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Highlights

Hypergeometric Function

Kinematic Wave Equation

Stiff Differential Equations

Countable Boolean Lattice

Discovering Thoughts, Inventing Future

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Characterization of Partial Lattices on Countable Boolean Lattice

By D.V.S.R. Anil Kumar, Y.V.Seshagiri Rao, Y Narasimhulu & Venkata Sundaranand Putcha
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Abstract - In this paper new concepts countable join property, countable meet property, P_σ – lattice and P_δ – lattice are introduced. We established that P_σ – lattice and P_δ – lattice are measureable partial lattices and characterized partial lattices of a lattice through countable join and meet properties. We also established some interesting result on the injective property of the lattice measurable functions defined over countable Boolean lattices.

Keywords : *lattice, partial lattice, σ - algebra, measure.*

GJSFR-F Classification : *ASM 03G10, 28A05*

CHARACTERIZATION OF PARTIAL LATTICES ON COUNTABLE BOOLEAN LATTICE

Strictly as per the compliance and regulations of :

Characterization of Partial Lattices on Countable Boolean Lattice

D.V.S.R. Anil Kumar [α], Y.V.Seshagiri Rao [σ], Y Narasimhulu [ρ] & Venkata Sundaranand Putcha [ω]

Abstract - In this paper new concepts countable join property, countable meet property, P_σ–lattice and P_δ–lattice are introduced. We established that P_σ–lattice and P_δ–lattice are measureable partial lattices and characterized partial lattices of a lattice through countable join and meet properties. We also established some interesting result on the injective property of the lattice measurable functions defined over countable Boolean lattices.

Keywords : *lattice, partial lattice, σ -algebra, measure.*

I. Introduction

The origin of a lattice concept can be traced back to Boole's analysis of thought and Dedekind's study of divisibility, Schroder and Pierce contributed substantially to this area. Though some of the work in this direction was done around 1930, much momentum was gained in 1967 with the contributions of Birkhoff's [2]. In 1963, Gabor szasz [9] introduced the generalization of the lattice measure concepts. To study σ-additive set functions on a lattice of sets, Gena A. DE Both [3] introduced σ - lattice in 1973. The concept of partial lattices was introduced by George Gratzer [5] in 1978. In 2000, Pao - Sheng Hus [8] characterized outer measures associated with lattice measure. The Hann decomposition theorem of a signed lattice measure by Jun Tanaka [10] defined a signed lattice measure on a lattice σ - algebras and the concept of sigma algebras are extensively studied by [4]. D.V.S.R. Anil Kumar etal [1] introduce the concept of measurable Borel lattices, σ - lattice and δ –lattice to characterize a class of Measurable Borel Lattices. This paper is organized as follows. Section 2 presents the preliminaries definitions and results. In Section 3 we proved that

P_σ – lattice and P_δ –lattice are measurable partial lattices and all partial lattices of a lattice satisfy both countable join and meet properties. Some interesting result on the injective property of the lattice measurable functions defined over countable Boolean lattices are established in Section 4.

Ref

2. Birkhoff. G, Lattice Theory 3rd ed., AMS Colloquim Publications, Providence, RI, 1967.

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II. Preliminaries

Consider a lattice (L,∧,∨) with the operations meet ∧ and join ∨ and usual ordering ≤, where L is a collection of subset of a non empty set X. Now this lattice (L,∧,∨) is denoted by L and satisfy the commutative law, the associative law and the absorption law. A lattice L is called distributive if the distributive law is satisfied. The zero and one elements of the lattice L are denoted by 0 and 1 respectively. A distributive lattice L is called a Boolean lattice if for any element x in L, there exists a unique complement x^c such that $x \vee x^c = 1$ and $x \wedge x^c = 0$. An operator C: L → L, where L is a lattice is called a lattice complement in L if the law of complementation, the law of contra positive and the law of double negation are satisfied. The following are very important examples of Boolean lattice.

Example2.1. Let ({0,1},≤) be the set consisting of the two elements 0,1 equipped with the usual order relation 0 ≤ 1.This poset is a Boolean lattice with respect to the operations presented in the tables below (at the left the lattice operations and at the right the complementation):

a	b	$a \wedge b$	$a \vee b$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

x	x^c
0	1
1	0

This is usually known as the two valued or two elements Boolean lattice, denoted by B= ({0,1},∨, ∧, ^{c},0,1).

Example2.2. The power set P(X) of a universe X a Boolean lattice if we choose the set theoretic complement $A^c = X \setminus A := \{x \in X:\ x \in X \text{ and } x \notin A\}$ as the complement of a given set A in the universe X. Such a Boolean lattice is P = (P(X), ∨, ∧, ^{c}, ϕ, X).

Example2.3. E = (2^X ,∨, ∧,^{c} 0, 1) is the collection 2^X of all two valued functional on the universe X is a Boolean lattice if we choose the functional $\chi^c = 1-\chi$ as the complement of a given functional χ.

Example2.4. Let (D, ∨, ∧, ^{c},1,70) is a Boolean lattice where D={1,2,5,7,10,14,35,70} is the set of all divisors of 70, x ∧ y = Greatest Common Devisor of x and y, x ∨ y = Least Common Multiple of x and y and $x^c = \dfrac{70}{x}$.

Definition2.1. A Boolean lattice L is called a countable Boolean lattice if L is closed under countable join and is denoted by σ (L).

Example2.5. {empty setϕ, X}, Power set of X, Let $X = \Re$, L = {measurable subsets of $\Re$} with usual ordering (≤) are all countable Boolean lattice.

Definitition2.2. The entire set X together with countable Boolean lattice is called lattice measurable space and is denoted by the ordered pair (X, σ (L)).

Notes

Example2.6. $X = \Re$, where $\Re$ is extended real number system and L = {All Lebesgue measurable sub sets of $\Re$ },($\Re$,σ (L)) is a lattice measurable space.

Definitition2.3. If μ: σ (L) → R ∪ {∞} satisfies the following properties (i) μ(ϕ) = μ (0) = 0 (ii) for all h, g ∈ σ (L), such that μ (h), μ (g) ≥ 0; h ≤ g ⇒ μ (h) ≤ μ (g) (iii) for all h, g ∈ σ (L): μ (h ∨ g) + μ (h ∧ g) = μ (h) + μ (g) (iv) If $h_n \in \sigma$ (L), n ∈ N such that $h_1 \leq h_2 \leq ... \leq h_n \leq ....$, then $\mu(\bigvee_{n=1}^{\infty} h_n) = \lim \mu(h_n)$ then μ is called a lattice measure on the countable Boolean lattice σ (L).

The following is definition given in [5]

Definition2.4. Let σ (L) be a countable Boolean lattice, H ⊆ σ (L), and restrict ∧ and ∨ to H as follows. For a, b, c ∈ H, if a ∧ b = c (dually, a ∨ b = c), then we say that in H, a ∧ b(dually a ∨ b) is defined and it equals c, if, for a, b ∈ H, a ∧ b ∉ H(dually a ∨ b ∉ H), then we say that a ∧ b(dually a ∨ b) is not defined in H. Thus (H, ∧,∨) is a set with two binary partial operations. (H, ∧,∨) is called a partial lattice, a relative sublattice of σ (L).

