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Highlights

Results Involving Certain

Generalized Fermat Points

Regular Recursion Trees

Fuzzy Multiset Approach

Discovering Thoughts, Inventing Future

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Regular Recursion Trees: Description and Theoretical Analysis

By Ulyanov M. V. & Goloveshkin V. A.
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Abstract- Recursive algorithms analysis by generated trees node counting refers to detailed study of their structures. In this connection the paper presents the specific description of peculiar to a number of recursive algorithms regular trees. We have developed the method which is based on introduced regular description and provides an analytical solution for a number of generated nodes at such regular tree each level. The results obtained make possible a theoretical time complexity analysis of recursive algorithms generating regular recursion trees.

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REGULAR RECURSION TREES DESCRIPTION AND THEORETICAL ANALYSIS

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1. Goloveshkin V. A., Ulyanov M. V. Recursion theory for programmers. — M.: FIZMATLIT. 2006. 296.

Regular Recursion Trees: Description and Theoretical Analysis

Ulyanov M. V. [α] & Goloveshkin V. A. [σ]

Abstract - Recursive algorithms analysis by generated trees node counting refers to detailed study of their structures. In this connection the paper presents the specific description of peculiar to a number of recursive algorithms regular trees. We have developed the method which is based on introduced regular description and provides an analytical solution for a number of generated nodes at such regular tree each level. The results obtained make possible a theoretical time complexity analysis of recursive algorithms generating regular recursion trees.

I. Introduction

The rational algorithm selection problem solution in software development as a rule is based on one of the main procedure quality characteristics called the time complexity. The time complexity is understood to be a number of assumed computing model elementary operations specified at the concrete input by the algorithm [1]. The estimate of higher value is a time complexity function with the size of algorithm input as the function argument. Meaningfully the time complexity function is considered in the best, worst and average cases [2]. Existing methods of algorithms analysis produce algorithm computational complexity – the time complexity function asymptotical estimate in the worst case and in some cases – the time complexity true function to solve the rational choice problem in real input sizes ranges. The time complexity true function derivation is a highly topical problem both for iterative and recursive algorithm solutions in which terms of theoretical study several methods are suggested at the present time. The methods produce analytical expressions of time complexity functions [1, 2, 3]. The generated recursion tree analysis technique is one of advanced methods in this direction. The most complicated step of it is obtaining of analytic expressions for number of generated recursion tree nodes at each tree level [1].

The main problem of recursion tree theoretical study is that in common case a number of generated nodes is the function of both the tree level number and a number of the generating node at this level. Moreover, in some cases such a function is not given analytically and the tree structure is markedly caused by specifics of concrete algorithm input, so the problem becomes significantly complicated. As an example, we consider a recursive algorithm of the solution of the classical problem of optimal packing by the dynamic programming method for which a generated tree is defined by both packing size

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and cargo types sizes [1]. Only solutions for complete m-ary trees are trivial to this problem.

Thus in the recursive algorithms study aspect the problem of generated recursion trees theoretical study in nontrivial cases is to be of evident interest.

II. Problem Setting Up

Notes

According to generated recursion tree nodes counting based time complexity analysis method [1, p. 181-185.] we will use the following notation:

— n is the depth of the recursion tree, i.e. the number of its levels, in addition the base of the tree is not numbered and has the specific index Root and the further numbering starts with zero;

— S is the formal description of generated recursion tree structure;

— $R_A(n,S)$ is the total node number for all n-deep levels of S structured recursion tree;

— $R_V(n,S)$ is the tree internal nodes total number;

— $R_L(n,S)$ is the terminal node number, i.e. the node number at the level n.

The article has two objects in view:

1. To offer the formal description S of the recursion tree having the regular structure that is typical of generating recursive algorithms, including the algorithms that implement dynamic programming method.
2. To develop a solution procedure that produces the analytical solution of the tree nodes number at the level $R(j,S)$ from the present tree structure formal description S and the given level number j. Thus if the current algorithm input produces the tree with the depth n and the structure S then the main tree characteristics which are necessary for the recursive algorithm time complexity analysis are calculated as

$$\begin{aligned} R_A(n,S) &= \sum_{j=0}^{n} R(j,S)+1; \\ R_V(n,S) &= \sum_{j=0}^{n-1} R(j,S)+1; \qquad (1) \\ R_L(n,S) &= R(n,S). \end{aligned}$$

Note that the vertex of the recursion tree root is accounted for the complementary unity.

III. Formal Description of the Regular Recursion Tree Structure

Let us start generating of the description of the structure from consideration of complete m-ary tree. It is plain, that in this case each node at each level (besides leaf level) produces evenly m new tops and the tree is completely self-similar starting from any node. Because features of some recursive algorithms are that the tops of the same level produce different amount of the next level nodes that is however less then some

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prescribed value we propose the following regular tree formal description as ordered m-ary tuple (S line) with non-ascending sorted unit values

$$S = (k_1; k_2; \ldots; k_j; \ldots; k_m),\ j = \overline{1,m};$$
$$k_1 \geq k_2 \geq \ldots \geq k_j \geq \ldots \geq k_m. \tag{2}$$

Notes

The S line structured tree is generated by the following algorithm using the specific node numeration within the level and the concept of node type. At the tree root level the **Root** node generates m tops of the level $n = 0$ using $1, 2, \ldots, m$ numeration. Let us assign the characteristic to each node – a number from the interval $[1, \ldots, m]$ – and call it the node type. We will say further that if a node has the type "j" then it generates k_j tops at the tree next level. At the level $n = 0$ the nodes generated by the root get the same of their number type, so the j-numbered node has the type "j" and thus generates k_j tops at the level 1. Next, at the next levels the generated nodes are numbered relative to the parent node starting from the number m from right to left in the order of numbers decreasing, and get the types that correspond to assigned numbers. Thus the nodes generated directly from the "j"-typed top (there are k_j of such nodes) are described by the following line

$$(0; 0; \ldots; k_l; k_{l+1}; \ldots; k_m),\ l = m - k_j + 1. \tag{3}$$

The proposed formal description as the line S (2) specifies the regular tree, i.e. the tree conforming to single at all levels regularity of nodes generating defined by the node type according to node numeration and generating way given in the formula (3). We note that in case of $k_1 = m$ the root has the type "1" and the line S is given by $S = (m; k_2; \ldots; k_j, \ldots; k_m)$ and describes a regular incomplete (truncated) m-ary tree.

For example, the formal description $S = (3; 3; 3)$ specifies a complete ternary tree and the description $S = (3; 3; 2)$ specifies the regular incomplete ternary tree illustrated in fig. 1. The tree nodes are marked by their numbers coinciding to their types in accordance to described numeration principle. Thus all "3"-typed nodes generate two tops at the next level because the formal description line $S = (3; 3; 2)$ includes $k_3 = 2$. Also let us remark here that the tree root level has the **Root** index and is not numbered and the next tree levels have numbers 0, 1 etc. We note that the node type introduced above corresponds to the top number in the formal description line $S = (k_1; k_2; \ldots; k_j; \ldots; k_m)$ and the "j"-typed node generates k_j tops which description line is specified in the formula (3).

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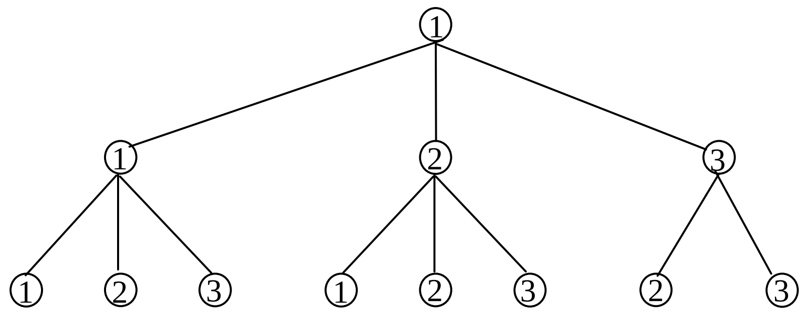

Fig. 1 : The fragment of the tree specified by the description $S = (3; 3; 2)$ with node types marked.

IV. Method of the Number of Regular Tree Nodes Problem Analytical Solving

Without losing generality we define the nodes number counting problem as the problem of the n-deep tree leaf number counting where the tree is generated by the description $S = (k_1; k_2; \ldots; k_j; \ldots; k_m)$, i.e. the counting of the regular tree nodes at the level n – the tree leaf level. With the preceding notation we solve the problem of determining $R(j,S)$ at $j = n$, i.e. the problem of obtaining the analytical solution for $R(n,S) = R_L(n,S)$.

Let us introduce the "j"-typed node characteristic $F(j)$. Meaningfully $F(j)$ is the type number of nodes generating the type"j" by the description S. $F(j)$ value is calculated from the formula

$$F(j) = \max_{1 \le i \le m} \{\, i : m - k_i + 1 \le j \,\}. \tag{4}$$

Let $w_{n,j}$ is the number of "j"-typed nodes at the tree level n. Let us introduce the vector $W_n = \{w_{n,1}; w_{n,2}; \ldots; w_{n,m}\}$ with dimension m. Thus the vector components W_n contain the number of each type nodes at the level n of recursion tree. Then with consideration of the characteristic $F(j)$ introduced above we obtain the following system of linear recurrence relations to calculate the $w_{n,j}$ values

$$\begin{cases} w_{\text{Root},1} = 1, \quad w_{\text{Root},j} = 0, \ j = \overline{2, m}, \quad n = \text{Root}; \\ w_{0,j} = 1, \ j = \overline{1, m}, \quad n = 0; \\ w_{n+1,j} = \displaystyle\sum_{i=1}^{F(j)} w_{n,i}, \quad n \ge 1. \end{cases} \tag{5}$$

We note that in accordance with the (5) $W_0 = \{1; 1; \ldots; 1\}$ and if $k_1 = m$ then $W_{\text{Root}} = \{1; 0; \ldots; 0\}$

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From the known method of linear recurrence relations solving [1, chapter 1], we will try solutions of the relation (5) in the following form

$$\left\{ \lambda \cdot h_{0,j} = \sum_{i=1}^{F(j)} h_{0,i}, \quad \forall j = \overline{1,m} . \right. \tag{7}$$

Our problem required solution – the vector W_n – is given by the linear combination of the computed solutions (6) with consideration of initial conditions from (5).

Let us formulate the suggested method stages for the recurrence relation (5) solving in the case when the number of eigenvectors H_0 is equal to m.

1. Evaluation of the node type characteristics $F(j)$ by the formula (4) from the offered formal description of the regular recursion tree in the form of the line S (2).
2. Formation of the linear recurrence relations system (5).
3. The derivation of the solutions in the form $H_n = \lambda^n H_0$.
   - 3.1. Construction of the equations set (7).
   - 3.2. Defining λ values by the condition of equality of zero and the system determinant (7).
   - 3.3. Finding of eigenvectors H_0 values for every value of λ.
4. Construction of the initial system (5) solutions W_n as a linear combination of the obtained solutions for H_0 from defining unknown coefficients of the linear combination by the values of initial conditions defined by the vector W_0.
5. Counting the number of nodes at the level n of the recursion tree under study

$$R(n,S) = \sum_{j=1}^{m} w_{n,j} . \tag{8}$$

6. Since obtained analytical solution for W_n holds for every value of n the formula (8) gives the solution also for $R(j,S)\,\forall j = \overline{0,n}$ what enables us to define the total number of generated nodes at all n-deep tree levels and the number of the internal nodes according to the formula (1).

We particularly note that in case of the number of eigenvectors H_0 is less than m the known special solving methods [1, 3, 4] is necessarily to be used.

V. Examples of Regular Recursion Tree Analysis

Later we will give the examples of regular trees analysis for both cases: when the number of eigenvectors H_0 is equal to or less than m.

a) At first we consider the example of the tree specified by the description $S = (3; 3; 2)$ which form fig. 1 shows. We point out in advance that the number of eigenvectors H_0 is equal to m in this case. We illustrate the method in details pointing

Ref

3. R. Graham, D. Knuth, Patashnik O. Concrete Mathematics. Foundation of computer science. Translated from English. — M.: Mir. 1998. 703.

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directly numbers and content of the stages of suggested solving method.

1. Evaluation of $F(j)$ by the description $S=(3;3;2)$:

$$F(1)=2,\ F(2)=3,\ F(3)=3.$$

2. Formation of the linear recurrence relations system (5):

$$\begin{cases} w_{0,j}=1,\ j=\overline{1,3}; \\ w_{n+1,1}=w_{n,1}+w_{n,2}; \\ w_{n+1,2}=w_{n,1}+w_{n,2}+w_{n,3}; \\ w_{n+1,3}=w_{n,1}+w_{n,2}+w_{n,3}. \end{cases}$$

3.1. Construction of the equations set (7):

$$\begin{cases} \lambda h_{0,1}=h_{0,1}+h_{0,2}; \\ \lambda h_{0,2}=h_{0,1}+h_{0,2}+h_{0,3}; \\ \lambda h_{0,3}=h_{0,1}+h_{0,2}+h_{0,3}. \end{cases} \tag{9}$$

3.2. Defining λ values by the condition of equality of zero and the system determinant:

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix}=0.$$

Computing the determinant we obtain the equation $\lambda\cdot(\lambda^2-3\lambda+1)=0$ which roots are:

$$\lambda_1=0;\quad \lambda_2=\frac{3-\sqrt{5}}{2};\quad \lambda_3=\frac{3+\sqrt{5}}{2}.$$

3.3. Finding of eigenvectors H_0 values for every value of λ.

Plugging in (9) the roots obtained and solving obtained systems we get eigenvectors H_0 which corresponds to the secular equation roots by the numeration:

$$H_0^{(1)}=\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix};\quad H_0^{(2)}=\begin{pmatrix} 1 \\ \frac{1-\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{pmatrix};\quad H_0^{(3)}=\begin{pmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \\ \frac{1+\sqrt{5}}{2} \end{pmatrix}.$$

4. Construction of the initial system (5) solutions W_n as a linear combination of the obtained solutions for H_0 with initial conditions W_0:

$$W_n=A\cdot 0^n\cdot H_0^{(1)}+B\cdot\left(\frac{3-\sqrt{5}}{2}\right)^n\cdot H_0^{(2)}+C\cdot\left(\frac{3+\sqrt{5}}{2}\right)^n\cdot H_0^{(3)},$$

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where by definition $0^0 = 1$ and A, B, C are linear combination unknown coefficients defined by the initial conditions W_0:

$$\begin{cases} A + B + C = 1; \\ -A + \dfrac{1-\sqrt{5}}{2}B + \dfrac{1+\sqrt{5}}{2}C = 1; \\ \dfrac{1-\sqrt{5}}{2}B + \dfrac{1+\sqrt{5}}{2}C = 1. \end{cases}$$

The solution of obtained system is of the form:

$$A = 0; \quad B = \frac{\sqrt{5}-1}{2\sqrt{5}}; \quad C = \frac{\sqrt{5}+1}{2\sqrt{5}},$$

that gives the terminal solution for the number of nodes of the regular tree under review

$$W_n = \frac{\sqrt{5}-1}{2\sqrt{5}} \cdot \left(\frac{3-\sqrt{5}}{2}\right)^n \cdot H_0^{(2)} + \frac{\sqrt{5}+1}{2\sqrt{5}} \cdot \left(\frac{3+\sqrt{5}}{2}\right)^n \cdot H_0^{(3)}.$$

5. Counting the number of nodes at the level n of the recursion tree under study
According to (8) the total nodes number at the level n is the sum of each type nodes:

$$R(n, S) = \sum_{j=1}^{3} w_{n,j}.$$

Making the necessary transformations we obtain the desired solution for $S = (3; 3; 2)$:

$$R(n, S) = \frac{7+3\sqrt{5}}{2\sqrt{5}} \cdot \left(\frac{3+\sqrt{5}}{2}\right)^n + \frac{3\sqrt{5}-7}{2\sqrt{5}} \cdot \left(\frac{3-\sqrt{5}}{2}\right)^n.$$

It may be verified by direct substitution that $R(1, S) = 8$, $R(2, S) = 21$.

б) The second example where the regular tree is given by the description $S = (3; 3; 1)$ also gives three different eigenvectors. In this case we will give the main results only. The values of λ obtained by the condition of equality of zero and the system determinant are

$$\lambda_1 = 0; \quad \lambda_2 = 1; \quad \lambda_3 = 2.$$

that gives the following solution for corresponding typed nodes number

$$w_{n,1} = 2^n; \quad w_{n,2} = 2^n; \quad w_{n,3} = 2 \cdot 2^n - 1.$$

Summing over all types we obtain the solution for the tree level

$$R(n, S) = 2^{n+2} - 1.$$

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в) The next example produces the case that needs the specific solving method. Let us consider the description $S=(m;\, m-1;\, m-2;\, \dots;1)$. Fig. 2 shows the tree that corresponds to this description at $m=3$ – $S=(3;\,2;\,1)$.

