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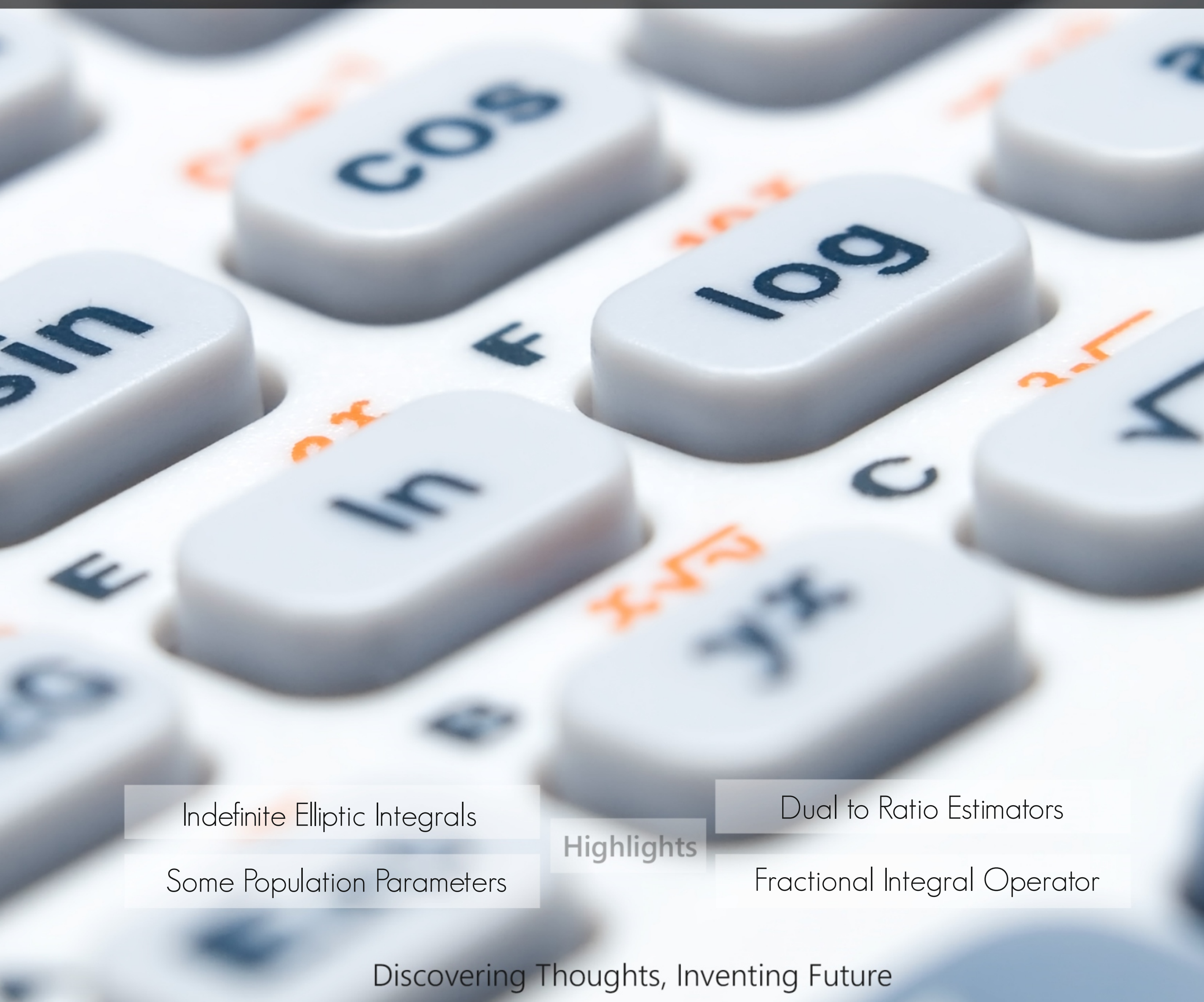

Highlights

- Indefinite Elliptic Integrals
- Some Population Parameters
- Dual to Ratio Estimators
- Fractional Integral Operator

Discovering Thoughts, Inventing Future

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A Common Fixed Point for Eight Mappings in an Intuitionistic M- Fuzzy Metric Space with Property 'E'

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Abstract - The aim of this paper is to introduce the concept of an intuitionistic M - fuzzy metric space with property 'E' and prove common fixed point theorem for eight weakly compatible mappings in intuitionistic M - fuzzy metric space with property 'E'.

Keywords : intuitionistic M-fuzzy metric space, compatible mapping, weak compatible mapping, property (E).

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5. O.Kramosil and J.Michalek: Fuzzy metric and statistical metric spaces, Ky-bernetics,11(1975), 330-334.

A Common Fixed Point for Eight Mappings in an Intuitionistic M-Fuzzy Metric Space with Property 'E'

Ranjeeta Jain [α] & N. Bajaj [σ]

Abstract - The aim of this paper is to introduce the concept of an intuitionistic M - fuzzy metric space with property 'E' and prove common fixed point theorem for eight weakly compatible mappings in intuitionistic M - fuzzy metric space with property 'E'.

Keywords : intuitionistic M - fuzzy metric space, compatible mapping, weak compatible mapping, property (E).

I. Introduction

In 1975, Kramosil and Michalek [5] introduced the concept of fuzzy metric space by generalizing the concept of probabilitic metric space to fuzzy situation. Many authors ([1],[2],[3],[5]) obtained common fixed point theorems involving fuzzy metric spaces.

In 2006, Sedghi and Shobe [6] introduced the concept of M – Fuzzy metric space as follows:

DEFINITION (1) : A 3-tuple (X,M,*) is called a M – Fuzzy metric space if X is an arbitrary (non-empty) set, * is a continuous t- norm, and M is a Fuzzy set on $X^3 x (0, \infty)$, satisfying the following condition : for each x, y, z, a $\in$ X and t, s > 0,

(1). M(x, y, z, t) > o, (2). M(x, y, z, t) = 1 if and only if x = y = z,

(3). M(x, y, z, t) = M(p{x, y, z}t, where p is a permutation function,

(4). $M(x, y, a\ t) * M(a, z, z, s) \leq M(x, y, z, t + s)$.

(5). $M(x, y, z) : (0, \infty) \rightarrow [0,1]$ is continuous.

As a generalization of fuzzy sets, Atanassov [4] introduce and studied the concept of intuitionistic fuzzy sets. park [3] and Alaca, Turkoglu and Yiliz [2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t–norm and continuous t–conorm as a generalization of fuzzy metric spaces due to George and Veeramani [1] and kramosil and Michalek [5] respectively.

In 2006, Sedghi and Shobe [6] defined M-Fuzzy metric space and proved a common fixed point theorem for four weakly compatible mappings in this space. In 2009, Seema Mehra and Meenakshi Gugnani [8] defined the notion of an intuitionistic M-Fuzzy metric space due to Sidgi and Shobe [6] and

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proved a common fixed point theorem for six mappings for property (E) in this newly defined space. Our result is an intuitionistic Fuzzy version of the results of Seema Mehra and Meenakshi Gugnani [8] result in M- Fuzzy metric space.

We introduce the concept of an intuitionistic M – Fuzzy metric space as follows.

DEFINITION (2): A binary operation $*:[o,1]\times[0,1]$ is a continuous t- norm of it satisfies the following condition

(1) $*$ is associative and commutative,

(2) $*$ is continuous

(3) $a * l = a$ for all $a \in [0,1]$

(4) $a * l = c * d$ whenever $a \leq c$ and $b \leq d$, for each a , b , c ,d $\in [\,o,1]$.

Two typical example of a continuous t – norm are $a * b = ab$ and $a * b = \min(a, b)$

DEFINITION (3): A binary operation $\Diamond$: $[0,1]\times[0,1] \rightarrow [\,0,1\,]$ is a continuous t–connorm if it satisfies the following conditions:

(1). $\Diamond$ is associative and commutative,

(2). $\Diamond$ is continuous,

(3). $a \Diamond o = a$ for all $a \in [\,0,I\,]$,

(4). $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$, for each a, b, c, d $\in [0, 1]$.

Two typical examples of a continuous t–conorm are $a \Diamond b = \min(1,a,+b)$ and $a \Diamond b = \max(a, b)$.

DETINITION (4): A 5–tuple $(X,M,N*,\Diamond)$ is called an intuitionistic M–fuzzy metric apace if X is an arbitrary (non-empty) set, * is a continuous t–norm , $\Diamond$ a continuous t–conorm and M, N are fuzzy sets on $X^3(0, \infty)$, satisfying the following conditions : for each x, y, z, a $\in$ X and t,s > 0,

(a) $M(x,y,z,t)+N(x,y,z,t) \leq 1$.

(b) $M(x,y,z,t) > 0$,

(c) $M(x,y,z,t,) = 1$ if and only if $x = y = z$,

(d) $M(x,y,z,t,) = M(p\{x,y,z,\}t)$, where p is a permutation function ,

(e) $M(x,y,a,t,) * M(a,z,z,s,) \leq M(x, y, z, t + s)$,

(f) $M(x,y,z,) : (0, \infty) \rightarrow [\,0,1]$ is continuous

(g) $N(x,y,z,t,) > 0$,

(h) $N(x,y,z,t,) = 0$, if and only if $x = y = z$,

(i) $N(x,y,z,t,) = N(p\{x,y,z,\}t)$, where p is a permutation function ,

(j) $N(x, y, a, t,) \Diamond N(a, z, z, s) \geq N(x, y, z, t + s)$,

(k) $N(x, y, z, .) : (0, \infty) \rightarrow [\,0, 1\,]$ is continuous.

Then (M, N) is called an intuitionistic M – Fuzzy metric on X.

Example(1) : Let X = R and $M(x, y, z, t,) = \dfrac{t}{t+|x-y|+|y+z|+|z-x|}$,

$$N(x, y, z, t,) = \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|},$$

Ref.

8. Mehra S.and Gugnani M.: A common fixed point for six mappings in an intruitionistiM-fuzzy metric space. Indian Journal of Mathematics, Vol. 51 No. 1, (2009) 23-47.

for every x, y, z and t > 0 Let A and B defined as Ax = 2x +1, Bx = x+2. consider the sequence $x_n = \frac{1}{n} + 1$, n = 1, 2 …….. Thus we have $\lim_{n\to\infty} M(Ax_n, 3, 3, t) = \lim_{n\to\infty} M(Bx_n, 3, 3, t) = 1$ and $\lim_{n\to\infty} N(Ax_n, 3, 3, t) = \lim_{n\to\infty} N(Bx_n, 3, 3, t) = 0$, for every t > 0.Then A and B Satisfying in the property (E).

In 2009, Seema Mehra and Meenakshi Gugnani [8] have proved the following theorem.

THEOREM (A) : Let P, Q, A, B, S and T be self mappings of X Satisfying the following conditions :

(i) P(X)⊂ST(X) and Q(X)⊂AB(X) and ST(X) or AB(X) or AB(X) is complete fuzzy metric subspace of X ,

(ii) AB = BA, ST = TS, PB = BP, TQ = QT,

(iii) The pair (P, AB) and (Q, ST) are weakly compatible and (P, AB) or (Q, ST) Satisfies the property (E),

(iv) If there exists a number K > 1 Such that

M(Px, Qy, Qz, t) ≥ ϕ{ (M(ABx, STy, STz, Kt), M(ABx, Qy, STz, Kt), M(ABx, STy, Qz, Kt), M(ABx, Qy, Qy, Kt) M(STy, Qy, Qz, Kt), M(STy, STy Qz, Kt), M(STy, Qy, Qy, Kt), M(STy, Qz, Qz, Kt), M(Qy, STy, STz, Kt), M(Qy, Qy, STz, Kt), M(Qy, STz, STz, Kt), M(STz Qz, Qz,, Kt)} and

N(Px, Qy, Qz, t) ≤ ϕ'{N(ABx, STy, STz, Kt), N(ABx, Qy, STz, Kt), N(ABx, STy, Qz, Kt), N(ABx, Qy, Qy, Kt),N(STy, Qy, Qz, Kt), N(STy, STy Qz, Kt), N(STy, Qy, Qy, Kt), N(STy, Qz, Qz, Kt) N(Qy, STy, STz, Kt), N(Qy, Qy, STz, Kt), N(Qy, STz, STz, Kt), N(STz Qz, Qz,, Kt) , for all x, y, z ∈ X and t > 0,

Then P, Q, A,B,S and T have unique common fixed point in X.

Where A class of implict relation: Let ψ denote a family of mappings and ϕ, ϕ' ∈ ψ, ϕ, ϕ': $[0,1]^{12} \to [0,1]$, and ϕ, ϕ' are continuous, increasing and decreasing respectively, in each co–ordinate variable. Also ϕ (s, s,…s) > s, ϕ'(s, s, …., s) < s for every s∈[0,1],

Example(3): Let ϕ, ϕ':$[0,1]^{12} \to [0,1]$ be define by $\phi(x_1, x_2 \ldots x_{12}) = (\min\{x_i\})^h$ for some 0 < h < l and $\phi'(x_1,x_2, \ldots, x_{12}) = (\max\{x_i\})^h$ for some h > l. Then ϕ, ϕ' ∈ ψ.

Here we generalized and extend the results of theorem (A) for eight mappings with property (E) in this newly defined space.

II. Main Result

Theorem 1 : Let P, Q, A, B, F, L, S and T be self mappings of X satisfying the following condition:

(1.2.1) P(X) ⊆ ST (X) ∪ F(X) and Q(X) ⊆ AB(X) ∪ L(X) and ST(X) or AB(X) and L(X) are complete fuzzy metric subspace of X.

(1.2.2) AB = BA, ST = TS, BP = PB, QT = TQ, FT = TF, LB = BL,

(1.2.3) The pair (P, AB), (P, L) and (Q, ST), (Q, F) are weak compatible and (P, AB) or (Q, ST) and (Q, F) satisfies the property (E)

(1.2.4) If there exists a number k > l such that

Ref.
8. Mehra S.and Gugnani M.: A common fixed point for six mappings in an intruitionistiM-fuzzy metric space. Indian Journal of Mathematics, Vol. 51 No. 1, (2009) 23-47.

$M(Px, Qy, Qz, t) \geq \phi\{ M(ABx, STy, Lx, kt), M(Lx, STy, STz, kt), M(ABx, STy, Fz, Kt), M(ABx, Qy, Fz, Kt),$

$M(ABx, Fy, Qz, Kt), M(STz, Qz, Fz, Kt), M(Fy, Qy, Qz, Kt), M(Lx, Qy, Fz, Kt),$

$M(Qy, STy, Fz, Kt), M(ABx, STy, STz, Kt), M(ABx, Qy, STz, Kt), M(ABx, STy, Qz, Kt)\}$

and

$N(Px, Qy, Qz, t) \leq \phi'\{N(ABx, STy, Lx, Kt), N(Lx, STy, STz, Kt), N(ABx, STy, Fz, Kt), N(ABx, Qy, Fz, Kt), N(ABx,$

$Fy, Qz, Kt), N(STz, Qz, Fz, Kt), N(Fy, Qy, Qz, Kt), N(Lx, Qy, Fz, Kt), N(Qy, STy, Fz, Kt),$

$N(ABx, Sty, STz, Kt), N(ABx, Qy, STz, Kt), N(ABx, STy, Qz, Kt)\}$

for all $x, y, z \in X$ and $t > 0$

Then P, Q, A, B, F, L, S and T have a unique common fixed point in X

Proof: Suppose that pair (Q, ST) and (Q, F) Satisfies the property (E). Hence there exists a sequence $\{x_n\}$. Such that

$$\lim_{n\to\infty} M(Qx_n, u, u, t) = \lim_{n\to\infty} M(STx_n, u, u, t) = 1$$

$$\lim_{n\to\infty} N(Qx_n, u, u, t) = \lim_{n\to\infty} N(STx_n, u, u, t) = 0 \text{ and}$$

$$\lim_{n\to\infty} M(Qx_n, u, u, t) = \lim_{n\to\infty} M(Fx_n, u, u, t) = 1$$

$$\lim_{n\to\infty} N(Qx_n, u, u, t) = \lim_{n\to\infty} N(Fx_n, u, u, t) = 0,$$ for some $u \in X$ and every $t > 0$. As $Q(X) \subset AB(X) \cup L(X)$, there exists a sequence $\{y_n\}$ such that $Qx_n = ABy_n = Ly_n = u$.

Hence,

$$\lim_{n\to\infty} M(ABy_n, u, u, t) = 1 \text{ and } \lim_{n\to\infty} N(ABy_n, u, u, t) = 0 \text{ and}$$

$$\lim_{n\to\infty} M(Ly_n, u, u, t) = 1 \text{ and } \lim_{n\to\infty} N(Ly_n, u, u, t) = 0$$

We prove that

$$\lim_{n\to\infty} M(Py_n, u, u, t) = 1, \quad \lim_{n\to\infty} N(Py_n, u, u, t) = 0$$

Step 1: Putting $x = y_n$, $y = x_n$, $z = x_{n+1}$ in (1.2.4), we obtain

$M(Py_n, Qx_n, Qx_{n+1}, t) \geq \phi\{ M(ABy_n, STx_n, Ly_n, Kt), M(Ly_n, STx_n, STx_{n+1}, Kt), M(ABy_n, STx_n,$

$Fx_{n+1}, Kt), M(ABy_n, Qx_n, Fx_{n+1}, Kt), M(ABy_n, Fx_n, Qx_{n+1}, Kt),$

$M(STx_{n+1}, Qx_{n+1}, Fx_{n+1}, Kt), M(Fx_n, Qx_n, Qx_{n+1}, Kt), M(Ly_n, Qx_n,$

$Fx_{n+1}, Kt), M(Qx_n, STx_n, Fx_n, Kt), M(ABy_n, STx_n, STx_{n+1}, Kt),$

$M(ABy_n, Qx_n, STx_{n+1}, Kt), M(ABy_n, STx_n, Qx_{n+1}, Kt), \}$

Notes

and

$$N(Py_n, Qx_n, Qx_{n+1}, t) \leq \phi' \{ N(ABy_n, STx_n, Ly_n, Kt), N(Ly_n, STx_n, STx_{n+1}, Kt), N(ABy_n, STx_n, Fx_{n+1}, Kt), N(ABy_n, Qx_n, Fx_{n+1}, Kt), N(ABy_n, Fx_n, Qx_{n+1}, Kt), N(STx_{n+1}, Qx_{n+1}, Fx_{n+1}, Kt), N(Fx_n, Qx_n, Qx_{n+1}, Kt), N(Ly_n, Qx_n, Fx_{n+1}, Kt), N(Qx_n, STx_n, Fx_n, Kt), N(ABy_n, STx_n, STx_{n+1}, Kt), N(ABy_n, Qx_n, STx_{n+1}, Kt), N(ABy_n, STx_n, Qx_{n+1}, Kt)\}$$

Letting $n \to \infty$ in the above inequality we get

$$\lim_{n\to\infty} M(Py_n, Qx_n, Qx_{n+1}, t) \geq \phi\{M(u, u, u, Kt), M(u, u, u, Kt), M(u, u, u, Kt), \ldots, M(u, u, u, Kt)\} = 1,$$

$$\lim_{n\to\infty} N(Py_n, Qx_n, Qx_{n+1}, t) \leq \phi'\{N(u, u, u, Kt), N(u, u, u, Kt), N(u, u, u, Kt), \ldots\ldots, N(u, u, u, Kt)\} = 0.$$

Therefore,

$$\lim_{n\to\infty} M(Py_n, u, u, t) = 1, \quad \lim_{n\to\infty} N(Py_n, u, u, t) = 0$$

Hence,

$$\lim_{n\to\infty} Py_n = \lim_{n\to\infty} ABy_n = \lim_{n\to\infty} Ly_n = \lim_{n\to\infty} Qx_n = \lim_{n\to\infty} STx_n = \lim_{n\to\infty} Fx_n = u.$$

Assume that AB(X) and L(X) are complete intutionstic M–Fuzzy metric space, then there exists $x \in X$ s.t $ABx = u$ and $Lx = u$.

Step 2: If $Px \neq u$, putting $y = x_n$, and $z = x_{n+1}$ in (1.2.4) then we have

$$M(Px, Qx_n, Qx_{n+1}, t) \geq \phi\{M(ABx, STx_n, Lx, Kt), M(Lx, STx_n, STx_{n+1}, Kt), M(ABx, STx_n, Fx_{n+1}, Kt), M(ABx, Qx_n, Fx_{n+1}, Kt), M(ABx, Fx_n, Qx_{n+1}, Kt), M(STx_{n+1}, Qx_{n+1}, Fx_{n+1}, Kt), M(Fx_n, Qx_n, Qx_{n+1}, Kt), M(Lx, Qx_n, Fx_{n+1}, Kt), M(Qx_n, STx_n, Fx_{n+1}, Kt), M(ABx, STx_n, STx_{n+1}, Kt), M(ABx, Qx_n, STx_{n+1}, Kt), M(ABx, STx_n, Qx_{n+1}, Kt)\}$$

and

$$N(Px, Qx_{n+1}, t) \leq \phi'\{N(ABx, STx_n, Lx, Kt), N(Lx, STx_n, STx_{n+1}, Kt), N(ABx, STx_n, Fx_{n+1}, Kt), N(ABx, Qx_n, Fx_{n+1}, Kt), N(ABx, Fx_n, Qx_{n+1}, Kt), N(STx_{n+1}, Qx_{n+1}, Fx_{n+1}, Kt), N(Fx_n, Qx_n, Qx_{n+1}, Kt), N(Lx, Qx_n, Fx_{n+1}, Kt), N(Qx_n, STx_n, Fx_{n+1}, Kt), N(ABx, STx_n, STx_{n+1}, Kt), N(ABx, Qx_n, STx_{n+1}, Kt), N(ABx, STx_n, Qx_{n+1}, Kt)\}$$

Letting $n \to \infty$ we get

$$M(Px, u, u, t) = 1, N(Px, u, u, t) = 0.$$

Notes

Hence,

$Px = u = ABx = Lx.$

If (P, AB) and (P, L) are weakly compatible , we have

$P(AB)x = (AB)Px$, so

$P\,Px= P(AB)x = (AB)Px = AB(AB)x,$

so we have Pu= ABu and PLx = LPx , Pu = u Hence Pu = ABu = Lu .

Step 3: As $p(X) \subset ST\,(X) \cup F(x)$,

there exists $v \in X$ such that Px = STv = Fv. We prove that STv= Qv

Case I: If STv ≠ Qv, Putting y = v, and z = v in (1.2.4) then we have

$$M(Px, Qv, Qv, t) \geq \phi\{M(ABx, STv, Lx, Kt), M(Lx, STv, STv, Kt), M(ABx, STv, Fv, Kt), M(ABx, Qv, Fv, Kt), M(ABx, Fv, Qv, Kt), M(STv, Qv, Fv, Kt), M(Fv, Qv, Qv, Kt), M(Lx, Qv, Fv, Kt), M(Qv, STv, Fv, Kt), M(ABx, STv, STv, Kt), M(ABx, Qv, STv, Kt), M(ABx, STv, Qv, Kt)\}$$

and

$$N(Px, Qv, Qv, t) \leq \phi'\{N(ABx, STv, Lx, Kt), N(Lx, STv, STv, Kt), N(ABx, STv, Fv, Kt), N(ABx, Qv, Fv, Kt), N(ABx, Fv, Qv, Kt), N(STv, Qv, Fv, Kt), N(Fv, Qv, Qv, Kt), N(Lx, Qv, Fv, Kt), N(Qv, STv, Fv, Kt), N(ABx, STv, STv, Kt), N(ABx, Qv, STv, Kt), N(ABx, STv, Qv, Kt)\}$$

Case II: If Qv ≠ u, then we have

$$M(Px, Qv, Qv, t) > M(Px, Qv, Qv, kt),$$

$$N(Px, Qv, Qv, t) < M(Px, Qv, Qv, kt),$$

which is a contradiction,

Thus STv = Qv = Px = Fv = u.