Observation2.1. Every subset of a countable Boolean lattice determines a partial lattice. Every sublattice of σ (L) is a partial lattice and the converse need not be true.

Definition2.5.[7] A set A is said to be measurable partial lattice, if A is in σ (L).

Example2.7. ($\Re$, σ (L)) be lattice measurable space. Then the interval (a, ∞) is a measurable partial lattice under usual ordering.

Example2.8. $[0, 1) < \Re$ is a measurable partial lattice under usual ordering.

Definition2.9. A P_σ - lattice is a poset for which sup exist for any countable collection of its partial lattices.

Example2.9. $R = \bigvee_{n=1}^{\infty} (-n, n)$ is a p_σ -lattice.

Definition2.10. A P_δ - lattice is a poset for which inf exist for any countable collection of its partial lattices.

Example2.11. (i) $\bigwedge_{n=1}^{\infty} (-n, n) = (-1, 1)$ and (ii) $\bigwedge_{n=1}^{\infty} \left(-\frac{1}{n}, \frac{1}{n}\right) = \{0\}$ are P_δ – lattices.

Definition2.11.Countable join property (CJP): If $\{E_k\}$ is monotonic increasing sequence of partial lattices of a lattice L and $E = \bigvee_{k=1}^{\infty} E_k$. Then $\mu(E) = \underset{n\to\infty}{Lt}\ \mu(E_n)$.

Definition2.12.Countable meet property (CMP): If $\{E_k\}$ is a monotonic decreasing sequence of partial lattices of a lattice L and $E = \bigwedge_{k=1}^{\infty} E_k$. Then $\mu(E) = \underset{n\to\infty}{Lt}\ m(E_n)$.

Result2.1.[1]. If E is measurable lattice so is E^c.

III. P_σ-LATTICE AND P_δ -LATTICE

Theorem3.1. Every P_σ - lattice is lattice measurable.

Proof. Let $E_1, E_2, ...$ are pair wise disjoint measurable partial lattices and $E = \bigvee_{k=1}^{\infty} E_k$,

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5. Gratzer. G, General Lattice Theory, Academic Press Inc., 1978.

Evidently,

$$\mu\,(\mathop{\vee}_{k=1}^{\infty} E_k) \le \sum_{k=1}^{\infty}\mu(E_k) \qquad (1)$$

and

$$\mu\,(\mathop{\vee}_{k=1}^{\infty} E_k) \ge \mu(\mathop{\vee}_{k=1}^{n} E_k) \qquad (2)$$

From definition 2.3. We have $\mu\,(E_1 \vee E_2) = \mu\,(E_1) + \mu\,(E_2)$. By the principle of mathematical induction on number of pair wise disjoint measurable partial lattices, n, we have $\mu\,(\mathop{\vee}_{k=1}^{n} E_k) = \sum_{k=1}^{n}\mu(E_k)$. As $n \to \infty$, from (2) it follows that

$$\mu\,(\mathop{\vee}_{k=1}^{\infty} E_k) \ge \sum_{k=1}^{\infty}\mu(E_k) \qquad (3)$$

From (1) and (3), we have $\mu\,(\mathop{\vee}_{k=1}^{\infty} E_k) = \sum_{k=1}^{\infty}\mu(E_k)$. Now $E = \mathop{\vee}_{k=1}^{\infty} E_k =$ $E_1 \vee (E_2 \wedge E_1^c) \vee \ldots \vee (E_k \wedge (\mathop{\vee}_{k=1}^{n-1} E_k^c) \vee \ldots$. Since $E_1, E_2 \wedge E_1^c, \ldots$ are disjoint measurable partial lattices, we have, $\mathop{\vee}_{k=1}^{\infty} E_k$ is a measurable partial lattice. Hence every P_σ - lattice is a lattice measurable.

Theorem3.2. Every P_σ - lattice satisfies CJP.

Proof. Suppose that $\{E_k\}$ is monotonic increasing sequence of partial lattices of a σ (L) and $E = \mathop{\vee}_{k=1}^{\infty} E_k$. Write $E = E_1 \vee (E_2 \wedge E_1^c) \vee \ldots \vee (E_k \wedge (\mathop{\vee}_{k=1}^{n-1} E_k^c) \vee \ldots$

So we have $E = E_1 \vee (\mathop{\vee}_{k=1}^{\infty}(E_{k+1} \wedge E_k^c)$ (a disjoint joint). By Theorem 3.1.

Now, $\mu\,(E) = \mu\,(E_1) + \sum_{k=1}^{\infty}\mu(E_{k+1} - E_k) = \mu\,(E_1) + \mathop{Lt}_{n\to\infty} \sum_{k=1}^{n} [\mu(E_{k+1}) - \mu(E_k)] = \mu\,(E_1)$ $+ \mathop{Lt}_{n\to\infty} [\mu(E_2) - \mu(E_1) + \ldots\ldots.. + \mu(E_n) - \mu(E_{n-1})] = \mu\,(E_1) + \mathop{Lt}_{n\to\infty} [-\mu(E_1) + \mu(E_n)] = \mu$ $(E_1) - \mu\,(E_1) + \mathop{Lt}_{n\to\infty} \mu(E_n) = \mathop{Lt}_{n\to\infty} \mu(E_n)$.

Theorem3.3. Every P_δ -lattice is lattice measurable.

Proof. Let $E_1, E_{2,} \ldots.$ are measurable partial lattices.

By theorem 3.1. $E = \mathop{\vee}_{k=1}^{\infty} E_k$ is a measurable partial lattice. Let $G = \mathop{\wedge}_{k=1}^{\infty} E_k$.

Then $G^c = (\mathop{\wedge}_{k=1}^{\infty} E_k)^c = \mathop{\vee}_{k=1}^{\infty} E_k^c$. Given that each E_k is a measurable partial lattice.

Hence by Result 2.1., each E_k^c is a measurable partial lattice. Which implies $\mathop{\vee}_{k=1}^{\infty} E_k^c$ is a measurable partial lattice (Every P_σ - lattice is a measurable partial lattice). This leads to G^c is measurable partial lattice. Hence G is measurable partial lattice (By Result 2.1.).

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Theorem 3.4. Every P_δ -lattice satisfies CMP**.**

Proof. Suppose that $\{E_k\}$ is a monotonic decreasing sequence of partial lattices of $\sigma(L)$ and $E = \bigwedge_{k=1}^{\infty} E_k$. Let $E = \bigwedge_{k=1}^{\infty} E_k$. Evidently $E_1 = E \vee (E_1 \wedge E_2^c) \vee (E_2 \wedge E_3^c) \vee \ldots$

Then $\mu(E_1) = \mu(E) + \sum_{k=1}^{\infty} \mu(E_k) - \mu(E_{k+1}) = \mu(E) + \underset{n\to\infty}{\mathrm{Lt}} \sum_{k=1}^{n} \mu(E_k) - \mu(E_{k+1})$

$= \mu(E) + \underset{n\to\infty}{\mathrm{Lt}} [\mu(E_1) - \mu(E_2) + \ldots\ldots\ldots + \mu(E_n) - \mu(E_{n+1})] = \mu(E) + \underset{n\to\infty}{\mathrm{Lt}} [\mu(E_1) - \mu(E_{n+1})]$

$= \mu(E) + \mu(E_1) - \underset{n\to\infty}{\mathrm{Lt}} \mu(E_{n+1})$. Which implies $\mu(E) = \underset{n\to\infty}{\mathrm{Lt}} \mu(E_n)$.