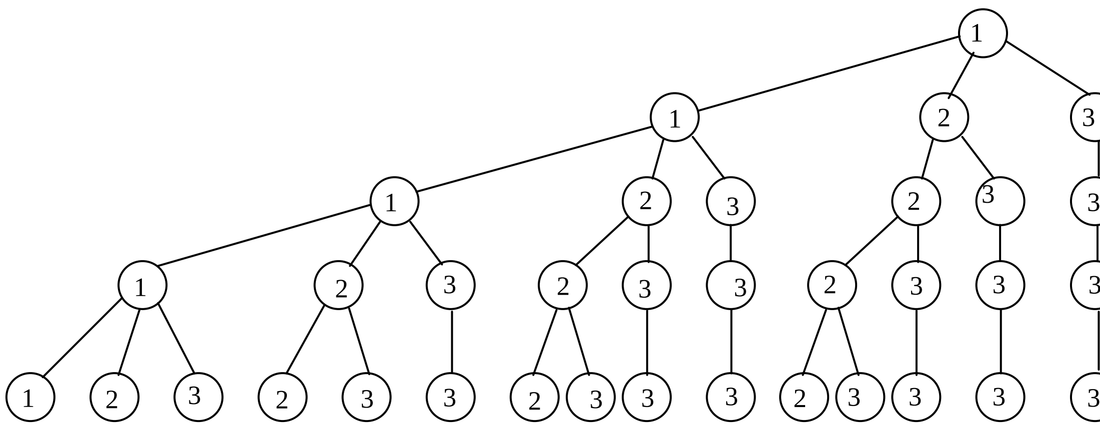

Fig. 2 : The fragment of the tree specified by the description $S=(3;\,2;\,1)$ with node types marked.

The initial stages of this case analytical solution are the same as for the method described above:

1. Evaluation of $F(j)$ by the description $S=(m;\, m-1;\, m-2;\, \dots;1)$:

$$F(1)=1,\, F(2)=2,\, F(3)=3, \dots, F(m)=m\,.$$

2. Formation of the linear recurrence relations system (5):

$$\begin{cases} w_{0,j}=1,\; j=\overline{1,3}; \\ w_{n+1,1}=w_{n,1}; \\ w_{n+1,2}=w_{n,1}+w_{n,2}; \\ \dots \\ w_{n+1,j}=w_{n,1}+w_{n,2}+\dots+w_{n,j}; \\ \dots \\ w_{n+1,m}=w_{n,1}+w_{n,2}+w_{n,3}+\dots+w_{n,m}; \end{cases} \tag{10}$$

3.1. Construction of the equations set (7):

$$\begin{cases} \lambda h_{0,1}=h_{0,1}; \\ \lambda h_{0,2}=h_{0,1}+h_{0,2}; \\ \lambda h_{0,3}=h_{0,1}+h_{0,2}+h_{0,3}; \\ \dots \\ \lambda h_{0,m}=h_{0,1}+h_{0,2}+h_{0,3}+\dots+h_{0,m}. \end{cases} \tag{11}$$

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3.2. Defining λ values by the condition of equality of zero and the system determinant:

Calculating the determinant of the system (11) we get the equation $(1-\lambda)^m = 0$ and thus the multiplicity of the root $\lambda = 1$ is m. In this problem the number of eigenvectors is less then m and we use the specific method that provides for the direct solving of the system (10).

Notes

4. The direct solving of the system (10).

4.1. Let us solve the recurrence relation for $j = 1$:

$$\begin{cases} w_{0,1} = 1; \\ w_{n+1,1} = w_{n,1}. \end{cases} \Rightarrow \forall n \; w_{n,1} = 1.$$

4.2. From the solution obtained we get the recurrence relation for $j = 2$:

$$\begin{cases} w_{0,2} = 1; \\ w_{n+1,2} = w_{n,2} + 1. \end{cases} \Rightarrow \forall n \; w_{n,2} = n + 1.$$

4.3. With consideration of this solution the recurrence relation for $j = 3$ is of the form

$$\begin{cases} w_{0,3} = 1; \\ w_{n+1,3} = w_{n,3} + (n+1) + 1. \end{cases} \tag{12}$$

We seek the specific solution in the form $w^*_{n,3} = An^2 + Bn + C$ and plugging it in (12) we get

$$2An + A + B = n + 2, \quad \Rightarrow \quad A = \frac{1}{2}, B = \frac{3}{2},$$

the initial condition gives the solution $C = 1$ and finally

$$w_{n,3} = \frac{1}{2}n^2 + \frac{3}{2}n + 1 = \frac{(n+1)(n+2)}{2}.$$

4.4. We note that $w_{n,1} = 1 = C_n^0$, $w_{n,2} = n + 1 = C_{n+1}^1$, $w_{n,3} = C_{n+2}^2$. Let us prove by induction that for an arbitrary value m

$$w_{n,m} = C_{n+m-1}^{m-1}. \tag{13}$$

The relation (13) is true for $m = 1, m = 2, m = 3$. By the induction hypothesis, the relation $w_{n,k} = C_{n+k-1}^{k-1}$ is true for all $k \le m$. Then by proceeding the linear recurrence relations system (10) for $m + 1$ we obtain the recurrence relation

$$\begin{aligned} w_{n+1,m+1} &= w_{n,1} + w_{n,2} + w_{n,3} + \ldots + w_{n,m} + w_{n,m+1} = \\ &= C_n^0 + C_{n+1}^1 + C_{n+2}^2 + \ldots + C_{n+m-1}^{m-1} + w_{n,m+1}. \end{aligned} \tag{14}$$

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We note that $C_n^0 = C_{n+1}^0$, $C_n^k + C_n^{k-1} = C_{n+1}^k$ [3], then

$$C_n^0 + C_{n+1}^1 + C_{n+2}^2 + \ldots + C_{n+m-1}^{m-1} = C_{n+2}^1 + C_{n+2}^2 + \ldots + C_{n+m-1}^{m-1} = C_{n+m}^{m-1}.$$

We show that

$$w_{n,m+1} = C_{n+m}^m$$

is the solution of the recurrence to n relation (14). Plugging this putative solution in (14) we get

$$w_{n+1,m+1} = C_{n+m}^{m-1} + C_{n+m}^m = C_{n+m+1}^m,$$

that corresponds to (13) with substituting for corresponding indexes. Thus the solution of the system (10) $\forall m \geq 1, \forall n \geq 1$ is of the form

$$w_{n,m} = C_{n+m-1}^{m-1},$$

that proves the inductive hypothesis (13).

From the solutions obtained we define the total nodes number at the tree level numbered n by summing the number of all type nodes at this level. We introduce the third argument m into the function of nodes number since we are concerned with generic description $S = (m;\, m-1;\, m-2;\, \ldots;1)$ and using the known relations for the binomial coefficients [3] we get:

$$R(n,m,S) = \sum_{j=1}^{m} w_{n,j} = \sum_{j=1}^{m} C_{n+j-1}^{j-1} = C_{n+m}^{m-1}. \tag{15}$$

From (15) let us define the total nodes number of the tree having description $S = (m;\, m-1;\, m-2;\, \ldots;1)$ by summing the number of nodes at all levels including the tree root

$$R_A(n,m,S) = 1 + \sum_{i=0}^{n} R(i,m,S) = \sum_{i=0}^{n} C_{i+m}^{m-1} = C_{n+m+1}^m. \tag{16}$$

Referring to the tree fragment shown in fig. 2 and having the description $S = (3;2;1)$, i.e. $m = 3$ we obtain the total nodes number at the level $n = 3$ by using the formula (15) – $R(3,3,S) = C_{3+3}^{3-1} = C_6^2 = 15$ – and define the nodes sum at all levels including the level $n = 3$ from the formula (16) – $R_A(3,3,S) = C_{3+3+1}^3 = C_7^3 = 35$. We note that the solution obtained by us is the same as the results of the generated recursion tree study of the algorithm of the solution of the optimal one-dimensional packing problem by dynamic programming method at the problem regular parametrization [1]. Notwithstanding the result contained in [1] is particular to this paper results because it gives the solution only for the trees having the description $S = (m;\, m-1;\, m-2;\, \ldots;1)$.

VI. Conclusion

For the sake of recursive algorithm analysis and study this paper suggests the specific description of regular recursion trees typical of some recursive algorithms. Such

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trees are generated for example by the algorithms that realize recursively the dynamic programming method with the special parametrization which regularizes the trees generated by recursive algorithms. Under this description the solution procedure is suggested that gives the number of generated nodes at each level of a such regular tree. The method devised by us produces the analytical solution for the number of generated nodes over every level of the recursion tree and thus the total number of nodes as the function of the tree depth.

Notes

The results obtained can be used for detailed theoretical analysis of recursive algorithms generating regular recursion trees and for getting true functions of time complexity and capacitory efficiency.

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Certain New Formulae Involving Modified Bessel Function of First Kind

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GJSFR-F Classification : *MSC 2010: 33C10*

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Certain New Formulae Involving Modified Bessel Function of First Kind

Salahuddin [α] & Intazar Husain [σ]

Abstract- In this paper we have developed certain new results involving Hypergeometric function, Modified Bessel function of first kind and exponential function. The results represent here are assume to be new.

Keywords: hypergeometric function, bessel function, modified bessel function of first kind, exponential function.

I. Introduction

a) *Bessel Function*

Bessel functions, first defined by the mathematician Daniel Bernoulli and generalized by Friedrich Bessel, are the canonical solutions y(x) of Bessel's differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \alpha^2)y = 0 \tag{1.1}$$

for an arbitrary complex number α (the order of the Bessel function). The most important cases are for α an integer or half-integer.

b) *Bessel Function of First Kind*

Bessel functions of the first kind, denoted as $J_\alpha(x)$, are solutions of Bessel's differential equation that are finite at the origin ($x = 0$) for integer or positive α, and diverge as x approaches zero for negative non-integer α. It is possible to define the function by its Taylor series expansion around $x = 0$.

$$J_\alpha(x) = \sum_{m=0}^{\infty}\frac{(-1)^m}{m!\ \Gamma(m+\alpha+1)}\left(\frac{x}{2}\right)^{2m+\alpha} \tag{1.2}$$

where Γz is the gamma function, a shifted generalization of the factorial function to non-integer values. The Bessel function of the first kind is an entire function if α is an integer. The graphs of Bessel functions look roughly like oscillating sine or cosine functions that decay proportionally to $\frac{1}{\sqrt{x}}$.

c) *Modified Bessel Function of First Kind*

The Bessel functions are valid even for complex arguments x, and an important special case is that of a purely imaginary argument. In this case, the solutions to the Bessel equation are called

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the modified Bessel functions (or occasionally the hyperbolic Bessel functions) of the first and second kind. The first kind of Bessel function is defined as

$$I_\alpha(x) = \iota^{-\alpha} J_\alpha(\iota x) = \sum_{m=0}^{\infty} \frac{1}{m!\,\Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha} \tag{1.3}$$

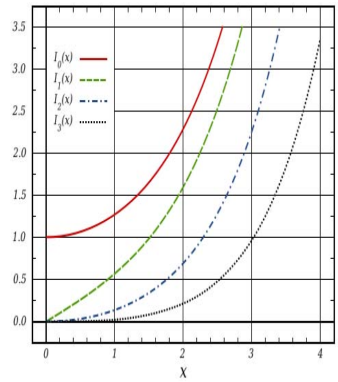

Figure 1 : Modified Bessel Function of First Kind

d) Exponential Function

In mathematics, the exponential function is the function e^x, where e is the number (approximately 2.718281828) such that the function e^x is its own derivative. The exponential function is used to model a relationship in which a constant change in the independent variable gives the same proportional change (i.e. percentage increase or decrease) in the dependent variable. The function is often written as $exp(x)$, especially when it is impractical to write the independent variable as a superscript. The exponential function is widely used in physics, chemistry, engineering, mathematical biology, economics and mathematics.
In particular the exponential function may be defined as

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots \tag{1.4}$$

e) Generalized Hypergeometric Functions

A generalized hypergeometric function ${}_pF_q(a_1, \ldots a_p; b_1, \ldots b_q; z)$ is a function which can be defined in the form of a hypergeometric series, i.e., a series for which the ratio of successive terms can be written

$$\frac{c_{k+1}}{c_k} = \frac{P(k)}{Q(k)} = \frac{(k+a_1)(k+a_2)\ldots(k+a_p)}{(k+b_1)(K+b_2)\ldots(k+b_q)(k+1)}\, z. \tag{1.5}$$

Where $k+1$ in the denominator is present for historical reasons of notation, and the resulting generalized hypergeometric function is written

$${}_pF_q\left[\begin{array}{ccc} a_1, a_2, \cdots, a_p & ; & \\ & & z \\ b_1, b_2, \cdots, b_q & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_p)_k z^k}{(b_1)_k (b_2)_k \cdots (b_q)_k k!} \tag{1.6}$$

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or

$$ {}_pF_q\left[\begin{array}{ccc}(a_p) & ; & \\ & & z \\ (b_q) & ; & \end{array}\right] = \sum_{k=0}^{\infty}\frac{((a_p))_k z^k}{((b_q))_k k!} \tag{1.7} $$

where the parameters $b_1, b_2, \cdots, b_q$ are neither zero nor negative integers and p, q are non-negative integers.

The ${}_pF_q$ series converges for all finite z if $p \leq q$, converges for $|\ z\ | < 1$ if $p \neq q+1$, diverges for all z , $z \neq 0$ if $p > q+1$.

The ${}_pF_q$ series absolutely converges for $|\ z\ | = 1$ if $R(\zeta) < 0$, conditionally converges for $|\ z\ | = 1, z \neq 0$ if $0 \leq R(\zeta) < 1$, diverges for $|\ z\ | = 1$, if $1 \leq R(\zeta)$, $\zeta = \sum_{i=1}^{p} a_i - \sum_{i=0}^{q} b_i$.

The function ${}_2F_1(a, b; c; z)$ corresponding to $p = 2, q = 1$, is the first hypergeometric function to be studied (and, in general, arises the most frequently in physical problems), and so is frequently known as "the" hypergeometric equation or, more explicitly, Gauss's hypergeometric function (Gauss 1812, Barnes 1908). To confuse matters even more, the term "hypergeometric function" is less commonly used to mean closed form, and "hypergeometric series" is sometimes used to mean hypergeometric function.

The hypergeometric functions are solutions of Gaussian hypergeometric linear differential equation of second order

$$ z(1-z)y'' + [c - (a+b+1)z]y' - aby = 0 \tag{1.8} $$

The solution of this equation is

$$ y = A_0\Big[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\Big] \tag{1.9} $$

This is the so-called regular solution,denoted

$$ {}_2F_1(a, b; c; z) = \Big[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\Big] = \sum_{k=0}^{\infty}\frac{(a)_k\ (b)_k z^k}{(c)_k k!} \tag{1.10} $$

which converges if c is not a negative integer for all of $|\ z\ | < 1$ and on the unit circle $|\ z\ | = 1$ if $R(c - a - b) > 0$.