Step 4: If (Q, ST) and (Q, F) is weakly compatible mappings then we get

$Q\,(ST)v = (ST)Qv$ so, $(ST)\,(ST)v = (ST)\,Qv. = Q.\,Qv,$

so STu= Qu. and QFv = FQv

we prove Pu = u, for Qu =Fu

$M(Pu, u\,u, t) = M(Pu, Qv, Qv, t)$

$$\geq \phi\{M(ABu, STv, Lu, Kt), M(Lu, STv, STv, Kt), M(ABu, STv, Fv, Kt), M(ABu, Qv, Fv, Kt), M(ABu, Fv, Qv, Kt), M(STv, Qv, Fv, Kt), M(Fv, Qv, Qv\,Kt), M(Lu\,Qv, Fv, Kt), M(Qv, STv, Fv, Kt), M(ABu, STv, STv, Kt), M(ABu, Qv, STv, Kt), M(ABu, STv, Qv, Kt)\}$$

and

$N(Pu, u,u\,t) = N(Pu, Qv, Qv, t)$

$$\leq \phi'\{N(ABu, STv, Lu, Kt), N(Lu, STv, STv, Kt), N(ABu, STv, Fv, Kt), N(ABu, Qv, Fv, Kt), N(ABu, Fv, Qv, Kt), N(STv, Qv, Fv, Kt), N(Fv, Qv, Qv\,Kt), N(Lu\,Qv, Fv, Kt), N(Qv, STv,Fv, Kt), N(ABu, STv, STv, Kt), N(ABu, Qv, STv, Kt), N(ABu, STv, Qv, Kt)\}$$

Notes

Notes

Step 5: If $Pu \neq u$, then we have

$$M(Pu, u, u, t) > M(Pu, u, u, Kt)$$

$$N(Pu, u, u, t) < N(Pu, u, u, Kt),$$

which is contradiction. Thus

$$Pu = u = ABu = Lu. \quad (1)$$

Step 6: Now we prove $Qu = u$. For

$$M(u, Qu, Qu, t) = M(Pu, Qu, Qu, t)$$
$$\geq \phi\{M(ABu,STu,Lu, Kt), M(Lu, STu, STu, Kt), M(ABu, STu, Fu, Kt), M(ABu, Qu, Fu, Kt),$$
$$M(ABu,Fu, Qu, Kt), M(STu, Qu, Fu, Kt), M(Fu, Qu, Qu\ Kt), M(Lu\ Qu, Fu, Kt),$$
$$M(Qu, STu, Fu, Kt), M(ABu,STu,STu, Kt), M(ABu,Qu,STu, Kt), M(ABu,STu,Qu, Kt\}$$

and

$$N(u, Qu, Qu, t) = N(Pu, Qu, Qu, t)$$
$$\leq \phi'\{N(ABu,STu,Lu, Kt), N(Lu, STu, STu, Kt), N(ABu, STu, Fu, Kt), N(ABu, Qu, Fu, Kt),$$
$$N(ABu, Fu, Qu, Kt), N(STu, Qu, Fu, Kt), N(Fu, Qu, Qu\ Kt), N(Lu\ Qu, Fu, Kt), N(Qu, STu, Fu, Kt),$$
$$N(ABu,STu,STu,Kt), N(ABu,Qu,STu,Kt), N(ABu, STu, Qu, Kt)\}$$

Step 7: If $Qu \neq u$ then we have,

$$M(u, Qu, Qu, t) > M(u, Qu, Qu, Kt)$$

$$N(u, Qu, Qu, t) < N(u, Qu, Qu, Kt),$$

which is contradiction. Thus

$$Pu = Qu = ABu = STu = Fu=Lu = u. \quad (2)$$

Step 8: Now we show that $Bu = u$ by putting $x = Bu$, $y = x_{2n+1}$ and $z = x_{2n}$ in (1.2.4)

If $Bu \neq u$ then

$$M(P(Bu), Qx_{2n+1}, Qx_{2n}, t) \geq \phi\{M(AB(Bu), STx_{2n+1}, L(Bu), Kt), M(L(Bu), STx_{2n+1}, STx_{2n}, Kt), M(AB(Bu), STx_{2n+1}, Fx_{2n},$$
$$Kt), M(AB(Bu), Qx_{2n+1}, Fx_{2n}, Kt), M(AB(Bu), Fx_{2n+1}, Qx_{2n}, Kt), M(STx_{2n}, Qx_{2n}, Fx_{2n}, Kt),$$
$$M(Fx_{2n+1}, Qx_{2n+1}, Qx_{2n}, Kt), M(L(Bu), Qx_{2n+1}, Fx_{2n}, Kt), M(Qx_{2n+1}, STx_{2n+1}, Fx_{2n}, Kt),$$
$$M(AB(Bu), STx_{2n+1}, STx_{2n}, Kt), M(AB(Bu), Qx_{2n+1}, STx_{2n}, Kt),$$
$$M(AB(Bu), STx_{2n+1}, Qx_{2n}, Kt)\}$$

and

$$N(P(Bu), Qx_{2n+1}, Qx_{2n}, t) \leq \phi'\{N(AB(Bu), STx_{2n+1}, L(Bu), Kt), N(L(Bu), STx_{2n-1}, STx_{2n}, Kt), N(AB(Bu), STx_{2n+1}, Fx_{2n}, Kt),$$
$$N(AB(Bu), Qx_{2n+1}, Fx_{2n}, Kt), N(AB(Bu), Fx_{2n+1}, Qx_{2n}, Kt), N(STx_{2n}, Qx_{2n}, Fx_{2n},$$
$$Kt), N(Fx_{2n+1}, Qx_{2n+1}, Qx_{2n}, Kt), N(L(Bu), Qx_{2n+1}, Fx_{2n}, Kt), N(Qx_{2n+1}\ STx_{2n+1},$$
$$Fx_{2n}, Kt), N(AB(Bu), STx_{2n+1}, STx_{2n}, Kt), N(AB(Bu), Qx_{2n+1}, STx_{2n}, Kt),$$
$$N(AB(Bu), STx_{2n+1}, Qx_{2n}, Kt)\}$$

since AB=BA, BP=PB and LB=BL ,

we have

$$P(Bu) = B(Pu) = Bu,\ AB\,(Bu) = BA\,(Bu) = Bu$$

$$L(Bu) = B(Lu) = Bu.$$

Letting $n \to \infty$, we have

$$M\,(Bu, u, u, t) > M\,(Bu, u, u, kt)$$

$$N\,(Bu, u, u, t) < N\,(Bu, u, u, kt),$$

which is contradiction.

Thus $Bu = u$.

Since $u = ABu$,

we have $u = Au$,

therefore,

$$u = Au = Bu = Pu = Lu. \qquad \ldots\ldots(3)$$

Step 9: Finally we show that $Tu = u$. By putting $x = u$, $y = Tu$ and $z = u$ in (1.2.4) If $Tu \neq u$, then

$$\begin{aligned} M(Pu, Q(Tu), Qu, t) \geq \phi\{ & M(ABu, ST(Tu), Lu, Kt), M(Lu, ST(Tu), STu, Kt), \\ & M(ABu, ST(Tu), Fu, Kt), M(ABu, Q(Tu), Fu, Kt), \\ & M(ABu, F(Tu), Qu,Kt), M(STu, Qu, Fu, Kt), M(F(Tu), Q(Tu), Qu, Kt), \\ & M(Lu,Q(Tu), Fu, Kt), M(Q(Tu), ST(Tu), Fu, Kt), M(ABu, ST(Tu), STu, Kt), \\ & M(ABu, Q(Tu), STu, Kt), M(ABu, ST(Tu), Qu, Kt)\} \text{ and} \end{aligned}$$

$$\begin{aligned} N(Pu,Q(Tu), Qu, t) \leq \phi'\{ & N(ABu, ST(Tu), Lu, Kt), N(Lu, ST(Tu), STu, Kt), N(ABu, ST(Tu), Fu, Kt), \\ & N(ABu, Q(Tu), Fu, Kt), N(ABu, F(Tu) ,Qu, Kt), N(STu, Qu , Fu, Kt), \\ & N(F(Tu), Q(Tu), Qu, Kt), N(Lu, Q(Tu), Fu, Kt), N(Q(Tu), ST(Tu) \\ & Fu, Kt), N(ABu, ST(Tu), STu, Kt), N(ABu, Q(Tu), STu, Kt), N(ABu, \\ & ST(Tu), Qu, Kt)\} \end{aligned}$$

Since $S\,T = TS$, $TQ = QT$ and $FT = TF$,

We have,

$$ST(Tu) = T(STu) = Tu,$$

$$QTu = TQu = Tu \text{ and}$$

$$FTu = TFu = Tu. \text{ Then,}$$

$$M(u, Tu, u, t) > M(u, Tu, u, Kt)$$

$$N(u, Tu, u, t) < N(u, Tu, u, Kt),$$

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which is a contradiction,

thus Tu = u ,

since u = STu,

we have u = Su = Tu . .....(4)

By combining the above result (1), (2), (3) and (4) we get

Au = Bu = Su = Tu = Fu = Lu = Pu = Qu = u. So P, Q, A, B, L, F, S and T have a common fixed point u.

Now to prove the uniqueness: suppose that $v \neq u$ is another common fixed point of P, Q, A, B, L, F, S and T , then

$$\begin{aligned} M(v, u, u, T) &= M(Pv, Qu, Qu, T) \\ &\geq \phi\{M(ABv, STu, Lv, Kt), M(Lv, STu, STu, STu, Kt), M(ABv, STu, Fu, Kt), M(ABv, Qu, Fu, Kt), M(ABv, Fu, Qu, Kt), M(STu, Qu, Fu, Kt), M(Fu, Qu, Qu, Kt), M(Lv, Qu, Fu, Kt), M(Qu, STu, Fu, Kt), M(ABv, STu, STu, Kt), M(ABv, Qu, STu, Kt), M(ABv, STu, Qu, Kt)\} \\ &> M(v, u, u, Kt). \end{aligned}$$

Similarly

$$\begin{aligned} N(v, u, u, t) &= N(Pv, Qu, Qu, t) \\ &\leq \phi'\{N(ABv, STu, Lv, Kt), N(Lv, STu, STu, STu, Kt), N(ABv, STu, Fu, Kt), N(ABv, Qu, Fu, Kt), N(ABv, Fu, Qu, Kt), N(STu, Qu, Fu, Kt), N(Fu, Qu, Qu, Kt), N(Lv, Qu, Fu, Kt), N(Qu, STu, Fu, Kt), N(ABv, STu, STu, Kt), N(ABv, Qu, STu, Kt), N(ABv, STu, Qu, Kt)\} \\ &< N(v, u, u, Kt), \end{aligned}$$

which is a contradiction ,

therefore v = u is common fixed point of P, Q, A, B, L, F, S and T.

COROLLARY: Let f, g be self mappings of X satisfying the following conditions

(i) $f(X) \subset g(X)$ and g (X) is complete fuzzy metric subspace of X .

(ii) The pair (f, g) is weakly compatible and (f, g) satisfies the property (E).

(iii) If there exists a number K > 1 S.t.

$$M(fx, fy, fz, t) \geq \phi\{M(gx, gy, fx, Kt), M(fx, gy, gz, Kt)), M(gx, gy, gz, Kt), M(gx, fy, gz, Kt), M(gx, gy, fz, Kt), M(gz, fz, gz, Kt), M(gy, fy, fz, Kt), M(fy, fx, gz, Kt), M(fy, gy, gz, Kt), M(gx, gy, gz, Kt), M(gx, fy, gz, Kt), M(gx, gy, fz, Kt)\}$$ and

$$N(fx, fy, fz, t) \leq \phi'\{N(gx, gy, fx, Kt), N(fx, gy, gz, Kt)), N(gx, gy, gz, Kt), N(gx, fy, gz, Kt), N(gx, gy, fz, Kt), N(gz, fz, gz, Kt), N(gy, fy, fz, Kt), N(fy, fx, gz, Kt), N(fy, gy, gz, Kt), N(gx, gy, gz, Kt), N(gx, fy, gz, Kt), N(gx, gy, fz, Kt)\}$$

for all $x, y, z \in X$ and $t > 0$ then f, g, have a unique common fixed point in X .

Example 3 : - Let X = [0,1] with the usual generalized metric D .

Define,

$$M(x, y, z, t) = \frac{t}{t+|x-y|+|y-z|+|z-x|},$$

$$N(x, y, z, t) = \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|}, \text{ for all } x, y, z \in X \text{ and } t > 0$$

$M(x, y, z, 0) = 0$, $N(x, y, z, 0) = 1$ for all $x, y, z \in X$ clearly (X, M, N, $*$, $\Diamond$) is a complete intuitionistic M–Fuzzy metric space where $*$ and $\Diamond$ are defined b $a * b = \min(a, b)$ and $a \Diamond b = \max(a, b)$. LetA, B, S, T, P, Q, L and F be defined as

$$Sx = x,\ \ Tx = \frac{x}{2},\ \ Ax = \frac{x}{5},\ \ Bx\frac{x}{3},\ \ Px = \frac{x}{6},\ \ Qx = 0,\ \ Fx = \frac{x}{4} \text{ and } Lx = \frac{x}{7} \text{ for all } x, y, z \in X.$$

Then

$$P(x) = \left[0, \frac{1}{6}\right] \subset \left[0, \frac{1}{2}\right] \cup \left[0, \frac{1}{4}\right] = ST(X) \cup F(X) \quad \text{and}$$

$$Q(X) = \{0\} \subset \left[0, \frac{1}{15}\right] \cup \left[0, \frac{1}{7}\right] = ABx(X) \cup L(X)$$

Clearly

AB = BA, ST = TS, PB = BP, TQ = QT, FT = TF, LB = BL.

Moreover, the pairs (P, AB), (Q, ST), (P, L) and (Q, F) are weakly compatible at 0 and the pair (Q, ST) and (Q, F) satisfies the property (E) if $\lim_{n \to \infty} x_n = 0$, where $\{ x_n \}$ is a sequence in X s.t.

$$\lim_{n \to \infty} M(Qx_n, u, u, t) = \lim_{n \ \infty} M(STx_n, u, u, t) = 1,$$

$$\lim_{n \to \infty} N(Qx_n, u, u, t) = \lim_{n \to \infty} N(STx_n, u, u, t) = 0 \text{ and}$$

$$\lim_{n \to \infty} M(Qx_n, u, u, t) = \lim_{n \to \infty} M(Fx_n, u, u, t) = 1, \text{ and}$$

$$\lim_{n \to \infty} N(Qx_n, u, u, t) = \lim_{n \to \infty} N(Fx_n, u, u, t) = 0$$

For $u = 0 \in X$ and $t > 0$ If we take $K = 2$ and $t = 1$, then conditions (1.2.4) of the main theorem is satisfied and 0 is the unique common fixed point of P, Q, A, S, L, F, S and T.

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Dual to Ratio Estimators of Population Mean in Post-Stratified Sampling using Known Value of Some Population Parameters

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Abstract - This paper extends the work carried out by Onyeka (2012), by proposing a class of dual to ratio combined estimators of the population mean in post-stratified sampling when using known value of some population parameters. The proposed estimators, under certain conditions, are shown to be more efficient than some existing estimators, including the usual poststratified estimator and the estimators proposed by Onyeka (2012). Properties of the proposed class of estimators, including conditions for optimal efficiency, are obtained up to first order approximation. The results are illustrated using empirical data.

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Keywords : *auxiliary information, general family of estimators, post-stratified sampling, mean squared errors.*

I. Introduction

Many authors have considered the use of some known population parameters of an auxiliary character in formulating estimators of population parameters of a variable of interest. A lot of theoretical and empirical studies have been carried out along this line. Some known population parameters of an auxiliary character, which have been considered for the purpose of constructing estimators for some population parameters of the study variate include coefficient of variation, (CV), used by Searls (1964) and Sisodia-Dwivedi (1981); coefficient of kurtosis, used by Singh et al. (1973) and Upadhyaya-Singh (1999); coefficient of skewness, used by G.N. Singh (2003); standard deviation, used by G.N. Singh (2003); and correlation coefficient, used by Singh and Tailor (2003). A general family of estimators of $\overline{Y}$ under the SRSWOR scheme was discussed by Khoshnevisan et.al. (2007), using known parameters of the auxiliary variable x, such as standard deviation, coefficient of variation, coefficient of skewness, kurtosis and correlation coefficient. Koyuncu and Kadilar (2009) also proposed a general family of combined estimators of $\overline{Y}$ in stratified random sampling. Onyeka (2012), motivated by the works carried out by Khoshnevisan et.al. (2007) and Koyuncu and Kadilar (2009), developed a general family of estimators of $\overline{Y}$ under the post-stratified sampling scheme using known values of some population parameters of an auxiliary character. The family of estimators discussed by Onyeka (2012), was found, under some optimum conditions, to be as efficient as the post-stratified regression estimator $\overline{y}_{psREG}$, but more efficient, in terms of having a smaller mean squared error, than the usual poststratified sampling estimator, $\overline{y}_{ps}$, and other particular cases of the proposed estimators. The present study is aimed at utilizing some variable transformation of an auxiliary character x, to extend the work carried out by Onyeka (2012) in poststratified sampling scheme. Srivenkataramana

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(1980) used the transformation, $x_i^* = \dfrac{N\bar{X} - nx_i}{N-n}$, $i = 1, 2, \cdots, N$, to obtain a dual to ratio estimate of $\bar{Y}$ in simple random sampling scheme. Authors, like Singh and Tailor (2005), Tailor and Sharma (2009), and Sharma and Tailor (2010) have used the same transformation to improve estimates under the simple random sampling scheme. Motivated by these studies, we intend, in the present work, to use the same transformation to extend the work carried out by Onyeka (2012) in poststratified sampling scheme.

Let y_{hi} (x_{hi}) denote the i^{th} observation in stratum h for the study (auxiliary) variate in post-stratified sampling scheme. Let a random sample of size n be drawn from a population of N units using SRSWOR method, and let the sampled units be allocated to their respective strata, where n_h (a random variable) is the number of units that fall into stratum h such that $\sum_{h=1}^{L} n_h = n$. It is assumed that n is large enough such that $P(n_h = 0) = 0, \forall\ h$. Onyeka (2012) proposed the following general family of combined estimators of the population mean $\bar{Y}$ in post-stratified sampling scheme:

$$\bar{y}_{pss} = \bar{y}_{ps}\left(\frac{a\bar{X} + b}{\alpha(a\bar{x}_{ps} + b) + (1-\alpha)(a\bar{X} + b)}\right)^g \qquad (1.1)$$

where,

$\bar{y}_{ps} = \sum_{h=1}^{L} \omega_h \bar{y}_h$ is the usual post-stratified estimator of $\bar{Y}$

$\bar{x}_{ps} = \sum_{h=1}^{L} \omega_h \bar{x}_h$ is the usual post-stratified estimator of $\bar{X}$

$\bar{X} = \sum_{h=1}^{L} \omega_h \bar{X}_h$ is the known population mean of the auxiliary variate x.

$a(\neq 0), b$ are either constants or functions of known population parameters of the auxiliary variate, such as standard deviation (σ_x), coefficient of variation (C_x), skewness $(\beta_1(x))$, kurtosis $(\beta_2(x))$, and correlation coefficient (ρ_{yx}).

$\omega_h = N_h / N$ is stratum weight, L is the number of strata in the population, N_h is the number of units in stratum h, N is the number of units in the population, $\bar{X}_h$ is the population mean of the auxiliary variate in stratum h, and $\bar{y}_h$ $(\bar{x}_h)$ is the sample mean of the study (auxiliary) variate in stratum h.

Under the unconditional argument, that is, for repeated samples of fixed size n, the variances and covariance of the estimators, $\bar{y}_{ps}$ and $\bar{x}_{ps}$, obtained up to first order approximation are:

$$V(\bar{y}_{ps}) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L} \omega_h S_{yh}^2, \qquad (1.2)$$

$$V(\bar{x}_{ps}) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L} \omega_h S_{xh}^2, \qquad (1.3)$$

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$$Cov(\bar{y}_{ps}, \bar{x}_{ps}) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{yxh}. \quad (1.4)$$

where $f = n/N$ is the population sampling fraction, $S_{yh}^2 (S_{xh}^2)$ is the population variance of y(x) in stratum h, and S_{yxh} is the population covariance of y and x in stratum h. Let

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$$e_0 = \frac{\bar{y}_{ps} - \bar{Y}}{\bar{Y}} \text{ and } e_1 = \frac{\bar{x}_{ps} - \bar{X}}{\bar{X}}. \quad (1.5)$$

Under the unconditional argument, it follows that

$$E(e_0) = E(e_1) = 0 \quad (1.6)$$

$$E(e_0^2) = \frac{V(\bar{y}_{ps})}{\bar{Y}^2} = \frac{1}{\bar{Y}^2}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{yh}^2 \quad (1.7)$$

$$E(e_1^2) = \frac{V(\bar{x}_{ps})}{\bar{X}^2} = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{xh}^2 \quad (1.8)$$

and

$$E(e_0 e_1) = \frac{Cov(\bar{y}_{ps}, \bar{x}_{ps})}{\bar{Y}\bar{X}} = \frac{1}{\bar{Y}\bar{X}}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{yxh} \quad (1.9)$$

Accordingly, Onyeka (2012) obtained the unconditional bias and mean squared error of $\bar{y}_{pss}$, up to first order approximation, respectively as

$$B(\bar{y}_{pss}) = \frac{\alpha\lambda g}{2\bar{X}}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h(\alpha\lambda(g+1)RS_{xh}^2 - 2S_{yxh}) \quad (1.10)$$

and

$$MSE(\bar{y}_{pss}) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h(S_{yh}^2 + \alpha^2\lambda^2 g^2 R^2 S_{xh}^2 - 2\alpha\lambda g R S_{yxh}) \quad (1.11)$$

where $\lambda = \frac{a\bar{X}}{a\bar{X}+b}$ and $R = \frac{\bar{Y}}{\bar{X}}$. The (optimum) choice of α that minimizes (1.11) is $\alpha_{opt} = \frac{\beta_0}{\lambda g R}$, and the resulting optimum unconditional mean squared error of $\bar{y}_{pss}$ is obtained as

$$MSE_{opt}(\bar{y}_{pss}) = \left(\frac{1-f}{n}\right)(1-\rho_0^2)\sum_{h=1}^{L}\omega_h S_{yh}^2 \quad (1.12)$$

where

$$\beta_0 = \frac{\sum_{h=1}^{L} \omega_h S_{yxh}}{\sum_{h=1}^{L} \omega_h S_{xh}^2}, \text{ and } \rho_0 = \frac{\sum_{h=1}^{L} \omega_h S_{yxh}}{\sqrt{\left(\sum_{h=1}^{L} \omega_h S_{yh}^2\right)\left(\sum_{h=1}^{L} \omega_h S_{xh}^2\right)}} \tag{1.13}$$

Notice that (1.12) is the same as the unconditional variance of the usual combined post-stratified regression estimator, $\bar{y}_{psREG} = \bar{y}_{ps} - \hat{\beta}_0(\bar{x}_{ps} - \bar{X})$. This implies that the efficiency of the general family of estimators, $\bar{y}_{pss}$, proposed by Onyeka (2012), may not be improved beyond the efficiency of the customary combined regression-type estimator in post-stratified sampling.

II. The Proposed Class of Estimators

Motivated by Onyeka (2012) and Srivenkataramana (1980), we propose a class of dual to ratio estimators of the population mean, $\bar{Y}$, in poststratified sampling, using known population parameters of an auxiliary character x, as:

$$\bar{y}_{pss}^{*} = \bar{y}_{ps}\left(\frac{\alpha(a\bar{x}_{ps}^{*} + b) + (1-\alpha)(a\bar{X} + b)}{a\bar{X} + b}\right)^{g} \tag{2.1}$$

where $\bar{x}_{ps}^{*}$ is a transformed sample mean of the auxiliary variable, x, based on the variable transformation, $x_{hi}^{*} = \dfrac{N\bar{X} - nx_{hi}}{N-n}$ and satisfying the relationship:

$$\bar{X} = f\bar{x}_{ps} + (1-f)\bar{x}_{ps}^{*} \tag{2.2}$$

The transformed sample mean, $\bar{x}_{ps}^{*}$, in poststratified sampling, is defined along the line of authors like Srivenkataramana and Srinath (1976), Srivenkataramana (1980), and Sharma and Tailor (2010). Using the transformation, $x_i^{*} = \dfrac{N\bar{X} - nx_i}{N-n}$, i = 1, 2, …, N, Srivenkataramana (1980) obtained a dual to ratio estimate of $\bar{Y}$ in simple random sampling scheme as

$$\bar{y}_R^{(d)} = \bar{y}\left(\frac{\bar{x}^{*}}{\bar{X}}\right) \tag{2.3}$$

This means that the proposed estimator in (2.1) is a type of dual to ratio estimator in poststratified sampling when using information on known parameters of an auxiliary character, x, provided the constant g is positive. The proposed estimator in (2.1) becomes a type of dual to product estimator if the constant g is negative. Notice that the transformed sample mean, $\bar{x}_{ps}^{*}$, in (2.2) can be written in terms of e_1 as

$$\bar{x}_{ps}^{*} = \bar{X}(1 - \pi e_1) \tag{2.4}$$

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where $\pi = \frac{f}{1-f} = \frac{n}{N-n}$. Consequently, the proposed class of estimators, $\bar{y}^*_{pss}$ in (2.1), can be rewritten in terms of e_0 and e_1 as

$$\bar{y}^*_{pss} = \bar{Y}(1+e_0)(1-\pi\alpha\lambda e_1)^g \quad (2.5)$$

Assuming $|\pi\alpha\lambda e_1| < 1$, so that the series $(1-\pi\alpha\lambda e_1)^g$ converges, and expanding (2.5) up to first order approximation in expected value, we obtain

$$(\bar{y}^*_{pss} - \bar{Y}) = \bar{Y}(e_0 - \pi\alpha\lambda g e_1 - \pi\alpha\lambda g e_0 e_1 + \tfrac{1}{2} g(g+1)\pi^2\alpha^2\lambda^2 e_1^2) \quad (2.6)$$

and

$$(\bar{y}^*_{pss} - \bar{Y})^2 = \bar{Y}^2(e_0^2 + \pi^2\alpha^2\lambda^2 g^2 e_1^2 - 2\pi\alpha\lambda g e_0 e_1) \quad (2.7)$$

To obtain the unconditional bias and mean squared error of the proposed estimators $\bar{y}^*_{pss}$ we take the unconditional expectations of (2.6) and (2.7), and use (1.6) – (1.9) to make the necessary substitutions. This gives the unconditional bias and mean squared error of the proposed class of estimators, $\bar{y}^*_{pss}$, up to first order approximation, respectively as

$$B(\bar{y}^*_{pss}) = \left(\frac{1-f}{n}\right)\left(\frac{\pi\alpha\lambda g}{2\bar{X}}\right)\sum_{h=1}^{L}\omega_h(\pi\alpha\lambda(g+1)RS_{xh}^2 - 2S_{yxh}) \quad (2.8)$$

and

$$MSE(\bar{y}^*_{pss}) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h(S_{yh}^2 + \pi^2\alpha^2\lambda^2 g^2 R^2 S_{xh}^2 - 2\pi\alpha\lambda g R S_{yxh}) \quad (2.9)$$

Applying the least squares method, the (optimum) choice of α that minimizes (2.9), is obtained as

$$\alpha_{opt} = \frac{\beta_0}{\pi\lambda g R} \quad (2.10)$$

and the resulting optimum unconditional mean squared error of $\bar{y}^*_{pss}$ is obtained as

$$MSE_{opt}(\bar{y}^*_{pss}) = \left(\frac{1-f}{n}\right)(1-\rho_0^2)\sum_{h=1}^{L}\omega_h S_{yh}^2 \quad (2.11)$$

We observe that the optimum mean square error of $\bar{y}^*_{pss}$, given in (2.11), is the same as the unconditional variance of the usual post-stratified regression estimator, $\bar{y}_{psREG} = \bar{y}_{ps} - \hat{\beta}_0(\bar{x}_{ps} - \bar{X})$, indicating that the efficiency of the proposed class of estimators, $\bar{y}^*_{pss}$, just like the estimators, $\bar{y}_{pss}$, proposed by Onyeka (2012), may not be improved beyond the efficiency of the customary regression-type estimator in post-stratified sampling.

