-order statistics is in the spirit of using attributes which are derived from the original data. Consequently, higher-order statistics now provide new attributes to be used in signal classification problems including detection.

By minimizing a distance measure in the cumulant domain Tsatsanis et al built shift-invariant and noise-insensitive classifiers which are summarize in table 2. For more details, the readers are referred to [23]

IV. Application to Communication Systems

Wireless single input- single output (SISO) techniques like BPSK, OQPSK, MSK, GMSK are used commonly in modern communication [24]. Most communication signals are corrupted by noise, thus it seems particularly important to detect signals more effectively at low SNR. There are many algorithms of signal detection and recognition based on higher-order statistics now, but few at low SNR. Furthermore, there are fewer algorithms based on higher-order statistics at low SNR with non-Gaussian background.

The bi-spectrum has been used in a technique [24] to suppress Gaussian noise for increasing the channel capacity (bit/s) as well as to test the inhibition effect of higher order statistics and the detection effect of BPSK, OQPSK, MSK, GMSK signals based on bi-spectrum in different SNR. Using the third order cumulant, Zhong Zhang et all used Matlab software [24] for finding the detection rate, by assuming following parameters:

Table 2 : Summary of Object CLASSIFYING BASED on Higher-Order Statistics

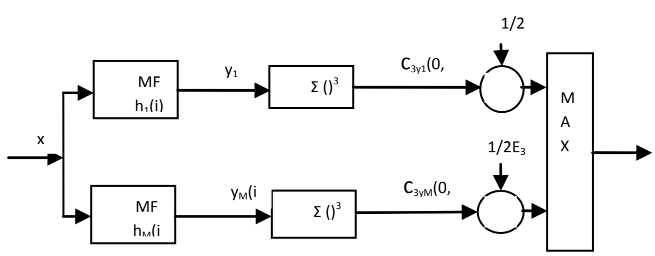
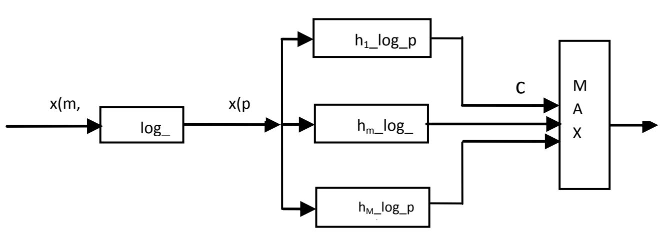
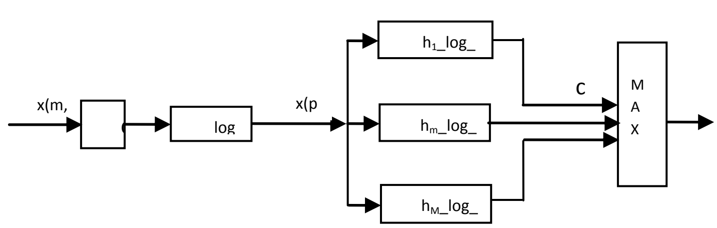
Classification	Block diagram	Algorithm's procedure
Shift-invariant Classification		Step 1. Build a bank of filters $h_m\ i\ {}^N_i$, where each one is matched to a template $h_m\ i =$ $s_m\ N-i\ , m\ \ldots\ M.$ Step 2. Filter $x\ i$ through these filters and compute $y\ _m$ at the output of each filter. Step 3. Add the bias term $-E\ _m$ at the output of each matched filter to compensate for templates with unequal triple correlation energies. Step 4. Select the maximum, say m^*, to declare H_m .
Rotation- and Scaling- invariant Classification		Step 1. Transform all templates to log-polar grid and design a bank of filters matched to the transformed templates. Step 2. Transform the input image to log-polar grid. Step 3. Apply the translation invariant algorithm to the transformed image.
Shift, Rotation and Scaling-invariant Classification		Step 1. Compute the zeroth slice $x\ \ i$ of the triple correlation of x_i. Step 2. Sample it in a log-polar grid to form $x\ (\ \ \rho\ \theta)$. Step 3. Apply the rotation-scaling invariant algorithm to $x\ (\ \ \rho\ \theta)$.

Table 3 : Bi-Sectrum Processing Effect of BPSK, OQPSK, MSK, GMSK Signal at Different SNR

Communication Signal Processing	SNR in dB	Detection Rate
BPSK (Binary Phase Shift Keying)	0	100%
	-4	100%
	-8	100%
	-12	95%
	-14	65%
OQPSK (Orthogonal Quadratic Phase Shift Keying)	0	100%
	-4	100%
	-8	100%
	-12	80%
	-14	60%
MSK (Minimum Phase Shift Keying)	0	100%
	-4	100%
	-8	100%
	-12	75%
	-14	60%
GMSK (Gaussian Minimum Shift Keying)	0	100%
	-4	100%
	-8	100%
	-12	70%
	-14	65%

Sample points: 3840, Carrier frequency: 150000 Hz, Sampling frequency: 1200000Hz, Code rate: 125000, SNR: -14db, which is shown in table 3.