It is known as Gauss hypergeometric function in terms of Pochhammer symbol $(a)_k$ or generalized factorial function.

f) Whittaker Function

In mathematics, a Whittaker function is a special solution of Whittaker's equation, a modified form of the confluent hypergeometric equation introduced by Whittaker (1904) to make the formulas involving the solutions more symmetric. More generally, Jacquet (1966, 1967) introduced Whittaker functions of reductive groups over local fields, where the functions studied by Whittaker are essentially the case where the local field is the real numbers and the group is $SL_2(R)$. Whittaker's equation is

$$ \frac{d^2w}{dz^2} + \Big(-\frac{1}{4} + \frac{k}{z} + \frac{\frac{1}{4} - \mu^2}{z^2}\Big)w = 0 \tag{1.11} $$

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It has a regular singular point at 0 and an irregular singular point at ∞. Two solutions are given by the Whittaker functions $M_{k,\mu}(z), W_{k,\mu}(z)$, defined in terms of Kummer's confluent hypergeometric functions M and U by

$$M_{k,\mu}(z) = exp\big(-\frac{z}{2}\big)z^{\mu+\frac{1}{2}}M\Big(\mu-k+\frac{1}{2}, 1+2\mu; z\Big) \tag{1.12}$$

$$W_{k,\mu}(z) = exp\big(-\frac{z}{2}\big)z^{\mu+\frac{1}{2}}U\Big(\mu-k+\frac{1}{2}, 1+2\mu; z\Big) \tag{1.13}$$

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Whittaker functions appear as coefficients of certain representations of the group $SL_2(R)$, called Whittaker models.

g) Associated Laguerre Polynomials

The Rodrigues representation for the associated Laguerre polynomials is

$$L_n^k(x) = \frac{e^x x^{-k}}{n!}\frac{d^n}{dx^n}(e^{-x}x^{n+k}) \tag{1.14}$$

$$= \sum_{m=0}^{n}(-1)^m\frac{(n+k)!}{(n-m)!(k+m)!m!}x^m \tag{1.15}$$

The first few associated Laguerre polynomials are

$$\begin{cases} L_0^k(x) = 1 \\ L_1^k(x) = -x+k+1 \\ L_2^k(x) = \frac{1}{2}[x^2-2(k+2)x+(k+1)(k+2)] \end{cases} \tag{1.16}$$

II. Main Results

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+1;x) = 2^{2a-1}x^{\frac{1}{2}-a}\ \Gamma\big(a+\frac{1}{2}\big)\ I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)- \\ -2^{2a-1}x^{\frac{1}{2}-a}\ \Gamma\big(a+\frac{1}{2}\big)\ I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big) \tag{2.1}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-1;x) = 2^{2a-3}(4a+x-2)x^{\frac{1}{2}-a}\ \Gamma\big(a-\frac{1}{2}\big)\ I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)+ \\ +2^{2a-3}x^{\frac{3}{2}-a}\ \Gamma\big(a-\frac{1}{2}\big)\ I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big) \tag{2.2}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+2;x) = \frac{2^{2a}x^{\frac{1}{2}-a}\Gamma(a+\frac{3}{2})I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)}{a+1}- \\ -\frac{2^{2a}x^{-\frac{1}{2}-a}(2a+x)\Gamma(a+\frac{3}{2})I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big)}{a+1} \tag{2.3}$$

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$$e^{-\frac{x}{2}}{}_1F_1(a;2a-2;x)=\frac{2^{2a-4}(8a^2+4ax-12a+x^2-2x+4)x^{\frac{1}{2}-a}\Gamma(a-\frac{1}{2})I_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right)}{a-1}+$$

$$+\frac{2^{2a-4}(2a+x-2)x^{-\frac{3}{2}-a}\Gamma(a-\frac{1}{2})I_{\frac{1}{2}(2a+1)}\left(\frac{x}{2}\right)}{a-1} \tag{2.4}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+3;x)=2^{2a+1}x^{-\frac{1}{2}-a}(a+x)\ \Gamma\left(a+\frac{3}{2}\right)\ I_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right)-$$

$$-\frac{2^{2a+1}x^{-\frac{3}{2}-a}(4a^2+3ax+2a+x^2)\ \Gamma\left(a+\frac{3}{2}\right)\ I_{\frac{1}{2}(2a+1)}\left(\frac{x}{2}\right)}{a+2} \tag{2.5}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-3;x)+$$

$$=\frac{2^{2a-5}(4a^2+3ax-10a+x^2-3x+6)x^{\frac{3}{2}-a}\ \Gamma\left(a-\frac{3}{2}\right)\ I_{\frac{1}{2}(2a+1)}\left(\frac{x}{2}\right)}{a-1}+$$

$$+\frac{2^{2a-5}(16a^3+12a^2x-48a^2+5ax^2-18ax+44a+x^3-3x^2+6x-12)}{a-1}\times$$

$$\times\ x^{\frac{1}{2}-a}\ \Gamma\left(a-\frac{3}{2}\right)\ I_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right) \tag{2.6}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+4;x)=\frac{2^{2a+2}x^{-\frac{3}{2}-a}(2a^2+2ax+2a+x^2)\ \Gamma\left(a+\frac{5}{2}\right)\ I_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right)}{(a+2)(a+3)}-$$

$$-\frac{2^{2a+2}x^{-\frac{5}{2}-a}(8a^3+8a^2x+12a^2+4ax^2+4ax+4a+x^3)\ \Gamma\left(a+\frac{5}{2}\right)\ I_{\frac{1}{2}(2a+1)}\left(\frac{x}{2}\right)}{(a+2)(a+3)} \tag{2.7}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-4;x)=$$

$$=\frac{2^{2a-6}x^{\frac{3}{2}-a}(8a^3+8a^2x-36a^2+4ax^2-20ax+52a+x^3-4x^2+12x-24)}{(a-2)(a-1)}\times$$

$$\times\ \Gamma\left(a-\frac{3}{2}\right)\ I_{\frac{1}{2}(2a+1)}\left(\frac{x}{2}\right)+2^{2a-6}x^{\frac{1}{2}-a}\Gamma\left(a-\frac{3}{2}\right)\ I_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right)\times$$

$$\times\left[\frac{(32a^4+32a^3x-160a^3+18a^2x^2-96a^2x+280a^2)}{(a-2)(a-1)}+\right.$$

$$\left.+\frac{(6ax^3-30ax^2+88ax-200a+x^4-4x^3+12x^2-24x+48)}{(a-2)(a-1)}\right] \tag{2.8}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+5;x)=$$

$$=\frac{2^{2a+3}x^{-\frac{5}{2}-a}(4a^3+5a^2x+10a^2+3ax^2+5ax+6a+x^3)\ \Gamma\left(a+\frac{5}{2}\right)\ I_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right)}{(a+3)(a+4)}-$$

$$-2^{2a+3}x^{-\frac{7}{2}-a}\ \Gamma\left(a+\frac{5}{2}\right)\ I_{\frac{1}{2}(2a+1)}\left(\frac{x}{2}\right)\left[\frac{(16a^4+20a^3x+48a_1^3 3a^2x^2+30a^2x+44a^2)}{(a+3)(a+4)}+\right.$$

$$\left.+\frac{(5ax^3+7ax^2+10ax+12a+x^4)}{(a+3)(a+4)}\right] \tag{2.9}$$

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$$e^{-\frac{x}{2}}{}_1F_1(a;2a-5;x)=$$
$$=2^{2a-7}x^{\frac{3}{2}-a}\,\Gamma\Big(a-\frac{5}{2}\Big)\,I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big)\Bigg[\frac{(16a^4+20a^3x-112a^3+13a^2x^2-90a^2x+284a^2)}{(a-2)(a-1)}+$$
$$+\frac{(5ax^3-33ax^2+130ax-308a+x^4-5x^3+20x^2-60x+120)}{(a-2)(a-1)}\Bigg]+$$
$$+2^{2a-7}x^{\frac{1}{2}-a}\,\Gamma\Big(a-\frac{5}{2}\Big)\,I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)\Bigg[\frac{(64a^5+80a^4x-480a^4+56a^3x^2-400a^3x+1360a^3)}{(a-2)(a-1)}+$$
$$+\frac{(25a^2x^3-180a^2x^2+700a^2x-1800a^2+7ax^4-45ax^3+184ax^2-500ax)}{(a-2)(a-1)}+$$
$$+\frac{(1096a+x^5-5x^4+20x^3-60x^2+120x-240)}{(a-2)(a-1)}\Bigg] \tag{2.10}$$

III. Special Cases

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+1;0)=e^{-\frac{x}{2}}\frac{\Gamma(1-a)\Big(\Gamma(1+2a)L_{-a}^{2a}(0)\Big)}{\Gamma(1+a)} \tag{3.1}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+1;1)=)=e^{-\frac{x}{2}}\frac{\Gamma(1-a)\Big(\Gamma(1+2a)L_{-a}^{2a}(1)\Big)}{\Gamma(1+a)} \tag{3.2}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+1;2)=)=e^{-\frac{x}{2}}\frac{e^1M_{\frac{1}{2},a}(2)}{2^{\frac{1}{2}(1+2a)}} \tag{3.3}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-1;1)=)=e^{-\frac{x}{2}}\frac{\sqrt{e}\;M_{-\frac{1}{2},\frac{1}{2}(-2+2a)}(1)}{1^{\frac{1}{2}(-1+2a)}} \tag{3.4}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-1;2)=)=e^{-\frac{x}{2}}\frac{e^1\;M_{-\frac{1}{2},\frac{1}{2}(-2+2a)}(2)}{2^{\frac{1}{2}(-1+2a)}} \tag{3.5}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-1;3)=)=e^{-\frac{x}{2}}\frac{e^{\frac{3}{2}}\;M_{-\frac{1}{2},\frac{1}{2}(-2+2a)}(3)}{3^{\frac{1}{2}(-1+2a)}} \tag{3.6}$$

$$e^{-\frac{x}{2}}{}_1F_1(1;1;-1)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-x)^{k_2}}{2^{k_2}\;k_1!k_2!} \tag{3.7}$$

$$e^{-\frac{x}{2}}{}_1F_1(1;3;-1)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}(-x)^{k_1}(1)_{k_2}}{2^{k_1}\;k_1!k_2!(3)_{k_2}} \tag{3.8}$$

IV. Applications

The results are applied in all branches of applied sciences.These are used for solving scientific problems such as diffusion problems on a lattice, solving for patterns of acoustical radiation,Modes of vibration of a thin circular (or annular) artificial membrane (such as a drum or other membranophone),Heat conduction in a cylindrical object.

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Accuracy in Collaborative Robotics: An Intuitionistic Fuzzy Multiset Approach

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ACCURACY IN COLLABORATIVE ROBOTICS AN INTUITIONISTIC FUZZY MULTISET APPROACH

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Accuracy in Collaborative Robotics: An Intuitionistic Fuzzy Multiset Approach

Shinoj T K, Sunil Jacob John

Abstract - In this paper, an application of Intuitionistic Fuzzy Multiset in Robotics is discussed. The basic operations on Intuitionistic Fuzzy Multisets such as union, intersection, addition, multiplication etc. are discussed. Accuracy of Collaborative Robots using the concept of Intuitionistic Fuzzy Multiset is discussed.

Keywords: intuitionistic fuzzy set, multiset, intuitionistic fuzzy multiset, robotics.

I. Introduction

Modern set theory formulated by George Cantor is fundamental for the whole Mathematics. One issue associated with the notion of a set is the concept of vagueness. Mathematics requires that all mathematical notions including set must be exact. This vagueness or the representation of imperfect knowledge has been a problem for a long time for philosophers, logicians and mathematicians. However, recently it became a crucial issue for computer scientists, particularly in the area of artificial intelligence. To handle situations like this, many tools were suggested. They include Fuzzy sets, Multi sets, Rough sets, Soft sets and many more.

Considering the uncertainty factor, Lofti Zadeh [1] introduced Fuzzy sets in 1965, in which a membership function assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. In 1983, Krassimir. T. Atanassov [2,3] introduced the concept of Intuitionistic Fuzzy sets (IFS) by introducing a non-membership fuction together with the membership function of the fuzzy set. Among the various notions of higher-order Fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainity and vagueness. IFS reflect better the aspects of human behavior.

A human being who expresses the degree of belongingness of a given element to a set, does not often expresses the corresponding degree of non-belongingness as the complement. This psychological fact states that linguistic negation does not always coincides with logical negation. This idea of Intuitionistic fuzzy sets, which is a natural generalization of a standard Fuzzy set, seems to be useful in modelling many real life situations, like negotiation processes, psychological investigations, reasoning etc. The relation between Intuitionistic Fuzzy sets and other theories modeling imprecision can be seen in [4,5].

Many fields of modern mathematics have been emerged by violating a basic principle of a given theory only because useful structures could be defined this way. Set is a well-defined collection of distinct objects, that is, the elements of a set are pair wise

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1. L. A. Zadeh (1965) Fuzzy sets, *Inform. and Control* 8 338-353.

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different. If we relax this restriction and allow repeated occurrences of any element, then we can get a mathematical structure that is known as Multisets or Bags. For example, the prime factorization of an integer n>0 is a Multiset whose elements are primes. The number 120 has the prime factorization $120 = 2^3 3^1 5^1$ which gives the Multiset {2, 2, 2, 3, 5}. A complete account of the development of multiset theory can be seen in [6,7]. As a generalization of multiset, Yager [8] introduced the concept of Fuzzy Multiset (FMS). An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values.

This paper explains how the concept of Intuitionist Fuzzy Multisets can be applied in the field of Robotics. Robots are machines which reduces human effort. Robots can be given intelligence to perform tasks that humans can and cannot do. They can be programmed for doing a task monotonously or they can work intelligently or dynamically according to the situations around them. Some of the applications of a mobile Robot include mine detection, surveillance, bomb detection, remote surgery, welding, cleaning small pipes, window panes and glass doors of buildings using snake-like Robots etc.

A Robot mainly contains: sensors, actuators and a controller. An accelerometer sensor is used for detecting shock/vibration, a temperature sensor can detect the temperature variations, an ultrasonic sensor/Infra-Red sensor/PIR sensor is used to detect obstacles, bump sensor senses a bump (collision), cliff sensor senses the presence of a cliff and so on. With the help of these sensors a Robot moves easily through its programmed path. iRobot Create is one such mobile robot as shown in Figure 1.

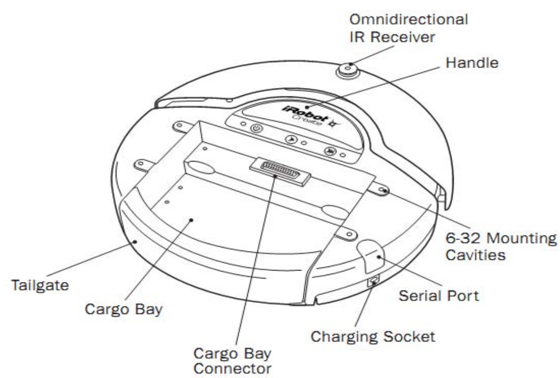

Figure 1 : iRobot Create.

When multiple Robots are used for completing a task, the system is called a multi-Robot system. Task accomplished by multiple Robots saves time and cost. To explain the concept of IFMS, a multi Robot scenario is considered consisting of a central server and a group of mobile Robots patrolling a given area for surveillance application.

Using the distance function, the sensor readings were properly interpreted for proper identification of the problem faced by the Robot.

II. Preliminaries

2.1 Definition [1] Let X be a nonempty set. A *Fuzzy set A* drawn from X is defined as $A = \{< x : \mu_A(x) > : x \in X\}$.Where : $X \rightarrow [0,1]$ is the membership function of the Fuzzy Set A.

2.2. Definition [8] Let X be a nonempty set. A *Fuzzy Multiset* (FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A such that $CM_A : X \rightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval

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[0,1]. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from [0,1]. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $(\mu^1_A(x), \mu^2_A(x),..., \mu^P_A(x))$ where $\mu^1_A(x) \geq \mu^2_A(x) \geq,..., \geq \mu^P_A(x)$.

A complete account of the applications of Fuzzy Multisets in various fields can be seen in [9].

2.3 Definition [3] Let X be a nonempty set. An *Intuitionistic Fuzzy Set* (IFS) A is an object having the form $A = \{< x : \mu_A(x), \nu_A(x) > : x \epsilon X\}$, where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ define respectively the degree of membership and the degree of non membership of the element $x \in X$ to the set A with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \epsilon X$.

2.4 Remark Every Fuzzy set A on a nonempty set X is obviously an IFS having the form

$$A = \{ < x : \mu_A(x), 1 - \mu_A(x) > : x \epsilon X \}$$

Using the definition of FMS and IFS, a new generalized concept called Intuitionistic Fuzzy Multiset (IFMS) is defined in [10].

III. Intuitionistic Fuzzy Multiset

3.1 Definition Let X be a nonempty set. An *Intuitionistic Fuzzy Multiset* A denoted by IFMS drawn from X is characterized by two functions: 'count membership' of A (CM_A) and 'count non membership' of A (CN_A) given respectively by $CM_A : X \to Q$ and $CN_A : X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval [0, 1] such that for each $x \epsilon X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CMA(x)$ which is denoted by $(\mu^1_A(x), \mu^2_A(x),..., \mu^P_A(x))$ where $(\mu^1_A(x) \geq \mu^2_A(x) \geq,... \geq, \mu^P_A(x)$ and the corresponding non membership sequence will be denoted by $(\nu^1_A(x), \nu^2_A(x),...,\nu^P_A(x))$ such that $0 \leq \mu^i_A(x) + \nu^i_A(x) \leq 1$ for every $x \epsilon X$ and $i = 1,2,...,$p.