III. Efficiency Comparisons

Here, we shall compare the efficiency of the proposed class of dual to ratio estimators, $\bar{y}^{*}_{pss}$, with those of some existing estimators of $\bar{Y}$, including the usual poststratified sampling estimator, $\bar{y}_{ps}$, and the estimator, $\bar{y}_{pss}$, proposed by Onyeka (2012).

a) *Efficiency Comparison of* $\bar{y}^{*}_{pss}$ *and* $\bar{y}_{ps}$

To compare the efficiencies of the proposed dual to ratio estimator, $\bar{y}^{*}_{pss}$, and the usual poststratified sampling estimator, $\bar{y}_{ps}$, we let $A_0 = \sqrt{\sum_{h=1}^{L}\omega_h S^2_{yh}}$ and $A_1 = \sqrt{\sum_{h=1}^{L}\omega_h S^2_{xh}}$. Then, we can rewrite (1.2) and (2.9), respectively as:

$$V(\bar{y}_{ps}) = \left(\frac{1-f}{n}\right)A_0^2 \tag{3.1}$$

and

$$MSE(\bar{y}^{*}_{pss}) = \left(\frac{1-f}{n}\right)\left(A_0^2 + \pi^2\alpha^2\lambda^2 g^2 R^2 A_1^2 - 2\pi\alpha\lambda g R\rho_0 A_0 A_1\right) \tag{3.2}$$

so that

$$V(\bar{y}_{ps}) - MSE(\bar{y}^{*}_{pss}) = \left(\frac{1-f}{n}\right)\left(2\pi\alpha\lambda g R\rho_0 A_0 A_1 - \pi^2\alpha^2\lambda^2 g^2 R^2 A_1^2\right) \tag{3.3}$$

This shows that the proposed class of estimators, $\bar{y}^{*}_{pss}$ is more efficient than the estimator, $\bar{y}_{ps}$, in terms of having a smaller mean squared error, if

$$\frac{\beta_0}{\pi\alpha\lambda g R} > \frac{1}{2} \tag{3.4}$$

provided $a \neq 0$, $\alpha \neq 0$ and $g \neq 0$. Note that if $a = 0$, $\alpha = 0$ and $g = 0$ separately, the proposed estimator, $\bar{y}^{*}_{pss}$ in (2.1) reduces to the usual poststratified estimator, $\bar{y}_{ps}$.

b) *Efficiency Comparison of* $\bar{y}^{*}_{pss}$ *and* $\bar{y}^{(R)}_{ps}$

Here, we compare the efficiencies of the proposed estimator, $\bar{y}^{*}_{pss}$ and the ratio-type combined estimator in poststratified sampling, given by

$$\bar{y}^{(R)}_{ps} = \frac{\bar{y}_{ps}}{\bar{x}_{ps}}\bar{X} \tag{3.5}$$

with mean squared error, approximated up to first order, as

$$MSE(\bar{y}^{(R)}_{ps}) = \left(\frac{1-f}{n}\right)\left(A_0^2 + R^2 A_1^2 - 2R\rho_0 A_0 A_1\right) \tag{3.6}$$

Notes

Using (3.2) and (3.6), it can be shown that the proposed class of estimators, $\bar{y}^*_{pss}$ is more efficient than the ratio-type estimator, $\bar{y}^{(R)}_{ps}$, in terms of having a smaller mean squared error, if

$$\frac{\beta_0(1-\pi\alpha\lambda g)}{R} < \frac{1}{2} \tag{3.7}$$

Notes

c) Efficiency Comparison of $\bar{y}^*_{pss}$ *and* $\bar{y}^{(P)}_{ps}$

Here, we compare the efficiencies of the proposed estimator, $\bar{y}^*_{pss}$ and the product-type combined estimator in poststratified sampling, given by

$$\bar{y}^{(P)}_{ps} = \frac{\bar{y}_{ps}\bar{x}_{ps}}{\bar{X}} \tag{3.8}$$

with mean squared error, approximated up to first order, as

$$MSE(\bar{y}^{(P)}_{ps}) = \left(\frac{1-f}{n}\right)\left(A_0^2 + R^2A_1^2 + 2R\rho_0A_0A_1\right) \tag{3.9}$$

Using (3.2) and (3.9), it can be shown that the proposed class of estimators, $\bar{y}^*_{pss}$ is more efficient than the product-type estimator, $\bar{y}^{(P)}_{ps}$, in terms of having a smaller mean squared error, if

$$\frac{\beta_0(1+\pi\alpha\lambda g)}{R} > -\frac{1}{2} \tag{3.10}$$

Note that the ratio-type and product-type estimators, $\bar{y}^{(R)}_{ps}$ and $\bar{y}^{(P)}_{ps}$, are both members of the family of combined-type estimators, $\bar{y}_{pss}$, proposed by Onyeka (2012).

d) Efficiency Comparison of $\bar{y}^*_{pss}$ *and* $\bar{y}_{pss}$

Here, we compare the efficiencies of the proposed estimator, $\bar{y}^*_{pss}$ and the estimator, $\bar{y}_{pss}$, proposed by Onyeka (2012), whose mean squared error can be rewritten from (1.11) as:

$$MSE(\bar{y}_{pss}) = \left(\frac{1-f}{n}\right)\left(A_0^2 + \alpha^2\lambda^2g^2R^2A_1^2 - 2\alpha\lambda gR\rho_0A_0A_1\right) \tag{3.11}$$

Using (3.2) and (3.11), it can be shown that the proposed class of estimators, $\bar{y}^*_{pss}$ is more efficient than the estimator, $\bar{y}_{pss}$, in terms of having a smaller mean squared error, if

$$\frac{\beta_0(1-\pi)}{\alpha\lambda gR} < \frac{1}{2} \tag{3.12}$$

provided $a \neq 0$, $\alpha \neq 0$ and $g \neq 0$, as expected. However, it is worthy of note that the estimators, $\bar{y}^*_{pss}$ and $\bar{y}_{pss}$ have equal efficiency under certain optimality conditions, namely, if we choose $\alpha_{opt} = \dfrac{\beta_0}{\lambda g R}$ for $\bar{y}_{pss}$ and $\alpha_{opt} = \dfrac{\beta_0}{\pi \lambda g R}$ for $\bar{y}^*_{pss}$. Under these conditions, both estimators have the same optimum mean squared error, (1.12) and (2.11), which is easily recognized as the variance of the usual poststratified regression-type estimator, $\bar{y}_{psREG}$.

IV. Empirical Illustration

100 Notes

Here, we use the data given in Onyeka (2012) to illustrate the properties of the estimators proposed in the present study. The data statistics, consisting mainly of population parameters, are shown in Table 1, while Table 2 shows the percentage relative efficiencies (PRE) of the proposed class of estimators, $\bar{y}^*_{pss}$ and the estimator, $\bar{y}_{pss}$, proposed by Onyeka (2012), over the usual poststratified estimator $\bar{y}_{ps}$ of $\bar{Y}$ in poststratified sampling scheme. We shall consider special cases of the proposed estimator, $\bar{y}^*_{pss}$, corresponding to the same special cases of $\bar{y}_{pss}$ discussed in Onyeka (2012).

Table 1 : Data Statistics

POPULATION	M ALES = STRATUM 1	FEMALES = STRATUM 2
$N = 96$	$N_1 = 72$	$N_2 = 24$
$n = 20$	$n_1 = 8$	$n_2 = 12$
$\bar{X} = 68.13$	$\bar{X}_1 = 68.11$	$\bar{X}_2 = 68.17$
$\bar{Y} = 2.44$	$\bar{Y}_1 = 2.44$	$\bar{Y}_2 = 2.46$
$S_x = 7.03$	$S_{x1} = 7.28$	$S_{x2} = 6.36$
$S^2_x = 49.37$	$S^2_{x1} = 52.97$	$S^2_{x2} = 40.41$
$S_y = 0.57$	$S_{y1} = 0.60$	$S_{y2} = 0.50$
$S^2_y = 0.33$	$S^2_{y1} = 0.35$	$S^2_{y2} = 0.25$
$S_{yx} = 3.26$	$S_{yx1} = 3.43$	$S_{yx2} = 2.75$
$\rho_{yx} = 0.82$	$\rho_{yx1} = 0.80$	$\rho_{yx2} = 0.90$
$\rho^2_{yx} = 0.67$	$^2_{yx1} = 0.64$	$\rho^2_{yx2} = 0.80$
$C_x = 0.10$	$C_{x1} = 0.11$	$C_{x2} = 0.09$
$C_y = 0.23$	$C_{y1} = 0.24$	$C_{y2} = 0.20$
$\beta_1(x) = -1.10$	$\beta_{11}(x) = -1.23$	$\beta_{12}(x) = 0.50$
$\beta_1(y) = -0.11$	$\beta_{11}(y) = -0.14$	$\beta_{12}(y) = 0.14$
$\beta_2(x) = 3.83$	$\beta_{21}(x) = 4.33$	$\beta_{22}(x) = 1.34$
$\beta_2(y) = 1.27$	$\beta_{21}(y) = 1.40$	$\beta_{22}(y) = 0.31$
$= 0.04$	$\gamma_1 = 0.05$	$\gamma_2 = 0.16$
– –	$\omega_1 = 0.75$	$\omega_2 = 0.25$
– –	$\omega^2_1 = 0.56$	$\omega^2_2 = 0.06$

Notes

Table 2 : PRE of $\bar{y}^*_{pss}$ and $\bar{y}_{pss}$ over $\bar{y}_{ps}$

ESTIMATORS	Constants & Parameters				$\bar{y}_{pss}$		$\bar{y}^*_{pss}$	
	α	g	a	b	MSE	PRE	MSE	PRE
1. Usual poststratified estimator, $\bar{y}_{ps}$	–	–	–	–	0.012864	100	0.012864	100
2. Ratio-type estimator,	1	1	1	0	0.006151	209.14	0.011308	113.76
3. Sisodia-Dwivedi (1981) estimator,	1	1	1	C_x	0.006158	208.90	0.011309	113.75
4. Singh-Kakran (1993) estimator (1),	1	1	1	$\beta_2(x)$	0.006381	201.60	0.011347	113.37
5. Upadhyaya-Singh (1999) estimator (1),	1	1	$\beta_2(x)$	C_x	0.006153	209.07	0.011308	113.76
6. Upadhyaya-Singh (1999) estimator (2),	1	1	C_x	$\beta_2(x)$	0.007984	161.12	0.011666	110.27
7. Singh-Tailor (2003) estimator (1),	1	1	1	ρ_{yx}	0.006202	207.42	0.011316	113.68
8. Product-type estimator,	1	–1	1	0	0.024637	52.21	0.016173	79.54
9. Pandey-Dubey (1988) estimator,	1	–1	1	C_x	0.024616	52.26	0.016167	79.57
10. Upadhyaya-Singh (1999) estimator (3),	1	–1	$\beta_2(x)$	C_x	0.024632	52.22	0.016171	79.55
11. Upadhyaya-Singh (1999) estimator (4),	1	–1	C_x	$\beta_2(x)$	0.019818	64.91	0.014781	87.03
12. G.N. Singh (2003) estimator (1),	1	–1	1	σ_x	0.023322	55.16	0.015789	81.47
13. G.N. Singh (2003) estimator (2),	1	–1	$\beta_1(x)$	σ_x	0.026145	49.20	0.016616	77.42
14. G.N. Singh (2003) estimator (3),	1	–1	$\beta_2(x)$	σ_x	0.024264	53.02	0.016064	80.08
15. Singh-Tailor (2003) estimator (2),	1	–1	1	ρ_{yx}	0.024468	52.57	0.016123	79.79
16. Singh-Kakran (1993) estimator (2),	1	–1	1	$\beta_2(x)$	0.023883	53.86	0.015953	80.64
17. Regression-type (Optimum) estimators					**0.004422**	**290.91**	**0.004422**	**290.91**

Table 2 shows that the estimators in the proposed class of estimators, $\bar{y}^*_{pss}$ are not always more efficient than the usual poststratified estimator $\bar{y}_{ps}$. The proposed class of estimators, $\bar{y}^*_{pss}$ is more efficient than the usual poststratified estimator $\bar{y}_{ps}$ only if the efficiency condition (3.4) is satisfied. The table also shows that the proposed dual to ratio-type estimator, $\bar{y}_{pss}^{(R*)} = \bar{y}_{ps}\left(\frac{\bar{x}_{ps}^{*}}{\bar{X}}\right)$ with PRE of 113.76%, is more efficient than the usual poststratified estimator $\bar{y}_{ps}$, while the proposed dual to product-type estimator, $\bar{y}_{pss}^{(P*)} = \bar{y}_{ps}\left(\frac{\bar{X}}{\bar{x}_{ps}^{*}}\right)$ with PRE of 79.54%, is less efficient than the usual poststratified estimator $\bar{y}_{ps}$. In fact, table 2 reveals that all the dual to ratio-type estimators (for all g > 0) perform better than the usual poststratified estimator $\bar{y}_{ps}$, while the dual to product-type estimators (for all g < 0) are less efficient than the usual poststratified estimator $\bar{y}_{ps}$. Onyeka (2012) noted that this is expected since the given data set shows a strong positive correlation $(\rho_{yx} = 0.82, \text{Table 1})$, between the study and auxiliary variables. The dual to product-type estimators are expected to perform better than $\bar{y}_{ps}$ and the dual to ratio-type estimators when there is a strong negative correlation between the study and auxiliary variables. Using table 2 to further compare the general performance of the proposed class of estimators, $\bar{y}_{pss}^{*}$ and the estimator, $\bar{y}_{pss}$ proposed by Onyeka (2012), we observed that for dual to ratio-type estimators, the estimator $\bar{y}_{pss}$ performs better than the estimator $\bar{y}_{pss}^{*}$, while for dual to product-type estimators, the estimator $\bar{y}_{pss}^{*}$ performs better than the estimator $\bar{y}_{pss}$, in terms of having a smaller mean squared error. This is equally in line with the efficiency condition in (3.12). With the understanding that product-type estimators perform well when there is a strong negative correlation between the study and auxiliary variates, it therefore follows that the proposed estimator $\bar{y}_{pss}^{*}$ should be preferred to the estimator $\bar{y}_{pss}$, proposed by Onyeka (2012), when there is highly negative correlation between the study and auxiliary characters and we are using the dual to product-type estimators (instead of dual to ratio-type estimators) within the proposed class of combined estimators, $\bar{y}_{pss}^{*}$.

V. Concluding Remark

We have extended the work carried out by Onyeka (2012) by considering a general family of dual to ratio-type (and/or dual to product-type) combined estimators of $\bar{Y}$, in poststratified sampling (PSS) scheme, using information on some known parameters of an auxiliary character. The proposed class of estimators is found, under some optimum conditions, to be as efficient as the poststratified regression estimator $\bar{y}_{psREG}$. We also obtained conditions under which the proposed estimator performs better (in terms of having a smaller mean squared error) than the usual poststratified estimator and the estimator proposed by Onyeka (2012). Properties of the proposed general family of estimators are obtained up to first order approximation and supported with some empirical illustration.

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Development of a Summation Formula in Connection with Hypergeometric and Gamma Function

By Salahuddin, R. K. Khola & S. R. Yadav
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Abstract - The aim of this paper is to derive a summation formula based on half argument in connection with Hypergeometric function and involving recurrence relation and Gauss summation theorem.

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GJSFR-F Classification : MSC 2010: 33C05, 33C20, 33C45, 33C70

DEVELOPMENT OF A SUMMATION FORMULA IN CONNECTION WITH HYPERGEOMETRIC AND GAMMA FUNCTION

Strictly as per the compliance and regulations of :

Notes

Development of a Summation Formula in Connection with Hypergeometric and Gamma Function

Salahuddin [α], R. K. Khola [σ] & S. R. Yadav [ρ]

Abstract - The aim of this paper is to derive a summation formula based on half argument in connection with Hypergeometric function and involving recurrence relation and Gauss summation theorem.
Keywords and Phrases : contiguous relation, gauss second summation theorem, recurrence relation.

I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$${}_AF_B\left[\begin{array}{lll} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!}$$

or

$${}_AF_B\left[\begin{array}{lll} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{lll} (a_j)_{j=1}^A & ; & \\ & & z \\ (b_j)_{j=1}^B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1}$$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers and $\mid z \mid = 1$.

Contiguous Relation is defined by

[Andrews p.363(9.16)]

$$(a-b)\ {}_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ c\ \ ; & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ll} a+1,\ b\ ; & \\ c\ \ ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ll} a,\ b+1\ ; & \\ c\ ; & z \end{array}\right] \tag{2}$$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.8)]

$${}_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ \frac{a+b+1}{2}\ ; & \frac{1}{2} \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3}$$

$$= \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4}$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov.,p.491(7.3.7.8)]

$${}_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ \frac{a+b-1}{2}\ ; & \frac{1}{2} \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{5}$$

Now using Legendre's duplication formula and Recurrence relation for Gamma function,

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the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a, \; b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Recurrence relation is defined by

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{7} $$

Notes

II. Main Formula

$$ {}_2F_1\left[\begin{matrix} a, \; b & ; & \frac{1}{2} \\ \frac{a+b+37}{2} & ; & \end{matrix}\right] = \frac{2^b\,\Gamma(\frac{a+b+37}{2})}{(a-b)\,\Gamma(b)} \times $$

$$ \times\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{131072a(-6332659870762850625 + 15188465029114325025a)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + \right.\right. $$

$$ + \frac{131072a(-14354510691610713240a^2 + 7524314127912551832a^3 - 2523698606200763196a^4)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

$$ + \frac{131072a(585146416702456764a^5 - 98283050207112680a^6 + 12319487399406824a^7)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

$$ + \frac{131072a(-1174199725349222a^8 + 86014818744998a^9 - 4862169489320a^{10} + 211577650856a^{11})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

$$ + \frac{131072a(-7020044668a^{12} + 174281212a^{13} - 3132760a^{14} + 38488a^{15} - 289a^{16} + a^{17})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

$$ + \frac{131072a(25321878164717979075b - 19523841512219551440ab + 4761199831691493007 2a^2b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

$$ + \frac{131072a(-12330825664600006416a^3b + 7687192319327829444a^4b - 1038346142047282320a^5b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

$$ + \frac{131072a(283129024934512456a^6b - 22414624986818768a^7b + 3231412550832642a^8b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

$$ + \frac{131072a(-155206622884720a^9b + 12794409439592a^{10}b - 366157152816a^{11}b + 17543988644a^{12}b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

$$ + \frac{131072a(-274185520a^{13}b + 7297080a^{14}b - 47600a^{15}b + 595a^{16}b + 2162023563730570920b^2)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]} + $$

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$$+\frac{131072a(6454317274328070036 0ab^2-1110717619199679492 0a^2b^2+2663883856003821756 0a^3b^2)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(-2867948454968860760a^4b^2+1845548308154811400a^5b^2-124702534849141480a^6b^2)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(35260676281141080a^7b^2-1500336516820680a^8b^2+222764240366360a^9b^2)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(-5784150923320a^{10}b^2+484991616200a^{11}b^2-6995348360a^{12}b^2+334423320a^{13}b^2)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(-2042040a^{14}b^2+52360a^{15}b^2+204377243290661301 84b^3+257551524003751588 8ab^3)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(3336387249195486208 8a^2b^3-2090930383100586720a^3b^3+4873159786850521320a^4b^3)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(-2581511566193375 20a^5b^3+163023689214444520a^6b^3-5972150284654400a^7b^3)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(166437933747932 0a^8b^3-38955947128560a^9b^3+5678665839000a^{10}b^3-75925522400a^{11}b^3)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(6182616440a^{12}b^3-35709520a^{13}b^3+1623160a^{14}b^3+26105571522811305 00b^4)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(1557215473353983646 0ab^4+7324822944680010 00a^2b^4+5851298044645884600a^3b^4)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(-1636461179578225 00a^4b^4+368261307782880820a^5b^4-10339842738560720a^6b^4)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(625694918568104 0a^7b^4-125626624472580a^8b^4+33613458015060a^9b^4-406746041240a^{10}b^4)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(56687092280a^{11}b^4-305965660a^{12}b^4+23535820a^{13}b^4+217255099873004466 0b^5)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(1004608127102243440ab^5+3242956850341887448a^2b^5+760552353026102 56a^3b^5)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072a(4307887963632135960a^4b^5-5907351875594400a^5b^5+127816399912148640a^6b^5)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(-192523576889952a^7b^5+110161047202668a^8b^5-1135650386640a^9b^5+287146418328a^{10}b^5)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(-1401879024a^{11}b^5+183579396a^{12}b^5+185576437854776920b^6+768237818623401560ab^6)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(1167354441335266800a^2b^6+263248376733566840a^3b^6+3399221138266800a^4b^6)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(14691849210062640a^5b^6-101267395503120a^6b^6+209987898508080a^7b^6)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(-1593776507400a^8b^6+855056340600a^9b^6-3530373000a^{10}b^6+834451800a^{11}b^6)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(59177652660443128b^7+371222705883252960ab^7+80953716224732296a^2b^7)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(5503690017256640a^3b^7+9557288389416240a^4b^7+69140320048800a^5b^7)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(238397117389200a^6b^7-782781595200a^7b^7+1551234029400a^8b^7-4639918800a^9b^7)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(2319959400a^10b^7+3287994950239450b^8+11248058823729750ab^8+22943949958657200a^2b^8)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(3442692988837960a^3b^8+115095771016380a^4b^8+161870114844900a^5b^8+616153923000a^6b^8)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(1745291809800a^7b^8-2149374150a^8b^8+4059928950a^9b^8+525728261810290b^9)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(3686676467012000ab^9+6480924526661200a^2b^9+565912478762400a^3b^9+647920260787800a^4b^9)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(1040587825200a^5b^9+1221799794600a^6b^9+1910554800a^7b^9+4537567650a^8b^9)}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

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$$+\frac{131072a(18727536011800b^{10}+56336707180600ab^{10}+12392461389000a^2b^{10}+14735070827400a^3b^{10})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(571214351400a^4b^{10}+526590436680a^5b^{10}+3247943160a^6b^{10}+3247943160a^7b^{10})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(1658243409592b^{11}+1171241432144ab^{11}+1773637762904a^2b^{11}+151878786080a^3b^{11})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(136066994280a^4b^{11}+1925658000a^5b^{11}+1476337800a^6b^{11}+36288133700b^{12})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(98497273420ab^{12}+19917501240a^2b^{12}+20082473320a^3b^{12}+584116260a^4b^{12})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(417225900a^5b^{12}+1818469940b^{13}+1160821200ab^{13}+1556610440a^2b^{13}+94143280a^3b^{13})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(70607460a^4b^{13}+22016360b^{14}+54237480ab^{14}+7652040a^2b^{14}+6724520a^3b^{14}+593096b^{15})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072a(272272ab^{15}+324632a^2b^{15}+2975b^{16}+6545ab^{16}+35b^{17})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072b(-6332659870762850625+25321878164717979075a+21620235637305709200a^2)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(20437724329066130184a^3+26105571522811305000a^4+21725509987300446600a^5)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(18557643785477692000a^6+5917765266044312800a^7+3287994950239450a^8)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(525728261810290a^9+18727536011800a^{10}+1658243409592a^{11}+36288133700a^{12})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(1818469940a^{13}+22016360a^{14}+593096a^{15}+2975a^{16}+35a^{17}+151884650291143250250b)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-19523841512219551440ab+64543172743280700360a^2b+25755152400375158880a^3b)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(1557215473353983646 0a^4b+1004608127102243440a^5b+768237818623401560a^6b)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(371222705883252 96a^7b+112480588237297 50a^8b+368667646701200a^9b)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(56336707180600a^{10}b+1171241432144a^{11}b+98497273420a^{12}b+1160821200a^{13}b)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(54237480a^{14}b+272272a^{15}b+6545a^{16}b-1435451069161071324 0b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(47611998316914930072ab^2-11107176191996794920a^2b^2+3336387249195486208 8a^3b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(7324822944680010 00a^4b^2+3242956850341887448a^5b^2+1167354441335266 80a^6b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(80953716224732296a^7b^2+2294394995865720a^8b^2+648092452666120a^9b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(12392461389000a^{10}b^2+1773637762904a^{11}b^2+19917501240a^{12}b^2+1556610440a^{13}b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(7652040a^{14}b^2+324632a^{15}b^2+7524314127912551832b^3-1233082566460000641 6ab^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(26638838560038217560a^2b^3-2090930383100586720a^3b^3+585129804464588460 0a^4b^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(76055235302610256a^5b^3+263248376733566840a^6b^3+5503690017256640a^7b^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(3442692988837960a^8b^3+56591247876240a^9b^3+14735070827400a^{10}b^3+151878786080a^{11}b^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(20082473320a^{12}b^3+94143280a^{13}b^3+6724520a^{14}b^3-2523698606200763196b^4)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072b(7687192319327829444ab^4-2867948454968860760a^2b^4+4873159786850521320a^3b^4)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