IV. The Injective and Projective Properties of Lattice Measurable Functions

Definition4.1. An extended real value function f defined on a lattice measurable E is said to be lattice measurable function if the set $\{x \in E / f(x) > \alpha\}$ is lattice measurable for all real numbers α.

Example4.1. Constant functions, Continuous functions and Characteristic functions are lattice measurable functions.

Result4.1. If f and g are lattice measurable functions then $f \vee g$ and $f \wedge g$ are also lattice measurable functions.

Proof. For any real number α we have $\{x \in L / (f \vee g)(x) > \alpha\} = \{x \in L / f(x) > \alpha\} \vee \{x \in L / g(x) > \alpha\}$ and $\{x \in L / (f \wedge g)(x) > \alpha\} = \{x \in L / f(x) > \alpha\} \wedge \{x \in L / g(x) > \alpha\}$. Since $\{x \in L / f(x) > \alpha\}$ and $\{x \in L / g(x) > \alpha\}$ are lattice measurable sets implies the sets of RHS are lattice measurable implies $f \vee g$ and $f \wedge g$ are lattice measurable functions.

The following interesting property can easily be verified from the works of [6] by considering lattice measurable functions f and g defined over countable Boolean lattice.

Property4.1. A Countable Boolean lattice A is a Retrace of Countable Boolean lattice B if there exist homomorphism g: $A \rightarrow B$ and f: $B \rightarrow A$ such that fg is the identity on A. Here g and f are necessarily a monomorphism (injection) and epimorphism(projective) respectively. That is A Countable Boolean lattice is Retrace injective if it is a Retrace of every Countable Boolean lattice that contains it

V. Conclusion

New concepts like countable join property, countable meet property, P_σ–lattice and P_δ–lattice are introduced. Characterized partial lattices of a lattice through countable join and meet properties and proved that P_σ–lattice and P_δ–lattice are measureable partial lattices. Interesting result on the injective property of the lattice measurable functions defined over Countable Boolean lattices are established.

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Numerical Approach for Solving Stiff Differential Equations: A Comparative Study

By Sharaban Thohura & Azad Rahman
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Abstract - In this paper our attention is directed towards the discussion of phenomenon of stiffness and towards general purpose procedures for the solution of stiff differential equations. Our aim is to identify the problem area and the characteristics of the stiff differential equations for which the equations are distinguishable. Most realistic stiff systems do not have analytical solutions so that a numerical procedure must be used. Computer implementation of such algorithms is widely available e.g. DIFSUB, GEAR, EPISODE etc. The most popular methods for the solution of stiff initial value problems for ordinary differential equations are the backward differentiation formulae (BDFs). In this study we focus on a particularly efficient algorithm which is named as EPISODE, based on variable coefficient backward differentiation formula. Through this study we find that though the method is very efficient it has certain problem area for a new user. All those problem area have been detected and recommended for further modification.

GJSFR-F Classification : MSC 2010: 12H20

NUMERICAL APPROACH FOR SOLVING STIFF DIFFERENTIAL EQUATIONS A COMPARATIVE STUDY

Notes

Numerical Approach for Solving Stiff Differential Equations: A Comparative Study

Sharaban Thohura [α] & Azad Rahman [σ]

Abstract - In this paper our attention is directed towards the discussion of phenomenon of stiffness and towards general purpose procedures for the solution of stiff differential equations. Our aim is to identify the problem area and the characteristics of the stiff differential equations for which the equations are distinguishable. Most realistic stiff systems do not have analytical solutions so that a numerical procedure must be used. Computer implementation of such algorithms is widely available e.g. DIFSUB, GEAR, EPISODE etc. The most popular methods for the solution of stiff initial value problems for ordinary differential equations are the backward differentiation formulae (BDFs). In this study we focus on a particularly efficient algorithm which is named as EPISODE, based on variable coefficient backward differentiation formula. Through this study we find that though the method is very efficient it has certain problem area for a new user. All those problem area have been detected and recommended for further modification.

I. Introduction

A very important special class of differential equations taken up in the initial value problems termed as stiff differential equations result from the phenomena with widely differing time scales. There is no universally accepted definition of stiffness.Stiffness is a subtle, difficult and important concept in the numerical solution of ordinary differential equations. It depends on the differential equation, the initial condition and the interval under consideration.

The initial value problems with stiff ordinary differential equation systems occur in many fields of engineering science, particularly in the studies of electrical circuits, vibrations, chemical reactions and so on. Stiff differential equations are ubiquitous in astrochemical kinetics, many control systems and electronics, but also in many non-industrial areas like weather prediction and biology.

A set of differential equations is "stiff" when an excessively small step is needed to obtain correct integration. In other words we can say a set of differential equations is "stiff" when it contains at least two "time constants" (where time is supposed to be the joint independent variable) that differ by several orders of magnitude. A more rigorous definition of stiffness was also given by Shampine and Gear: "By a stiff problem we mean one for which no solution component is unstable (no eigenvalue of the Jacobian matrix has a real part which is at all large and positive) and at least some component is very stable (at least one eigenvalue has a real part which is large and negative). Further, we

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will not call a problem stiff unless its solution is slowly varying with respect to most negative part of the eigenvalues. Consequently a problem may be stiff for some intervals and not for others."

When solving the (vector) system of equations

$$y' = f(t, y), \qquad y(t_0) = a \text{ (given)} \tag{1}$$

we must consider the behavior of solutions near to the one we seek. This is because as we step along from $y_n = y(x_n)$ to y_{n+1} approximating $y(x_n + h)$ we make inevitable errors causing us to move from the desired integral curve to a nearby one. If we make no further errors, we follow this new curve so that the resulting error depends on the relative behavior of the two solution curves. Let us consider the example of the single equation

$$y' = A(y - p(t)) + p'(t), \qquad y(t_o) = a \tag{2}$$

where A is constant. The analytical solution is

$$y(t) = (a - p(0))\exp(At) + p(t) \tag{3}$$

If A is large and positive, the solution curves for the various a fan out and we say the problem is unstable. Such a problem, obviously, is difficult for any general numerical method, which proceeds in a step-by-step fashion. When A is small in magnitude, the curves are more or less parallel and such neutrally stable problems are easily handled by conventional means. When A is large and negative, the solution curves converge very quickly. In fact, whatever be the value of $y(t_o)$, the solution curve is virtually identical to the particular solution $p(t)$ after a short distance called an initial transient. This super-stable situation is ideal for the propagation of error in a numerical scheme. The last class of problems is called stiff.