An IFMS A is denoted by

$$A = \{ < x: (\mu^1_A(x), \mu^2_A(x),..., \mu^P_A(x)), (\nu^1_A(x), \nu^2_A(x), ... ,\nu^P_A(x)) > : x \epsilon X \}$$

3.2. Remark We arrange the membership sequence in decreasing order but the corresponding non membership sequence may not be in decreasing or increasing order.

3.3. Definition Length of an element x in an IFMS A is defined as the Cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq \mu^j_A(x) + \nu^j_A(x) \leq 1$ and it is denoted by $L(x: A)$. That is

$$L(x:A) = |CM_A(x)| = |CN_A(x)|$$

3.4 Definition If A and B are IFMSs drawn from X then $L(x:A,B) = Max\{L(x : A), L(x : B)\}$. Alternatively we use $L(x)$ for $L(x :A, B)$.

33.5.Example Consider the set $X = \{x, y, x, w\}$ with A= {< x : (0.3, 0.2), (0.4, 0.5) >, < y : (1,0.5,0.5), (0.0.5,0.2) >, < z : (0.5, 0.4, 0.3, 0.2), (0.4, 0.6, 0.6, 0.7)>}, B= {< x : (0.4), (0.2) >, < y : (1,0.3, 0.2), (0,0.4, 0.5) >, < w : (0.2, 0.1), (0.7, 0.8) >}.

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Now we define basic operations on IFMS. Note that we can make L(x : A) = L(x : B) by appending sufficient number of 0's and 1's with the membership and non membership values respectively.

3.6 Definition Let A and B be two IFMS. The distance function is defined as

$$d(A,B) = \left(\frac{1}{2}\sum_i((\mu_A^i(x) - \mu_B^i(x))^2 + (\nu_A^i(x) - \nu_B^i(x))^2 + (\Pi_A^i(x) - \Pi_B^i(x))^2)\right)^{\frac{1}{2}}$$

where $\Pi_A^i = 1 - \mu_A^i(x) - \nu_A^i(x)$ called the IFMS index or hesitation margin.

3.7 Definition For any two IFMSs A and B drawn from a set X, the following operations and relations will hold. Let $A = \{< x : (\mu^1{}_A(x), \mu^2{}_A(x),\ldots,\mu^P{}_A(x)), (\nu^1{}_A(x), \nu^2{}_A(x),\ldots,\nu^P{}_A(x)) > : x \in X\}$ and $B = \{< x : (\mu^1{}_B(x), \mu^2{}_B(x),\ldots, \mu^P{}_B(x)), (\nu^1{}_B(x), \nu^2{}_B(x),\ldots,\nu^P{}_B(x)) > : x \in X\}$ then

1. Inclusion

$A \subset B \Leftrightarrow \mu^j{}_A(x) \leq \mu^j{}_A(x)$ and $\nu^j{}_A(x) \geq \nu^j{}_B(x)$;

$j = 1,2,\ldots, L(x), x \in X$

$A = B \Leftrightarrow A \subset B$ and $B \subset A$

2. Complement

$\neg A = \{< x : (\nu^1{}_A(x),\ldots,\nu^P{}_A(x)), (\mu^1{}_A(x),\ldots\ldots\ldots, \mu^P{}_A(x)) > : x \in X\}$

3. Union $(A \cup B)$

In $A \cup B$ the membership and non membership values are obtained as follows.

$\mu^j{}_{A\cup B}(x) = \mu^j{}_A(x) \vee \mu^j{}_B(x)$

$\nu^j{}_{A\cup B}(x) = \nu^j{}_A(x) \wedge \nu^j{}_B(x)$

$j = 1, 2,\ldots,L(x), x \in X.$

4. Intersection $(A \cap B)$

In $A \cap B$ the membership and non membership values are obtained as follows.

$\mu^j{}_{A\cap B}(x) = \mu^j{}_A(x) \wedge \mu^j{}_B(x)$

$\nu^j{}_{A\cap B}(x) = \nu^j{}_A(x) \vee \nu^j{}_B(x)$

$j = 1, 2,\ldots,L(x), x \in X.$

5. Addition $(A \oplus B)$

In $A \oplus B$ the membership and non membership values are obtained as follows.

$\mu^j{}_{A\oplus B}(x) = \mu^j{}_A(x) + \mu^j{}_B(x) - \mu^j{}_A(x).\ \mu^j{}_B(x)$

$\nu^j{}_{A\oplus B}(x) = \nu^j{}_A(x).\ \nu^j{}_B(x)$

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$j = 1, 2,...,L(x), x \in X.$

6. Multiplication (A ⊗ B)

In $A \otimes B$ the membership and nonmembership values are obtained as follows.

$$\mu^j_{A\otimes B}(x) = \mu^j_A(x).\ \mu^j_B(x)$$

$$\nu^j_{A\otimes B}(x) = \nu^j_A(x) + \nu^j_B(x) - \nu^j_A(x).\ \nu^j_B(x)$$

$j = 1, 2,...,L(x), x \in X.$

here ∨, ∧ , . , +, - denotes maximum, minimum, multiplication, addition, subtraction of real numbers respectively.

IV. IFMS Theory for Multi Robot System

Most of human reasoning involves the use of variables whose values are fuzzy sets. This is the basis for the concept of a linguistic variable, that is, a variable whose values are words rather than numbers. But in some situations like decision making problems (such as Medical diagnosis, Sales analysis, Marketing etc.) the description by a linguistic variable in terms of membership function only is not adequate. There is chance of existing a non-null complement. IFS can be used in this context as a proper tool for representing both membership and non-membership of an element to a set. Such situations are explained in [11]. But there are situations that each element has different membership values. In such situations IFMS is more adequate. Here we present IFMS as a tool for reasoning such a situation.

An example of a multi Robot system is presented. The multi Robot system [12] considered consists of a central controller and four patrolling Robots in a large area. The total area is divided into four equal parts and assigned to each Robot. The Robot patrols in its assigned area. Each Robot is equipped with ultrasonic sensor, accelerometer sensor, cliff sensor, bump sensor and temperature sensor and is wirelessly controlled by the controller. The controller makes decisions depending upon the sensor readings. For example, if the cliff sensor value in Robot1 indicates the presence of a cliff, the controller can change the commands that are sent to the Robot1; that is, the controller can direct the Robot1 towards the right, left or backward directions. Similar is the case with every other sensor reading.

Let R = {R1, R2, R3, R4} be a set of four Robots, C = {Fire, Obstacle, Bump, Cliff, Vibration} be a set of situations or conditions and S = {Temperature sensor, Ultrasonic sensor, Bump sensor, Cliff sensor, Accelerometer sensor} be a set of sensors deployed on each Robot. A single Robot can be assigned different membership and non membership values for the five different sensor readings. This is where IFMS comes into picture.

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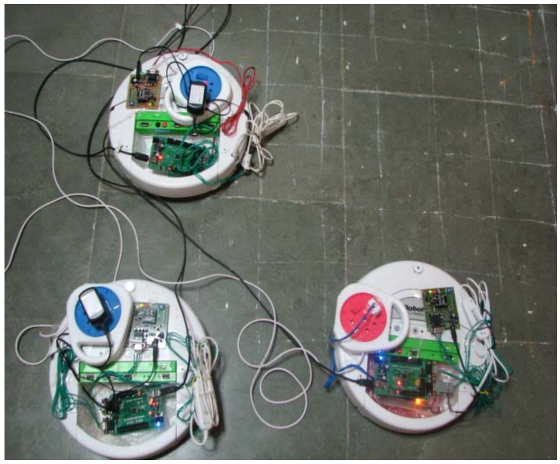

Figure 2 : A multi robot system with three patrolling robots

Whether from a single reading can we conclude what are the situations faced by the Robots? The sensor readings from the Robots have to be monitored for a particular time, say for three minutes. If for example, the ultrasonic sensor in Robot1 indicates an obstacle, it sends a message to the controller so that the corrective measure could be taken. The controller has to make sure whether the Robot1 is really faced with an obstacle or not. For that purpose, the controller monitors the ultrasonic sensor reading for three minutes. Depending upon the consistency of the readings, the controller identifies the situation.

To understand IFMS theory, let us consider the situation where the Robot1 faces an obstacle, Robot2 experiences a shock/vibration, Robot3 faces a bump and Robot4 detects a cliff. Thus whenever the ultrasonic sensor detects an obstacle and the accelerometer sensor detects a vibration, alert is sent to the controller and the controller monitors the situation for 3 minutes.

In Table-I each sensor reading is described by three numbers: Membership μ, non-membership ν and hesitation margin Π.

Table II shows the Robots and the corresponding membership functions to the sensor values.

Table 1

	Fire	Obstacle	Bump	Cliff	Shock/Vibration
Temperature sensor	(0.8,0..1,0.1)	(0.2,0.7,0.1)	(0.1,0.7,0.2)	(0.2,0.5,0.3)	(0.5,0.2,0.3)
Ultrasonic sensor	(0.2,0..7,0.1)	(0.8,0.1,0.1)	(0.6,0.3,0.1)	(0.2,0..7,0.1)	(0.1,0.7,0.2)
Bump sensor	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.9,0.1,0.0)	(0.1,0.7,0.2)	(0.2,0.5,0.3)
Cliff sensor	(0.2,0.5,0.3)	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.7,0.1,0.2)	(0.1,0.7,0.2)
Accelerometer sensor	(0.1,0.7,0.2)	(0.2,0.5,0.3)	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.8,0.2,0.0)

The objective is to make a proper decision for each Robot. Hence the readings are monitored for a particular interval time (3 minutes).

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Table II

	Temperature sensor	Ultrasonic sensor	Bump sensor	Cliff sensor	Accelerometer sensor
R_1	(0.8,0.1, 0.1)	(0.8, 0.1, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.8,0.0)	(0.3, 0.6, 0.1)
R_2	(0.4, 0.5, 0.1)	(0.3, 0.7,0.0)	(0.1, 0.7, 0.2)	(0.2, 0.6, 0.2)	(0.8, 0.1, 0.1)
R_3	(0.1, 0.8, 0.1)	(0.6, 0.4,0.0)	(0.8, 0.1, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.7, 0.1)
R_4	(0.1, 0.7, 0.2)	(0.3, 0.6, 0.1)	(0.2, 0.7, 0.1)	(0.7, 0.2, 0.1)	(0.1, 0.7, 0.2)

Table III shows the sensor readings monitored for 3 minutes, one reading per minute. Table IV shows the distances of each Robot to the situation considered. Thus, using this distance function, IFMS theory is able to make out the correct situation of each Robot.

Table III

	Temperature sensor	Ultrasonic sensor	Bump sensor	Cliff sensor	Accelerometer sensor
R_1	(0.8,0.7, 0.9) (0.1, 0.2, 0.0) (0.1, 0.1, 0.1)	(0.8, 0.8, 0.9) (0.1, 0.1, 0.1) (0.1, 0.1, 0.0)	(0.1, 0.2, 0.0) (0.9, 0.7, 0.8) (0.0, 0.1, 0.2)	(0.2, 0.1, 0.0) (0.8, 0.6, 0.7) (0.0, 0.3, 0.3)	(0.3, 0.3, 0.4) (0.6, 0.4, 0.4) (0.1, 0.3, 0.2)
R_2	(0.4, 0.3, 0.3) (0.5, 0.4, 0.6) (0.1, 0.3, 0.1)	(0.3, 0.2, 0.3) (0.7, 0.6, 0.1) (0, 0.2, 0.7)	(0.1, 0.2, 0.4) (0.7, 0.6, 0.4) (0.2, 0.2, 0.2)	(0.2, 0.5, 0.2) (0.6, 0.4, 0.7) (0.2, 0.1, 0.1)	(0.8, 0.7, 0.6) (0.1, 0.2, 0.3) (0.1, 0.1, 0.1)
R_3	(0.1, 0.2, 0.1) (0.8, 0.6, 0.9) (0.1, 0.2, 0.0)	(0.6, 0.2, 0.1) (0.4, 0.0, 0.7) (0, 0.8, 0.2)	(0.8, 0.7, 0.8) (0.1, 0.1, 0.1) (0.1, 0.2, 0.1)	(0.1, 0.2, 0.2) (0.9, 0.7, 0.6) (0.0, 0.1, 0.2)	(0.2, 0.3, 0.2) (0.7, 0.7, 0.7) (0.1, 0.0, 0.1)
R_4	(0.1, 0.4, 0.5) (0.7, 0.4, 0.3) (0.2, 0.2, 0.2)	(0.3, 0.3, 0.4) (0.6, 0.3, 0.5) (0.1, 0.4, 0.1)	(0.2, 0.1, 0.0) (0.7, 0.6, 0.7) (0.1, 0.3, 0.3)	(0.8, 0.6, 0.9) (0.2, 0.3, 0.0) (0, 0.1, 0.1)	(0.1, 0.5, 0.4) (0.7, 0.4, 0.3) (0.2, 0.1, 0.3)

Table IV

	Fire	Obstacle	Bump	Cliff	Shock/Vibration
R_1	0.72	**0.65**	1.07	1.06	0.90
R_2	0.84	0.79	0.97	0.83	**0.52**
R_3	1.07	0.89	**0.50**	1.03	1.02
R_4	0.79	0.84	1.07	**0.46**	0.87

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In the above table the lowest distance point gives the accuracy of the Robot. Robot $\mathbf{R_1}$ is near an obstacle, R2 experiences a vibration, R3 is bumped and R4 is near a cliff.

V. Conclusions

In this paper, we have discussed the various basic operations of Intuitionistic Fuzzy Multiset and its application in Robotics. In the proposed method, we measured the distances of each Robot from each situation by considering the sensor readings. The concept of multiness is incorporated by taking the samples from the same Robot for a particular time.

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Computation of Some Wonderful Results Involving Certain Polynomials

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Abstract- In this paper we have developed some indefinite integrals involving certain polynomials in the form of Hypergeometric function. The results represent here are assume to be new.

Keywords: hypergeometric function, lucas polynomials, gegenbaur polynomials, harmonic number, bernoulli polynomial, hermite polynomials.

GJSFR-F Classification : MSC 2010 11B39, 11B68, 33C05, 33C45, 33D50, 33D60

COMPUTATION OF SOME WONDERFUL RESULTS INVOLVING CERTAIN POLYNOMIALS

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Computation of Some Wonderful Results Involving Certain Polynomials

Salahuddin[α] & R.K. Khola[σ]

Abstract- In this paper we have developed some indefinite integrals involving certain polynomials in the form of Hypergeometric function. The results represent here are assume to be new.

Keywords: hypergeometric function, lucas polynomials, gegenbaur polynomials, harmonic number, bernoulli polynomial, hermite polynomials.

I. Introduction

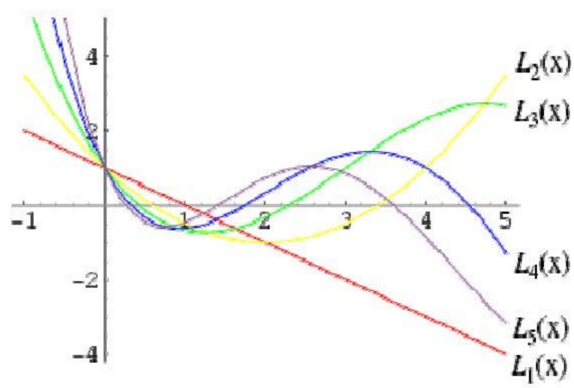
a) We have the generalized Gaussian hypergeometric function of one variable

$$ {}_AF_B(a_1,a_2,\ldots,a_A;b_1,b_2,\ldots b_B;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k\ldots(a_A)_k\, z}{(b_1)_k(b_2)_k\ldots(b_B)_k\, k!} \tag{1}$$

where the parameters $b_1, b_2, \ldots, b_B$ are neither zero nor negative integers and A , B are non negative integers.