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$$+\frac{131072b(-163646117957822500a^4b^4+430788796363213596a^5b^4+3399221138266800a^6b^4)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(95572883894162 40a^7b^4+115095771016380a^8b^4+64792026078780a^9b^4+571214351400a^{10}b^4)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(136066994280a^{11}b^4+584116260a^{12}b^4+70607460a^{13}b^4+585146416702456764b^5)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-1038346142047282320ab^5+1845548308154811400a^2b^5-2581511566193375 20a^3b^5)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(368261307782880820a^4b^5-5907351875594400a^5b^5+14691849210062640a^6b^5)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(69140320048800a^7b^5+161870114844900a^8b^5+1040587825200a^9b^5+526590436680a^{10}b^5)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(1925658000a^{11}b^5+417225900a^{12}b^5-98283050207112680b^6+283129024934512456ab^6)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-124702534849141480a^2b^6+163023689214444520a^3b^6-103398427385607 20a^4b^6)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(12781639991214864a^5b^6-101267395503120a^6b^6+238397117389200a^7b^6)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(616153923000a^8b^6+1221799794600a^9b^6+3247943160a^{10}b^6+1476337800a^{11}b^6)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(12319487399406824b^7-22414624986818768ab^7+35260676281141080a^2b^7)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-5972150284654400a^3b^7+6256949185681040a^4b^7-192523576889952a^5b^7)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(209987898508080a^6b^7-782781595200a^7b^7+1745291809800a^8b^7+1910554800a^9b^7)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(3247943160a^{10}b^7-1174199725349222b^8+3231412550832642ab^8-1500336516820680a^2b^8)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(1664379337479320a^3b^8-125626624472580a^4b^8+110161047202668a^5b^8)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-1593776507400a^6b^8+1551234029400a^7b^8-2149374150a^8b^8+4537567650a^9b^8)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(860148187449 98b^9-155206622884720ab^9+222764240366360a^2b^9-38955947128560a^3b^9)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(33613458015060a^4b^9-1135650386640a^5b^9+855056340600a^6b^9-4639918800a^7b^9)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(4059928950a^8b^9-4862169489320b^{10}+12794409439592ab^{10}-5784150923320a^2b^{10})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(5678665839000a^3b^{10}-406746041240a^4b^{10}+287146418328a^5b^{10}-3530373000a^6b^{10})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(2319959400a^7b^{10}+211577650856b^{11}-366157152816ab^{11}+484991616200a^2b^{11})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-75925522400a^3b^{11}+56687092280a^4b^{11}-1401879024a^5b^{11}+834451800a^6b^{11})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-7020044668b^{12}+17543988644ab^{12}-6995348360a^2b^{12}+6182616440a^3b^{12})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-305965660a^4b^{12}+183579396a^5b^{12}+174281212b^{13}-274185520ab^{13}+334423320a^2b^{13})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(-35709520a^3b^{13}+23535820a^4b^{13}-3132760b^14+7297080ab^{14}-2042040a^2b^{14})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072b(1623160a^3b^{14}+38488b^{15}-47600ab^{15}+52360a^2b^{15}-289b^{16}+595ab^{16}+b^{17})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{262144(6332659870762850625+25321878164717979075a-21620235637305709 20a^2)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(20437724329066130184a^3-26105571522811305 00a^4+2172550998730044660a^5)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

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$$+\frac{262144(-185576437854776920a^{6}+59177652660443128a^{7}-3287994950239450a^{8})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(525728261810290a^{9}-18727536011800a^{10}+1658243409592a^{11}-36288133700a^{12})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(1818469940a^{13}-22016360a^{14}+593096a^{15}-2975a^{16}+35a^{17}+151884650291143250025b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(195238415122195514440ab+64543172743280700360a^{2}b-2575515240037515888a^{3}b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(15572154733539836460a^{4}b-1004608127102243440a^{5}b+768237818623401560a^{6}b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(-37122270588325296a^{7}b+11248058823729750a^{8}b-368667646701200a^{9}b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(56336707180600a^{10}b-1171241432144a^{11}b+98497273420a^{12}b-1160821200a^{13}b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(54237480a^{14}b-272272a^{15}b+6545a^{16}b+14354510691610713240b^{2})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(47611998316914930072ab^{2}+11107176191996794920a^{2}b^{2}+33363872491954862088a^{3}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(-7324822944680010000a^{4}b^{2}+3242956850341887448a^{5}b^{2}-116735444133526680a^{6}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(80953716224732296a^{7}b^{2}-2294394995865720a^{8}b^{2}+648092452666120a^{9}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(-12392461389000a^{10}b^{2}+1773637762904a^{11}b^{2}-19917501240a^{12}b^{2}+1556610440a^{13}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(-7652040a^{14}b^{2}+324632a^{15}b^{2}+7524314127912551832b^{3}+12330825664600006416ab^{3})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(26638838560038217560a^{2}b^{3}+2090930383100586720a^{3}b^{3}+5851298044645884600a^{4}b^{3})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(-760552353026102 56a^5b^3+263248376733566840a^6b^3-5503690017256640a^7b^3)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(3442692988837960a^8b^3-56591247876240a^9b^3+14735070827400a^{10}b^3-151878786080a^{11}b^3)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(20082473320a^{12}b^3-94143280a^{13}b^3+6724520a^{14}b^3+2523698606200763196b^4)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(7687192319327829444ab^4+2867948454968860760a^2b^4+487315978685052132 0a^3b^4)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(163646117957822500a^4b^4+43078879636321359 6a^5b^4-3399221138266800a^6b^4)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(9557288389416240a^7b^4-115095771016380a^8b^4+64792026078780a^9b^4-571214351400a^{10}b^4)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(136066994280a^{11}b^4-584116260a^{12}b^4+70607460a^{13}b^4+585146416702456764b^5)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(1038346142047282320ab^5+1845548308154811400a^2b^5+258151156619337520a^3b^5)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(368261307782880820a^4b^5+5907351875594400a^5b^5+14691849210062640a^6b^5)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(-69140320048800a^7b^5+161870114844900a^8b^5-1040587825200a^9b^5+526590436680a^{10}b^5)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(-1925658000a^{11}b^5+417225900a^{12}b^5+98283050207112680b^6+283129024934512456ab^6)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(124702534849141480a^2b^6+163023689214444520a^3b^6+10339842738560720a^4b^6)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(12781639991214864a^5b^6+101267395503120a^6b^6+238397117389200a^7b^6-616153923000a^8b^6)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(1221799794600a^9b^6-3247943160a^{10}b^6+1476337800a^{11}b^6+12319487399406824b^7)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

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$$+\frac{262144(22414624986818768ab^{7}+35260676281141080a^{2}b^{7}+5972150284654400a^{3}b^{7})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(6256949185681040a^{4}b^{7}+192523576889952a^{5}b^{7}+209987898508080a^{6}b^{7}+782781595200a^{7}b^{7})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(1745291809800a^{8}b^{7}-1910554800a^{9}b^{7}+3247943160a^{10}b^{7}+11741997253349222b^{8})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(3231412550832642ab^{8}+15003365168206080a^{2}b^{8}+1664379337479320a^{3}b^{8})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(125626624472580a^{4}b^{8}+110161047202668a^{5}b^{8}+1593776507400a^{6}b^{8}+1551234029400a^{7}b^{8})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(2149374150a^{8}b^{8}+4537567650a^{9}b^{8}+860148187449 98b^{9}+155206622884720ab^{9})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(222764240366360a^{2}b^{9}+38955947128560a^{3}b^{9}+33613458015060a^{4}b^{9}+1135650386640a^{5}b^{9})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(855056340600a^{6}b^{9}+4639918800a^{7}b^{9}+4059928950a^{8}b^{9}+4862169489320b^{10})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(12794409439592ab^{10}+5784150923320a^{2}b^{10}+5678665839000a^{3}b^{10}+406746041240a^{4}b^{10})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(287146418328a^{5}b^{10}+3530373000a^{6}b^{10}+2319959400a^{7}b^{10}+211577650856b^{11})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(366157152816ab^{11}+484991616200a^{2}b^{11}+75925522400a^{3}b^{11}+56687092280a^{4}b^{11})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(1401879024a^{5}b^{11}+834451800a^{6}b^{11}+7020044668b^{12}+17543988644ab^{12}+6995348360a^{2}b^{12})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(6182616440a^{3}b^{12}+305965660a^{4}b^{12}+183579396a^{5}b^{12}+174281212b^{13}+274185520ab^{13})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(334423320a^{2}b^{13}+35709520a^{3}b^{13}+23535820a^{4}b^{13}+3132760b^{14}+7297080ab^{14})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{262144(2042040a^2b^{14}+1623160a^3b^{14}+38488b^{15}+47600ab^{15}+52360a^2b^{15}+289b^{16})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(595ab^{16}+b^{17})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{262144(6332659870762850625+15188465029114325025a+14354510691610713240a^2)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(7524314127912551832a^3+2523698606200763196a^4+585146416702456764a^5)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(98283050207112680a^6+12319487399406824a^7+1174199725349222a^8+86014818744998a^9)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(4862169489320a^{10}+211577650856a^{11}+7020044668a^{12}+174281212a^{13}+3132760a^{14})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(38488a^{15}+289a^{16}+a^{17}+2532187816471797907 5b+19523841512219551440ab)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(4761199831691493007 2a^2b+12330825664600006416a^3b+7687192319327829444a^4b)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(1038346142047282320a^5b+283129024934512456a^6b+22414624986818768a^7b)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(3231412550832642a^8b+155206622884720a^9b+12794409439592a^{10}b+366157152816a^{11}b)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(17543988644a^{12}b+274185520a^{13}b+7297080a^{14}b+47600a^{15}b+595a^{16}b)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-2162023563730570920b^2+64543172743280700360ab^2+11107176191996794920a^2b^2)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(26638838560038217560a^3b^2+2867948454968860760a^4b^2+1845548308154811400a^5b^2)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(124702534849141480a^6b^2+35260676281141080a^7b^2+1500336516820680a^8b^2)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

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$$+\frac{262144(222764240366360a^{9}b^{2}+5784150923320a^{10}b^{2}+484991616200a^{11}b^{2}+6995348360a^{12}b^{2})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(334423320a^{13}b^{2}+2042040a^{14}b^{2}+52360a^{15}b^{2}+2043772432906613018 4b^{3})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-2575515240037515888ab^{3}+3336387249195486208 8a^{2}b^{3}+2090930383100586720a^{3}b^{3})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(4873159786850521320a^{4}b^{3}+258151156619337520a^{5}b^{3}+16302368921444452 0a^{6}b^{3})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(5972150284654400a^{7}b^{3}+1664379337479320a^{8}b^{3}+38955947128560a^{9}b^{3})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(5678665839000a^{10}b^{3}+75925522400a^{11}b^{3}+6182616440a^{12}b^{3}+35709520a^{13}b^{3}+1623160a^{14}b^{3})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-2610557152281130500b^{4}+15572154733539836460ab^{4}-7324822944680010 00a^{2}b^{4})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(5851298044645884600a^{3}b^{4}+1636461179578225 00a^{4}b^{4}+3682613077828808 20a^{5}b^{4})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(10339842738560720a^{6}b^{4}+6256949185681040a^{7}b^{4}+125626624472580a^{8}b^{4})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(33613458015060a^{9}b^{4}+406746041240a^{10}b^{4}+56687092280a^{11}b^{4}+305965660a^{12}b^{4})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(23535820a^{13}b^{4}+217255099873004466 0b^{5}-1004608127102243440ab^{5})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(3242956850341887448a^{2}b^{5}-76055235302610256a^{3}b^{5}+430788796363213596a^{4}b^{5})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(5907351875594400a^{5}b^{5}+12781639991214864a^{6}b^{5}+192523576889952a^{7}b^{5})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(110161047202668a^{8}b^{5}+1135650386640a^{9}b^{5}+287146418328a^{10}b^{5}+1401879024a^{11}b^{5})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(183579396a^{12}b^{5}-1855764378547776920b^{6}+7682378186234015660ab^{6})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-1167354441335266680a^{2}b^{6}+263248376733566840a^{3}b^{6}-3399221138266800a^{4}b^{6})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(14691849210062640a^{5}b^{6}+101267395503120a^{6}b^{6}+209987898508080a^{7}b^{6}+1593776507400a^{8}b^{6})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(855056340600a^{9}b^{6}+3530373000a^{10}b^{6}+834451800a^{11}b^{6}+591776526604431 28b^{7})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-371222705883252 96ab^{7}+809537162247322 96a^{2}b^{7}-5503690017256640a^{3}b^{7})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(9557288389416240a^{4}b^{7}-69140320048800a^{5}b^{7}+238397117389200a^{6}b^{7}+782781595200a^{7}b^{7})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(1551234029400a^{8}b^{7}+4639918800a^{9}b^{7}+2319959400a^{10}b^{7}-328799495023945 0b^{8})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(11248058823729750ab^{8}-2294394995865720a^{2}b^{8}+3442692988837960a^{3}b^{8})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-115095771016380a^{4}b^{8}+161870114844900a^{5}b^{8}-616153923000a^{6}b^{8}+1745291809800a^{7}b^{8})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(2149374150a^{8}b^{8}+4059928950a^{9}b^{8}+5257282618102 90b^{9}-368667646701200ab^{9})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(648092452666120a^{2}b^{9}-56591247876240a^{3}b^{9}+64792026078780a^{4}b^{9}-1040587825200a^{5}b^{9})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(1221799794600a^{6}b^{9}-1910554800a^{7}b^{9}+4537567650a^{8}b^{9}-187275360118 00b^{10})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(56336707180600ab^{10}-12392461389000a^{2}b^{10}+14735070827400a^{3}b^{10}-571214351400a^{4}b^{10})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(526590436680a^{5}b^{10}-3247943160a^{6}b^{10}+3247943160a^{7}b^{10}+1658243409592b^{11})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

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$$+\frac{262144(-1171241432144ab^{11}+1773637762904a^2b^{11}-151878786080a^3b^{11}+136066994280a^4b^{11})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-1925658000a^5b^{11}+1476337800a^6b^{11}-36288133700b^{12}+98497273420ab^{12})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-19917501240a^2b^{12}+20082473320a^3b^{12}-584116260a^4b^{12}+417225900a^5b^{12})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(1818469940b^{13}-1160821200ab^{13}+1556610440a^2b^{13}-94143280a^3b^{13}+70607460a^4b^{13})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(-22016360b^{14}+54237480ab^{14}-7652040a^2b^{14}+6724520a^3b^{14}+593096b^{15}-272272ab^{15})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{262144(324632a^2b^{15}-2975b^{16}+6545ab^{16}+35b^{17})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}\Bigg\}\Bigg] \quad (8)$$

III. Evaluation of Main Summation Formula (8)

Substituting $c=\frac{a+b+37}{2}$ and $z=\frac{1}{2}$ in equation (2), and after that involving Gauss theorem , we get

$$L.H.S = a\ \frac{2^b\ \Gamma(\frac{a+b+37}{2})}{\Gamma(b)}\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{131072(-6332659870762850625+15188465029114325025a)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(-14354510691610713240a^2+7524314127912551832a^3-2523698606200763196a^4)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(585146416702456764a^5-98283050207112680a^6+12319487399406824a^7)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(-1174199725349222a^8+86014818744998a^9-4862169489320a^{10}+211577650856a^{11})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(-7020044668a^{12}+174281212a^{13}-3132760a^{14}+38488a^{15}-289a^{16}+a^{17})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(2532187816471797907 5b-19523841512219551440ab+4761199831691493007 2a^2b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(-12330825664600006416a^3b+7687192319327829444a^4b-1038346142047282320a^5b)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(283129024934512456a^{6}b-22414624986818768a^{7}b+3231412550832642a^{8}b)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-155206622884720a^{9}b+12794409439592a^{10}b-366157152816a^{11}b+17543988644a^{12}b)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-274185520a^{13}b+7297080a^{14}b-47600a^{15}b+595a^{16}b+2162023563730570920b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(6454317274328070036 0ab^{2}-1110717619199679492 0a^{2}b^{2}+2663883856003821756 0a^{3}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-2867948454968860760a^{4}b^{2}+1845548308154811400a^{5}b^{2}-124702534849141480a^{6}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(35260676281141080a^{7}b^{2}-1500336516820680a^{8}b^{2}+222764240366360a^{9}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-5784150923320a^{10}b^{2}+484991616200a^{11}b^{2}-6995348360a^{12}b^{2}+334423320a^{13}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-2042040a^{14}b^{2}+52360a^{15}b^{2}+2043772432906613018 4b^{3}+2575515240037515888ab^{3})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(3336387249195486208 8a^{2}b^{3}-2090930383100586720a^{3}b^{3}+4873159786850521320a^{4}b^{3})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-258151156619337520a^{5}b^{3}+16302368921444452 0a^{6}b^{3}-5972150284654400a^{7}b^{3})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(166437933747932 0a^{8}b^{3}-38955947128560a^{9}b^{3}+5678665839000a^{10}b^{3}-75925522400a^{11}b^{3})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(6182616440a^{12}b^{3}-35709520a^{13}b^{3}+1623160a^{14}b^{3}+2610557152281130500b^{4})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(15572154733539836460ab^{4}+7324822944680010000a^{2}b^{4}+5851298044645884600a^{3}b^{4})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-1636461179578225000a^{4}b^{4}+368261307782880820a^{5}b^{4}-10339842738560720a^{6}b^{4})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

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$$+\frac{131072(6256949185681040a^{7}b^{4}-125626624472580a^{8}b^{4}+33613458015060a^{9}b^{4}-406746041240a^{10}b^{4})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(56687092280a^{11}b^{4}-305965660a^{12}b^{4}+23535820a^{13}b^{4}+2172550998730044660b^{5})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(10046081271022434440ab^{5}+3242956850341887448a^{2}b^{5}+760552353026102556a^{3}b^{5})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(430788796363213596a^{4}b^{5}-5907351875594400a^{5}b^{5}+12781639991214864a^{6}b^{5})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-192523576889952a^{7}b^{5}+110161047202668a^{8}b^{5}-1135650386640a^{9}b^{5}+287146418328a^{10}b^{5})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-1401879024a^{11}b^{5}+183579396a^{12}b^{5}+1855764378547769920b^{6}+7682378186234015560ab^{6})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(1167354441335266680a^{2}b^{6}+2632483767335666840a^{3}b^{6}+3399221138266800a^{4}b^{6})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(14691849210062640a^{5}b^{6}-101267395503120a^{6}b^{6}+209987898508080a^{7}b^{6})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-1593776507400a^{8}b^{6}+855056340600a^{9}b^{6}-3530373000a^{10}b^{6}+834451800a^{11}b^{6})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(591776526604431228b^{7}+371222705883252296ab^{7}+809537162247322296a^{2}b^{7})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(550369001725664040a^{3}b^{7}+95572883894162240a^{4}b^{7}+691403200488000a^{5}b^{7})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(238397117389200a^{6}b^{7}-782781595200a^{7}b^{7}+1551234029400a^{8}b^{7}-4639918800a^{9}b^{7})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(2319959400a^{1}0b^{7}+32879949502394450b^{8}+112480588237297750ab^{8}+229439499586572020a^{2}b^{8})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(344269298883796960a^{3}b^{8}+115095771016380a^{4}b^{8}+1618701148449000a^{5}b^{8}+616153923000a^{6}b^{8})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(1745291809800a^7b^8-2149374150a^8b^8+4059928950a^9b^8+5257282618102900b^9)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(368667646701200ab^9+648092452666120a^2b^9+56591247876240a^3b^9+64792026078780a^4b^9)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(1040587825200a^5b^9+1221799794600a^6b^9+1910554800a^7b^9+4537567650a^8b^9)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(18727536011800b^{10}+56336707180600ab^{10}+12392461389000a^2b^{10}+14735070827400a^3b^{10})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(571214351400a^4b^{10}+526590436680a^5b^{10}+3247943160a^6b^{10}+3247943160a^7b^{10})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(1658243409592b^{11}+1171241432144ab^{11}+1773637762904a^2b^{11}+151878786080a^3b^{11})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(136066994280a^4b^{11}+1925658000a^5b^{11}+1476337800a^6b^{11}+362881133700b^{12})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(98497273420ab^{12}+19917501240a^2b^{12}+20082473320a^3b^{12}+584116260a^4b^{12})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(417225900a^5b^{12}+1818469940b^{13}+1160821200ab^{13}+1556610440a^2b^{13}+94143280a^3b^{13})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(70607460a^4b^{13}+22016360b^{14}+54237480ab^{14}+7652040a^2b^{14}+6724520a^3b^{14}+593096b^{15})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(272272ab^{15}+324632a^2b^{15}+2975b^{16}+6545ab^{16}+35b^{17})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+2}{2})}\Bigg\{\frac{131072(6332659870762850625+25321878164717979075a-2162023563730570920a^2)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}$$

$$+\frac{131072(20437724329066130184a^3-2610557152281130500a^4+2172550998730044660a^5)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(-185576437854776920a^6+59177652660443128a^7-3287994950239450a^8)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

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$$+\frac{131072(525728261810290a^{9}-18727536011800a^{10}+1658243409592a^{11}-36288133700a^{12})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(1818469940a^{13}-22016360a^{14}+593096a^{1}5-2975a^{16}+35a^{17}+151884650291143250 25b)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(19523841512219551440ab+6454317274328070036 0a^{2}b-25755152400375158 88a^{3}b)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(15572154733539836460a^{4}b-1004608127102243440a^{5}b+76823781862340 1560a^{6}b)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-37122270588325296a^{7}b+11248058823729750a^{8}b-368667646701200a^{9}b)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(56336707180600a^{10}b-1171241432144a^{11}b+98497273420a^{12}b-1160821200a^{13}b)}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(54237480a^{14}b-272272a^{15}b+6545a^{16}b+14354510691610713240b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(47611998316914930072ab^{2}+11107176191996794920a^{2}b^{2}+33363872491954862088a^{3}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-7324822944680010 00a^{4}b^{2}+3242956850341887448a^{5}b^{2}-1167354441335266 80a^{6}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(80953716224732296a^{7}b^{2}-2294394995865720a^{8}b^{2}+648092452666120a^{9}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-12392461389000a^{10}b^{2}+1773637762904a^{11}b^{2}-19917501240a^{12}b^{2}+1556610440a^{13}b^{2})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-7652040a^{14}b^{2}+324632a^{15}b^{2}+752431412791255 1832b^{3}+12330825664600006416ab^{3})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(26638838560038217560a^{2}b^{3}+20909303831005867 20a^{3}b^{3}+5851298044645884600a^{4}b^{3})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-760552353026102 56a^{5}b^{3}+26324837673356684 0a^{6}b^{3}-5503690017256640a^{7}b^{3})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

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$$+\frac{131072(3442692988837960a^{8}b^{3}-56591247876240a^{9}b^{3}+14735070827400a^{10}b^{3}-151878786080a^{11}b^{3})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(20082473320a^{12}b^{3}-94143280a^{13}b^{3}+6724520a^{14}b^{3}+2523698606200763196b^{4})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(7687192319327829444ab^{4}+2867948454968860760a^{2}b^{4}+487315978685052132 0a^{3}b^{4})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(163646117957822500a^{4}b^{4}+43078879636321359 6a^{5}b^{4}-3399221138266800a^{6}b^{4})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(9557288389416240a^{7}b^{4}-115095771016380a^{8}b^{4}+64792026078780a^{9}b^{4}-571214351400a^{10}b^{4})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(136066994280a^{11}b^{4}-584116260a^{12}b^{4}+70607460a^{13}b^{4}+5851464167024567 64b^{5})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(1038346142047282320ab^{5}+1845548308154811400a^{2}b^{5}+258151156619337520a^{3}b^{5})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(36826130778288082 0a^{4}b^{5}+5907351875594400a^{5}b^{5}+14691849210062640a^{6}b^{5})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-69140320048800a^{7}b^{5}+161870114844900a^{8}b^{5}-1040587825200a^{9}b^{5}+526590436680a^{10}b^{5})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(-1925658000a^{11}b^{5}+417225900a^{12}b^{5}+98283050207112680b^{6}+283129024934512456ab^{6})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(124702534849141480a^{2}b^{6}+163023689214444520a^{3}b^{6}+10339842738560720a^{4}b^{6})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(12781639991214864a^{5}b^{6}+101267395503120a^{6}b^{6}+238397117389200a^{7}b^{6}-616153923000a^{8}b^{6})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(1221799794600a^{9}b^{6}-3247943160a^{10}b^{6}+1476337800a^{11}b^{6}+12319487399406824b^{7})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

$$+\frac{131072(22414624986818768ab^{7}+35260676281141080a^{2}b^{7}+5972150284654400a^{3}b^{7})}{\left[\prod_{\varepsilon=1}^{17}\left\{a-b-(2\varepsilon-1)\right\}\right]\left[\prod_{\zeta=1}^{18}\left\{a-b+(2\zeta-1)\right\}\right]}+$$

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$$+\frac{131072(6256949185681040a^4b^7+192523576889952a^5b^7+209987898508080a^6b^7+782781595200a^7b^7)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(1745291809800a^8b^7-1910554800a^9b^7+3247943160a^{10}b^7+1174199725349222b^8)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(3231412550832642ab^8+1500336516820680a^2b^8+1664379337479320a^3b^8)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(125626624472580a^4b^8+110161047202668a^5b^8+1593776507400a^6b^8+1551234029400a^7b^8)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(2149374150a^8b^8+4537567650a^9b^8+860148187449 98b^9+155206622884720ab^9)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(222764240366360a^2b^9+38955947128560a^3b^9+33613458015060a^4b^9+1135650386640a^5b^9)}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(855056340600a^6b^9+4639918800a^7b^9+4059928950a^8b^9+4862169489320b^{10})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(12794409439592ab^{10}+5784150923320a^2b^{10}+5678665839000a^3b^{10}+406746041240a^4b^{10})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(287146418328a^5b^{10}+3530373000a^6b^{10}+2319959400a^7b^{10}+211577650856b^{11})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(366157152816ab^{11}+484991616200a^2b^{11}+75925522400a^3b^{11}+56687092280a^4b^{11})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(1401879024a^5b^{11}+834451800a^6b^{11}+7020044668b^{12}+17543988644ab^{12}+6995348360a^2b^{12})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(6182616440a^3b^{12}+305965660a^4b^{12}+183579396a^5b^{12}+174281212b^{13}+274185520ab^{13})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(334423320a^2b^{13}+35709520a^3b^{13}+23535820a^4b^{13}+3132760b^{14}+7297080ab^{14})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

$$+\frac{131072(2042040a^2b^{14}+1623160a^3b^{14}+38488b^{15}+47600ab^{15}+52360a^2b^{15}+289b^{16})}{\left[\prod_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}+$$

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$$+\frac{131072(595ab^{16}+b^{17})}{\left[\prod\limits_{\varepsilon=1}^{17}\{a-b-(2\varepsilon-1)\}\right]\left[\prod\limits_{\zeta=1}^{18}\{a-b+(2\zeta-1)\}\right]}\Bigg\}\Bigg]-$$

$$-\frac{2^{b+1}\;\Gamma(\frac{a+b+37}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\left\{\frac{131072(6332659870762850625+15188465029114325025a)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+\right.\right.$$

$$+\frac{131072(14354510691610713240a^2+7524314127912551832a^3+2523698606200763196a^4)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(585146416702456764a^5+98283050207112680a^6+12319487399406824a^7)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(1174199725349222a^8+86014818744998a^9+4862169489320a^{10}+211577650856a^{11})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(7020044668a^{12}+174281212a^{13}+3132760a^{14}+38488a^15+289a^{16}+a^{17})}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(2532187816471797907 5b+19523841512219551440ab+4761199831691493007 2a^2b)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(12330825664600006416a^3b+7687192319327829444a^4b+1038346142047282320a^5b)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(283129024934512456a^6b+22414624986818768a^7b+3231412550832642a^8b)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(155206622884720a^9b+12794409439592a^{10}b+366157152816a^{11}b+17543988644a^{12}b)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(274185520a^{13}b+7297080a^{14}b+47600a^{15}b+595a^{16}b-21620235637305709 20b^2)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(64543172743280700360ab^2+111071761919967949 20a^2b^2+26638838560038217560a^3b^2)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(28679484549688607 60a^4b^2+18455483081548114 00a^5b^2+1247025348491414 80a^6b^2)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(3526067628114108 0a^7b^2+1500336516820680a^8b^2+222764240366360a^9b^2)}{\left[\prod\limits_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod\limits_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