On the other hand, table 3. shows that the detection rate decreases with SNR decreased. On the other hand, four signals can achieve the desired detection effect at 0,-4,-8 db and detection rate starts to decline from -12db. According to the different signals and different intensity of background noise, an effective detection rate can be obtained using higher-order cumulant.

Li Taijie et al proposed a Genetic algorithm which is independent of both noise source and reference signal as well as HOS which can preserve information of non-Gaussian stationary random process [25].Compare with the adaptive filters based on second-order statistics, the HOS and GA-based filter can reject the interference more efficiently and is independent of uncorrelated Gaussian noise. So it is important to obtain an algorithm which is independent of both uncorrelated noise sources and the statistics of the reference signal. For this purpose, D.C. Shin and C.L. Nikias [26] have introduced an adaptive scheme based on HOS for narrow band interference rejection.

Besides, Spread spectrum is itself a powerful technique to mitigate multipath propagation but when, as in Cellular CDMA (Code Division Multiple Access), bandwidth limitations are consequent to system design considerations, other counter measures must be adopted [30], 31], [33]. Multipath propagation degrades system performance not only because it introduces intersymbol interference (ISI) but also because it deteriorates the orthogonality property of the spreading codes. To overcome this limitation, Massimiliano Martone [27] studied a cellular direct sequence spread spectrum CDMA system which seems to be attractive due to numerical stability and computational complexity based on fourth order cumulants with respect to TDMA and FDMA.

In many applications, it is desired to extract some information from a signal corrupted by multiplicative noise. One such application lies in mobile communications where the transmitted signals are bandlimited, frequency or phase-modulated signals, $T_x(t)$ is given by

$$T_x(t) = Ae^{j(2\pi f_0 t + \phi(t))} \tag{56}$$

where $\phi(t)$ represents the baseband signal, f_0 is the carrier frequency and A is the constant amplitude of the transmitted signal. The mobile radio channel is a time-varying multipath channel and is subject to physical propagation path loss [28].The time-variations are caused by the medium changes as the vehicles moves. The propagation losses are related to both the atmospheric propagation and the terrain configuration. The multipath aspect is caused by different scatterers and reflectors such as buildings or trees that surround the mobile unit which is shown in Figure 16.

As a result of these propagation phenomena in a narrow-band transmission, where narrow-band is defined with respect to the coherence bandwidth of the channel [22], the received signal $R_x(t)$ is given by:

$$R_x(t) = Ae(t)e^{j(2\pi f_0 t + \phi(t) + \psi_r(t))} \tag{57}$$

where $e(t)$ and $\psi_r(t)$ are respectively a Rayleigh distributed random amplitude modulation signal and a uniformly distributed random phase.

Both $e(t)$ and $\psi_r(t)$ affect the performance of the receiver which results in an increase of bit-error rate. Current techniques for reducing the effect of the Rayleigh multiplicative noise are based on the use of diversity techniques,[28]. As a matter of fact, a new cost-effective method based on Time-varying Higher Order Spectra (TV-HOS), particularly the Wigner-Ville Trispectrum (WVT) is proposed for optimal recovery of the information contained in the signal without use of diversity techniques in [29].

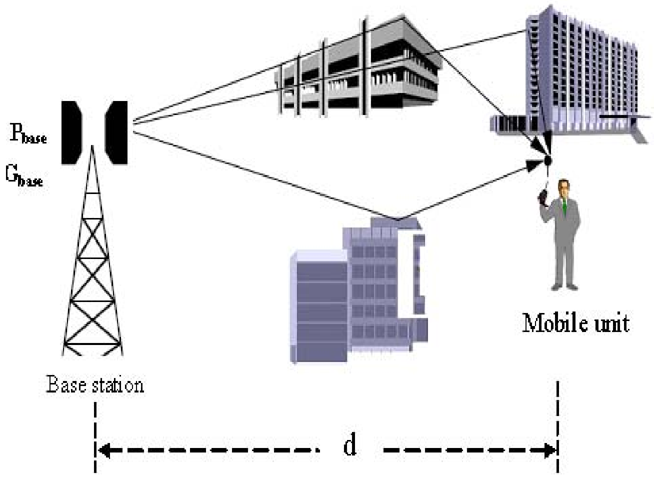


Figure 16 : Multipath Fading Phenomenon

For non-stationary random signal, Time-Varying Higher-Order Spectra (TV-HOS) extends the class of Polynomial Wigner-Ville Distributions (PWVD). The k-th order Wigner-Ville spectrum of a random signal $x(t)$ is defined as