The series converges for all finite z if $A \leq B$, converges for $|z|<1$ if $A=B+1$, diverges for all z, $z \neq 0$ if $A > B+1$.

b) Lucas Polynomials



The Lucas polynomials are the w-polynomials obtained by setting p(x) = x and q(x) = 1 in the Lucas polynomials sequence. It is given explicitly by

$$L_n(x) = 2^{-n}[(x-\sqrt{x^2+4})^n + (x+\sqrt{x^2+4})^n] \tag{2}$$

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The first few are

$$\left.\begin{aligned} L_1(x) &= x \\ L_2(x) &= x^2+2 \\ L_3(x) &= x^3+3x \\ L_4(x) &= x^4+4x^2+2 \end{aligned}\right\} \tag{3}$$

Notes

c) *Generalized Harmonic Number*

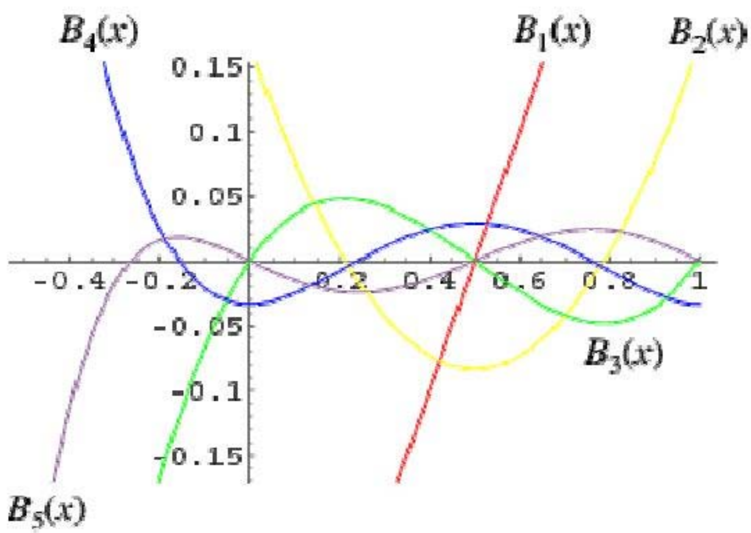
The generalized harmonic number of order n of m is given by

$$H_n^{(m)} = \sum_{k=1}^{n} \frac{1}{k^m} \tag{4}$$

In the limit of $n \to \infty$ the generalized harmonic number converges to the Riemann zeta function

$$\lim_{n\to\infty} H_n^{(m)} = \varsigma(m) \tag{5}$$

d) *Bernoulli Polynomial*



In mathematics, the Bernoulli polynomials occur in the study of many special functions and in particular the Riemann zeta function and Hurwitz zeta function. This is in large part because they are an Appell sequence, i.e. a Sheffer sequence for the ordinary derivative operator. Unlike orthogonal polynomials, the Bernoulli polynomials are remarkable in that the number of crossing of the x-axis in the unit interval does not go up as the degree of the polynomials goes up. In the limit of large degree, the Bernoulli polynomials, appropriately scaled, approach the sine and cosine functions.

Explicit formula of Bernoulli polynomials is

$$B_n(x) = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} b_{n-k} x^k$$, for n≥0, where b_k are the Bernoulli numbers.

The generating function for the Bernoulli polynomials is

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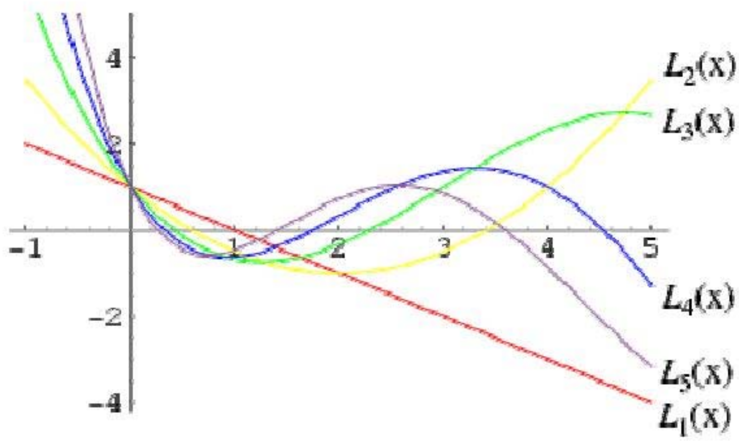
Notes

e) Gegenbauer polynomials

In Mathematics, Gegenbauer polynomials or ultraspherical polynomials $C_n^{(\alpha)}(x)$ are orthogonal polynomials on the interval [-1,1] with respect to the weight function $(1-x^2)^{\alpha-\frac{1}{2}}$. They generalize Legendre polynomials and Chebyshev polynomials, and are special cases of Jacobi polynomials. They are named after Leopold Gegenbauer.
Explicitly ,

$$C_n^{(\alpha)}(x)=\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor}(-1)^k\frac{\Gamma(n-k+\alpha)}{\Gamma(\alpha)k!(n-2k)!}(2z)^{(n-2k)} \tag{7}$$

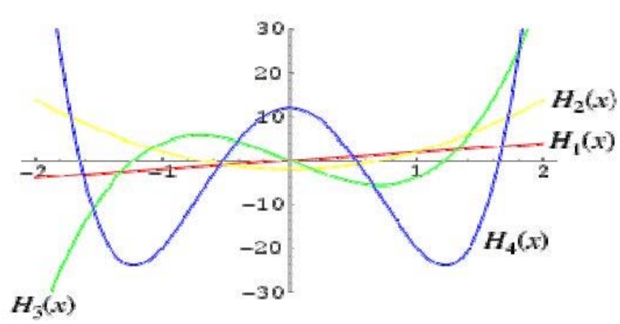
f) Laguerre polynomials



The Laguerre polynomials are solutions $L_n(x)$ to the Laguerre differential equation $xy''+(1-x)y'+\lambda y=0$, which is a special case of the more general associated Laguerre differential equation, defined by
$xy''+(\nu+1-x)y'+\lambda y=0$, where λ and ν are real numbers with ν=0.
The Laguerre polynomials are given by the sum

$$L_n(x)=\sum_{k=0}^{n}\frac{(-1)^k}{k!}\frac{n!}{k!(n-k)!}x^k \tag{8}$$

g) Hermite polynomials



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The Hermite polynomials $H_n(x)$ are set of orthogonal polynomials over the domain (-∞,∞) with weighting function e^{-x^2}.

The Hermite polynomials $H_n(x)$ can be defined by the contour integral

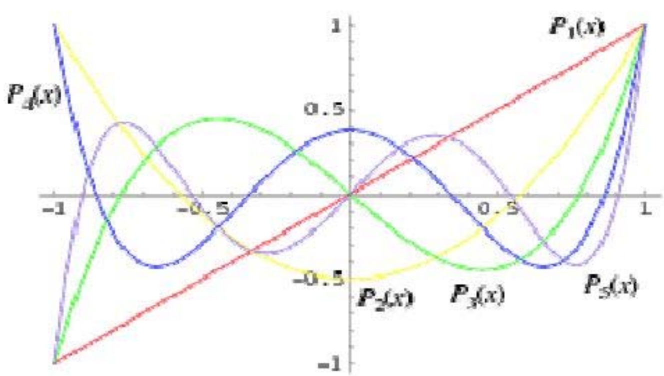
$$H_n(z) = \frac{n!}{2\pi i} \oint e^{-r^2+2rz} t^{-n-1} dt,$$

Where the contour incloses the origin and is traversed in a counterclockwise direction (Arfken 1985, p. 416).

The first few Hermite polynomials are

$$\left.\begin{aligned} H_0(x) &= 1 \\ H_1(x) &= 2x \\ H_2(x) &= 4x^2 - 2 \\ H_3(x) &= 8x^3 - 12x \\ H_4(x) &= 16x^4 - 48x^2 + 12 \end{aligned}\right\} \tag{9}$$

h) Legendre function of the first kind



The Legendre polynomials, sometimes called Legendre functions of the first kind, Legendre coefficients, or zonal harmonics (Whittaker and Watson 1990, p. 302), are solutions to the Legendre differential equation. If l is an integer, they are polynomials. The Legendre polynomials $P_n(x)$ are illustrated above for $x \in [-1, 1]$ and n=1, 2, ..., 5.

The Legendre polynomials $P_n(x)$ can be defined by the contour integral

$$P_n(z) = \frac{1}{2\pi i} \oint (1 - 2tz + t^2)^{-\frac{1}{2}} t^{-n-1} dt, \tag{10}$$

where the contour encloses the origin and is traversed in a counterclockwise direction (Arfken 1985, p. 416).

Notes

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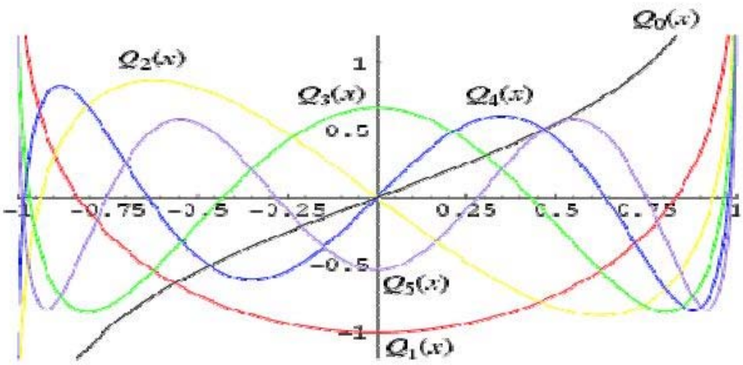
i) Legendre function of the second kind

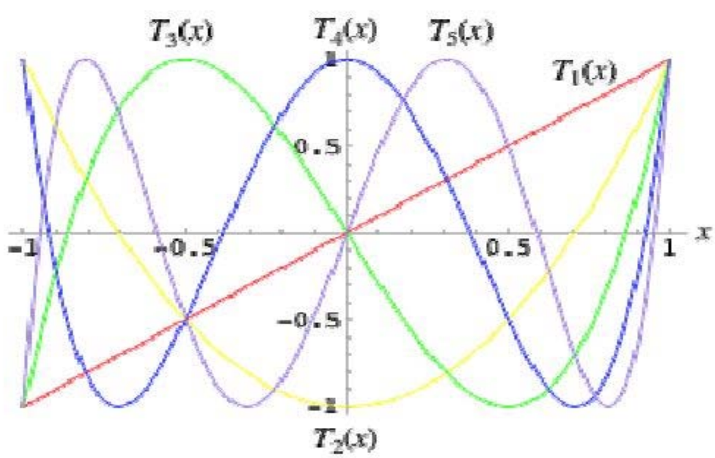


Notes

The second solution $Q_1(x)$ to the Legendre differential equation. The Legendre functions of the second kind satisfy the same recurrence relation as the Legendre polynomials.
The first few are

$$\left.\begin{aligned} Q_0(x) &= \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \\ Q_1(x) &= \frac{x}{2}\ln\left(\frac{1+x}{1-x}\right)-1 \\ Q_2(x) &= \frac{3x^2-1}{4}\ln\left(\frac{1+x}{1-x}\right)-\frac{3x}{2} \end{aligned}\right\} \tag{11}$$

j) Chebyshev polynomial of the first kind



The Chebyshev polynomials of the first kind are a set of orthogonal polynomials defined as the solutions to the Chebyshev differential equation and denoted $T_n(x)$. They are used as an approximation to a least squares fit, and are a special case of the Gegenbauer polynomial with $\alpha = 0$. They are also intimately connected with trigonometric multiple-angle formulas.

The Chebyshev polynomial of the first kind $T_n(z)$ can be defined by the contour integral

$$T_n(z) = \frac{1}{4\pi i}\oint \frac{(1-t^2)t^{-n-1}}{1-2tz+t^2}dt\ , \tag{12}$$

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where the contour encloses the origin and is traversed in a counterclockwise direction (Arfken 1985, p. 416).

The first few Chebyshev polynomials of the first kind are

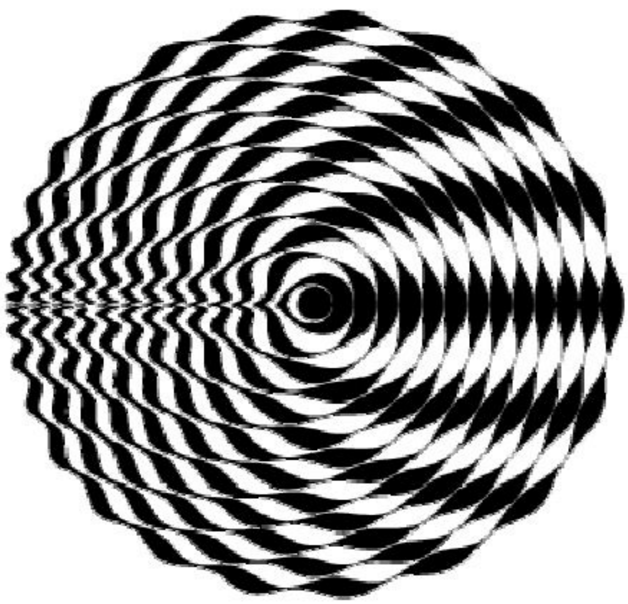
$T_0(x) = 1$

$T_1(x) = x$

$T_2(x) = 2x^2 - 1$

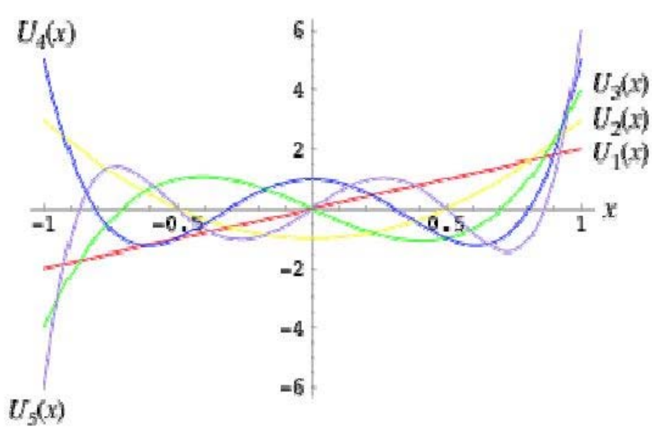
$T_3(x) = 4x^3 - 3x$

A beautiful plot can be obtained by plotting $T_n(x)$ radially, increasing the radius for each value of n, and filling in the areas between the curves (Trott 1999, pp. 10 and 84).



The Chebyshev polynomials of the first kind are defined through the identity $T_n(\cos\theta)=\cos n\theta$.

k) Chebyshev polynomial of the second kind



A modified set of Chebyshev polynomials defined by a slightly different generating function. They arise in the development of four-dimensionalspherical harmonics in angular momentum

Notes

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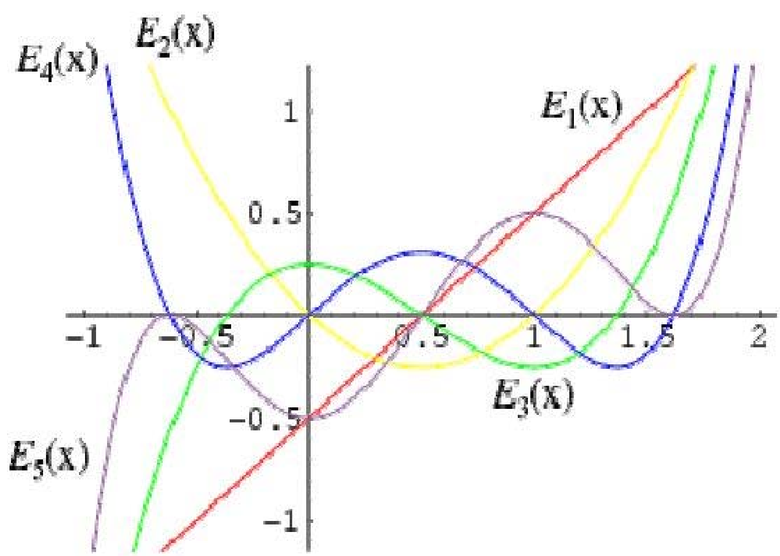
theory. They are a special case of the Gegenbauer polynomial with $\alpha = 1$. They are also intimately connected with trigonometric multiple-angle formulas.