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$$+\frac{131072(5784150923320a^{10}b^{2}+484991616200a^{11}b^{2}+6995348360a^{12}b^{2}+334423320a^{13}b^{2})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(2042040a^{14}b^{2}+52360a^{15}b^{2}+204377243290661301184b^{3}-257551524003751588 8ab^{3})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(3336387249195486 2088a^{2}b^{3}+2090930383100586720a^{3}b^{3}+487315978685052132 0a^{4}b^{3})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(258151156619337520a^{5}b^{3}+16302368921444452 0a^{6}b^{3}+5972150284654400a^{7}b^{3})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(1664379337479320a^{8}b^{3}+38955947128560a^{9}b^{3}+5678665839000a^{10}b^{3}+75925522400a^{11}b^{3})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(6182616440a^{12}b^{3}+35709520a^{13}b^{3}+1623160a^{14}b^{3}-26105571522811305 00b^{4})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(15572154733539836460ab^{4}-732482294468001000a^{2}b^{4}+585129804464588460 0a^{3}b^{4})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(1636461179578225 00a^{4}b^{4}+36826130778288082 0a^{5}b^{4}+10339842738560720a^{6}b^{4})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(6256949185681040a^{7}b^{4}+125626624472580a^{8}b^{4}+33613458015060a^{9}b^{4}+406746041240a^{10}b^{4})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(56687092280a^{11}b^{4}+305965660a^{12}b^{4}+23535820a^{13}b^{4}+2172550998730044660b^{5})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-1004608127102243440ab^{5}+324295685034188744 8a^{2}b^{5}-76055235302610256a^{3}b^{5})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(43078879636321359 6a^{4}b^{5}+5907351875594400a^{5}b^{5}+12781639991214864a^{6}b^{5})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(192523576889952a^{7}b^{5}+110161047202668a^{8}b^{5}+1135650386640a^{9}b^{5}+287146418328a^{10}b^{5})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(1401879024a^{11}b^{5}+183579396a^{12}b^{5}-18557643785477692 0b^{6}+768237818623401560ab^{6})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-116735444133526680a^2b^6+263248376733566840a^3b^6-3399221138266800a^4b^6)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(14691849210062640a^5b^6+101267395503120a^6b^6+209987898508080a^7b^6)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(1593776507400a^8b^6+855056340600a^9b^6+3530373000a^{10}b^6+834451800a^{11}b^6)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(5917765266044312 8b^7-371222705883252 96ab^7+809537162247322 96a^2b^7)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-5503690017256640a^3b^7+9557288389416240a^4b^7-69140320048800a^5b^7)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(238397117389200a^6b^7+782781595200a^7b^7+1551234029400a^8b^7+4639918800a^9b^7)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(2319959400a^10b^7-3287994950239450b^8+11248058823729750ab^8-2294394995865720a^2b^8)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(3442692988837960a^3b^8-115095771016380a^4b^8+161870114844900a^5b^8-616153923000a^6b^8)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(1745291809800a^7b^8+2149374150a^8b^8+4059928950a^9b^8+5257282618102 90b^9)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-368667646701200ab^9+648092452666120a^2b^9-56591247876240a^3b^9+64792026078780a^4b^9)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-1040587825200a^5b^9+1221799794600a^6b^9-1910554800a^7b^9+4537567650a^8b^9)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-187275360011800b^{10}+56336707180600ab^{10}-12392461389000a^2b^{10}+14735070827400a^3b^{10})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-571214351400a^4b^{10}+526590436680a^5b^{10}-3247943160a^6b^{10}+3247943160a^7b^{10})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(1658243409592b^{11}-1171241432144ab^{11}+1773637762904a^2b^{11}-151878786080a^3b^{11})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

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$$+\frac{131072(136066994280a^4b^{11}-1925658000a^5b^{11}+1476337800a^6b^{11}-36288133700b^{12})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(98497273420ab^{12}-19917501240a^2b^{12}+20082473320a^3b^{12}-584116260a^4b^{12})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(417225900a^5b^{12}+1818469940b^{13}-1160821200ab^{13}+1556610440a^2b^{13}-94143280a^3b^{13})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(70607460a^4b^{13}-22016360b^{14}+54237480ab^{14}-7652040a^2b^{14}+6724520a^3b^{14}+593096b^{15})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-272272ab^{15}+324632a^2b^{15}-2975b^{16}+6545ab^{16}+35b^{17})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{131072(-6332659870762850625+25321878164717979075a+21620235637305709 20a^2)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}$$

$$+\frac{131072(20437724329066130184a^3+26105571522811305 00a^4+2172550998730044660a^5)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(185576437854776920a^6+5917765266044312 8a^7+3287994950239450a^8+525728261810290a^9)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(18727536011800a^{10}+1658243409592a^{11}+36288133700a^{12}+1818469940a^{13}+22016360a^{14})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(593096a^{15}+2975a^{16}+35a^{17}+15188465029114325025b-19523841512219551440ab)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(6454317274328070036 0a^2b+2575515240037515888a^3b+1557215473353983646 0a^4b)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(100460812710224344 0a^5b+768237818623401560a^6b+37122270588325296a^7b)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(11248058823729750a^8b+368667646701200a^9b+56336707180600a^{10}b+1171241432144a^{11}b)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(98497273420a^{12}b+1160821200a^{13}b+54237480a^{14}b+272272a^{15}b+6545a^{16}b)}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-1435451069161071324 0b^2+4761199831691493007 2ab^2-111071761919967949 20a^2b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(3336387249195486208 8a^3b^2+7324822944680010 00a^4b^2+3242956850341887448a^5b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(116735444133526680a^6b^2+8095371622473229 6a^7b^2+2294394995865720a^8b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(648092452666120a^9b^2+12392461389000a^{10}b^2+1773637762904a^{11}b^2+19917501240a^{12}b^2)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(1556610440a^{13}b^2+7652040a^{14}b^2+324632a^{15}b^2+752431412791255183 2b^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-1233082566460000641 6ab^3+2663883856003821756 0a^2b^3-209093038310058672 0a^3b^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(5851298044645884600a^4b^3+760552353026102 56a^5b^3+263248376733566840a^6b^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(5503690017256640a^7b^3+344269298883796 0a^8b^3+56591247876240a^9b^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(14735070827400a^{10}b^3+151878786080a^{11}b^3+20082473320a^{12}b^3+94143280a^{13}b^3)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(6724520a^{14}b^3-2523698606200763196b^4+7687192319327829444ab^4)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-2867948454968860760a^2b^4+4873159786850521320a^3b^4-1636461179578225 00a^4b^4)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(430788796363213596a^5b^4+3399221138266800a^6b^4+9557288389416240a^7b^4)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(115095771016380a^8b^4+64792026078780a^9b^4+571214351400a^{10}b^4+136066994280a^{11}b^4)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(584116260a^{12}b^4+70607460a^{13}b^4+585146416702456764b^5-1038346142047282320ab^5)}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

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$$+\frac{131072(18455483081548114 00a^{2}b^{5}-2581511566193375 20a^{3}b^{5}+368261307782880820a^{4}b^{5})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-5907351875594400a^{5}b^{5}+1469184921006264 0a^{6}b^{5}+69140320048800a^{7}b^{5})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(161870114844900a^{8}b^{5}+1040587825200a^{9}b^{5}+526590436680a^{10}b^{5}+1925658000a^{11}b^{5})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(417225900a^{12}b^{5}-982830502071126 80b^{6}+28312902493451245 6ab^{6})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-124702534849141480a^{2}b^{6}+163023689214444520a^{3}b^{6}-103398427385607 20a^{4}b^{6})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(12781639991214864a^{5}b^{6}-101267395503120a^{6}b^{6}+238397117389200a^{7}b^{6}+616153923000a^{8}b^{6})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(1221799794600a^{9}b^{6}+3247943160a^{10}b^{6}+1476337800a^{11}b^{6}+123194873994068 24b^{7})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-22414624986818768ab^{7}+35260676281141080a^{2}b^{7}-5972150284654400a^{3}b^{7})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(6256949185681040a^{4}b^{7}-192523576889952a^{5}b^{7}+209987898508080a^{6}b^{7}-782781595200a^{7}b^{7})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(1745291809800a^{8}b^{7}+1910554800a^{9}b^{7}+3247943160a^{10}b^{7}-1174199725349222b^{8})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(3231412550832642ab^{8}-1500336516820680a^{2}b^{8}+1664379337479320a^{3}b^{8})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-125626624472580a^{4}b^{8}+110161047202668a^{5}b^{8}-1593776507400a^{6}b^{8}+1551234029400a^{7}b^{8})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(-2149374150a^{8}b^{8}+4537567650a^{9}b^{8}+860148187744998b^{9}-155206622884720ab^{9})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(222764240366360a^{2}b^{9}-38955947128560a^{3}b^{9}+33613458015060a^{4}b^{9}-1135650386640a^{5}b^{9})}{\left[\prod_{\varpi=1}^{18}\left\{a-b-(2\varpi-1)\right\}\right]\left[\prod_{\varrho=1}^{17}\left\{a-b+(2\varrho-1)\right\}\right]}+$$

$$+\frac{131072(855056340600a^6b^9-4639918800a^7b^9+4059928950a^8b^9-4862169489320b^{10})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(12794409439592ab^{10}-5784150923320a^2b^{10}+5678665839000a^3b^{10}-406746041240a^4b^{10})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(287146418328a^5b^{10}-3530373000a^6b^{10}+2319959400a^7b^{10}+2115776508560b^{11})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-366157152816ab^{11}+484991616200a^2b^{11}-75925522400a^3b^{11}+56687092280a^4b^{11})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-1401879024a^5b^{11}+834451800a^6b^{11}-70200446680b^{12}+17543988644ab^{12}-6995348360a^2b^{12})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(6182616440a^3b^{12}-305965660a^4b^{12}+183579396a^5b^{12}+174281212b^{13}-274185520ab^{13})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(334423320a^2b^{13}-35709520a^3b^{13}+23535820a^4b^{13}-3132760b^{14}+7297080ab^{14})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(-2042040a^2b^{14}+1623160a^3b^{14}+38488b^{15}-47600ab^{15}+52360a^2b^{15}-289b^{16})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}+$$

$$+\frac{131072(595ab^{16}+b^{17})}{\left[\prod_{\varpi=1}^{18}\{a-b-(2\varpi-1)\}\right]\left[\prod_{\varrho=1}^{17}\{a-b+(2\varrho-1)\}\right]}\Bigg\}\Bigg]$$

On simplification the result (8) is derived.

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Some New Properties of Generalized Polynomials and $\bar{H}$-Function Associated with Feynman Integrals

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Abstract - In the present paper we study the integrals involving generalized polynomials (multivariable) and the $\bar{H}$-function. The $\bar{H}$-function was proposed by Inayat-Hussain which contain a certain class of Feynman integrals, the exact partition function of the Gaussian model in statistical mechanics and several other functions as its particular cases. Our integrals are unified in nature and act as key formulae from which we can derive as particular cases, integrals involving a large number of simpler special functions and polynomials. For the sake of illustration, we give here some particular cases of our main integral which are also new and of interest by themselves. At the end, we give applications of our main findings by interconnecting them with the Riemann–Liouville type of fractional integral operator. The results obtained by us are basic in nature and are likely to find useful applications in several fields notably electricals networks, probability theory and statistical mechanics.

Keywords : *feynman integrals, $\bar{H}$ - function, generalized polynom ials, fractional integral operator.*

GJSFR-F Classification : *MSC 2010: 08A40, 81Q30*

SOME NEW PROPERTIES OF GENERALIZED POLYNOMIALS AND H-FUNCTION ASSOCIATED WITH FEYNMAN INTEGRALS

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Some New Properties of Generalized Polynomials and $\bar{H}$-Function Associated with Feynman Integrals

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Abstract - In the present paper we study the integrals involving generalized polynomials (multivariable) and the $\bar{H}$ -function. The $\bar{H}$- function was proposed by Inayat-Hussain which contain a certain class of Feynman integrals, the exact partition function of the Gaussian model in statistical mechanics and several other functions as its particular cases. Our integrals are unified in nature and act as key formulae from which we can derive as particular cases, integrals involving a large number of simpler special functions and polynomials. For the sake of illustration, we give here some particular cases of our main integral which are also new and of interest by themselves. At the end, we give applications of our main findings by interconnecting them with the Riemann–Liouville type of fractional integral operator. The results obtained by us are basic in nature and are likely to find useful applications in several fields notably electricals networks, probability theory and statistical mechanics.

Keywords : *feynman integrals, $\bar{H}$ -function, generalized polynom ials, fractional integral operator.*

I. Introduction

Feynman path integrals are reformulation of quantum mechanics and are more fundamental than the conventional one in terms of operators because in the domain of quantum cosmology the conventional formulation may fail but Feynman path integrals still apply [6]. Inayat-Hussain [9] has pointed out the usefulness of Feynman integrals in the study and development of simple and multiple variable hypergeometric series which in turn are very useful in statistical mechanics. Hussain has introduced in another paper [10] the $\bar{H}$-function which is a new generalization of the familiar H-function of Fox [4]. The $\bar{H}$-function contains the exact partition function of the Gaussian model in statistical mechanics, functions useful in testing hypothesis and several others as its special cases.

The $\bar{H}$-function has been defined and represented as follows [2].

$$\bar{H}_{P,Q}^{M,N}[z] = \bar{H}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right] = \frac{1}{2\pi\omega}\int_{-i\infty}^{+i\infty}\phi(\xi)z^{\xi}d\xi \tag{1.1}$$

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Where
$$\phi(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=M+1}^{Q}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

which contains fractional powers of some of the gamma functions. Here, and throughout the paper $a_j(j=1,...,P)$, and $b_j(j=1,...,Q)$ are complex parameters, $\alpha_j\geq 0(j=1,...,P)$, $\beta_j\geq 0(j=1,...,Q)$ (not all zero simultaneously) and the exponents $A_j(j=1,...,N)$ and $B_j(j=M+1,...,Q)$ can take non-integer values.

The contour in (1.1) is along imaginary axis $\text{Re}(\xi)=0$. It is suitably indented in order to avoid the singularities of the gamma functions and to keep those singularities on appropriate sides. Again, for $A_j(j=1,...,N)$ not an integer, the poles of the gamma functions of the numerator in (1.2) are converted to branch points. However, as long as there is no coincidence of poles from any $\Gamma(b_j-\beta_j\xi)$ (j = 1,…,M) and $\Gamma(1-a_j-\alpha_j\xi)$ (j = 1,…,N) pair, the branch cuts can be chosen so that the path of integration can be distorted in the usual manner.

Evidently, when the exponents A_j and B_j all take an integer values, the $\bar{H}$-function reduces to the well known Fox's H-function [4].

The following sufficient conditions for the absolute convergence of the defining integral for

$\bar{H}$-function given by equation (1.1) have been given by Buschman and Srivastava[2].

$$\theta=\sum_{j=1}^{M}|\beta_j|+\sum_{j=1}^{N}|A_j\alpha_j|-\sum_{j=M+1}^{Q}|B_j\beta_j|-\sum_{j=N+1}^{P}|\alpha_j|>0, \tag{1.3}$$

and
$$|\arg z|<\frac{1}{2}\theta\pi\,. \tag{1.4}$$

where θ is given by (1.3).

The behaviour of the $\bar{H}$-function for small values of $|z|$ follows easily from a result recently given by Rathie [13, p. 306, eq. (6.9)], we have

$$\bar{H}_{P,Q}^{M,N}[z]=o\left(|z|^{\alpha}\right),\ \alpha=\underset{1\leq j\leq M}{Min.}\left\{\text{Re}\left(b_j/\beta_j\right)\right\}\text{ for small }|z|. \tag{1.5}$$

Investigations of the convergence conditions, all possible types of contours, type of critical points of the integrand of (1.1), etc. can be made by an interested reader by following analogous techniques given in the well known works of Braaksma [1], Hai and Yakubovich [8]. We however omit the details.

Srivastava ([14], P.185,eq.(7)) has defined and introduced the generalized polynomials (multivariable)

Ref.

4. Fox C., The G and H functions as symmetrical Fourier kernels, Trans. Amer. Math. Soc., 98(1961), 395-429.

$$S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[x_1,\ldots,x_r\right]=\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1k_1}}{k_1!}\cdots\frac{(-n_r)_{m_rk_r}}{k_r!}A[n_1,k_1;\ldots;n_r,k_r]x^{k_1}\ldots,x^{k_r} \tag{1.6}$$

Notes

where $n_i = 0,1,2,\ldots (I = 1,\ldots,r)$, $m_1,\ldots,m_r$ are an arbitrary positive integers and the coefficients $A[n_1,k_1;\ldots;n_r,k_r]$ are arbitrary constants, real or complex .

II. Integrals Required

The following integrals will be required in our results

$$\int_0^b x^{\lambda-1}(b-x)^{\eta-1}\,dx=b^{\lambda+\eta-1}B(\lambda,\eta)\quad ;\quad \mathrm{Re}(\lambda)>0,\ \mathrm{Re}(\eta)>0 \tag{2.1}$$

$$\int_0^u x^{\mu-1}(u-x)^{\upsilon-1}e^{\alpha x}\,dx= B(\upsilon,\mu)u^{\mu+\upsilon-1}\,{}_1F_1(\mu;\mu+\upsilon;\alpha u)\quad ; \quad \mathrm{Re}(\mu)>0,\ \mathrm{Re}(\upsilon)>0 \tag{2.2}$$

$$\int_0^u x^{-\mu-1}(u-x)^{\mu-1}e^{-\alpha/x}\,dx=\alpha^{-\mu}u^{\mu-1}\Gamma(\mu)e^{-\alpha/u}\quad ;\quad \mathrm{Re}(\mu)>0,\ u>0 \tag{2.3}$$

III. Main Integrals

a) First Integral

We shall establish the following integral formulas :

$$\int_0^b x^{\rho-1}(b-x)^{\sigma-1}\bar{H}_{P,Q}^{M,N}\left[zx^u(b-x)^v\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]\times$$

$$S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[z_1x^{\lambda_1}(b-x)^{\mu_1},\ldots,z_rx^{\lambda_r}(b-x)^{\mu_r}\right]\mathrm{d}x$$

$$=b^{\rho+\sigma+\sum_{i=1}^r(\lambda_i+\mu_i)k_i-1}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1k_1}}{k_1!}\cdots\frac{(-n_r)_{m_rk_r}}{k_r!}A[n_1,k_1;\ldots;n_r,k_r]\prod_{i=1}^r z_i^{k_i}$$

$$\bar{H}_{P+2,Q+1}^{M,N+2}\left[zb^{u+v}\left|\begin{matrix}\left(1-\rho-\sum_{i=1}^r\lambda_ik_i,u;1\right),\left(1-\sigma-\sum_{i=1}^r\mu_ik_i,v;1\right),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},\left(1-\rho-\sigma-\sum_{i=1}^r(\lambda_i+\mu_i)k_i,u+v;1\right)\end{matrix}\right.\right] \tag{3.1}$$

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valid under the conditions

(i) $u \geq 0, v \geq 0$ (not both zero simultaneously

(ii) $\text{Re}(\rho) + \sum_{i=1}^{r} \lambda_i \left(\frac{n_i}{m_i}\right) + u \min_{1 \leq j \leq M} \left[\text{Re}(b_j / \beta_j)\right] > 0$

$$\text{Re}(\sigma) + \sum_{i=1}^{r} \mu_i \left(\frac{n_i}{m_i}\right) + v \min_{1 \leq j \leq M} \left[\text{Re}(b_j / \beta_j)\right] > 0$$

(iii) The $\bar{H}$ - function occurring in (3.1) satisfy conditions corresponding appropriately to those given by (1.3) and (1.4).

b) Second Integral

$$\int_0^b x^{\rho-1} (b-x)^{\sigma-1} e^{\alpha x} \bar{H}_{P,Q}^{M,N} \left[zx^u (b-x)^v e^{\delta x} \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{matrix} \right. \right] \times$$

$$S_{n_1,\dots,n_r}^{m_1,\dots,m_r} \left[z_1 x^{\lambda_1} (b-x)^{\mu_1}, \dots, z_r x^{\lambda r} (b-x)^{\mu_r} \right] \text{dx}$$

$$= b^{\rho+\sigma+\sum_{i=1}^{r}(\lambda_i+\mu_i)k_i - 1} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1 k_1}}{k_1!} \dots \frac{(-n_r)_{m_r k_r}}{k_r!} \frac{b^t}{t!} A[n_1, k_1; \dots; n_r, k_r] \prod_{i=1}^{r} z_i^{k_i}$$

$$\bar{H}_{P+3,Q+2}^{M,N+3} \left[zb^{u+v} \left| \begin{matrix} \left(1-\rho-\sum_{i=1}^{r}\lambda_i k_i - t, u; 1\right), \left(1-\sigma-\sum_{i=1}^{r}\mu_i k_i, v; 1\right), (-\alpha, \delta; r), (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (1-\alpha, \delta; r), \left(1-\rho-\sigma-\sum_{i=1}^{r}(\lambda_i+\mu_i)k_i - t, u+v; 1\right) \end{matrix} \right. \right]$$

(3.2)

where the $\bar{H}$-function occurring in the left hand side of (3.2) stands for the new generalized H-function defined by (1.1) and $s_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[x_1,\dots,x_r\right]$ stands for the generalized polynomials given in(1.6).

The above integral holds true under the following conditions:-

(i) Re(ρ,σ)>0, u,v ≥ 0,

(ii) when min(μ_i, λ_i) ≥ 0 for all I = 1,…,r (not all zero simulteneously).

I $\text{Re}(\rho) + \sum_{i=1}^{r} \lambda_i \left[\frac{n_i}{m_i}\right] + u \min_{1 \leq j \leq M} \text{Re}(b_j / B_j) > 0$

Notes

$$\text{II} \quad \mathrm{Re}(\sigma)+\sum_{i=1}^{r}\mu_i\left[\frac{n_i}{m_i}\right]+v\min_{1\le j\le M}\mathrm{Re}(b_j/B_j)>0$$

(iii) when $\max(\mu_i,\lambda_i)<0$ for all I = 1,…,r (not all zero simulteneously).

$$\text{I} \quad \mathrm{Re}(\rho)+\sum_{i=1}^{r}\lambda_i\left[\frac{n_i}{m_i}\right]+u\min_{1\le j\le M}\mathrm{Re}(b_j/B_j)>0$$

$$\text{II} \quad \mathrm{Re}(\sigma)+\sum_{i=1}^{r}\mu_i\left[\frac{n_i}{m_i}\right]+v\min_{1\le j\le M}\mathrm{Re}(b_j/B_j)>0$$

(iv) when $\lambda_i\ge 0$ and $\mu_i<0$ inequalities I and IV are satisfied.

(v) when $\lambda_i<0$ and $\mu_i\ge 0$ inequalities II and III are satisfied.

c) Third Integral

$$\int_0^b x^{-\rho-1}(b-x)^{\rho-1}e^{-\alpha/x}\bar{H}_{P,Q}^{M,N}\left[ze^{-\delta/x}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},\end{matrix}\right.\right]\times$$

$$S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[z_1x^{-\lambda_1}(b-x)^{\lambda_1},\dots,z_rx^{-\lambda r}(b-x)^{\lambda_r}\right]\mathrm{dx}$$

$$= b^{\rho+\sum_{i=1}^{r}\lambda_ik_i-1}e^{-\alpha/b}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1k_1}}{k_1!}\cdots\frac{(-n_r)_{m_rk_r}}{k_r!}A[n_1,k_1;\dots;n_r,k_r]\prod_{i=1}^{r}z_i^{k_i}\Gamma(\rho+\sum_{i=1}^{r}\lambda_ik_i)\times$$

$$\bar{H}_{P+1,Q+1}^{M,N+1}\left[zb^{u+v}\left|\begin{matrix}\left(1-\alpha,\delta;\rho+\sum_{i=1}^{r}\lambda_ik_i\right),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(-\alpha,\delta;\rho+\sum_{i=1}^{r}\lambda_ik_i)\end{matrix}\right.\right] \quad (3.3)$$

The above result is valid under the following conditions :-

(i) $\mathrm{Re}(\alpha)>0,\delta>0$

(ii) when $\lambda_i>0,\rho>0$

when $\lambda_i<0,\rho+\sum_{i=1}^{r}\lambda_i\left[\frac{n_i}{m_i}\right]>0$

PROOF :- To establish the integral (3.1), we express the generalized polynomials occurring in the left hand side in the series form given by (1.6) and the $\bar{H}$-function in terms of Mellin-Barnes contour integral given by (1.1) and then interchanging the order of summation and integration (which is permissible under the conditions stated with (3.1)) so that the left hand side of (3.1) (say Δ) assume the following after little simplification

$$\Delta = \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1k_1}}{k_1!} \cdots \frac{(-n_r)_{m_rk_r}}{k_r!} A[n_1,k_1;\ldots;n_r,k_r] \prod_{i=1}^{r} z_i^{k_i} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \theta(s) z^s$$

$$\left\{ \int_0^b x^{\rho+\sum_{i=1}^{r}\lambda_i k_i + us - 1} (b-x)^{\sigma + \sum_{i=1}^{r}\mu_i k_i + vs - 1} dx \right\} ds \qquad (3.4)$$

On evaluating the inner integral occurring in (3.4) by using Eulerian integral (2.1) and on reinterpreting the Mellin-Barnes contour integral in terms of the $\bar{H}$- function given by (1.1), we easily arrive at the desired result (3.1).

Similarly the integrals (3.2) and (3.3) can also be established in the same manner by using the integral (2.2) and the integral (2.3) respectively.