$$ {}^{k}_{x}(t\ f)\quad \int_{-\infty}^{\infty} m_x^k(t\ \tau)\, e^{-j\ \ \pi f\tau}\, d\tau \qquad (58)$$

where $m_x^k(t,\tau)$ represents a specific slice of the time-varying k-th order moment function of $x(t)$ and is expressed as

$$ m_x^k(t\ \tau)\quad E\left(x^*(t\quad \lambda\ \tau)\prod_{i}^{k-} P_i\quad [x(t\quad \lambda_i\quad \tau)]\right) $$

(59)

where $(\lambda_1, \ldots, \lambda_k)$ determines the slice, * represents the complex-conjugate operator, $P_{i+1}[.]$ is the complex conjugate operator and identity operator if *i* is odd and even respectively, and $E(.)$ is expectation function.

For $k = 4$, the k-th order Wigner-Ville spectrum converts to Wigner-Ville Trispectrum which is defined as

$$ M_x(t\ f)\quad E(W_x\ (t\ f)) $$
$$ \int_{-\infty}^{\infty} E\left[x(t\quad {}^{\tau}/\ )\ x^*(t - {}^{\tau}/\ )\ \right] e^{j\ \ \pi f\tau}\, d\tau \qquad (60)$$

where $W_x^4(t,f)$ is the fourth-order Polynomial Wigner-Ville Distribution (PWVD) function of $x(t)$and $M_x^4(t,f)$ is referred to as moment-based Wigner-Ville Trispectrum (WVT).

In [29] this moment based WVT which is described in equation (48), in particular, is successful to recover the spectral characteristics as well as optimum estimation of the instantaneous frequency (IF) of the signals for increasing the system performance.

V. Potential of Hos for Further Applications

SISO systems are used commonly in modern communication system owing to their reduced cost and complexity. But most communication signals encounter degradations in channel (e. g. scatters and reflectors such as buildings and trees) and it is particularly important to detect signals effectively at low SNR.

MIMO systems show promise in this context. It may be possible to use HOS to analyze MIMO system performance taking mutual electromagnetic coupling of antenna elements into consideration.

Non-stationary and cyclostationary signal analysis with HOS-based time frequency representations can reveal information on sub-manifolds in higher dimensional frequency spaces that offer greater SNR pathways in available channels. It can lead to design of coding techniques that may be superior to those based on second order information theory concepts and channel capacity limits.

VI. Conclusion

Regardless of the use of diversity or spatial multiplexing technique, the main drawback of any MIMO system is increased complexity and cost. Due to this reason, there is now great interest in Higher Order Spectra based schemes. HOS can be applied in transmitter section, receiver section or on both sides and have the potential to retain the performance improvement similar to that of a full complexity MIMO system at reduced complexity and cost. In addition, spectrum estimation techniques have proved essential to the creation of advanced communication systems. These techniques only use second-order statistical information, which means that we have been assuming that the signals are inherently Gaussian. Most real – world signals are non Gaussian. It is no wonder, therefore, that spectral techniques often have serious difficulties in practice. For this reason, there is a need for a thorough literature review and original innovations in this area with emphasis on wireless communication and pattern recognition and so on.

There is much more information in a stochastic non-Gaussian or deterministic signal than conveyed by its auto- correlation or spectrum. Higher-order spectra, which are defined in terms of the higher-order statistics of a signal, contain this additional information.

Signal processing algorithms based on higher-order spectra are now available for use in commercial and military applications. The emergence of low cost very high speed hardware chips and the ever growing availability of fast computers now demand that we extract more information than we have been doing in the past from signals, so that better decisions can be made. All of the new algorithms that have been developed using higher-order spectra are application driven. That is why higher order spectra (HOS) are an attractive scheme for the above mentioned field.

Acknowledgement

The material presented in this paper is based on the textbook by C.L Nikias and A.P. Petropulo, Higher- Order spectral Analysis, Oppenheim Series of Signal Processing, Prentice-Hall, Inc. 1993 as well as the tutorials by C.L. Nikias and M.R. Raghuveer, “Bispectrum Estimation: A Digital Processing Framework”, Proc. IEEE, Vol. 75, july 1987.

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Before start writing a good quality Computer Science Research Paper, let us first understand what is Computer Science Research Paper? So, Computer Science Research Paper is the paper which is written by professionals or scientists who are associated to Computer Science and Information Technology, or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

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11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

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15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

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20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including <u>definite statistics</u> - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
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Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
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ISSN 9755896