If A is very negative and $p(t)$ is slowly varying, equation (3) represents a stiff problem after the transient e^{At} has died out (that is, e^{At} is below the error tolerance of interest) but it is not be stiff in the transient region. If (1) is linear with a constant Jacobian J (where $\boldsymbol{J} = \partial \mathbf{f} / \partial \mathbf{y}$ is the associated Jacobian matrix), it will not be stiff in the initial transient, but will be stiff after the fastest transient has died out. We see that in case of stiff differential equation problem the solution being sought is varying slowly, but there are nearby solutions that vary rapidly, so the numerical method must take small steps to obtain satisfactory results. Stiffness is an efficiency issue. If we were not concerned with how much time a computation takes, we would not be concerned about stiffness. Nonstiff methods can solve stiff problems, but take a long time to do it.

II. Numerical Solution of Stiff Differential Equation

As stiff differential equations occur in many branches of engineering and science, it is required to solve efficiently. Most realistic stiff systems do not have analytical solutions so that a numerical procedure must be used. Conventional methods such as Euler, explicit Runge-Kutta and Adams –Moulton are restricted to a very small step size in order to that the solution be stable. This means that a great deal of computer time could be required.

In a number of areas, particularly in chemical applications one often encounters systems of ordinary differential equations which, although mathematically well conditioned, are virtually impossible to solve with traditional numerical methods because

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of the severe step size constraint imposed by numerical stability. These stiff equations can be characterized by the presence of transient components which, although negligible relative to the numerical solution, constrain the step size of traditional numerical methods to be of the order of the smallest time constant of the problem.

Over the last three decades, there has been significant progress in the development of numerical stiff ODE solvers both in the areas of ODE solution algorithms and the associated linear algebra. Consequently, a wide variety of very efficient and reliable ODE solvers have been developed. In order to take full advantage of the available state-of-the-art solvers, and to handle computationally demanding various models in the different field both accurately and efficiently, a great deal of understanding is required for the formulation of the problem. The numerical solution algorithm of a standard stiff ODE solver package comprises two major components: one is the numerical solution method for the systems of ODEs and the other is for the solution of the resulting linear algebraic system that arises due to the ODEs solution technique. The structure of the resulting matrix associated with the linear system has significant computational consequence.

To better understand the advanced ODE solvers and their differences, we first need to briefly consider the solution methods underlying stiff systems of ODEs and their corresponding linear algebra. For the solution of a system of ODEs of size N of the form (1) and a given initial condition, $\mathbf{y}\ (t_0) = \mathbf{a}$, some classes of multistep methods are generally used. To advance the solution in time t from one mesh point to the next, considering a discrete time mesh $\{t_0, t_1, \ldots\ldots t_n \ldots\ldots\}$, multistep methods make use of several past values of the variable $\mathbf{y}$ and its rate of change $\mathbf{f}$ with respect to time t (i.e. the past values of the abundances and the rate equations). The general form of a k-step multistep method is

$$\sum_{i=0}^{k} \alpha_i \mathbf{y}_{n-i} = h \sum_{i=0}^{k} \beta_i \mathbf{f}_{n-i} \tag{4}$$

where α_i and β_i are constants depending on the order the method, h is the step size in time and n denotes the mesh number. The well-known Adams methods which use mostly the past values of $\mathbf{f}$,

$$\mathbf{y}_n = \mathbf{y}_{n-1} + h \sum_{i=0}^{k} \beta_i \mathbf{f}_{n-i} \tag{5}$$

are the best-known multistep methods for solving nonstiff problems. Each step requires the solution of a nonlinear system and often a simple functional iteration with an initial guess, or predictor estimate, is used to advance the integration, which is terminated by a convergence test. For stiff problems, where sudden changes in the variables can occur (i.e. there are strong dependencies of the rate equations $\mathbf{f}$ upon abundances $\mathbf{y}$ in small time intervals say), simple iteration leads to unacceptable restriction of the step size and functional iteration fails to converge. Thus, stiffness forces the use of implicit methods with infinite stability regions when there is no restriction on the step size. The backward difference formulae (BDF) methods with unbounded region of absolute stability were the first numerical methods to be proposed for solving stiff ODEs (Curtiss and Hirschfelder, 1952). The BDF used in ODE solvers, are of the general form

$$\mathbf{y}_n = \sum_{i=0}^{k} \alpha_i \mathbf{y}_{n-i} + h\beta_0 \mathbf{f}_n \tag{6}$$

where α_i and β_0 are coefficients of k th order, k-step BDF methods. As mentioned earlier, a simple functional iteration will usually fail to converge when problems are stiff and some form of Newton iteration is usually used for the solution of the resulting nonlinear system. The Newton iteration involves the solution of an $N \times N$ matrix, $\boldsymbol{P}$,

Notes

$$\boldsymbol{P} \approx \boldsymbol{I} - h\beta_0 \boldsymbol{J}, \tag{7}$$

where $\boldsymbol{J} = \partial \mathbf{f} / \partial \mathbf{y}$ is the associated Jacobian matrix, I is an $N \times N$ identity matrix. The solution to this linear algebraic system contributes significantly to the total computational time for the solution of stiff problems, as well as affecting the accuracy of the solution (and hence, also affecting the computational time). For stiff problems the ODE solvers use a modified Newton iteration that allows time saving strategies for the computation, storage and the use of the Jacobian matrix. When solving a linear algebraic system, there are generally two classes of solution methods, direct methods and iterative methods. The most common direct method used to solve linear systems is the Gaussian elimination method based on factorization of the matrix in lower and upper triangular factors. The GEAR, LSODE and VODE solvers all use such a method for the solution of the resulting linear system. The simplest iterative scheme used to solve linear systems is the Jacobi iteration, although the more sophisticated iterative solution methods of Krylov subspace methods, based on a sequence of orthogonal vectors and matrix-vector multiplications, have been widely used in practical applications such as computational fluid dynamics (Saad, 2003a). The ODE solvers, LSODPK and VODPK implement Preconditioned Krylov iterative techniques for the solution of the resulting linear system.
Some of the more readily available methods for stiff equations include:

- Variable- order methods based on backward differentiation multistep formulas, originally analysed and implemented by Gear (1969,1971) and later modified and studied by Hindmarsh (1974) and Byrne and Hindmarsh (1975).
- Methods based on trapezoidal rule, such as those proposed by Dahlquist (1963) and subsequently studied by Lindberg (1971, 1972).
- Implicit Runge-Kutta methods suitable for stiff equations, such as those based on the formulas of Butcher (1964) and studied by Ehle (1968).
- Methods based on the use of preliminary mathematical transformations to remove stiffness and the solution of the transformed problem by traditional techniques, such as those studied and implemented by Lawson and Ehle (1972).
- Methods based on second derivative multistep formulas, such as those developed by Linger and Willoughby (1967) and Enright (1974).

Unfortunately, although a number of methods have been developed, and many more basic formulas suggested for stiff equations, until recently there has been little advice or guidance to help a practitioner choose a good method for his problem.