IV. Special Case

(i) If we take $A(n_1,k_1;\ldots;n_r,k_r) = \prod_{i=1}^{r} A(n_i,k_i)$ in the definition of generalized polynomials occurring in the left hand side of the integral (3.1), we get

$$\int_0^b x^{\rho-1}(b-x)^{\sigma-1} \bar{H}_{P,Q}^{M,N}\left[z x^u (b-x)^v \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] \times \prod_{i=1}^{r} S_{n_i}^{m_i}\left[z_i x^{\lambda_i}(b-x)^{\mu_i} \right] \mathrm{dx}$$

$$= b^{\rho+\sigma+\sum_{i=1}^{r}(\lambda_i+\mu_i)k_i - 1} \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1k_1}}{k_1!} \cdots \frac{(-n_r)_{m_rk_r}}{k_r!} \prod_{i=1}^{r} A(n_i,k_i) \prod_{i=1}^{r} z_i^{k_i}$$

$$\bar{H}_{P+2,Q+1}^{M,N+2}\left[z b^{u+v} \left| \begin{matrix} \left(1-\rho-\sum_{i=1}^{r}\lambda_i k_i, u; 1\right), \left(1-\sigma-\sum_{i=1}^{r}\mu_i k_i, v; 1\right), (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q}, \left(1-\rho-\sigma-\sum_{i=1}^{r}(\lambda_i+\mu_i)k_i, u+v; 1\right) \end{matrix} \right. \right] \qquad (4.1)$$

(a) Taking i = 2 in our result (4.1), we obtain the result discussed by Gupta and Soni [7, p.100, eq.(2.1)].

Ref.

7. Gupta K.C. and Soni R.C. New Properties of a generalization of Hypergeometric Series Associated with Feynman Integrals, KYUNGPOOK Math. J. 41(2001), 97-104.

$$\int_0^b x^{\rho-1}(b-x)^{\sigma-1}\bar{H}_{P,Q}^{M,N}\left[zx^u(b-x)^v\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]\times S_{n_1}^{m_1}\left[z_1x^{\lambda_1}(b-x)^{\mu_1}\right]S_{n_2}^{m_2}\left[z_2x^{\lambda_2}(b-x)^{\mu_2}\right]dx$$

$$=b^{\rho+\sigma-1}\sum_{k_1=0}^{[n_1/m_1]}\sum_{k_2=0}^{[n_2/m_2]}\frac{(-n_1)_{m_1k_1}}{k_1!}\frac{(-n_2)_{m_2k_2}}{k_2!}A[n_1,k_1;n_2,k_2]z_1^{k_1}z_2^{k_2}b^{(\lambda_1+\mu_1)k_1+(\lambda_2+\mu_2)k_2}$$

$$\bar{H}_{P+2,Q+1}^{M,N+2}\left[zb^{u+v}\left|\begin{matrix}(1-\rho-\lambda_1k_1-\lambda_2k_2,u;1),(1-\sigma-\mu_1k_1-\mu_2k_2,v;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(1-\rho-\sigma-(\lambda_1+\mu_1)k_1-(\lambda_2+\mu_2)k_2,u+v;1)\end{matrix}\right.\right]$$

(4.1.1)

(b) Taking i = 1 in the result (4.1), we obtain the result discussed by Gupta and Soni [7, p.101, eq.(3.1)].

$$\int_0^b x^{\rho-1}(b-x)^{\sigma-1}\bar{H}_{P,Q}^{M,N}\left[zx^u(b-x)^v\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]\times S_{n_1}^{m_1}\left[z_1x^{\lambda_1}(b-x)^{\mu_1}\right]dx$$

$$=b^{\rho+\sigma-1}\sum_{k_1=0}^{[n_1/m_1]}\frac{(-n_1)_{m_1k_1}}{k_1!}A[n_1,k_1]z_1^{k_1}b^{(\lambda_1+\mu_1)k_1}$$

$$\bar{H}_{P+2,Q+1}^{M,N+2}\left[zb^{u+v}\left|\begin{matrix}(1-\rho-\lambda_1k_1,u;1),(1-\sigma-\mu_1k_1,v;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(1-\rho-\sigma-(\lambda_1+\mu_1)k_1,u+v;1)\end{matrix}\right.\right]$$

(4.1.2)

In the similar manner if we put i = 2 and i = 1 in both the integrals (3.2) and (3.3), we obtain the known results given by Mishra Rupakshi [11, p.42, eq.(1.3.1)] and [11, p.43, eq. (1.3.2)].

(iv) Taking the exponents $A_j = B_j = 1$ in the $\bar{H}$- function occurring in the left hand side of the integrals (3.1), (3.2) and (3.3) we get the results in terms of well known Fox's H-function.

The importance of the main integral of the present paper lies in its many fold generality. Again several integrals obtained by various authors and lying scattered in the literature also follow as simple special cases of our findings. Thus, if we reduce the $\bar{H}$-

Ref.

7. Gupta K.C. and Soni R.C. New Properties of a generalization of Hypergeometric Series Associated with Feynman Integrals, KYUNGPOOK Math. J. 41(2001), 97-104.

function occurring on the left hand side of (4.1.1) to the Fox's H function and the generalized polynomials $s_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x_1,\dots,x_r]$ occurring therein to unity, we get a known integral [5,p.202].

V. Applications

We shall define the Rieman – Liouville fractional derivative of a function f(x) of order σ (or, alternatively, $-\sigma^{\text{th}}$ order fractional integral) [3,p.181;12,p.49] by (5.1)

$$ {}_aD_x^{\sigma}\{f(x)\} = \begin{cases} \dfrac{1}{\Gamma(-\sigma)}\displaystyle\int_a^x (x-t)^{-\sigma-1} f(t)\,dt, \operatorname{Re}(\sigma)<0, \\ \dfrac{d^q}{dx^q} D_x^{\sigma-q}\{f(x)\}, (q-1)\le \operatorname{Re}(\sigma)<q, \end{cases} \tag{5.1} $$

where q is a positive integer and the integral exists.

For simplicity the special case of the fractional derivative operator ${}_aD_x^{\sigma}$ when a = 0 will be written as D_x^{σ}. Thus we have

$$ D_x^{\sigma} \equiv {}_0D_x^{\sigma} \tag{5.2} $$

Now by setting b = x and x = t in the main integral (3.1), it can be written as the following fractional integral formula :

$$ D_x^{-\sigma}\left\{ t^{\rho-1}\bar{H}_{P,Q}^{M,N}\left[zt^u(x-t)^v \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}, \end{matrix} \right.\right] \times S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[z_1t^{\lambda_1}(x-t)^{\mu_1}\dots z_rt^{\lambda_r}(x-t)^{\mu_r}\right]\right\} $$

$$ = \frac{x^{\rho+\sigma-1}}{\Gamma(\sigma)}\sum_{k_1=0}^{[n_1/m_1]}\dots\sum_{k'=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1k_1}\dots(-n_r)_{m'k'}}{k_1!\dots k_r!}A[n_1,k_1;\dots;n_r,k_r]\prod_{i=1}^{r} z_i^{k_i}x^{(\lambda_i+\mu_i)k_i}\times $$

$$ \bar{H}_{P+2,Q+1}^{M,N+2}\left[zx^{u+v}\left| \begin{matrix} \left(1-\rho-\sum_{i=1}^{r}\lambda_ik_i,u;1\right),\left(1-\sigma-\sum_{i=1}^{r}\mu_ik_i,v;1\right),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},\left(1-\rho-\sigma-\sum_{i=1}^{r}(\lambda_i+\mu_i)k_i,u+v;1\right) \end{matrix}\right.\right] \tag{5.3} $$

where $\operatorname{Re}(\sigma)>0$ and all the conditions of validity mentioned with (3.1) are satisfied.

The fractional integral formula given by (5.3) is also quite general in nature and can easily yield Riemann-Liouville fractional integrals of a large number of simpler functions polynomials merely by specializing the parameters of $\bar{H}$-function and $S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x_1,\dots,x_r]$, occurring in it which may find applications in electromagnetic theory and probability.

Ref.

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(1, 2) - Domination in Some Harmonius Graphs

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Abstract - In this paper we discuss (1, 2) - domination in some harmonious graphs namely ladder graph, wheel graph and tetrahedral graph.

Keywords : dominating set, domination number, (1,2) - dominating set, (1,2) - domination number.

GJSFR-F Classification : MSC 2010: 05C10, AMS: 05C69

1, 2 - DOMINATION IN SOME HARMONIUS GRAPHS

Strictly as per the compliance and regulations of :

(1, 2) - Domination in Some Harmonius Graphs

N. Murugesan [α] & Deepa.S.Nair [σ]

Abstract - In this paper we discuss (1, 2) - domination in some harmonious graphs namely ladder graph, wheel graph and tetrahedral graph.

Keywords : dominating set, domination number, (1,2) - dominating set, (1,2) - domination number.

I. Introduction

Let G = (V,E) be a simple graph. A subset D of V is a *dominating set* of G if every vertex of V- D is adjacent to a vertex of D. The *domination number* of G, denoted by γ(G), is the minimum cardinality of a dominating set of G.

A (1,2) - dominating set in a graph G = (V,E) is a set S having the property that for every vertex v in V - S there is atleast one vertex in S at distance 1 from v and a second vertex in S at distance almost 2 from v. The order of the smallest (1,2) - dominating set of G is called the *(1,2) - domination number* of G and we denote it by $\gamma_{(1,2)}$.

A harmonius graph is a connected labeled graph with n graph edges in which all graph vertices can be labeled with distinct integers (mod n) so that the sums of the pairs of numbers at the ends of each graph edge are also distinct (mod n). The ladder graph and wheel graph are harmonius. The n-ladder graph can be defined as $P_2 \square P_n$, where P_n is a path graph. It is therefore equal to the 2×n grid graph. This graph looks like a ladder, having two rails and n rungs between them. A wheel graph Wn of order n, contains a cycle of order n-1, and for which every graph vertex in the cycle is connected to one other graph vertex. *The tetrahedral graph* is the platonic graph that is the unique polyledral graph on four nodes which is also the complete graph K_4 and therefore the wheel graph W_4.

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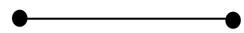
II. (1,2) - Domination in Ladder Graphs

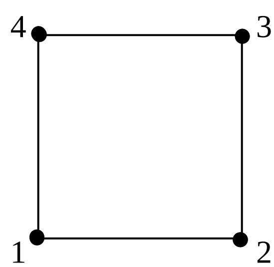
In this section we consider ladder graphs of order upto 10 and find out their domination number and (1,2) - domination number.

i) For n = 1,



This is a graph of order 2. (1,2) - domination number is defined for graphs of order atleast 3.

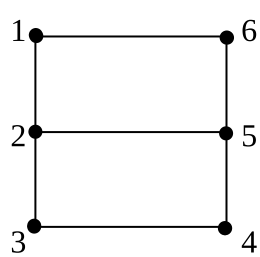
For n = 2,



{1,2} is a dominating set and also a (1,2) - dominating set.{1,2} is a dominating set.

∴ $\gamma_{(1,2)} = 2$ and $\gamma = 2$.

For n = 3,



{1,2,3} is a (1,2) - dominating set.{2,5} is a dominating set.

∴ $\gamma_{(1,2)} = 3$ and $\gamma = 2$.

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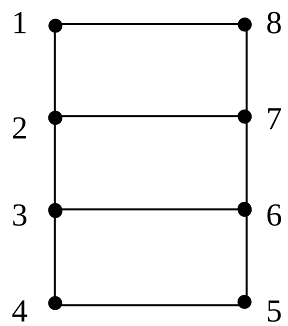
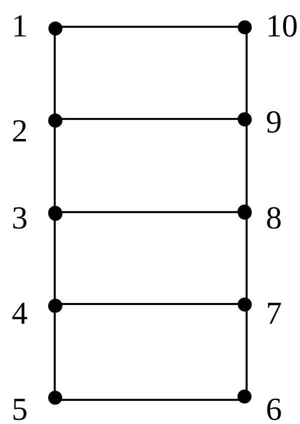
For n = 4,



{1,2,3,4} is a (1,2) - dominating set.{1,3,5,7} is a dominating set.

$\gamma_{(1,2)} = 4$ and $\gamma = 4$.

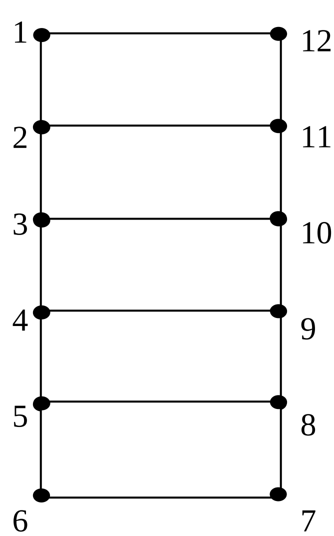
For n = 5,



{1,2,3,4,5} is a (1,2) - dominating set.{1,3,6,8} is a dominating set.

$\gamma_{(1,2)} = 5$ and $\gamma = 4$.

For n = 6,

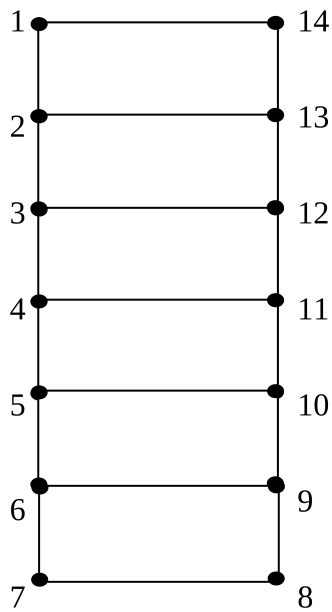


{1,2,3,4,5,6} is a (1,2) - dominating set.{1,3,5,7,9,11} is a dominating set.

$\gamma_{(1,2)} = 6$ and $\gamma = 6$.

Notes

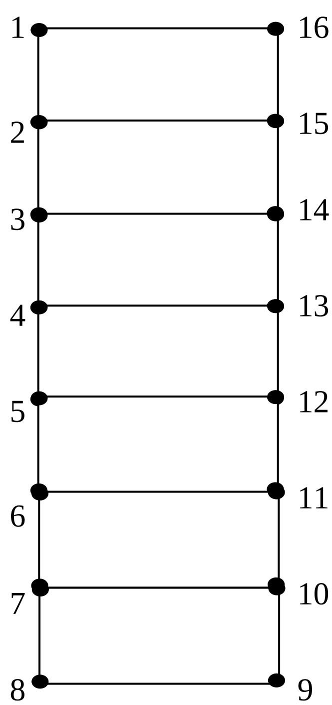
For n = 7,



{1,2,3,4,5,6,7} is a (1,2) - dominating set.{1,4,6,8,11,13} is a dominating set.

$\gamma_{(1,2)} = 7$ and $\gamma = 6$.

For n = 8,

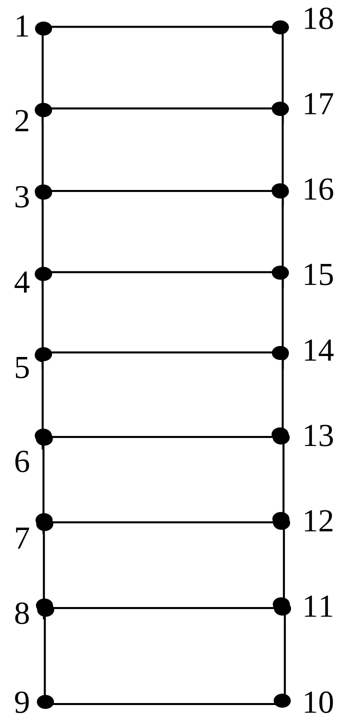


{1,2,3,4,5,6,7,8} is a (1,2) - dominating set.{1,3,5,7,9,11,13,15} is a dominating set.

$\gamma_{(1,2)} = 8$ and $\gamma = 8$.

Notes

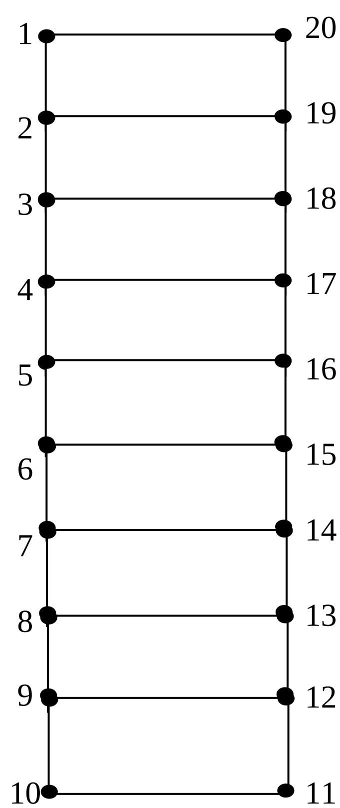
For n = 9,



{1,2,3,4,5,6,7,8,9} is a (1,2) - dominating set.{1,3,5,7,10,13,14,15} is a dominating set.

$\gamma_{(1,2)} = 9$ and $\gamma = 8$.

For n = 10,



{1,2,3,4,5,6,7,8,9,10} is a (1,2) - dominating set.{1,3,5,7,9,11,13,15,17,19} is a dominating set.

$\gamma_{(1,2)} = 10$ and $\gamma = 10$.

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From the above examples we have the following theorems.

Theorem 2.1

For a ladder graph L_n, (1,2)- domination number is n. That is, $\gamma_{(1,2)}(L_n) = n$.

Proof :

For a ladder graph L_n, there are 3n-2 edges and 2n vertices. Also there are n vertices in both the rails. Suppose a vertex v_1 in the first rail is adjacent to a vertex u_1 in the second rail. Then all the remaining vertices in the first rail will be at distance greater than 1 from u_1. So to form a (1,2) - dominating set we have to include all the vertices in one rail. So the (1,2) - domination number is n.

Theorem 2.2

For a ladder graph L_n with n even, $\gamma(L_n)= n$.

Proof :

Each L_n has 3n-2 edges and 2n vertices. If n is even, the vertices in the inner rungs, that is, $\frac{n}{2}$ rungs can form a dominating set. So the number of vertices in the dominating set will be n, since each rung contains two vertices. Hence $\gamma(L_n)= n$.

Theorem 2.3

For a ladder graph L_n with n odd, $\gamma(L_n)= n-1$.

Proof :

Each L_n has 3n-2 edges and 2n vertices. Since n is odd, the vertices in the middle rung will be at equal distance from the vertices in the outer rungs. So if we take the two vertices of the middle rung and one vertex each from the alternate rungs, that set will form a dominating set. So since there are n rungs, the set will consist of n-1 vertices. So the domination number is n-1. Hence $\gamma(L_n)= n-1$.

III. Relation Between Domination Number and (1,2)-Domination Number of Ladder Graphs

Lemma 3.1([5],p.782)

In a graph G, domination number is less than or equal to (1,2)-domination number.

Ref.

5. Murugesan N. and Deepa S. Nair, (1,2) - domination in Graphs, J. Math. Comput. Sci., Vol.2, 2012, No.4, 774-783.

Notes

Proof:

Let G be a graph and D be its dominating set. Then every vertex in V-D is adjacent to a vertex in D. That is, in D, for every vertex u, there is a vertex which is at distance 1 from u. But it is not necessary that there is a second vertex at distance atmost 2 from u. So if we find a (1,2)- dominating set ,it will contain more vertices or atleast equal number of vertices than the dominating set. So the domination number is less than or equal to (1,2)- domination number.

This is true for ladder graphs also.

From the examples discussed in section 2 we have the following theorems

Theorem 3.1

For a ladder graph L_n with n even, the domination number and (1,2) - domination number are equal.

Proof :

In a ladder graph, there are 2n vertices and 3n-2 edges. The n vertices in one rail form a (1.2) - dominating set. If n is even the number of inner rungs will be $\frac{n}{2}$ even. And the vertices of these inner rungs form a dominating set. Since each rung contains 2 vertices, the dominating set will consist of n vertices. Hence the domination number and (1,2) - domination number are equal.

Theorem 3.2

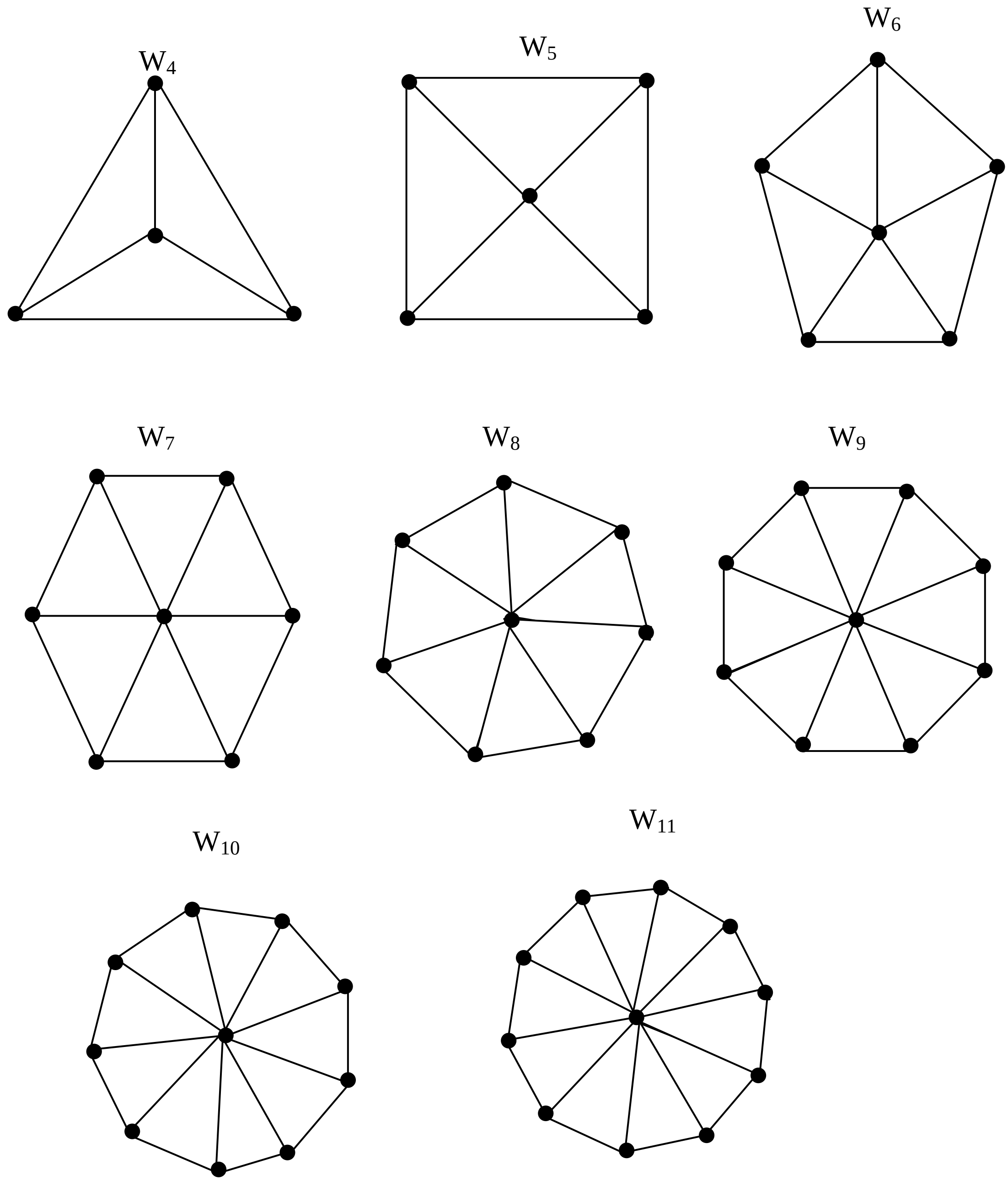
For n odd, the domination number of a ladder graph L_n is less than the (1.2) - domination number.

Proof :

For a ladder graph with n odd, the number of inner rungs will be (n-2), odd. The vertices of the middle rung and one vertex each from the alternate rungs will form a dominating set. So altogether we will get (n-1) vertices. That is, the domination number is (n-1). But the (1,2) - domination number is n. Hence the domination number is less than the (1,2) - domination number.

IV. (1,2) - Domination in Wheel Graphs

Consider the following wheel graphs.



Theorem 4.1

The domination number of a wheel graph is 1. That is, $\gamma(W_n) = 1$

Proof :

In a wheel graph it contains a cycle of order n - 1 every graph vertex in the cycle is connected to one other graph vertex. In a wheel W_n, there is a vertex with degree n-1. So that vertex is adjacent to all other vertices. Hence the domination number is one.

Notes

Theorem 4.2

For a wheel graph W_n, (1,2) - domination number is 2.

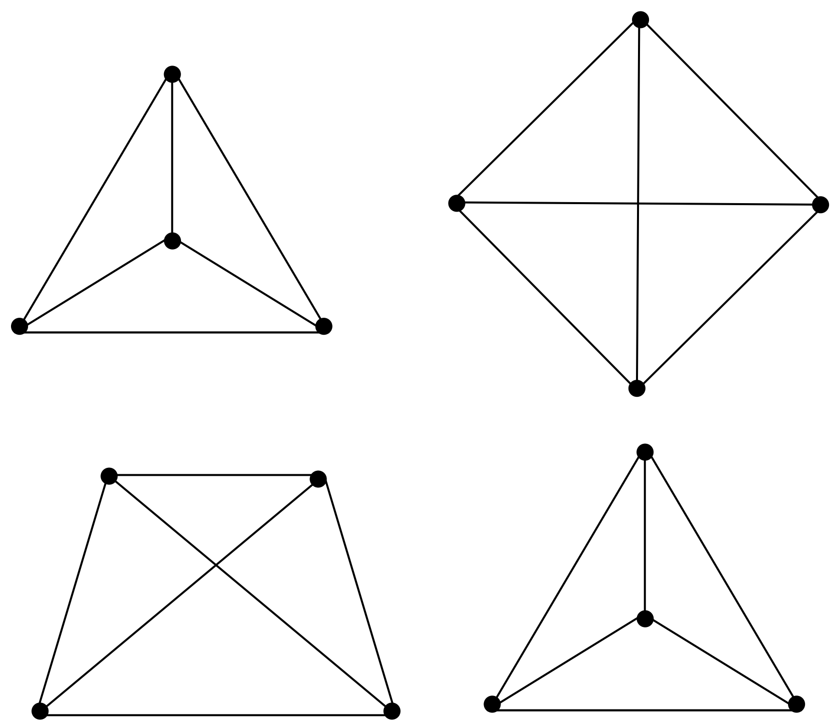
That is, $\gamma_{(1,2)}(W_n) = 2$.