The first few Chebyshev polynomials of the second kind are

Notes

$$\left.\begin{aligned} U_0(x) &= 1 \\ U_1(x) &= 2x \\ U_2(x) &= 4x^2 - 1 \\ U_3(x) &= 8x^3 - 4x \end{aligned}\right\} \tag{13}$$

l) Euler polynomial



The Euler polynomial $E_n(x)$ is given by the Appell sequence with

$$g(t) = \frac{1}{2}(e^t + 1) \ ,$$

giving the generating function

$$\frac{2e^{xt}}{e^t + 1} \equiv \sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!} \ . \tag{14}$$

The first few Euler polynomials are

$$\left.\begin{aligned} E_0(x) &= 1 \\ E_1(x) &= x - \frac{1}{2} \\ E_2(x) &= x^2 - x \\ E_3(x) &= x^3 - \frac{3}{2}x^2 + \frac{1}{4} \end{aligned}\right\} \tag{15}$$

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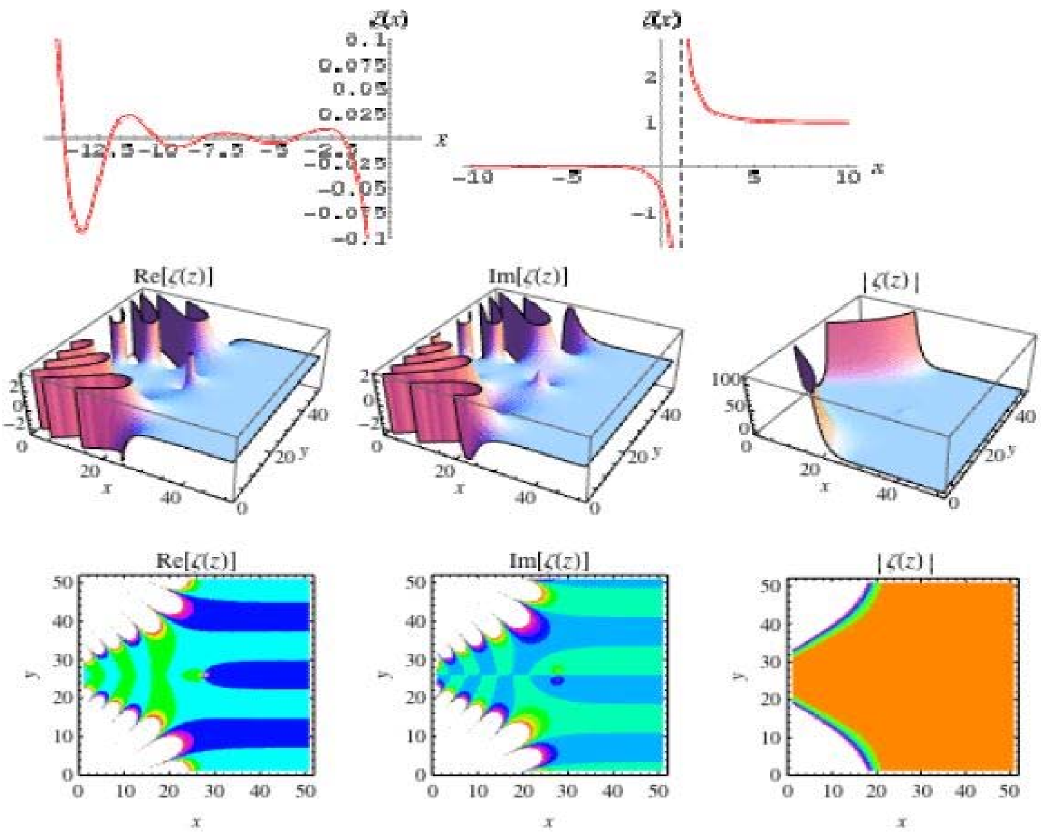
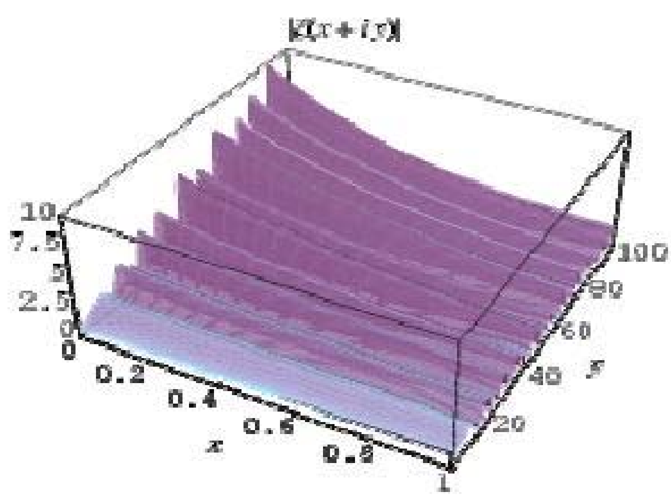
m) Generalized Riemann zeta function



The Riemann zeta function is an extremely important special function of mathematics and physics that arises in definite integration and is intimately related with very deep results surrounding the prime number theorem. While many of the properties of this function have been investigated, there remain important fundamental conjectures (most notably the Riemann hypothesis) that remain unproved to this day. The Riemann zeta function $\zeta(s)$ is defined over the complex plane for one complex variable, which is conventionally denoted s (instead of the usual z) in deference to the notation used by Riemann in his 1859 paper that founded the study of this function (Riemann 1859).



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The plot above shows the "ridges" of $|\zeta(x+iy)|$ for $0<x<1$ and $1<y<100$. The fact that the ridges appear to decrease monotonically for $0 \le x \le 1/2$ is not a coincidence since it turns out that monotonic decrease implies the Riemann hypothesis (Zvengrowski and Saidak 2003; Borwein and Bailey 2003, pp. 95-96).

On the real line with $x>1$, the Riemann zeta function can be defined by the integral

$$\varsigma(x) \equiv \frac{1}{\Gamma(x)} \int_0^\infty \frac{u^{x-1}}{e^u - 1} du$$, where $\Gamma(x)$ is the gamma function.

Notes

n) Complex infinity

Complex infinity is an infinite number in the complex plane whose complex argument is unknown or undefined. Complex infinity may be returned by *Mathematica*, where it is represented symbolically by ComplexInfinity. The Wolfram Functions Site uses the notation $\tilde{\infty}$ to represent complex infinity.

II. Main Results

$$\int \frac{L_1(x)}{\sqrt{1-x^n}} dx = \frac{1}{2} x^2 \, {}_2F_1\left(\frac{1}{2}, \frac{2}{n}; \frac{n+2}{n}; x^n\right) + C \tag{17}$$

$$\int \frac{H_1(x)}{\sqrt{1-x^n}} dx = x \, {}_2F_1\left(\frac{1}{2}, \frac{1}{n}; \frac{n+1}{n}; x^n\right) + C \tag{18}$$

$$\int \frac{B_1(x)}{\sqrt{1-x^n}} dx = \frac{1}{2} x \left[x \, {}_2F_1\left(\frac{1}{2}, \frac{2}{n}; \frac{n+2}{n}; x^n\right) - {}_2F_1\left(\frac{1}{2}, \frac{1}{n}; \frac{n+1}{n}; x^n\right)\right] + C \tag{19}$$

$$\int \frac{C_1(x)}{\sqrt{1-x^n}} dx = x^2 \, {}_2F_1\left(\frac{1}{2}, \frac{2}{n}; \frac{n+2}{n}; x^n\right) + C \tag{20}$$

$$\int \frac{F_1(x)}{\sqrt{1-x^n}} dx = \frac{1}{2} x^2 \, {}_2F_1\left(\frac{1}{2}, \frac{2}{n}; \frac{n+2}{n}; x^n\right) + C \tag{21}$$

$$\int \frac{L_1[x]}{\sqrt{1-x^n}} dx = x \, {}_2F_1\left(\frac{1}{2}, \frac{1}{n}; \frac{n+1}{n}; x^n\right) - \frac{1}{2} x^2 \, {}_2F_1\left(\frac{1}{2}, \frac{2}{n}; \frac{n+2}{n}; x^n\right) + C \tag{22}$$

$$\int \frac{H_1(x)}{\sqrt{1-x^n}} dx = x^2 \, {}_2F_1\left(\frac{1}{2}, \frac{2}{n}; \frac{n+2}{n}; x^n\right) + C \tag{23}$$

$$\int \frac{P_1(x)}{\sqrt{1-x^n}} dx = \frac{1}{2} x^2 \, {}_2F_1\left(\frac{1}{2}, \frac{2}{n}; \frac{n+2}{n}; x^n\right) + C \tag{24}$$

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$$\int \frac{T_1(x)}{\sqrt{1-x^n}}\,dx = \frac{1}{2}\,x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right) + C \tag{25}$$

$$\int \frac{U_1(x)}{\sqrt{1-x^n}}\,dx = x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right) + C \tag{26}$$

$$\int \frac{E_1(x)}{\sqrt{1-x^n}}\,dx = \frac{1}{2}\,x\left[x\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right) - {}_2F_1\left(\frac{1}{2},\frac{1}{n};\frac{n+1}{n};x^n\right)\right] + C \tag{27}$$

$$\int \frac{\zeta(1,x)}{\sqrt{1-x^n}} = \tilde{\infty} + C \tag{28}$$

Notes

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The Metric Characterization of the Generalized Fermat Points

By Sándor N. Kiss

Abstract- We build on the sides of a triangle ABC, outwards and inwards, similar triangles with one another. We prove some relations referring to the generalized Fermat points. We deduce these results based on trigonometrical identities.

GJSFR-F Classification : MSC 2010: 11J83, 11D41

THE METRIC CHARACTERIZATION OFTHEGENERALIZED FERMATPOINTS

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The Metric Characterization of the Generalized Fermat Points

Sándor N. Kiss

Abstract- We build on the sides of a triangle ABC, outwards and inwards, similar triangles with one another. We prove some relations referring to the generalized Fermat points. We deduce these results based on trigonometrical identities.

I. Introduction

If we build outwards on the sides BC, CA, AB of an arbitrary triangle ABC three equilateral triangles BCX_1, CAY_1, ABZ_1 then

$$AX_1 = BY_1 = CZ_1 = \pm AF_1 \pm BF_1 \pm CF_1, \tag{1}$$

where $F_1 = AX_1 \cap BY_1 \cap CZ_1$ is *the first (outward) Fermat point of triangle ABC* (Figure 1). A minus sign being taken if the angle of triangle ABC at that vertex exceeds 120°.

If we build inwards on the sides BC, CA, AB of an arbitrary triangle ABC three equilateral triangles BCX_2, CAY_2, ABZ_2 then

$$AX_2 = BY_2 = CZ_2 = \pm AF_2 \pm BF_2 \pm CF_2, \tag{2}$$

where $F_2 = AX_2 \cap BY_2 \cap CZ_2$ is *the second (inward) Fermat point of triangle ABC* (Figure 2). A minus sign being taken at each vertex where the angle of triangle ABC is larger than 60°.

In this paper we generalize these results: on the sides of triangle ABC we build, outwards resp. inwards, three similar triangles.

We mention but in this article we don't discuss the Fermat-Steiner problem:

Given a triangle ABC, how can we find a point P in its plane for which $PA+PB+PC$ is minimal?

The solution and some generalization of this problem was a deal for many mathematicians (see [2], [4], [5]). The historical survey of the Fermat-Steiner problem could be found in [3].

Let be given a triangle ABC with the usual notations: A, B, C the angles, a, b, c the sides, R the circumradius, Δ the area of the triangle ABC, S the twice of the area of the triangle $(S = 2\Delta)$.

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Ref

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II. The First Generalized Fermat Point of the Triangle

We build on the sides of the triangle *ABC* outwards the similar triangles *BCD, CAE* and *ABF* in that way, that

$BAF\angle = \alpha = CAE\angle,\ CBD\angle = \beta = ABF\angle,\ ACE\angle = \gamma = BCD\angle$ (Figure 3). Since $A+B+C=\pi=\alpha+\beta+\gamma$, therefore $BDC\angle = \alpha,\ CEA\angle = \beta,\ AFB\angle = \gamma$.

We work with barycentric coordinates with reference to the triangle *ABC*. According to the Conway's formulas, the barycentric coordinates of the points *D, E, F* are: $D=\left(-a^2 : S_\gamma + S_C : S_\beta + S_B\right)$, $E=\left(S_\gamma + S_C : -b^2 : S_\alpha + S_A\right)$, $F=\left(S_\beta + S_B : S_\alpha + S_A : -c^2\right)$, where $S_\theta = S\cdot ctg\theta$. Consequently $S_A = bc\cos A$, $S_B = ca\cos B$, $S_C = ab\cos C$.

The equations of the lines *AD, BE, CF* are: $-\left(S_\beta + S_B\right)y + \left(S_\gamma + S_C\right)z = 0$,

$$\left(S_\alpha + S_A\right)x - \left(S_\gamma + S_C\right)z = 0,$$

$$-\left(S_\alpha + S_A\right)x + \left(S_\beta + S_B\right)y = 0.$$

Swmming random by two of the above equations, we get the third equation, the lines *AD, BE* and *CF* meet each other in a point $K = AD\cap BE\cap CF$. This point *K* will be called **the first generalized Fermat point of the triangle**.

The barycentric coordinates of the point *K* are:

$$K=\left(\left(S_\beta+S_B\right)\left(S_\gamma+S_C\right):\left(S_\gamma+S_C\right)\left(S_\alpha+S_A\right):\left(S_\alpha+S_A\right)\left(S_\beta+S_B\right)\right)$$
$$=\left(\frac{1}{S_\alpha+S_A}:\frac{1}{S_\beta+S_B}:\frac{1}{S_\gamma+S_C}\right).$$

We introduce the following notations:

$$\lambda=\lambda\left(A,B,C,\alpha,\beta,\gamma\right)=$$
$$=\sin A\sin B\sin C\left(ctgA\sin^2\alpha+ctgB\sin^2\beta+ctgC\sin^2\gamma+2\sin\alpha\sin\beta\sin\gamma\right)$$
$$=\frac{1}{4R^2}\left(S_A\sin^2\alpha+S_B\sin^2\beta+S_C\sin^2\gamma+2S\sin\alpha\sin\beta\sin\gamma\right).$$
$$\lambda_0=\lambda\left(\alpha,\beta,\gamma,A,B,C\right)=$$
$$=\sin\alpha\sin\beta\sin\gamma\left(ctg\alpha\sin^2 A+ctg\beta\sin^2 B+ctg\gamma\sin^2 C+2\sin A\sin B\sin C\right)$$
$$=\frac{\sin\alpha\sin\beta\sin\gamma}{4R^2}\left(a^2ctg\alpha+b^2ctg\beta+c^2ctg\gamma+2S\right)$$
$$=\frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\left(a^2S_\alpha+b^2S_\beta+c^2S_\gamma+2S^2\right).$$

We will proove that $\lambda_0=\lambda$:

$$\sin\alpha\sin\beta\sin\gamma\left(ctg\alpha\sin^2 A+ctg\beta\sin^2 B+ctg\gamma\sin^2 C+2\sin A\sin B\sin C\right)=$$
$$=\sin A\sin B\sin C\left(ctgA\sin^2\alpha+ctgB\sin^2\beta+ctgC\sin^2\gamma+2\sin\alpha\sin\beta\sin\gamma\right)\Leftrightarrow$$

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$$\Leftrightarrow \cos\alpha\sin\beta\sin\gamma\sin^2 A+\sin\alpha\cos\beta\sin\gamma\sin^2 B+\sin\alpha\sin\beta\cos\gamma\sin^2 C=$$
$$=\cos A\sin B\sin C\sin^2\alpha+\sin A\cos B\sin C\sin^2\beta+\sin A\sin B\cos C\sin^2\gamma\Leftrightarrow$$
$$\Leftrightarrow \cos\alpha\left(\cos\alpha+\cos\beta\cos\gamma\right)\sin^2 A+\cos\beta\left(\cos\beta+\cos\alpha\cos\gamma\right)\sin^2 B+$$
$$+\cos\gamma\left(\cos\gamma+\cos\alpha\cos\beta\right)\sin^2 C=\cos A\left(\cos A+\cos B\cos C\right)\sin^2\alpha+$$
$$+\cos B\left(\cos B+\cos A\cos C\right)\sin^2\beta+\cos C\left(\cos C+\cos A\cos B\right)\sin^2\gamma\Leftrightarrow$$
$$\Leftrightarrow \sin^2 A\cos^2\alpha+\sin^2 B\cos^2\beta+\sin^2 C\cos^2\gamma+\cos\alpha\cos\beta\cos\gamma\left(\sin^2 A+\sin^2 B+\sin^2 C\right)=$$
$$=\sin^2\alpha\cos^2 A+\sin^2\beta\cos^2 B+\sin^2\gamma\cos^2 C+\cos A\cos B\cos C\left(\sin^2\alpha+\sin^2\beta+\sin^2\gamma\right)\Leftrightarrow$$
$$\Leftrightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma-\left(\cos^2 A+\cos^2 B+\cos^2 C\right)+$$
$$+2\cos\alpha\cos\beta\cos\gamma\left(1+\cos A\cos B\cos C\right)-2\cos A\cos B\cos C\left(1+\cos\alpha\cos\beta\cos\gamma\right)=0\Leftrightarrow$$
$$\Leftrightarrow 1-2\cos\alpha\cos\beta\cos\gamma-1+2\cos A\cos B\cos C+2\cos\alpha\cos\beta\cos\gamma-2\cos A\cos B\cos C=0,$$

and this is true.

a) The length of the segments AD, BE and CF

In absolute barycentric coordinates the distance between two points $P=(x,y,z)$ and $P'=(x',y',z')$ is given by $|PP'|=\sqrt{(x-x')^2 S_A+(y-y')^2 S_B+(z-z')^2 S_C}$.