In case of stiff differential equations stability requirements force the solver to take a lot of small time steps; this happens when we have a system of coupled differential equations that have two or more very different scales of the independent variable over which we are integrating. Another way of thinking about this to consider what must happen when two different parts of the solution require very different time steps. For example, suppose our solution is the combination of two exponential decay curves, one that decays away very rapidly and one that decays away very slowly. Except for the few time steps away from the initial condition, the slowly decaying curve dominate since the rapid curve will have decayed away. But because the variable time step routine to meet stability requirements for both components, we will be locked into small time steps even though the dominant component would allow much lager time steps. This is what we mean by stiff equations, we get locked into taking very small time steps for a component of the solution that makes infinitesimally small contributions to the solution. In other words, we are forced to move slowly when we could be leaping along to a solution.

The specific methods that we assess in this study are the methods based on backward differentiation formulas, DIFSUB (Gear (1971a, 1971b)), GEAR. Rev. 3 (Hindmarsh (1974)) and EPISODE (Byrne and Hindmarsh (1975)). As general ODE packages, GEAR and EPISODE are quite useful for both Stiff and nonstiff problems. In the nonstiff case, with the nonstiff method option, they seem to perform competitively in comparison with other sophisticated nonstiff system solvers. In the stiff case, these codes allow for the use of the Jacobian matrix, and contain routines for solving the associated linear systems, in full matrix form.

EPISODE is very similar to a package called GEAR [8], which is a heavily modified form of C.W. Gear's well-Known code, DIFSUB [9]. The GEAR package is based on fixed step formulas (Adams and BDF), and achieves changes in step size (when required) by interpolating to generate the multipoint data needed at the new spacing. In contrast, EPISODE is based on formulas that are truly variable-step, and step size changes can occurring as frequently as every step, with no interpolation involved. Like Gear, Episode varies its order in a dynamic way, as well as its step size, in an effort to complete the integration with a minimum number of steps. Lida A. M. Nejad [13] have summarized the functions of various packages in the following table

Table 1 : An abridged list of general-purpose solver packages available for solving systems of ODEs

Solver	Comments
GEAR (1974) – standard (supersedes DIFSUB – Gear 1968) GEARB – for Banded Jacobian GEARS – Sparse Jacobian	For stiff and nonstiff problems; for nonstiff problems-Adams methods, for stiff problems – fixed-coefficient form of BDF methods.
LSODE(1982) – standard LSODES – Sparse Jacobian	LSODE (Livermore Solver for ODEs) Combines the capabilities of GEAR and GEARB. Fixed-coefficient formulation of BDF methods.
LSODPK – with preconditioned Krylov iteration methods	LSODPK – uses a preconditioned Krylov iteration method for the solution of the linear system.
VODE (1989) – standard (supersedes EPISODE and EPISODEB)	VODE – variable-coefficient and fixed leading coefficient form of BDF for stiff systems.
VODPK (1992) – with preconditioned Krylov iteration methods	VODPK – uses preconditioned Krylov iteration methods for the solution of the linear system.
CVODE – in ANSI standard C	CVODE – with VODE and VODPK options written in C.
PVODE (1995) – Parallel VODE in ANSI standard C with preconditioned Krylov iteration methods.	PVODE – implements functional iteration, Newton iteration with a diagonal approximate Jacobian and Newton iteration with the iterative method SPGMR (Scaled Preconditioned Generalized Minimal Residual).

Ref

8. Gear, C.W. (1969) "*The automatic integration of stiff ordinary differential equations*". ***Information Processing 68***, A. J. H. Morrell, Ed., North Holland, Amsterdam, pp. 187-193.

III. Solver Package Episode

The EPISODE program is a package of FORTRAN subroutines aimed at the automatic solution of problems, with a minimum effort required in the face of possible difficulties in the problem. The program implements both a generalized Adams method, well suited for nonstiff problems, and a generalized backward differentiation formula (BDF), well suited for stiff problems. Both methods are of implicit multistep type. In solving stiff problems, the package makes the heavy use of the $N \times N$ Jacobian matrix,

$$J = \frac{\partial \mathbf{f}}{\partial \mathbf{y}} = \left(\frac{\partial \mathbf{f}_i}{\partial \mathbf{y}_j} \right)_{i,j=1}^{N}$$

the $\mathbf{f}_i$ and $\mathbf{y}_j$ are the vector components of **f** and **y,** respectively.

A complete discussion of the use of EPISODE is given in [11]. However, a few basic parameter definitions are needed here, in order to present the examples. Beyond the specification of the problem itself, represented by example 1 and perhaps example 2, the most important input parameter to EPISODE is the method flag, MF. This has eight values-10, 11, 12, 13, 20, 21, 22, and 23. The first digit of MF, called METH, indicates the two basic methods to be used namely implicit Adams and BDF. The second digit, called MITER, indicates the method of iterative solution of the implicit equations arising from the chosen formula. The parameter MITER takes four different values (0, 1, 2, 3) to indicate the following respectively

- Functional (or fixed-point) iteration (no Jacobian matrix used.).
- A chord method (or generalized Newton method, or semi-stationary Newton iteration) with Jacobian given by a subroutine supplied by the user.
- A chord method with Jacobian generated internally by finite differences.
- A chord method with a diagonal approximation to the Jacobian, generated internally (at less cost in storage and computation, but with reduced effectiveness).

The EPISODE package is used by making calls to a driver subroutine, EPSODE, which in turn calls other routines in the package to solve the problem. The function **f** is communicated by way of a subroutine, DIFFUN, which the user must write. A subroutine for the Jacobian, PEDERV, must also be written. Calls to EPSODE are made repeatedly, once for each of the user's output points. A value of t at which output is desired is put in the argument TOUT to EPSODE, and when TOUT is reached, control returns to the calling program with the value of y at t =TOUT. Another argument to EPSODE, called INDEX, is used to convey whether or not the call is the first one for the problem (and thus whether to initialize various variables). It is also used as an output argument, to convey the success or failure of the package in performing the requested task. Two other input parameters. EPS and IERROR, determine the nature of the error control performed within EPISODE.

The EPISODE package consists of eight FORTRAN subroutines, to be combined with the user's calling program and Subroutines DIFFUN and PEDERV. As discussed earlier, only Subroutine EPSODE is called by the user; the others are called within the package. The functions of the eight package routines can be briefly summarized as follows:

Ref.

11. Hindmarsh, A. C. and Byrne, G. D. (1975) "Episode. An experimental package for the integration of systems of ordinary differential equations", *L.L.L. Report UCID-30112, L.L.* (www.netlib.org/ode/epsode.f)

Notes

i) EPSODE sets up storage, makes calls to the core integrator, TSTEP, checks for and deals with error returns, and prints error message as needed.
ii) INTERP computes interpolated values of $\mathbf{y}(t)$ at the user specified output points, using an array of multistep history data.
iii) TSTEP performs a single step of the integration, and does the control of local error (which entails selection of the step size and order) for that step.
iv) COSET sets coefficients that are used by TSTEP, both for the basic integration step and for error control.
v) ADJUST adjusts the history array when the order is reduced.
vi) PSET sets up the matrix $p = I - h\beta_0 J$, where I is the identity matrix, h is the step size, β_0 is a scalar related to the method, and J is the Jacobian matrix. It then processes P for subsequent solution of linear algebraic system with P for subsequent solution of linear algebraic systems with P as coefficient matrix.
vii) DEC performs an LU (lower-upper triangular) decomposition of an $N \times N$ matrix.
viii) SOL solves linear algebraic systems for which the matrix was factored by DEC.