Proof :

The dominating set of a wheel graph consists of only one vertex. By the definition of (1,2) - dominating set, it should contain atleast two vertices. So if we take the central vertex and any one of the vertex from the cycle, that will form a (1,2) - dominating set. The cardinality of the (1,2) - dominating is 2. Hence $\gamma_{(1,2)}(W_n) = 2$.

V. (1,2)- Domination in the Tetrahedral Graphs

Consider the following graphs



Theorem 5.1

For a tetrahedral graphs, there does not exist any (1,2) - dominating set.

Proof :

A tetrahedral graph is also a complete graph K_4. We proved in paper [5] that (1,2) - domination is not possible in complete graphs. We cannot find a (1,2) - dominating set in tetrahedral graphs.

Ref.
5. Murugesan N. and Deepa S. Nair, (1,2) - domination in Graphs, J. Math. Comput. Sci., Vol.2, 2012, No.4, 774-783.

VI. Conclusion

Here we discussed the (1,2)-domination in three types of harmonius graphs. The domination number of ladder graphs is less than or equal to (1.2) - domination number which agrees to the result of previous paper [5]. (1,2) - domination is not possible in tetrahedral graphs.

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Pathway Fractional Integral Operator Concerning to Polynomials

By Saroj Kumari
Singhania University

Abstract - We have made an attempt to study a pathway fractional integral operator concerning to pathway model and pathway probability density for product of some special functions with a general class of polynomials. Our results are quite general in nature and hence compass a large number of results hitherto in the literature.

Keywords : pathway fractional integral operator, fox h-function, m-series, a general class of polynomials, mittag-leffler function.

GJSFR-F Classification : MSC 2010: 26A33, 11B83

PATHWAY FRACTIONAL INTEGRAL OPERATOR CONCERNING TO POLYNOMIALS

Ref.
13. Nair, Seema S., Pathway fractional integration operator, Fract. Cal. Appl. Anal., 12(3) (23009), 237-259.

Pathway Fractional Integral Operator Concerning to Polynomials

Saroj Kumari

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Keywords : pathway fractional integral operator, fox h-function, m-series, a general class of polynomials, mittag-leffler function.

I. Introduction

The pathway fractional integral operator introduced by Nair [13] as follows

$$(P_{0+}^{(\eta,\alpha)}f)(x) = x^{\eta}\int_0^{\left[\frac{x}{a(1-\alpha)}\right]}\left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{1-\alpha}} f(t)\,dt \tag{1.1}$$

where $f(x) \in L\ (a, b)$, $\eta \in C$, $a > 0$, $R(\eta) > 0$, $\alpha < 1$ (pathway parameter).

The pathway model introduced by Mathai [ ] and studied by Mathai and Haubold ([10], [11]). For real α, the pathway model for scalar random variables is represented by the following probability density function (p.d.f.)W.

$$f(x) = c\,|x|^{\gamma-1}\,[1-a\,(1-\alpha)|x|^{\delta}]^{\frac{\beta}{1-\alpha}}, \tag{1.2}$$

$\gamma > 0, \delta > 0, \beta > 0,\ x \in (-\infty,\infty), [1-a(1-\alpha)|x|^{\delta}] > 0,$ C is the normalizing constant and α is called the pathway parameter. For real α, the normalizing constant is as follows:

$$c = \frac{1}{2}\frac{\delta[a(1-\alpha)]^{\frac{\gamma}{\delta}}\,\Gamma\left(\frac{\gamma}{\delta}+\frac{\beta}{1-\alpha}+1\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{1-\alpha}+1\right)}, \quad \alpha < 1, \tag{1.3}$$

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$$= \frac{1}{2} \frac{\delta [a(1-\alpha)]^{\frac{\gamma}{\delta}} \Gamma\left(\frac{\beta}{\alpha-1}\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{\alpha-1}-\frac{\gamma}{\delta}\right)}, \text{ for } \frac{1}{\alpha-1}-\frac{\gamma}{\delta} > 0, \alpha < 1, \tag{1.4}$$

$$= \frac{1}{2} \frac{\delta (a\beta)^{\frac{\gamma}{\delta}}}{\Gamma\left(\frac{\gamma}{\delta}\right)} \quad \text{for} \quad \alpha \to 1. \tag{1.5}$$

For $\alpha < 1$, it is a finite range density with $[1 - a(1-\alpha) — x —^{\beta}] > 0$ and (1.2) remains in the extended generalized type - 1 beta family. The pathway density in (1.2) for $\alpha < 1$, includes the extended type - 1 beta density, the triangular density, the uniform density and many other p.d.f. For $\alpha > 1$, we have

$$f(x) = c|x|^{\gamma-1}[1+a(\alpha-1)|x|^{\delta}]^{-\frac{\beta}{\alpha-1}}, \tag{1.6}$$

where $\alpha > 1, \delta > 0, \beta \geq 0, x \in (-\infty, \infty)$, which is extended generalized type 2 beta model for real x. It includes the type – 2 beta density. The F-density, the Student- t density, the Cauchy density and many more.

Here it is considered only the case of pathway parameter $\alpha < 1$. For $\alpha \to 0$ (1.2) and (1.6) take the exponential form, since

$$\lim_{\alpha \to 1} c|x|^{\gamma-1}[1-a(1-\alpha|x|^{\delta}]^{\frac{\eta}{1-\alpha}} = \lim_{x \to 1} c|x|^{\gamma-1}[1+a(\alpha-1)|x|^{\delta}]^{-\frac{\eta}{\alpha-1}}$$

$$= c|x|^{\gamma-1} e^{-a\eta|x|^{\delta}}. \tag{1.7}$$

This includes the generalized Gamma-, the Weibull -, the Chi-square, the Laplace, and the Maxwell-Boltzmann and other related densities,

$$\text{when } \alpha \to 1 \left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{1-\alpha}} \to d^{-\frac{a\eta}{x}t} U,$$

the operator (1.1) reduces to the Laplace integral transform of f with parameter $\frac{a\eta}{x}$

$$(P_{0+}^{(\eta,1)} f)(x) = x^{\eta} \int_0^{\infty} e^{-\frac{a\eta}{x}t} f(t)\, dt$$

$$= x^{\eta} Lt\left(\frac{a\eta}{x}\right).$$

1. Chaurasia, V.B.L. and Ghiya, Neeti, Pathway fractional integral operator pertaining to special functions, Global J. Sci. Front. Res., 10(6) (ver. 1.0) (2010), 79-83.

when $\alpha = 0, a = 1$, then replacing η by $\eta-1$ in (1.1) the integral operator reduces to the Riemann-Liouville fractional integral operator.
Srivastava [15] introduced the general class of polynomials

$$S_n^m[x] = \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, x^{\ell},$$

$$= \psi_1(\ell) \quad \ell = 0,1,2,... \tag{1.9}$$

when m is an arbitrary positive integer and the coefficients $A_{n,\ell}(n,\ell \geq 0)$ are arbitrary constants, real or complex.

The following generalized M-series was introduced by Sharma and Jain [16]

$$_{\rho}\overset{\alpha',\beta'}{M}{}_{\sigma}(z) = \sum_{k=0}^{\infty} \frac{(a_1')_k \ldots (a_\rho')_k\, z^k}{(b_1')_k \ldots (b_\sigma')_k\, \Gamma(\alpha' k + \beta')},$$

$$= \psi_2(k) \tag{1.10}$$

where $z, \alpha', \beta' \in C, Re(\alpha') > 0, \forall z$ if $\rho \leq \sigma, |z| < (\alpha')^{\alpha'}$, for other details see [16].
The series representation of Fox H-function [6] was studied by Skinbinski [14]

$$H_{P,Q}^{M,N}\left[z \,\middle|\, \begin{matrix}(e_P, E_P)\\(f_Q, F_Q)\end{matrix}\right] = \sum_{h=1}^{N} \sum_{v=0}^{\infty} \frac{(-1)^v \chi(\xi)}{v!\, E_h} \left(\frac{1}{z}\right)^{\xi},$$

where
$$\xi = \frac{e_h - v - 1}{E_h} \text{ and } (h = 1, ..., N) \tag{1.11}$$

and

$$\chi(\xi) = \frac{\prod_{j=1}^{M} \Gamma(f_j + F_j \xi) \prod_{\substack{j=1\\j\neq h}}^{N} \Gamma(1 - e_j - E_j \xi)}{\prod_{j=m+1}^{Q} \Gamma(1 - f_j - F_j \xi) \prod_{j=N+1}^{P} \Gamma(e_j + \xi E_j)} \tag{1.12}$$

For convergence conditions and other details see [ ].
For the sake of brevity

$$T_1 = \sum_{1}^{N} E_1 - \sum_{N+1}^{P} E_i + \sum_{1}^{M} F_i - \sum_{N+1}^{Q} F_i \tag{1.13}$$

$$T_2 = \sum_{1}^{n} \alpha_i - \sum_{n+1}^{p} \alpha_i + \sum_{1}^{m} \beta_i - \sum_{m+1}^{q} \beta_i \tag{1.14}$$

Ref.

15. Srivastava, H.M., A contour integral involving Fox's H-function, Indian J. Math., 14 (1972), 1-6.

II. Main Results

Theorem 1. Let $\eta, \omega \in C, \alpha < 1, c, b, \in R, \mathrm{Re}(\beta) > 0, \mathrm{Re}(\delta) > 0, \mathrm{Re}\left(1+\frac{h}{1-\alpha}\right) > 0,$

$\mathrm{Re}\left(\omega+\delta\frac{f_j}{F_j}\right) > 0, \mathrm{Re}\left(\omega+\beta\frac{b'_j}{\beta'_j}\right) > 0, |\arg c| < \frac{1}{2}T_1\pi, |\arg b| < \frac{1}{2}T_2\pi, T_1, T_2 > 0, \rho \leq \sigma,$

$|d| < (\alpha')^{\alpha'}, \beta^* > 0, m'$ is an arbitrary positive integer and the coefficients $A_{n',R}(n', \ell \geq 0)$ are arbitrary constants, real or complex. Then

$$P_{0+}^{(\eta,\alpha)}\left\{t^{\omega-1}\,{}_{\rho}M_{\sigma}^{\alpha',\beta'}[d\,t^{-\beta^*}]S_{n'}^{m'}[d'\,t^{-\beta''}]H_{P,Q}^{M,N}\left[c\,t^{\delta}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]H_{p,q}^{m,n}\left[b\,t^{\beta}\middle|\begin{matrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{matrix}\right]\right\}$$

$$=\frac{\psi_1(k)\,\psi_2(\ell)(d')^{\ell}\,d^k\,x^{\eta+\omega+\beta^*k-(\beta'')^{\ell}}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{\omega-\beta^*k-(\beta'')^{\ell}}\,\Gamma(\alpha k+\beta)}H_{P,Q}^{M,N}\left[\frac{c\,x^{\delta}}{a(1-\alpha)^{\delta}}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]$$

$$.\,H_{p+1,q+1}^{m,n+1}\left[\frac{b\,x^{\beta}}{a(1-\alpha)^{\beta}}\middle|\begin{matrix}(1-\omega+\delta+\beta^*k+(\beta'')\ell,\beta),(a_p,\alpha_p)\\(b_q,\beta_q),(-\omega+\delta+\beta^*k+(\beta'')\ell-\frac{\eta}{1-\alpha},\beta)\end{matrix}\right]. \qquad (2.1)$$

Proof. Making use of (1.9), (1.10), (1.11) and (1.1) with applying a known result [1], we find the required result.

Theorem 2. Let $\eta, \gamma, \delta, \beta, T_1, T_2 > 0, \mathrm{Re}(\eta) > 0, \mathrm{Re}(\gamma) > 0, \mathrm{Re}(\omega) > 0,$

$$\mathrm{Re}\left(1+\frac{\eta}{1-\alpha}\right) > \mathrm{Max.}[0, -\mathrm{Re}(\omega)], b, c \in R, \alpha < 1, \mathrm{Re}\left(\omega+\delta\frac{f_j}{F_j}\right) > 0,\ j = 1, \ldots, M,$$

$|\arg c| < \frac{1}{2}T_1\pi, \rho \leq \sigma, |d| < (\alpha')^{\alpha'}, \beta^*, \beta'' > 0, m'$ is an arbitrary positive integer and the coefficients $A_{n',\ell}(n', \ell \geq 0)$ are arbitrary constants, real or complex. Then

$$P_{0+}^{(\eta,\alpha)}\left\{t^{\omega-1}\,{}_{\rho}M_{\sigma}^{\alpha',\beta'}[d\,t^{-\beta^*}]S_{n'}^{m'}[d'\,t^{-\beta''}]H_{P,Q}^{M,N}\left[c\,t^{\delta}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]E_{\beta,\rho}^{\gamma}(b\,t^{\beta})\right\}$$

$$=\frac{\psi_1(k)\,\psi_2(\ell)(d')^{\ell}\,d^k\,x^{\eta+\omega+\beta^*k-(\beta'')^{\ell}}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{\Gamma(\gamma)\,\Gamma[a(1-\alpha)]^{\omega-\beta^*k-(\beta'')^{\ell}}\,\Gamma(\alpha k+\beta)}H_{P,Q}^{M,N}\left[\frac{c\,x^{\delta}}{a(1-\alpha)^{\delta}}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]$$

$$.\,{}_2\Psi_2\left[\frac{b\,x^{\beta}}{a(1-\alpha)^{\beta}}\middle|\begin{matrix}(\gamma,1),(1-\omega+\delta+\beta^*k+(\beta'')\ell,\beta)\\(\omega,\beta),(1+\omega+\frac{\eta}{1-\alpha}-\delta-\beta^*k-(\beta'')\ell\beta)\end{matrix}\right]. \qquad (2.2)$$

Ref.

1. Chaurasia, V.B.L. and Ghiya, Neeti, Pathway fractional integral operator pertaining to special functions, Global J. Sci. Front. Res., 10(6) (ver. 1.0) (2010), 79-83.

where $E^{\gamma}_{\beta,\omega}(b)$ is the generalized Mittag-Leffler function (see [8],[10]).

Proof. The result in (2.2) can be obtained from Theorem 1 by putting m = 1 = η, p = 1, $q=2, b_1=0, \beta_1=1, b_2=1-\omega, \beta_2=\beta, \alpha_1=1-\gamma$ and $\alpha_1=1$. We get the desired result

Theorem 3. Let $\eta,\ \gamma,\ v \in C, \delta > 0,\ \alpha<1,\ \rho\le\sigma,\ |d|<(\alpha')^{\alpha'}, \mathrm{Re}(\eta)>0, c\in R,$

$$\mathrm{Re}(\gamma+v)>0, \mathrm{Re}\left(1+\frac{\eta}{1-\alpha}\right)>0, \mathrm{Re}\left(\gamma+\delta\frac{f_j}{F_j}\right)>0,\ j=1,\ldots,M, |\arg c|<\frac{1}{2}T_1\pi, T_1>0,$$

$\beta^*, \beta''>0, m'$ is an arbitrary positive integer and the coefficients $A_{n',\ell}(n',\ell\ge 0)$ are arbitrary constants, real or complex. Then

$$P^{(\eta,\alpha)}_{0+}\left\{\left(\frac{t}{2}\right)^{\gamma-1} {}_{\rho}M^{\alpha',\beta'}_{\sigma}\left[d\left(\frac{t}{2}\right)^{\beta*}\right] S^{m'}_{n'}\left[d'\left(\frac{t}{2}\right)^{\beta''}\right] H^{M,N}_{P,Q}\left[c\left(\frac{t}{2}\right)^{\delta}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right] J_v(t)\right\}$$

$$=\frac{\psi_1(k)\,\psi_2(\ell)\,d^k\,(d')^{\ell}\ x^{\eta+\gamma+v-\beta*k-(\beta'')^{\ell}}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{\Gamma[a(1-\alpha)]^{\gamma+v-\beta*k-(\beta'')^{\ell}}\ 2^{\gamma+v+\eta-\beta*k-(\beta'')^{\ell}}}\, H^{M,N}_{P,Q}\left[\frac{c\,x^{\delta}}{a(1-\alpha)^{\delta}}\middle|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right]$$

$$\cdot\ {}_1\psi_2\left[-\frac{x^2}{4a^2(1-\alpha)^2}\middle|\begin{matrix}(\gamma+v-\delta-\beta*k+(\beta'')\ell,2)\\(1+\gamma+v-\delta-\beta*k-(\beta'')\ell+\frac{\eta}{1-\alpha},2),(v+1,1)\end{matrix}\right] \tag{2.3}$$

Here ${}_p\psi_q$ denotes the generalized Wright hypergeometric function ([17], [18]).

Proof. The result in (2.3) can be established by letting p = 0, q = - 2, n = 0, m = 1, b_1= 0, $\beta_1=1, b_2=-v, \beta_2=1, \omega=\gamma+v, b'=1, \beta=2$ and replacing t by $\frac{t}{2}$ after a little simplification, we get the required result.

III. Special Cases

1. Putting $\beta^*\to 0, \delta\to 0$ in the result (2.1), we find a result recently derived by Chaurasia and Ghiya [1] when making ρ, ρ_1 and $\rho_2\to 0$.
2. Letting $\beta^*\to 0, n'\to 0$ in (2.1) through (2.3), we get the results recently obtained by Chaurasia and Gill [2].
3. Taking $n'\to 0$ in the results (2.1) through (2.3), we get the results recently established by Chaurasia and Singh [4].
4. Giving suitable values to the parameters in the results (2.1) through (2.3), we have the results recently derived by Nair [13].

A large number of simpler corresponding results involving simpler functions can be obtained easily merely by specializing the parameters in then.

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Notes

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The Integration of Certain Products of Special Functions

By Saroj Kumari
Singhania University

Abstract - The aim of the present paper is to obtain a finite integral involving a product of Fujiwara's polynomial [7], M-series [15], a general class of polynomial [10], with the H-function of several complex variables [11]. The results are quite general in nature hence encompass many new, known and unknown results hitherto in the literature.

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10. Srivastava, H.M., A contour integral involving Fox's function, Indian J. Math., 14 (1972), 1-6.

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Saroj Kumari

Abstract - The aim of the present paper is to obtain a finite integral involving a product of Fujiwara's polynomial [7], M-series [15], a general class of polynomial [10], with the H-function of several complex variables [11]. The results are quite general in nature hence encompass many new, known and unknown results hitherto in the literature.

I. Introduction

Srivastava [10] introduced a general class of polynomials (see also Srivastava and Singh [14])

$$S_q^p[x] = \sum_{s=0}^{[q/p]} \frac{(-q)_{ps}}{s!} A_{q,s}\, x^s,$$

$$= \Phi_3(s) \quad q = 0,1,2,... \tag{1.1}$$

where p is an arbitrary positive integer and the coefficients $A_{q,s}\,(q,s \geq 0)$ are arbitrary coefficients, real or complex.

The series representation of the multivariable H-function (Srivastava and Panda [11]) studied by Olkha and Chaurasia ([8], [9]) is given as follows:

$$H[z_1,...,z_r] = H^{0,\lambda':(u',v');...;(u^{(r)},v^{(r)})}_{A',C':[B',D'];...;[B^{(r)},D^{(r)}]}$$

$$\left[\begin{matrix}[(a):\theta',...,\theta^{(r)}]\;:[b':\phi'];...;[b^{(r)}:\phi^{(r)}]; \\ [(c):\psi_1,...,\psi^{(r)}]:[d':\delta'];...;[d^{(r)}:\delta^{(r)}];\end{matrix}\; z_1,...,z_r\right]$$

$$= \sum_{m_i=1}^{u^{(i)}} \sum_{n_i=0}^{\infty} \Phi_1 \Phi_2 \frac{\prod_{i=1}^{r} (z_i)^{U_i} (-1)^{\sum_{i=1}^{r}(n_i)}}{\prod_{i=1}^{r} (\delta^{(i)}_{m_i})\; n_i!} \tag{1.2}$$

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where

$$\Phi_1 = \frac{\prod_{j=1}^{\lambda'} \Gamma\left[1-a_j+\sum_{i=1}^{r} \theta_j^{(i)} U_i\right]}{\prod_{j=\lambda'+1}^{A'} \Gamma\left[a_j-\sum_{i=1}^{r} \theta_j^{(i)} U_i\right] \prod_{j=1}^{C'} \left[1-c_j+\sum_{i=1}^{r} \psi_j^{(i)} U_i\right]}, \tag{1.3}$$

$$\Phi_2 = \frac{\prod_{\substack{j=1 \\ j\neq m_i}}^{u^{(i)}} \Gamma(d_j^{(i)}-\delta_j^{(i)} U_i) \prod_{j=1}^{v^{(i)}} \Gamma(1-b_j^{(i)}+\phi_j^{(i)} U_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma(1-d_j^{(i)}+\delta_j^{(i)} U_i) \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma(b_j^{(i)}-\phi_j^{(i)} U_i)} \tag{1.4}$$

and

$$U_i = \frac{d_{m_i}^{(i)}+n_i}{\delta_{m_i}^{(i)}}, \; i=1,\ldots,r \tag{1.5}$$

which is valid under the conditions

$$\delta_{m_i}^{(i)} [d_j^{(i)}+p_i] \neq \delta_j^{(i)} [d_{m_i}^{(i)}+n_i] \tag{1.6}$$

$$\text{for } j \neq m_i, m_i = 1,\ldots,u^{(i)}; p_i, n_i = 0,1,2,\ldots; z \neq 0 \tag{1.7}$$

$$\nabla_i = \sum_{j=1}^{\lambda'} \theta_j^{(i)} - \sum_{j=1}^{C'} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} < 0, \; \forall i = 1,\ldots,r \tag{1.8}$$

Srivastava and Panda [12] introduced the multivariable H-function as follows:

$$H[y_1,\ldots,y_R] = H_{A,C:[M',N'];\ldots;[M^{(R)},N^{(R)}]}^{0,\lambda:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})} \left[\begin{matrix} [(g):\gamma';\ldots;\gamma^{(R)}]:[q':\eta'];\ldots;[q^{(R)},\eta^{(R)}]; \\ (f):\xi';\ldots;\xi^{(R)}]:[p',\epsilon');\ldots;[p^{(R)},\epsilon^{(R)}]; \end{matrix} \; y_1,\ldots,y_R \right] \tag{1.9}$$

For the sake of brevity

$$T_i = \sum_{j=1}^{\lambda} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \epsilon_j^{(i)} \leq 0, \tag{1.10}$$

$$\Omega_i = \sum_{j=\lambda+1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{\beta^{(i)}} \eta_j^{(i)} - \sum_{j=\beta^{(i)}+1}^{M^{(i)}} \eta_j^{(i)} + \sum_{j=1}^{\alpha^{(i)}} \epsilon_j^{(i)} - \sum_{j=\alpha^{(i)}+1}^{N^{(i)}} \epsilon_j^{(i)} > 0 \tag{1.11}$$

Ref.

12. Srivastava, H.M. and Panda, R., Some bilateral generating functions for a class of generalized hypergeometric polynomials, J. Reine Angew. Math. 283/284 (1976),265-274.

$$|\arg(y_i)| < \frac{1}{2}\Omega_i\,\pi,\ \forall = 1,\ldots,R \tag{1.12}$$

$$ {}_{p'}\overset{\alpha}{M}{}_{q'}[y] = \sum_{s'=0}^{\infty} \frac{(a_1)_{s'}\ldots(a_{p'})_{s'}}{(b_1)_{s'}\ldots(b_{q'})_{s'}} \frac{y^{s'}}{\Gamma(\alpha s'+1)} \tag{1.13}$$

Here $\alpha \in C, \mathrm{Re}(\alpha) > 0, (a_j)_{k'}, (b_j)_{k'}$ are the Pochammer symbols. The series in (1.13) is defined when none of the parameters b_js, j = 1,2,...,q, is a negative integer or zero. If any numerator parameter a_j is negative integer or zero, then the series terminates to a polynomial in y. The series is convergent if p' ≤ q' and — y — < 1. For other details see [ ].

II. MAIN THEOREM

The transformation is valid under the following conditions:

(i) $h_i, h_i', T_i, \Omega_i, D^* = \tau(b-a), k > 0, i = 1,\ldots,R, i' = 1,\ldots,r, k' > 0$

(ii) $\mathrm{Re}(\rho) > -1, \mathrm{Re}\left(\sigma + \sum_{i=1}^{R} h_i \frac{p_j^{(i)}}{\epsilon_j^{(i)}} + \sum_{i'=1}^{r} h_{i'}' \frac{d^{(i)}}{\delta_j^{(i)}}\right) > -1$

(iii) $F_n(\rho,\omega;t)$ is Fujiwar's polynomial[7].

(iv) p is an arbitrary positive integer and the coefficients $A_{q,s}(q,s \geq 0)$ are arbitrary coefficients, real or complex.

(v) $|\arg(y_i)| < \frac{1}{2}\Omega_i, T_i, \Omega_i$ are given in (1.10) and (1.11).

(vi) $p' \leq q', |y| < 1$.

Thus, the following transformation holds

$$\int_a^b (t-a)^{\rho}(b-t)^{\sigma} F_n(\rho,\omega;t) S_q^p[x(b-t)^k]\, {}_{p'}\overset{\alpha}{M}{}_{q'}[y(b-t)^{k'}]$$

$$.\ H[z_1(b-t)^{h'},\ldots,z_r(b-t)^{h'_r}]H[y_1(b-t)^h, y_R(b-t)^{h_R}]dt$$

$$= \sum_{m_i=1}^{u^{(i)}} \sum_{n_i=0}^{\infty} \Phi_1\Phi_2\Phi_3(s)\Phi_4(s')\Gamma(1+\rho+n)(b-a)^{\rho+\sigma+1+\sum_{i=1}^{r} h_i' U_i + k_s + k's'}$$

$$.\ \frac{(-1)^{\sum_{i=1}^{r}(n_i)+n+ks+k's'} \prod_{i=1}^{r}(z_i)^{U_i}(D^*)^{\eta}}{\prod_{i=1}^{r}((\delta_{m_i}^{(i)})n_i!)\,n!}$$

Ref.