In order to calculate the lenght of the segments *AD, BE* and *CF* we introduce the following notations:

$$t_\alpha=S_\beta+S_\gamma=Sctg\beta+Sctg\gamma=S\left(\frac{\cos\beta}{\sin\beta}+\frac{\cos\gamma}{\sin\gamma}\right)=\frac{S}{\sin\beta\sin\gamma}\sin\left(\beta+\gamma\right)=\frac{S\sin\alpha}{\sin\beta\sin\gamma},$$

$$t_\beta=S_\gamma+S_\alpha=\frac{S\sin\beta}{\sin\gamma\sin\alpha},\quad t_\gamma=S_\alpha+S_\beta=\frac{S\sin\gamma}{\sin\alpha\sin\beta}.$$

The sum of the coordinates of the point *D, E* resp. *F* is t_α, t_β resp. t_γ. The length of the segment *AD* is:

$$AD=\frac{1}{t_\alpha}\sqrt{\left(a^2+t_\alpha\right)^2 S_A+\left(S_\gamma+S_C\right)^2 S_B+\left(S_\beta+S_B\right)^2 S_C}$$
$$=\frac{1}{t_\alpha}\sqrt{\left(a^2+2t_\alpha\right)\left(a^2 S_A+S_{BC}\right)+S_A t_\alpha^2+S_B S_\gamma^2+S_C S_\beta^2}$$
$$=\frac{1}{t_\alpha}\sqrt{\left(a^2+2t_\alpha\right)S^2+S_A t_\alpha^2+S_B S_\gamma^2+S_C S_\beta^2}$$
$$=\frac{1}{t_\alpha}\sqrt{\left(a^2+2t_\alpha\right)S^2+S_A S^2\frac{\sin^2\alpha}{\sin^2\beta\sin^2\gamma}+S_B S^2\frac{\cos^2\gamma}{\sin^2\gamma}+S_C S^2\frac{\cos^2\beta}{\sin^2\beta}}$$
$$=\frac{1}{\sin\alpha}\sqrt{a^2\sin^2\beta\sin^2\gamma+2S\sin\alpha\sin\beta\sin\gamma+S_A\sin^2\alpha+S_B\sin^2\beta\cos^2\gamma+S_C\cos^2\beta\sin^2\gamma}$$
$$=\frac{1}{\sin\alpha}\sqrt{S_A\sin^2\alpha+S_B\sin^2\beta+S_C\sin^2\gamma+2S\sin\alpha\sin\beta\sin\gamma}$$

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$$=\frac{2R\sqrt{\lambda}}{\sin\alpha}=\frac{2R\sqrt{\lambda_0}}{\sin\alpha}=\frac{\sqrt{\sin\alpha\sin\beta\sin\gamma\left(a^2ctg\alpha+b^2ctg\beta+c^2ctg\gamma+2S\right)}}{\sin\alpha}.$$

For this reason:

$$AD=\sqrt{\frac{\sin\beta\sin\gamma}{\sin\alpha}\left(a^2ctg\alpha+b^2ctg\beta+c^2ctg\gamma+2S\right)}, \tag{3}$$

$$BE=\sqrt{\frac{\sin\gamma\sin\alpha}{\sin\beta}\left(a^2ctg\alpha+b^2ctg\beta+c^2ctg\gamma+2S\right)}, \tag{4}$$

$$CF=\sqrt{\frac{\sin\alpha\sin\beta}{\sin\gamma}\left(a^2ctg\alpha+b^2ctg\beta+c^2ctg\gamma+2S\right)}. \tag{5}$$

Based on the formulas (3), (4) and (5):

$$AD\cdot\sin\alpha=BE\cdot\sin\beta=CF\cdot\sin\gamma=\sqrt{\sin\alpha\sin\beta\sin\gamma\left(a^2ctg\alpha+b^2ctg\beta+c^2ctg\gamma+2S\right)}. \tag{6}$$

b) The distance of the first generalized Fermat point from the vertices of the reference triangle

In what follows we calculate the lengts of the segments *AK*, *BK* and *CK*. The sum of coordinates of the point K is:

$$\Phi=\left(S_\beta+S_B\right)\left(S_\gamma+S_C\right)+\left(S_\gamma+S_C\right)\left(S_\alpha+S_A\right)+\left(S_\alpha+S_A\right)\left(S_\beta+S_B\right)$$
$$=S_{BC}+S_{CA}+S_{AB}+S_{\beta\gamma}+S_{\gamma\alpha}+S_{\alpha\beta}+\left(S_B+S_C\right)S_\alpha+\left(S_C+S_A\right)S_\beta+\left(S_A+S_B\right)S_\gamma$$
$$=a^2S_\alpha+b^2S_\beta+c^2S_\gamma+2S^2.$$

Therefore

$$\lambda=\frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\left(a^2S_\alpha+b^2S_\beta+c^2S_\gamma+2S^2\right)=\frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\Phi.$$

The length of the segment AK is:

$$AK=\frac{1}{|\Phi|}\sqrt{\left[\Phi-\left(S_\beta+S_B\right)\left(S_\gamma+S_C\right)\right]^2S_A+\left(S_\gamma+S_C\right)^2\left(S_\alpha+S_A\right)^2S_B+\left(S_\alpha+S_A\right)^2\left(S_\beta+S_B\right)^2S_C}$$
$$=\frac{|S_\alpha+S_A|}{|\Phi|}\sqrt{\left(S_B+S_C+S_\beta+S_\gamma\right)^2S_A+\left(S_\gamma+S_C\right)^2S_B+\left(S_\beta+S_B\right)^2S_C}$$
$$=\frac{|S_\alpha+S_A|}{|\Phi|}\sqrt{\left(a^2+t_\alpha\right)^2S_A+\left(S_\gamma+S_C\right)^2S_B+\left(S_\beta+S_B\right)^2S_C}$$
$$=\frac{|S_\alpha+S_A|}{|\Phi|}AD\cdot t_\alpha=\frac{|S_\alpha+S_A|}{|\Phi|}\frac{2R\cdot t_\alpha\sqrt{\lambda}}{\sin\alpha}=\frac{\sin\alpha}{2R\sqrt{\lambda}}|S_\alpha+S_A|=\frac{S}{2R\sqrt{\lambda}\sin A}|\sin(A+\alpha)|$$
$$=\frac{S}{a\sqrt{\lambda}}|\sin(A+\alpha)|=\frac{bc}{2R\sqrt{\lambda}}|\sin(A+\alpha)|.$$

Therefore:

$$AK=\frac{bc}{2R\sqrt{\lambda}}|\sin(A+\alpha)|, \tag{7}$$
$$BK=\frac{ca}{2R\sqrt{\lambda}}|\sin(B+\beta)|, \tag{8}$$

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$$CK = \frac{ab}{2R\sqrt{\lambda}}\left|\sin\left(C+\gamma\right)\right|. \tag{9}$$

c) Relations referring to the segments AK, BK and CK

Now we will demonstrate the following conditional identity:

$$bc\sin\alpha\sin(A+\alpha)+ca\sin\beta\sin(B+\beta)+ab\sin\gamma\sin(C+\gamma)=4R^2\lambda. \tag{10}$$

Notes

Indeed

$$bc\sin\alpha\sin\left(A+\alpha\right)+ca\sin\beta\sin\left(B+\beta\right)+ab\sin\gamma\sin\left(C+\gamma\right)=$$
$$=4R^2\sin B\sin C\sin\alpha\left(\sin A\cos\alpha+\sin\alpha\cos A\right)+4R^2\sin C\sin A\sin\beta\left(\sin B\cos\beta+\sin\beta\cos B\right)+$$
$$+4R^2\sin A\sin B\sin\gamma\left(\sin C\cos\gamma+\sin\gamma\cos C\right)=$$
$$=4R^2\sin A\sin B\sin C\left[ctgA\sin^2\alpha+ctgB\sin^2\beta+ctgC\sin^2\gamma+\left(\sin 2\alpha+\sin 2\beta+\sin 2\gamma\right)/2\right]=$$
$$=4R^2\sin A\sin B\sin C\left(ctgA\sin^2\alpha+ctgB\sin^2\beta+ctgC\sin^2\gamma+2\sin\alpha\sin\beta\sin\gamma\right)=4R^2\lambda.$$

Since $A+\alpha+B+\beta+C+\gamma=2\pi$, from inequalities $A+\alpha>\pi$, $B+\beta>\pi$, $C+\gamma>\pi$ only one can be true. If $A+\alpha<\pi$, $B+\beta<\pi$, $C+\gamma<\pi$, then the K is an inside point of the triangle ABC and based on the (10) we have

$$AD\sin\alpha=BE\sin\beta=CF\sin\gamma=2R\sqrt{\lambda}=AK\sin\alpha+BK\sin\beta+CK\sin\gamma. \tag{11}$$

If for instance $A+\alpha>\pi$, then

$$AD\sin\alpha=BE\sin\beta=CF\sin\gamma=2R\sqrt{\lambda}=-AK\sin\alpha+BK\sin\beta+CK\sin\gamma. \tag{12}$$

III. The Second Generalized Fermat Point of the Triangle

We build on the sides of the triangle ABC inwards the similar triangles BCL, CAM and ABN in that way, that

$BAN\angle=\alpha=CAM\angle,\ CBL\angle=\beta=ABN\angle,\ ACM\angle=\gamma=BCL\angle$ (Figure 4). Since $A+B+C=\pi=\alpha+\beta+\gamma$, therefore $BLC\angle=\alpha,\ CMA\angle=\beta,\ ANB\angle=\gamma.$

According to the Conway's formulas, the barycentric coordinates of the points L, M, N are:

$$L=\left(a^2:S_\gamma-S_C:S_\beta-S_B\right),\ M=\left(S_\gamma-S_C:b^2:S_\alpha-S_A\right),\ N=\left(S_\beta-S_B:S_\alpha-S_A:c^2\right).$$

The equations of the lines AL, BM, CN are: $-\left(S_\beta-S_B\right)y+\left(S_\gamma-S_C\right)z=0$,

$$\left(S_\alpha-S_A\right)x-\left(S_\gamma-S_C\right)z=0,$$
$$-\left(S_\alpha-S_A\right)x+\left(S_\beta-S_B\right)y=0.$$

Since adding whatever two equation, we get the third equation, the lines AL, BM and CN meets each other in a point $T=AL\cap BM\cap CN$. This point T will be called **the second generalized Fermat point of the triangle**.

The barycentric coordinates of the point T are:

$$T=\left(\left(S_\beta-S_B\right)\left(S_\gamma-S_C\right):\left(S_\gamma-S_C\right)\left(S_\alpha-S_A\right):\left(S_\alpha-S_A\right)\left(S_\beta-S_B\right)\right)$$
$$=\left(\frac{1}{S_\alpha-S_A}:\frac{1}{S_\beta-S_B}:\frac{1}{S_\gamma-S_C}\right).$$

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We introduce the following notations:

$$\mu = \mu(A,B,C,\alpha,\beta,\gamma) =$$
$$= \sin A \sin B \sin C\left(ctgA\sin^2\alpha + ctgB\sin^2\beta + ctgC\sin^2\gamma - 2\sin\alpha\sin\beta\sin\gamma\right)$$
$$= \frac{1}{4R^2}\left(S_A\sin^2\alpha + S_B\sin^2\beta + S_C\sin^2\gamma - 2S\sin\alpha\sin\beta\sin\gamma\right).$$
$$\mu_0 = \mu(\alpha,\beta,\gamma,A,B,C) =$$
$$= \sin\alpha\sin\beta\sin\gamma\left(ctg\alpha\sin^2 A + ctg\beta\sin^2 B + ctg\gamma\sin^2 C - 2\sin A\sin B\sin C\right)$$
$$= \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)$$
$$= \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\left(a^2S_\alpha + b^2S_\beta + c^2S_\gamma - 2S^2\right).$$

In this case the equality $\mu_0 = \mu$ is valid too.

a) The length of the segments AL, BM and CN

The sum of the coordinates of the point L, M resp. N are t_α, t_β resp. t_γ. The length of the segment AL is:

$$AL = \frac{1}{t_\alpha}\sqrt{\left(a^2 - t_\alpha\right)^2 S_A + \left(S_\gamma + S_C\right)^2 S_B + \left(S_\beta + S_B\right)^2 S_C}$$
$$= \frac{1}{t_\alpha}\sqrt{\left(a^2 - 2t_\alpha\right)\left(a^2S_A + S_{BC}\right) + S_At_\alpha^2 + S_BS_\gamma^2 + S_CS_\beta^2}$$
$$= \frac{1}{t_\alpha}\sqrt{\left(a^2 - 2t_\alpha\right)S^2 + S_At_\alpha^2 + S_BS_\gamma^2 + S_CS_\beta^2}$$
$$= \frac{1}{t_\alpha}\sqrt{\left(a^2 - 2t_\alpha\right)S^2 + S_AS^2\frac{\sin^2\alpha}{\sin^2\beta\sin^2\gamma} + S_BS^2\frac{\cos^2\gamma}{\sin^2\gamma} + S_CS^2\frac{\cos^2\beta}{\sin^2\beta}}$$
$$= \frac{1}{\sin\alpha}\sqrt{a^2\sin^2\beta\sin^2\gamma - 2S\sin\alpha\sin\beta\sin\gamma + S_A\sin^2\alpha + S_B\sin^2\beta\cos^2\gamma + S_C\cos^2\beta\sin^2\gamma}$$
$$= \frac{1}{\sin\alpha}\sqrt{S_A\sin^2\alpha + S_B\sin^2\beta + S_C\sin^2\gamma - 2S\sin\alpha\sin\beta\sin\gamma}$$
$$= \frac{2R\sqrt{\mu}}{\sin\alpha} = \frac{2R\sqrt{\mu_0}}{\sin\alpha} = \frac{\sqrt{\sin\alpha\sin\beta\sin\gamma\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}}{\sin\alpha}.$$

For this reason:

$$AL = \sqrt{\frac{\sin\beta\sin\gamma}{\sin\alpha}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}, \tag{13}$$
$$BM = \sqrt{\frac{\sin\gamma\sin\alpha}{\sin\beta}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}, \tag{14}$$
$$CN = \sqrt{\frac{\sin\alpha\sin\beta}{\sin\gamma}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}. \tag{15}$$

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Based on the (13), (14) and (15) we obtain:

$$AL\cdot\sin\alpha = BM\cdot\sin\beta = CN\cdot\sin\gamma = \sqrt{\sin\alpha\sin\beta\sin\gamma\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}. \quad (16)$$

b) The distance of the second generalized Fermat point from the vertices of the reference triangle

The sum of coordinates of the point T is:

$$\Gamma = \left(S_\beta - S_B\right)\left(S_\gamma - S_C\right) + \left(S_\gamma - S_C\right)\left(S_\alpha - S_A\right) + \left(S_\alpha - S_A\right)\left(S_\beta - S_B\right)$$

$$= S_{BC} + S_{CA} + S_{AB} + S_{\beta\gamma} + S_{\gamma\alpha} + S_{\alpha\beta} - \left(S_B + S_C\right)S_\alpha - \left(S_C + S_A\right)S_\beta - \left(S_A + S_B\right)S_\gamma$$

$$= -a^2S_\alpha - b^2S_\beta - c^2S_\gamma + 2S^2 = -\left(a^2S_\alpha + b^2S_\beta + c^2S_\gamma - 2S^2\right).$$

Therefore:

$$\mu = \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\left(a^2S_\alpha + b^2S_\beta + c^2S_\gamma - 2S^2\right) = -\frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\Gamma.$$