The subroutine EPSODE based on variable coefficient backward differentiation formula can be used. The nonstiff option uses an Adams-Bashforth predictor and an Adams-Moulton corrector.

$$\text{Predictor: } y_{n+1} = y_n + h\sum_{i=1}^{k}\beta_i y'_{n+1-i} \quad \& \quad \text{Corrector: } y_{n+1} = y_n + h\sum_{i=0}^{k}\beta_i y'_{n+1-i}$$

The order may vary from one to seven.

IV. Numerical Implementation

In order to illustrate how the EPISODE package can be used to solve stiff initial value problems, we give here an example, chosen from the areas of chemical kinetics. For each example problem, the appropriate FORTRAN coding for its solution, with EPISODE, is given, followed by the output generated by that coding.

Example 1: A kinetics problem: The following kinetics problem, given by Robertson, is frequently used as an illustrative example. It involves the following three nonlinear rate equations:

$$y_1' = -.04y_1 + 10^4 y_2 y_3 \tag{8}$$

$$y_2' = .04y_1 - 10^4 y_2 y_3 - 3.10^7 y_2^2 \tag{9}$$

$$y_3' = 3.10^7 y_2^2 \tag{10}$$

The initial values at $t = 0$ are

$$y_1(0) = 1, \qquad y_2(0) = y_3(0) = 0 \tag{11}$$

Since $\sum y_i' = 0$, the solution must satisfy $\sum y_i = 1$, identically. This identity can be used as an error check.

Global Journal of Science Frontier Research (F) Volume XIII Issue VI Version I Year 2013

Here we intend to solve this problem with the BDF method and use the chord of iteration method with the user-supplied Jacobian (MITER=1). Suppose a local error bound of $\mathrm{EPS}=10^{-6}$, and control absolute error (IERROR=1). We choose an initial step size of $\mathrm{H0}=10^{-8}$. The use of MITER=1 requires that the Jacobian $J=\partial f/\partial y$ be calculated and programmed. This is given by

$$J=\begin{pmatrix} -.04 & 10^4 y_3 & 10^4 y_2 \\ .04 & -10^4 y_3 - 6.10^7 y_2 & -10^4 y_2 \\ 0 & 6.10^7 y_2 & 0 \end{pmatrix}$$

The final value of t is 40. So we consider taking output at $t=4\times10^k$, where $k=$ -1, 0, 1, 2, …. These will be the values of the argument TOUT.

The following coding, together with the EPISODE package, can be used to solve this problems with the options described above. The output of the above program in tabular form is as follows:

Table 2 : MF=21, EPS=10^{-6}

T	H	Y_1	Y_2	Y_3	SUM(Y)-
0.4E+00	0.16E+00	0.98517E+00	0.33864E-04	0.14794E-01	-0.4E-15
0.4E+01	0.56E+00	0.90552E+00	0.22405E-04	0.94462E-01	-0.5E-15
0.4E+02	0.23E+01	0.71582E+00	0.91851E-05	0.28417E+00	-0.3E-16
0.4E+03	0.20E+02	0.45051E+00	0.32228E-05	0.54949E+00	-0.5E-15
0.4E+04	0.24E+03	0.18320E+00	0.89423E-06	0.81680E+00	-0.2E-15
0.4E+05	0.33E+04	0.38986E-01	0.16219E-06	0.96101E+00	0.3E-15
0.4E+07	0.38E+06	0.50319E-03	0.20660E-08	0.99948E+00	0.3E-15
0.4E+10	0.12E+10	0.54561E-06	0.37316E-11	0.10000E+01	-0.1E-13

We see that the equilibrium values are $y_1=y_2=0$, $y_3=1$ and that the approach to equilibrium is quite slow. Here we note that the time step, H, rises steadily with time, T. We also observe that the code generated negative and thus physically incorrect answers during the last decade. This reflects instability, or a high sensitivity of the problem to numerical errors at late t, and will, if the integration is continued, lead to answers diverging to $\pm\infty$. The accuracy of the above result can be verified in the usual way- by re running the program with a smaller value of EPS=10^{-9} and nothing else changed, the output in the tabular form is as follows:

Notes

Table 3 : MF=21, EPS=10^{-9}

T	H	Y_1	Y_2	Y_3	SUM(Y)-
0.4E+00	0.34E-01	0.985172E+00	0.338641E-04	0.147940E-01	0.2E-15
0.4E+01	0.14E+00	0.905519E+00	0.224048E-04	0.944589E-01	0.5E-15
0.4E+02	0.13E+01	0.715827E+00	0.918552E-05	0.284164E+00	0.6E-15
0.4E+03	0.82E+01	0.450519E+00	0.322290E-05	0.549478E+00	0.8E-15
0.4E+04	0.76E+02	0.183202E+00	0.894237E-06	0.816797E+00	0.1E-14
0.4E+05	0.88E+03	0.389834E-01	0.162177E-06	0.961016E+00	0.9E-15
0.4E+07	0.20E+06	0.516813E-03	0.206835E-08	0.999483E+00	0.1E-14
0.4E+10	0.67E+09	0.522363E-06	0.208942E-11	0.999999E+00	0.1E-14

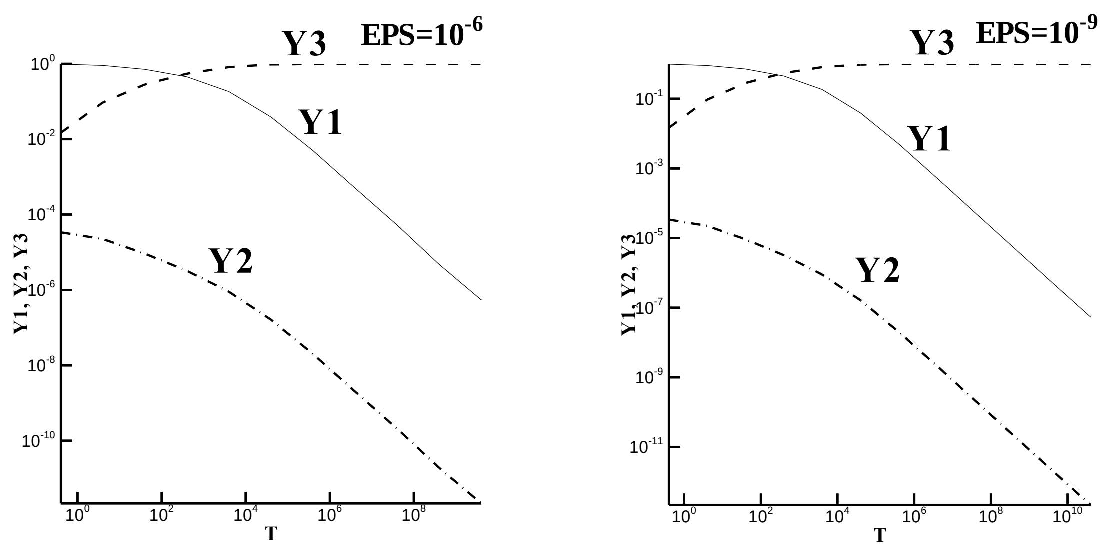

Fig. 1 : The graph of the approximated solution of Example 1 (by using log scale)

Now we consider another example of stiff system of differential equations which can be solved analytically.