7. Fujiwara, J., A unified presentation of classical orthogonal polynomials, Math. Japan, 11 (1966), 133-148.

$$. \; H^{0,\lambda+2 \; :(\alpha',\beta');...;(\alpha^{(R)},\beta^{(R)})}_{A+2,C+2:[M',N'];...;[M^{(R)},N^{(R)}]} \Bigg[$$

$$\left[\omega-\sigma-\sum_{i=1}^{r} h_i' U_i - ks - k's' : h_1,...,h_R\right], \left[-\sigma-\sum_{i=1}^{r} h_i' U_i - ks - k's' : h_1,...,h_r\right],$$

$$\left[\omega+n-\sigma-\sum_{i=1}^{r} h_i' U_i - ks - k's' : h_1,...,h_R\right], \left[-1-\rho-n-\sigma-\sum_{i=1}^{r} h_i' U_i - ks - k's' : h_1,...,h_R\right],$$

$$\begin{matrix}[(g):\gamma',...,\gamma^{(R)}]:[q':\eta'];...;[q^{(R)},\eta^{(R)}]; \\ [(f):\xi',...,\xi^{(R)}]:[p':\in'];...;[p^{(R)},\in^{(R)}];\end{matrix} \; y_1(b-a)^{h_1},...,y_R(b-a)^{h_R} \Bigg]. \qquad (2.1)$$

III. Proof

To derive (2.1), we express the general class of polynomials, M-series, the multivariable H-function in series form with the help of (1.2), (1.1) and (1.13) and then changing the order of integration and summation which is valid with the conditions stated and evaluating the remaining integral with the help of a known result of Chaurasia and Sharma ([2], p.269, eqn. (2.1)), we arrive at the desired result.

IV. Special Cases

(i) Assigning suitable values to the parameters with appealing to a known result ([11], p.139, eqn.(4.11)), after a little simplification, we have the following result

Theorem (A)

The transformation is valid under the following conditions

(a) $\mathrm{Re}(\rho) > -1, \mathrm{Re}(\sigma) > -1$

(b) $h_j > 0, h_{i'}' > 0, k > 0, k' > 0, \; j = 1,...,R, \; i' = 1,...,r, \; D^* = \tau(b-a)$

where

$$\Delta_j = 1 + \sum_{i=1}^{\mu} \xi_i^{(j)} + \sum_{i=1}^{B^{(j)}} \in_i^{(j)} - \sum_{i=1}^{\lambda} \gamma_i^{(j)} - \sum_{i=1}^{\alpha^{(j)}} \eta_i^{(j)} \quad (j = 1,...,R)$$

(c) The equality holds when — y_j — < L_j, j = 1,...,R with the L_j defined by equation (5.3), p.157 in [12].

(d) p is an positive integer and the coefficients $A_{q,s}\,(q,s \geq 0)$ are arbitrary coefficients, real or complex.

(e) $F_n(\rho,\omega;t)$ is Fujiwara polynomial [7].

(f) $p' \leq q'$ and $|y| < 1$.

$$\int_a^b (t-a)^{\rho}(b-t)^{\sigma} F_n(\rho,\omega;t)\, S_q^p[x(b-t)^k]\; {}_{p'}\overset{\alpha}{M}_{q'}[y(b-t)^{k'}]$$

Ref.

2. Chaurasia, V.B.L. and Sharma, S.C., An integral involving extended Jacobi polynomials and H-function of several complex variables, Vij. Pari. Anu. Pat. 27(3) (1984), 267-272.

Notes

$$\cdot\, F^{A:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}}\left[z_1(b-t)^{h'},\ldots,z_r(b-t)^{h'_r}\right]$$

$$\cdot\, F^{\lambda:\alpha';\ldots;\alpha^{(R)}}_{\mu:\beta';,,,;\beta^{(R)}}\left[y_1(b-t)^{h_1},\ldots,y_R(b-t)^{h_R}\right]dt$$

$$=\sum_{m_1,\ldots,m_r=0}^{\infty}\Phi_3(s)\,\Phi_4(s')\frac{\prod_{i=1}^{A}(a_i)_{m_1\theta_1+\ldots+m_r\theta_i^{(r)}}\prod_{i=1}^{B'}(b')_{m_1\phi_i'}\cdots}{\prod_{i=1}^{C}(c)_{m_1\psi_1+\ldots+m_r\psi_i^{(r)}}\prod_{i=1}^{D'}(d')_{m_1\delta_i'}\cdots}$$

$$\cdot\,\frac{\prod_{i=1}^{B^{(r)}}(b_i^{(r)})_{m_r\phi_i^{(r)}}}{\prod_{i=1}^{D^{(r)}}(d_i^{(r)})_{m_r\delta_i^{(r)}}}\frac{z_1^{m_1}}{m_1!}\cdots\frac{z_r^{m_r}}{m_r!}\frac{(-1)^n}{n!}(b-a)^{=r+s+1+\sum_{i=1}^{r}h'm_i+sk+s'k'}$$

$$\cdot\,\frac{\Gamma(1+\rho+n)\Gamma\left(1+\sigma+\sum_{i=1}^{r}h'm_i+sk+s'k')\right)}{\Gamma\left(1+\sigma-\omega-n+\sum_{i=1}^{r}h'm_i+sk+s'k')\right)}$$

$$\cdot\,\frac{\Gamma\left(1+\sigma-\omega+\sum_{i=1}^{r}h'm_i+sk+s'k'\right)}{\Gamma\left(1+\omega+n+\sigma+\sum_{i=1}^{r}h'm_i+sk+s'k'\right)}$$

$$\cdot\, F^{\lambda+2:\alpha';\ldots;\alpha^{(R)}}_{\mu+2:\beta';\ldots;\beta^{(R)}}\left[\begin{matrix}\left[1+\sigma+\sum_{i=1}^{r}h'm_i+sk+s'k':h_1,\ldots,h_R\right],\\ \left[1+\sigma-\omega-\eta+\sum_{i=1}^{r}h'm_i+sk+s'k':h_1,\ldots,h_R\right],\end{matrix}\right.$$

$$\left.\begin{matrix}\left[1+\sigma-\omega+\sum_{i=1}^{r}h'm_i+sk+s'k':h_1,\ldots,h_R\right],[(g):\gamma',\ldots,\gamma^{(R)}]:[(q'):\eta'];\ldots'[(q^{(R)}:\eta^{(R)}];\\ \left[2+\omega+n+\sigma+\rho+\sum_{i=1}^{r}h'm_i+sk+s'k':h_1,\ldots,h_R\right],[(f):\xi',\ldots,\xi^{(R)}]:[(p'):\in'];\ldots;[(p^{(R)}:\in^{(R)}];\end{matrix}\; y_1(b-a)^{h_1},\ldots,y_R(b-a)^{h_R}\right] \tag{4.1}$$

(ii) Taking $r = 1 = R$ in (2.1), we have the following result

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Theorem (B)

The transformation is valid under the following conditions

(a) $\mathrm{Re}(1+\rho)>0, h, h', k, k', T>0, |\arg(y)|<\frac{1}{2}T\pi, D^{*}=\tau(b-a)$

(b) $\mathrm{Re}\left(\sigma+h'\frac{p_j}{\in_j}+h\frac{d_{j'}}{\delta_{j'}}+1\right)>0, j=1,\ldots,u, j'=1,\ldots,\alpha.$

(c) p is an positive integer and the coefficient $A_{q,s}(q,s\geq 0)$ are arbitrary coefficients, real or complex.

(d) $F_n(\rho,\omega;t)$ is Fujiwara polynomial [7].

(e) $p'\leq q'$ and $|y|<1.$

Thus, the following transformation holds

$$\int_a^b (t-a)^{\rho}(b-t)^{\sigma}F_n(\rho,\omega;t)\,S_q^p[x(b-t)^k]\,{}_{p'}\overset{\alpha}{M}{}_{q'}[y(b-t)^{k'}]$$

$$.\ H_{B,D}^{u,v}\left[\begin{matrix}[b':\phi]\\ [d':\delta]\end{matrix}\middle| z(b-t)^{h'}\right]H_{M,N}^{\alpha',\beta'}\left[\begin{matrix}[q'':\eta\\ [p'':\in\end{matrix}\middle| y'(b-t)^{h}\right]dt$$

$$=\sum_{m_1=0}^{u}\sum_{n_1=0}^{\infty}\Phi_1^{*}\Phi_3(s)\Phi_4(s')(-1)^{n_1}z^{U}(D^{*})^{n}\frac{(b-a)^{\rho+\sigma+1+h'U+sk+s'k'}\Gamma(1+\rho+n)}{n!\ \ n_1!\ \ \delta n_1}$$

$$.\ H_{M+2,N+2}^{\alpha,\beta+2}\left[\begin{matrix}[\omega-\sigma-h'U-sk-s'k':h],[-\sigma-h'U-sk-s'k':h],[b':\phi];\\ [(d':\delta],[\omega+n-\sigma-\eta'U-sk-s'k':h],[-1-\rho-\eta-\sigma-h'U-sk-s'k':h];\end{matrix}\ y'(b-a)^{h}\right]. \qquad (4.2)$$

(iii) When $k'\to 0, q\to 0$, the result in (2.1), (4.1) and (4.2) reduce to the result obtained by Chaurasia and Chand [3].

(iv) Putting $q\to 0, h_i'\to 1, y\to 0, i=1,\ldots,r$ in (2), we have a result due to Chaurasia and Sharma [3].

(v) The results derived by the equations (3.2) and (3.3) in [2] can b e obtained from our results.

(vi) Setting $a=-1, b=1=\lambda, q\to 0, y\to 0, h_i'=1, i=1,\ldots,r$ in (2.1), we get a known result of Srivastava and Panda [11].

(vii) Taking $q\to 0, y\to 0, h_i'=1, i=1,\ldots,r$ the result in (4.1) reduces to a known result derived by Chaurasia and Sharma in [3].

(viii) The results (2.1),(4.1) and (4.2) established by Chaurasia and Singh in [4] can be reduced as a particular cases of our results.

A great number of interesting transformation formulae as special cases of our results can be derived, but we omit them here for lack of space.

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Notes

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On Certain Indefinite Elliptic Integrals

By Salahuddin & M. P. Chaudhary
P.D.M College of Engineering

Abstract - In this paper we have developed some formulae related to indefinite integrals in association with Hypergeometric functions.

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ON CERTAIN INDEFINITE ELLIPTIC INTEGRALS

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On Certain Indefinite Elliptic Integrals

Salahuddin [α] & M. P. Chaudhary [σ]

Abstract - In this paper we have developed some formulae related to indefinite integrals in association with Hypergeometric functions.

Keywords and Phrases : pochhammer symbol; gaussian hypergeometric function; complete elliptic integrals; kampé de fériet double hypergeometric function and sri- vastava's triple hypergeometric function.

I. Introduction and Preliminaries

The Pochhammer's symbol or Appell's symbol or shifted factorial or rising factorial or generalized factorial function is defined by

$$(b,\ k)=(b)_k=\frac{\Gamma(b+k)}{\Gamma(b)}=\begin{cases} b(b+1)(b+2)\cdots(b+k-1); & \text{if } k=1,2,3,\cdots \\ 1 \quad ; & \text{if } k=0 \\ k! \quad ; & \text{if } b=1,\ k=1,2,3,\cdots \end{cases}$$

where b is neither zero nor negative integer and the notation Γ stands for Gamma function.

a) *Generalized Gaussian Hypergeometric Function*

Generalized ordinary hypergeometric function of one variable is defined by

$${}_AF_B\left[\begin{array}{ccc} a_1,a_2,\cdots,a_A & ; & \\ & & z \\ b_1,b_2,\cdots,b_B & ; & \end{array}\right]=\sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\cdots(a_A)_k z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k k!}$$

or

$${}_AF_B\left[\begin{array}{ccc} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right]\equiv {}_AF_B\left[\begin{array}{ccc} (a_j)_{j=1}^A & ; & \\ & & z \\ (b_j)_{j=1}^B & ; & \end{array}\right]=\sum_{k=0}^{\infty}\frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1.1}$$

where denominator parameters $b_1,b_2,\cdots,b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

b) *Kampje de Fjeriet's General Double Hypergeometric Function*

In 1921, Appell's four double hypergeometric functions F_1,F_2,F_3,F_4 and their confluent forms $\Phi_1,\Phi_2,\Phi_3,\Psi_1,\Psi_2,\Xi_1,\Xi_2$ were unified and generalized by Kampé de Fériet.

We recall the definition of general double hypergeometric function of Kampé de Fériet in slightly modified notation of H.M.Srivastava and R.Panda:

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Author σ : International Scientific Research and Welfare Organization, New Delhi, India. E-mail : mpchaudhary 2000@yahoo.com

$$\mathrm{F}^{A:B;D}_{E:G;H}\left[\begin{array}{lll}(a_A):(b_B);(d_D) & ; & \\ & & x,\ y\\ (e_E):(g_G);(h_H) & ; & \end{array}\right] = \sum_{m,n=0}^{\infty}\frac{((a_A))_{m+n}\,((b_B))_m\,((d_D))_n\ x^m\,y^n}{((e_E))_{m+n}\,((g_G))_m\,((h_H))_n\,m!\,n!} \tag{1.2}$$

where for convergence

(i) $A+B<E+G+1,\ A+D<E+H+1\quad ;|x|<\infty,\quad |y|<\infty,$ or

(ii) $A+B=E+G+1,\ A+D=E+H+1,$ and

$$\begin{cases}|x|^{\frac{1}{(A-E)}}+|y|^{\frac{1}{(A-E)}}<1 & ,\text{if } E<A\\ \max\ \{|x|,|y|\}<1 & ,\text{if } E\geq A\end{cases}$$

c) Srivastava's General Triple Hypergeometric Function

In 1967, H. M. Srivastava defined a general triple hypergeometric function $F^{(3)}$ in the following form

$$F^{(3)}\left[\begin{array}{ll}(a_A)::(b_B);\,(d_D);\,(e_E):(g_G);\,(h_H);\,(l_L); & \\ & x,\ y,\ z\\ (m_M)::(n_N);\,(p_P);\,(q_Q):(r_R);\,(s_S);\,(t_T); & \end{array}\right]$$

$$=\sum_{i,j,k=0}^{\infty}\frac{((a_A))_{i+j+k}\ ((b_B))_{i+j}\ ((d_D))_{j+k}\ ((e_E))_{k+i}\ ((g_G))_i\ ((h_H))_j\ ((l_L))_k\ x^i\ y^j\ z^k}{((m_M))_{i+j+k}\ ((n_N))_{i+j}\ ((p_P))_{j+k}\ ((q_Q))_{k+i}\ ((r_R))_i\ ((s_S))_j\ ((t_T))_k\ i!\ j!\ k!} \tag{1.3}$$

d) Wright's Generalized Hypergeometric Function

$${}_p\Psi_q\left[\begin{array}{lll}(\alpha_1,A_1),\cdots,(\alpha_p,A_p) & ; & \\ & & x\\ (\lambda_1,B_1),\cdots,(\lambda_q,B_q) & ; & \end{array}\right]=\sum_{m=0}^{\infty}\frac{\Gamma(\alpha_1+mA_1)\Gamma(\alpha_2+mA_2)\cdots\Gamma(\alpha_p+mA_p)x^m}{\Gamma(\lambda_1+mB_1)\Gamma(\lambda_2+mB_2)\cdots\Gamma(\lambda_q+mA_q)m!} \tag{1.4}$$

$${}_p\Psi^*_q\left[\begin{array}{lll}(\alpha_1,A_1),\cdots,(\alpha_p,A_p) & ; & \\ & & x\\ (\lambda_1,B_1),\cdots,(\lambda_q,B_q) & ; & \end{array}\right]=\sum_{m=0}^{\infty}\frac{(\alpha_1)_{mA_1}(\alpha_2)_{mA_2}\cdots(\alpha_p)_{mA_p}x^m}{(\lambda_1)_{mB_1}(\lambda_2)_{mB_2}\cdots(\lambda_q)_{mB_q}m!} \tag{1.5}$$

II. Main Integrals

$$\int\frac{\mathrm{d}x}{\sqrt{(1+x\sinh x)}}=$$

$$=-\cosh x\sinh^{m+1}x(-\sinh^2 x)^{\frac{-m-1}{2}}\ F^{1;2}_{0;1}\left[\begin{array}{lll}\frac{1}{2}\ ;\frac{1}{2},\frac{1-m}{2} & ; & \\ & & -x,\ \cosh^2 x\\ -;\ \frac{3}{2} & ; & \end{array}\right]+Constant \tag{2.1}$$

$$\int\frac{\mathrm{d}x}{\sqrt{(1+x\cosh x)}}=-\frac{\sinh x\cosh^{m+1}x}{(m+1)\sqrt{-\sinh^2 x}}\ F^{1;2}_{0;1}\left[\begin{array}{lll}\frac{1}{2}\ ;\frac{1}{2},\frac{m+1}{2} & ; & \\ & & -x,\ \cosh^2 x\\ -;\ \frac{m+3}{2} & ; & \end{array}\right]+Constant \tag{2.2}$$

Notes

Notes

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x\tanh x)}} = \frac{\tanh^{m+1}x}{(m+1)}\; F^{1;2}_{0;1}\left[\begin{array}{ll} \frac{1}{2}\ ;1,\ \frac{m+1}{2} & ; \\ & \quad -x,\ \tanh^2 x \\ -;\ \frac{m+3}{2} & ; \end{array}\right] + Constant \qquad (2.3)$$

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x\coth x)}} = \frac{\coth^{m+1}x}{(m+1)}\; F^{1;2}_{0;1}\left[\begin{array}{ll} \frac{1}{2}\ ;1,\ \frac{m+1}{2} & ; \\ & \quad -x,\ \coth^2 x \\ -;\ \frac{m+3}{2} & ; \end{array}\right] + Constant \qquad (2.4)$$

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x\ \mathrm{sech}\ x)}} =$$

$$= \sinh x\cosh^2(x)^{\frac{m+1}{2}}\mathrm{sech}^{m+1}x\; F^{1;2}_{0;1}\left[\begin{array}{ll} \frac{1}{2}\ ;\frac{1}{2},\ \frac{1+m}{2} & ; \\ & \quad -x,\ -\sinh^2 x \\ -;\ \frac{3}{2} & ; \end{array}\right] + Constant \qquad (2.5)$$

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x\ \mathrm{cosech}\ x)}} =$$

$$= \cosh x(-\sinh^2(x))^{\frac{m+1}{2}}\mathrm{cosech}^{m+1}x\; F^{1;2}_{0;1}\left[\begin{array}{ll} \frac{1}{2}\ ;\frac{1}{2},\ \frac{1+m}{2} & ; \\ & \quad -x,\ \cosh^2 x \\ -;\ \frac{3}{2} & ; \end{array}\right] + Constant \qquad (2.6)$$

III. Derivation of Integrals

Derivation of integral (2.1)

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x\sinh x)}} = \int (1+x\sinh x)^{-\frac{1}{2}}\mathrm{d}x = \int \{1-(-x\sinh x)\}^{-\frac{1}{2}}\mathrm{d}x$$

$$\int \sum_{m=0}^{\infty}\frac{(\frac{1}{2})_m(-x)^m}{m!}\sinh^m x\ \mathrm{d}x = \sum_{\mathrm{m}=0}^{\infty}\frac{(\frac{1}{2})_{\mathrm{m}}(-\mathrm{x})^{\mathrm{m}}}{\mathrm{m}!}\int \sinh^{\mathrm{m}}\mathrm{x}\ \mathrm{dx}$$

$$= \sum_{m=0}^{\infty}\frac{(\frac{1}{2})_m(-x)^m}{m!}(-\cosh x)\ \sinh^{m+1}x(-\sinh^2 x)^{\frac{-m-1}{2}}{}_2F_1\left[\begin{array}{ll} \frac{1}{2},\ \frac{1-m}{2} & ; \\ & \cosh^2 x \\ \frac{3}{2} & ; \end{array}\right] + Constant$$

$$= -\cosh x\sinh^{m+1}x(-\sinh^2 x)^{\frac{-m-1}{2}}\; F^{1;2}_{0;1}\left[\begin{array}{ll} \frac{1}{2}\ ;\frac{1}{2},\ \frac{1-m}{2} & ; \\ & \quad -x,\ \cosh^2 x \\ -;\ \frac{3}{2} & ; \end{array}\right] + Constant$$

Derivation of integral (2.2)

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x\cosh x)}} = \int (1+x\cosh x)^{-\frac{1}{2}}\mathrm{d}x = \int \{1-(-x\cosh x)\}^{-\frac{1}{2}}\mathrm{d}x$$

$$\int \sum_{m=0}^{\infty}\frac{(\frac{1}{2})_m(-x)^m}{m!}\cosh^m x\ \mathrm{d}x = \sum_{\mathrm{m}=0}^{\infty}\frac{(\frac{1}{2})_{\mathrm{m}}(-\mathrm{x})^{\mathrm{m}}}{\mathrm{m}!}\int \cosh^{\mathrm{m}}\mathrm{x}\ \mathrm{dx}$$

$$= \sum_{m=0}^{\infty}\frac{(\frac{1}{2})_m(-x)^m}{m!}\frac{(-\sinh x)\cosh^{m+1}x}{(m+1)\sqrt{-\sinh^2 x}}{}_2F_1\left[\begin{array}{ll} \frac{1}{2},\ \frac{m+1}{2} & ; \\ & \cosh^2 x \\ \frac{m+3}{2} & ; \end{array}\right] + Constant$$

$$= -\frac{\sinh x \cosh^{m+1} x}{(m+1)\sqrt{-\sinh^2 x}} F_{0;1}^{1;2}\left[\begin{array}{ll} \frac{1}{2} \;;\frac{1}{2},\frac{m+1}{2} \;; & \\ & -x,\ \cosh^2 x \\ —;\ \frac{m+3}{2} \;; & \end{array}\right] + Constant$$

Derivation of integral (2.3)

$$\int \frac{dx}{\sqrt{(1+x\tanh x)}} = \int (1+x\tanh x)^{-\frac{1}{2}} dx = \int \{1-(-x\tanh x)\}^{-\frac{1}{2}} dx$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \tanh^m x \ dx = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \int \tanh^m x \ dx$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \frac{\tanh^{m+1} x}{(m+1)} {}_2F_1\left[\begin{array}{ll} 1, \frac{m+1}{2} \;; & \\ \frac{m+3}{2} \;; & \tanh^2 x \end{array}\right] + Constant$$

$$= \frac{\tanh^{m+1} x}{(m+1)} F_{0;1}^{1;2}\left[\begin{array}{ll} \frac{1}{2} \;;1,\frac{m+1}{2} \;; & \\ & -x,\ \tanh^2 x \\ —;\ \frac{m+3}{2} \;; & \end{array}\right] + Constant$$

Derivation of integral (2.4)

$$\int \frac{dx}{\sqrt{(1+x\coth x)}} = \int (1+x\coth x)^{-\frac{1}{2}} dx = \int \{1-(-x\coth x)\}^{-\frac{1}{2}} dx$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \coth^m x \ dx = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \int \coth^m x \ dx$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \frac{\coth^{m+1} x}{(m+1)} {}_2F_1\left[\begin{array}{ll} 1, \frac{m+1}{2} \;; & \\ \frac{m+3}{2} \;; & \coth^2 x \end{array}\right] + Constant$$

$$= \frac{\coth^{m+1} x}{(m+1)} F_{0;1}^{1;2}\left[\begin{array}{ll} \frac{1}{2} \;;1,\frac{m+1}{2} \;; & \\ & -x,\ \coth^2 x \\ —;\ \frac{m+3}{2} \;; & \end{array}\right] + Constant$$

Derivation of integral (2.5)

$$\int \frac{dx}{\sqrt{(1+x\ \mathrm{sech}\ x)}} = \int (1+x\ \mathrm{sech}\ x)^{-\frac{1}{2}} dx = \int \{1-(-x\ \mathrm{sech}\ x)\}^{-\frac{1}{2}} dx$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \mathrm{sech}^m x \ dx = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \int \mathrm{sech}^m x \ dx$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!} \sinh x\ \cosh^2(x)^{\frac{m+1}{2}} \mathrm{sech}^{m+1}\ x\ {}_2F_1\left[\begin{array}{ll} \frac{1}{2}, \frac{m+1}{2} \;; & \\ \frac{3}{2} \;; & -\sinh^2 x \end{array}\right] + Constant$$

$$= \sinh x \cosh^2(x)^{\frac{m+1}{2}} \mathrm{sech}^{m+1} x\ F_{0;1}^{1;2}\left[\begin{array}{ll} \frac{1}{2} \;;\frac{1}{2},\frac{1+m}{2} \;; & \\ & -x,\ -\sinh^2 x \\ —;\ \frac{3}{2} \;; & \end{array}\right] + Constant$$

Notes

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Derivation of integral (2.6)

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x\ \mathrm{cosech}\ x)}} = \int (1+x\ \mathrm{cosech}\ x)^{-\frac{1}{2}}\mathrm{d}x = \int \{1-(-x\ \mathrm{cosech}\ x)\}^{-\frac{1}{2}}\mathrm{d}x$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!}\mathrm{cosech}^m x\ \mathrm{dx} = \sum_{\mathrm{m}=0}^{\infty} \frac{(\frac{1}{2})_{\mathrm{m}}(-\mathrm{x})^{\mathrm{m}}}{\mathrm{m}!} \int \mathrm{cosech}^{\mathrm{m}}\mathrm{x}\ \mathrm{dx}$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(-x)^m}{m!}\ \cosh x\ (-\sinh^2(x))^{\frac{m+1}{2}}\mathrm{cosech}^{m+1}\ x\ {}_2F_1\left[\begin{matrix} \frac{1}{2}, \frac{m+1}{2} & ; \\ \frac{3}{2} & ; \end{matrix}\ \cosh^2 x\right] + Constant$$

$$= \cosh x(-\sinh^2(x))^{\frac{m+1}{2}}\mathrm{cosech}^{m+1}x\ F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2}\ ;\frac{1}{2}, \frac{1+m}{2} & ; \\ & & -x,\ \cosh^2 x \\ -;\ \frac{3}{2} & ; \end{matrix}\right] + Constant$$

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3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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