The length of the segment AT is:

$$AT = \frac{1}{|\Gamma|}\sqrt{\left[\Gamma - \left(S_\beta - S_B\right)\left(S_\gamma - S_C\right)\right]^2 S_A + \left(S_\gamma - S_C\right)^2\left(S_\alpha - S_A\right)^2 S_B + \left(S_\alpha - S_A\right)^2\left(S_\beta - S_B\right)^2 S_C}$$

$$= \frac{\left|S_\alpha - S_A\right|}{|\Gamma|}\sqrt{\left(S_\beta + S_\gamma - S_B - S_C\right)^2 S_A + \left(S_\gamma - S_C\right)^2 S_B + \left(S_\beta - S_B\right)^2 S_C}$$

$$= \frac{\left|S_\alpha - S_A\right|}{|\Gamma|}\sqrt{\left(-a^2 + t_\alpha\right)^2 S_A + \left(S_\gamma - S_C\right)^2 S_B + \left(S_\beta - S_B\right)^2 S_C}$$

$$= \frac{\left|S_\alpha - S_A\right|}{|\Gamma|}AL\cdot t_\alpha = \frac{\left|S_\alpha - S_A\right|}{|\Gamma|}\frac{2R\cdot t_\alpha\sqrt{\mu}}{\sin\alpha} = \frac{\sin\alpha}{2R\sqrt{\mu}}\left|S_\alpha - S_A\right| = \frac{S}{2R\sqrt{\mu}\sin A}\left|\sin\left(A-\alpha\right)\right|$$

$$= \frac{S}{a\sqrt{\mu}}\left|\sin\left(A-\alpha\right)\right| = \frac{bc}{2R\sqrt{\mu}}\left|\sin\left(A-\alpha\right)\right| = \frac{bc}{2R\sqrt{\mu}}\sin\left|A-\alpha\right|.$$

Therefore:

$$AT = \frac{bc}{2R\sqrt{\mu}}\sin\left|A-\alpha\right|, \quad (17) \qquad BT = \frac{ca}{2R\sqrt{\mu}}\sin\left|B-\beta\right|, \quad (18)$$

$$CT = \frac{ab}{2R\sqrt{\mu}}\sin\left|C-\gamma\right|. \quad (19)$$

c) Relations referring to the segments AT, BT and CT

Here is valid too the following conditional identity:

$$bc\sin\alpha\sin(A-\alpha) + ca\sin\beta\sin(B-\beta) + ab\sin\gamma\sin(C-\gamma) = -4R^2\mu. \quad (20)$$

Since $A-\alpha+B-\beta+C-\gamma=0$, only two of the $A>\alpha$, $B>\beta$, $C>\gamma$ inequalities

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can be true at the same time. For instance if A $<\alpha$, B $>\beta$, C $>\gamma$, then based on the formula (20)

$$AL\sin\alpha = BM\sin\beta = CN\sin\gamma = 2R\sqrt{\mu} = AT\sin\alpha - BT\sin\beta - CT\sin\gamma . \qquad (21)$$

Or if A $<\alpha$, B $<\beta$, C $>\gamma$, then

$$AL\sin\alpha = BM\sin\beta = CN\sin\gamma = 2R\sqrt{\mu} = AT\sin\alpha + BT\sin\beta - CT\sin\gamma . \qquad (22)$$

Notes

IV. Speciale Case

In this section we use the following relations:

$$abc(ctgA + ctgB + ctgC) = R(a^2 + b^2 + c^2), \qquad (23)$$

$$\sin A\sin B\sin C\left(ctgA + ctgB + ctgC + \sqrt{3}\right) = \\ = 1 + \sqrt{3}\sin A\sin B\sin C + \cos A\cos B\cos C, \qquad (24)$$

$$\sin A\sin B\sin C\left(ctgA + ctgB + ctgC - \sqrt{3}\right) = \\ = 1 - \sqrt{3}\sin A\sin B\sin C + \cos A\cos B\cos C. \qquad (25)$$

The proof of (23):

$$abc(ctgA + ctgB + ctgC) = abc\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}\right) = 2Rabc\left(\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}\right) = \\ = R(2bc\cos A + 2ca\cos B + 2ab\cos C) = R(b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2) = \\ = R(a^2 + b^2 + c^2).$$

The proof of (24) and (25):

$$\sin A\sin B\sin C\left(ctgA + ctgB + ctgC \pm \sqrt{3}\right) = \sin A\sin B\sin C\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C} \pm \sqrt{3}\right) = \\ = \cos A\sin B\sin C + \cos B\sin C\sin A + \cos C\sin A\sin B \pm \sqrt{3}\sin A\sin B\sin C = \\ = 1 \pm \sqrt{3}\sin A\sin B\sin C + \cos A\cos B\cos C.$$

If $\alpha = \beta = \gamma = \pi/3$, then $K \equiv F_1$ and $T \equiv F_2$, where F_1 resp. F_2 is the first resp. the second Fermat's point of the triangle ABC (X_{13} resp. X_{14} in [1]). In this case

$$\lambda^* = \lambda\left(A, B, C, \frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3}\right) = \frac{3}{4}\sin A\sin B\sin C\left(ctgA + ctgB + ctgC + \sqrt{3}\right) = \\ = \frac{3}{32R^2}\left(a^2 + b^2 + c^2 + 2\sqrt{3}S\right) = \frac{3}{4}\left(1 + \sqrt{3}\sin A\sin B\sin C + \cos A\cos B\cos C\right),$$

$$\mu^* = \mu\left(A, B, C, \frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3}\right) = \frac{3}{4}\sin A\sin B\sin C\left(ctgA + ctgB + ctgC - \sqrt{3}\right) = \\ = \frac{3}{32R^2}\left(a^2 + b^2 + c^2 - 2\sqrt{3}S\right) = \frac{3}{4}\left(1 - \sqrt{3}\sin A\sin B\sin C + \cos A\cos B\cos C\right).$$

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Based on (6) we have

$$AX_1 = BY_1 = CZ_1 = \frac{4R}{\sqrt{3}}\sqrt{\lambda^*} = \frac{\sqrt{a^2+b^2+c^2+2\sqrt{3}S}}{\sqrt{2}}. \qquad (26)$$

According to (7), (8) and (9)

$$AF_1 = \frac{bc}{2R\sqrt{\lambda^*}}\left|\sin\left(A+\frac{\pi}{3}\right)\right|, \qquad (27) \qquad BF_1 = \frac{ca}{2R\sqrt{\lambda^*}}\left|\sin\left(B+\frac{\pi}{3}\right)\right|, \qquad (28)$$

$$CF_1 = \frac{ab}{2R\sqrt{\lambda^*}}\left|\sin\left(C+\frac{\pi}{3}\right)\right|. \qquad (29)$$

If $A<2\pi/3,\ B<2\pi/3,\ C<2\pi/3$, then

$$AX_1 = BY_1 = CZ_1 = \frac{4R}{\sqrt{3}}\sqrt{\lambda^*} = AF_1 + BF_1 + CF_1. \qquad (30)$$

If for instance $A>2\pi/3$, then

$$AX_1 = BY_1 = CZ_1 = \frac{4R}{\sqrt{3}}\sqrt{\lambda^*} = -AF_1 + BF_1 + CF_1. \qquad (31)$$

Based on (16) we have

$$AX_2 = BY_2 = CZ_2 = \frac{4R}{\sqrt{3}}\sqrt{\mu^*} = \frac{\sqrt{a^2+b^2+c^2-2\sqrt{3}S}}{\sqrt{2}}. \qquad (32)$$

According to (17), (18) and (19)

$$AF_2 = \frac{bc}{2R\sqrt{\mu^*}}\sin\left|A-\frac{\pi}{3}\right|, \qquad (33) \qquad BF_2 = \frac{ca}{2R\sqrt{\mu^*}}\sin\left|B-\frac{\pi}{3}\right|, \qquad (34)$$

$$CF_2 = \frac{ab}{2R\sqrt{\mu^*}}\sin\left|C-\frac{\pi}{3}\right|. \qquad (35)$$

If for instance $A<\pi/3,\ B>\pi/3,\ C>\pi/3$, then

$$AX_2 = BY_2 = CZ_2 = \frac{4R}{\sqrt{3}}\sqrt{\mu^*} = AF_2 - BF_2 - CF_2. \qquad (36)$$

Or if A $<\pi/3$, B $<\pi/3$, C $>\pi/3$, then

$$AX_2 = BY_2 = CZ_2 = \frac{4R}{\sqrt{3}}\sqrt{\mu^*} = AF_2 + BF_2 - CF_2. \qquad (37)$$

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Other arrangements of the triangles *BCD, CAE* and *ABF* are possible. To investigate the valabilities of the above metric relations in this cases is a possible subject for further researches.

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[4] P. G. Spain, The Fermat Point of a Triangle, *Math. Mag.* **69**, 131-133, 1996.

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Notes

The Metric Characterization of the Generalized Fermat Points

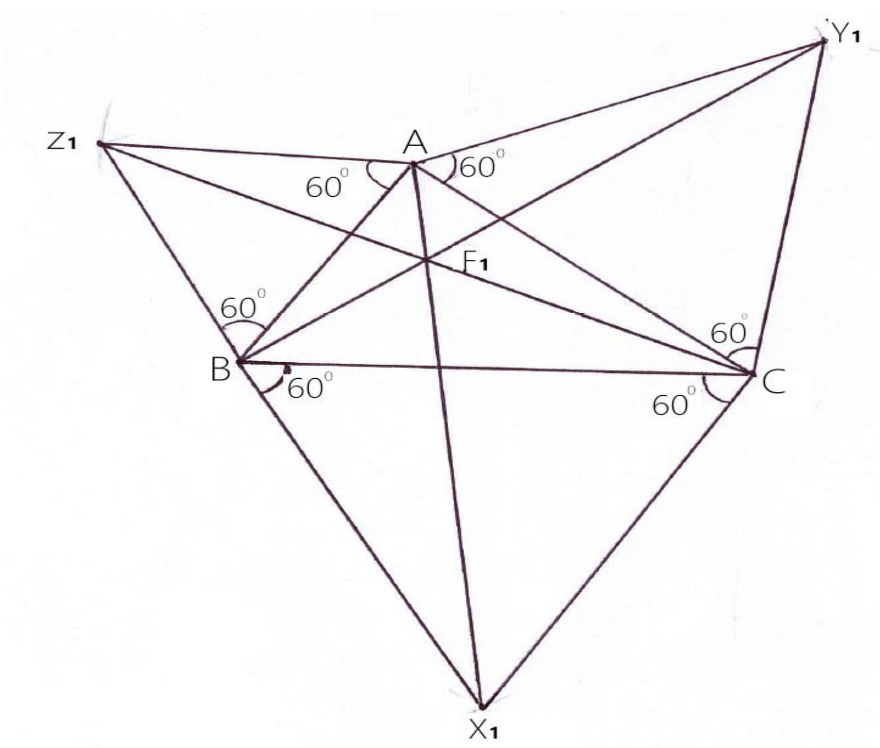

Figure 1

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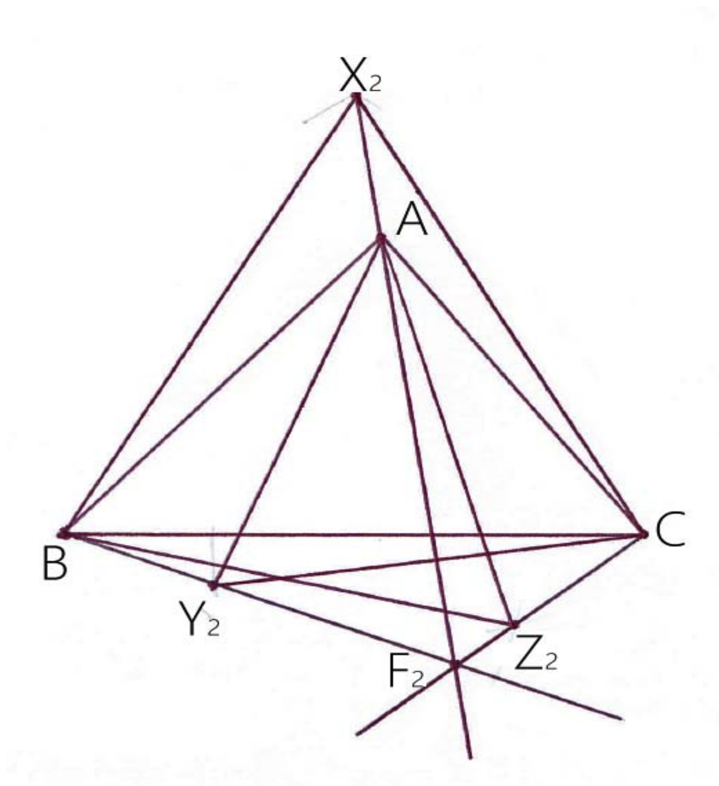

Figure 2

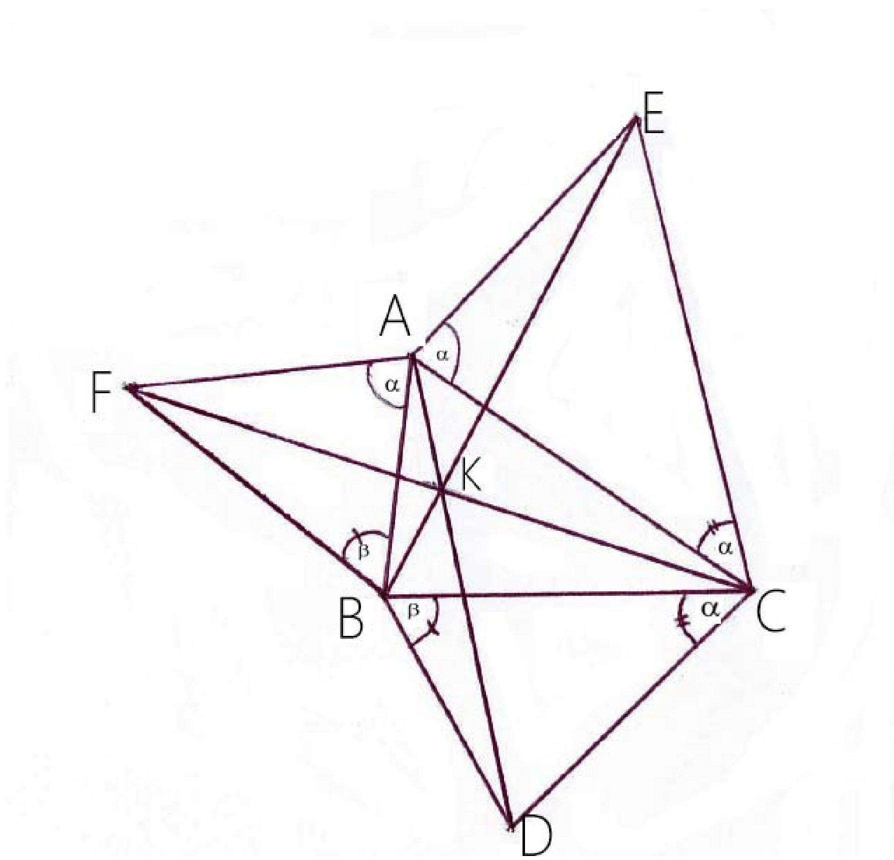

Figure 3

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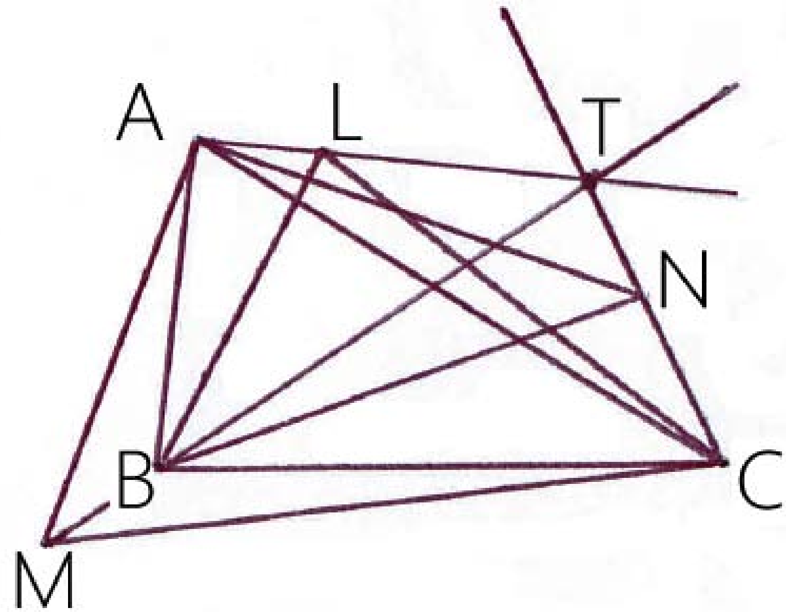

Figure 4

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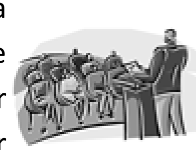

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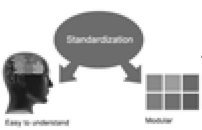

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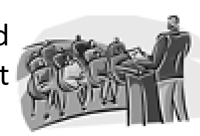

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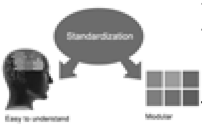

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References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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