Example 2: The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t, \qquad u_1(0) = \frac{4}{3} \tag{12}$$

$$u_2' = -24u_1 - 51u_2 - 95\cos t + \frac{1}{3}\sin t, \qquad u_1(0) = \frac{2}{3} \tag{13}$$

has the unique solution

$$u_1(t) = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t. \qquad (14)$$

$$u_2(t) = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t \qquad (15)$$

Notes

The transient term e^{-39t} in the solution causes this system to be stiff. The results, obtained by EPISODE are summarized in the following table.

Table 4

t	h	Approximated value of $u_1(t)$	Approximated value of $u_2(t)$	Exact value of $u_1(t)$	Exact value of $u_2(t)$
0.0	.40E-02	1.33333333	0.666666666	1.33333333	0.666666666
0 .1	.40E-02	1.79306146	-1.03200020	1.79306300	-1.03200200
0.2	.91E-02	1.42390205	-0.87468033	1.42390200	-0.87468100
0.3	.12E-01	1.13157624	-0.72499799	1.13157700	-0.72499860
0.4	.33E-01	0.90940824	-0.60821345	0.90940860	-0.60821420
0.5	.33E-01	0.73878794	-0.51565752	0.73878780	-0.51565770
0.7	.66E-01	0.49986115	-0.37740429	0.49986030	-0.37740380
1.0	.66E-01	0.27968063	-0.22989065	0.27967490	-0.22988780

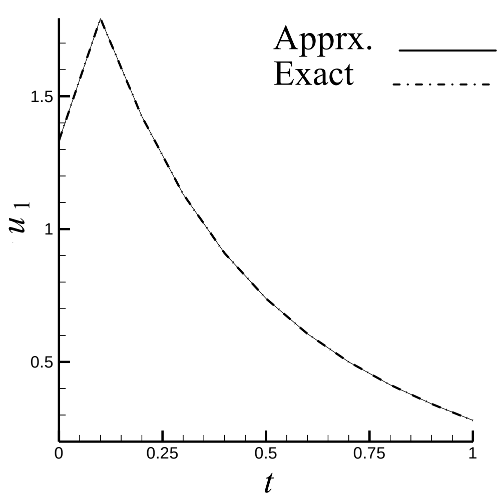

Fig. 2(a) : The graph of the solutions for u_1

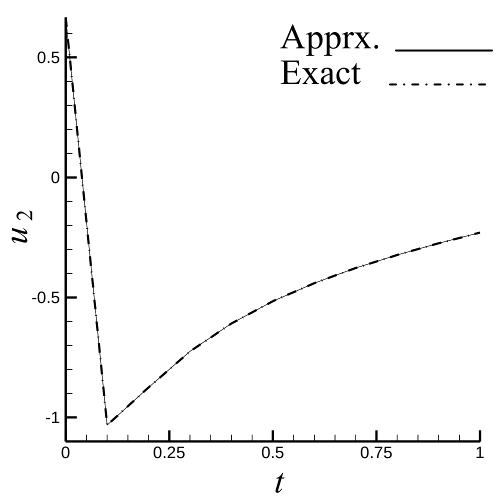

Fig. 2(b) : The graph of the solutions for u_2

Here we observe that the graph of the approximated solution and the graph of the exact solution coincide with each other.

V. Conclusion

Notes

In this paper our aim is to study the phenomenon of stiff differential equations and the general purpose procedures for the solution of stiff differential equations. There are effective codes available to solve these problems, but it is necessary that the user may have some idea how they work in order to take full advantage of them. Although a number of methods have been developed, and many more basic formulas are suggested for stiff equations, until now there has been little advice or guidance to help a user choose a good method for his problem. In our study we focus on a particularly efficient program which is named as EPISODE. We explain the capabilities of this code and present few practical examples for which it is effective. However, this experimental package EPISODE requires some explanation. First of all, the program is relatively new and has not been used extensively, and so its position in the field of existing available ordinary differential equation software is not yet clear. Secondly we have shown that, for some types of problems, the program spends more time on the linear system of the algorithm than we feel it should. This behavior is related to the extent to which the matrix during the solution of a problem, and in this area improvement of the efficiency of the algorithm is required.

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Some Indefinite Integrals in the Light of Hypergeometric Function

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GJSFR-F Classification : MSC NO: 33C05,33C45,33C15,33D50,33D60

SOME INDEFINITE INTEGRALS IN THE LIGHT OF HYPERGEOMETRIC FUNCTION

Strictly as per the compliance and regulations of :

Some Indefinite Integrals in the Light of Hypergeometric Function

Salahuddin [α] & Intazar Husain [σ]

Abstract - In this paper we have evaluated some indefinite integrals associated to Hypergeometric function. The results represent here are assume to be new.

Keywords : pochhammer symbol; gaussian hypergeometric function; kampé de Fériet double hypergeometric function and srivastava's triple hypergeometric function.

I. Introduction and Preliminaries

The Pochhammer's symbol or Appell's symbol or shifted factorial or rising factorial or generalized factorial function is defined by

$$(b,\ k) = (b)_k = \frac{\Gamma(b+k)}{\Gamma(b)} = \begin{cases} b(b+1)(b+2)\cdots(b+k-1); & \text{if } \ k = 1, 2, 3, \cdots \\ 1 \qquad ; & \text{if } \ k = 0 \\ k! \qquad ; & \text{if } \ b = 1,\ k = 1, 2, 3, \cdots \end{cases}$$

where b is neither zero nor negative integer and the notation Γ stands for Gamma function.

a) *Generalized Gaussian Hypergeometric Function*

Generalized ordinary hypergeometric function of one variable is defined by

$${}_AF_B\left[\begin{matrix} a_1, a_2, \cdots, a_A & ; \\ & & z \\ b_1, b_2, \cdots, b_B & ; \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!}$$

or

$${}_AF_B\left[\begin{matrix} (a_A) & ; \\ & & z \\ (b_B) & ; \end{matrix}\right] \equiv {}_AF_B\left[\begin{matrix} (a_j)_{j=1}^{A} & ; \\ & & z \\ (b_j)_{j=1}^{B} & ; \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \qquad (1.1)$$

where denominator parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

b) *Kampé de Fjeriet's General Double Hypergeometric Function*

In 1921, Appell's four double hypergeometric functions F_1, F_2, F_3, F_4 and their confluent forms $\Phi_1, \Phi_2, \Phi_3, \Psi_1, \Psi_2, \Xi_1, \Xi_2$ were unified and generalized by Kampé de Fériet.

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We recall the definition of general double hypergeometric function of Kampé de Fériet in slightly modified notation of H.M.Srivastava and R.Panda:

$$\mathrm{F}^{A:B;D}_{E:G;H}\left[\begin{array}{ll}(a_A):(b_B);(d_D) & ; \\ & x,\ y \\ (e_E):(g_G);(h_H) & ;\end{array}\right] = \sum_{m,n=0}^{\infty}\frac{((a_A))_{m+n}\,((b_B))_m\,((d_D))_n\, x^m\, y^n}{((e_E))_{m+n}\,((g_G))_m\,((h_H))_n\, m!\, n!} \tag{1.2}$$

where for convergence

(i) $A+B<E+G+1,\ A+D<E+H+1\quad ;|x|<\infty,\quad |y|<\infty,$ or

(ii) $A+B=E+G+1,\ A+D=E+H+1,$ and

$$\begin{cases}|x|^{\frac{1}{(A-E)}}+|y|^{\frac{1}{(A-E)}}<1 & ,\text{if } E<A\\ \max\ \{|x|,|y|\}<1 & ,\text{if } E\geq A\end{cases}$$

c) Srivastava's General Triple Hypergeometric Function

In 1967, H. M. Srivastava defined a general triple hypergeometric function $F^{(3)}$ in the following form

$$F^{(3)}\left[\begin{array}{ll}(a_A)::(b_B);(d_D);(e_E):(g_G);(h_H);(l_L); & \\ & x,\ y,\ z\\ (m_M)::(n_N);(p_P);(q_Q):(r_R);(s_S);(t_T); & \end{array}\right]$$

$$=\sum_{i,j,k=0}^{\infty}\frac{((a_A))_{i+j+k}\ ((b_B))_{i+j}\ ((d_D))_{j+k}\ ((e_E))_{k+i}\ ((g_G))_i\ ((h_H))_j\ ((l_L))_k\ x^i\ y^j\ z^k}{((m_M))_{i+j+k}\ ((n_N))_{i+j}\ ((p_P))_{j+k}\ ((q_Q))_{k+i}\ ((r_R))_i\ ((s_S))_j\ ((t_T))_k\ i!\ j!\ k!} \tag{1.3}$$

d) Wright's Generalized Hypergeometric Function

$${}_p\Psi_q\left[\begin{array}{ll}(\alpha_1,A_1),\cdots,(\alpha_p,A_p) & ;\\ & x\\ (\lambda_1,B_1),\cdots,(\lambda_q,B_q) & ;\end{array}\right]=\sum_{m=0}^{\infty}\frac{\Gamma(\alpha_1+mA_1)\Gamma(\alpha_2+mA_2)\cdots\Gamma(\alpha_p+mA_p)x^m}{\Gamma(\lambda_1+mB_1)\Gamma(\lambda_2+mB_2)\cdots\Gamma(\lambda_q+mA_q)m!} \tag{1.4}$$

$${}_p\Psi^*_q\left[\begin{array}{ll}(\alpha_1,A_1),\cdots,(\alpha_p,A_p) & ;\\ & x\\ (\lambda_1,B_1),\cdots,(\lambda_q,B_q) & ;\end{array}\right]=\sum_{m=0}^{\infty}\frac{(\alpha_1)_{mA_1}(\alpha_2)_{mA_2}\cdots(\alpha_p)_{mA_p}x^m}{(\lambda_1)_{mB_1}(\lambda_2)_{mB_2}\cdots(\lambda_q)_{mB_q}m!} \tag{1.5}$$

II. Main Integrals

$$\int\frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\sin^3 y]}}=$$

$$=-\cos y\sin^{3m+1}y\ (\sin^2 y)^{\frac{-1-3m}{2}}\ F^{1;2}_{0;1}\left[\begin{array}{ll}\frac{1}{2}\ ;\frac{1}{2},\ \frac{1-3m}{2} & ;\\ & \frac{1+x}{2},\ \cos^2 y\\ \text{—};\ \frac{3}{2} & ;\end{array}\right]+Constant \tag{2.1}$$

Notes

Global Journal of Science Frontier Research (F) Volume XIII Issue VI Version I Year 2013

$$\int \frac{dy}{\sqrt{[1-(\frac{1+x}{2})\cos^3 y]}} =$$

$$= \frac{\sqrt{-\sin^2 y} \ \ \mathrm{cosec}\ y \cos^{3m+1} y}{3m+1} F^{1;2}_{0;1} \left[\begin{array}{ccc} \frac{1}{2} & ;\frac{1}{2}, \frac{3m+1}{2} & ; \\ & & \frac{1+x}{2}, \cos^2 y \\ \text{—}; & \frac{3m+3}{2} & ; \end{array} \right] + Constant \qquad (2.2)$$

Notes

$$\int \frac{dy}{\sqrt{[1-(\frac{1+x}{2})\tan^3 y]}} = \frac{\tan^{3m+1} y}{(3m+1)} F^{1;2}_{0;1} \left[\begin{array}{ccc} \frac{1}{2} & ;1, \frac{3m+1}{2} & ; \\ & & \frac{1+x}{2}, -\tan^2 y \\ \text{—}; & \frac{3m+3}{2} & ; \end{array} \right] + Constant \quad (2.3)$$

$$\int \frac{dy}{\sqrt{[1-(\frac{1+x}{2})\cot^3 y]}} = -\frac{\cot^{3m+1} y}{(3m+1)} F^{1;2}_{0;1} \left[\begin{array}{ccc} \frac{1}{2} & ;1, \frac{3m+1}{2} & ; \\ & & \frac{1+x}{2}, -\cot^2 y \\ \text{—}; & \frac{3m+3}{2} & ; \end{array} \right] + Constant$$

(2.4)

$$\int \frac{dy}{\sqrt{[1-(\frac{1+x}{2})\sec^3 y]}} =$$

$$= \sin(y)\cos^2(y)^{\frac{3m+1}{2}} \sec^{3m+1}(y)\ F^{1;2}_{0;1} \left[\begin{array}{ccc} \frac{1}{2} & ;\frac{1}{2}, \frac{1+3m}{2} & ; \\ & & \frac{1+x}{2}, \sin^2 y \\ \text{—}; & \frac{3}{2} & ; \end{array} \right] + Constant \qquad (2.5)$$

$$\int \frac{dx}{\sqrt{(1-(\frac{1+x}{2})\ \mathrm{cosech}^3\ y)}} =$$

$$= -\cos y (\sin^2(y))^{\frac{3m+1}{2}} \mathrm{cosec}^{3m+1} y\ F^{1;2}_{0;1} \left[\begin{array}{ccc} \frac{1}{2} & ;\frac{1}{2}, \frac{1+3m}{2} & ; \\ & & \frac{1+x}{2}, \cos^2 y \\ \text{—}; & \frac{3}{2} & ; \end{array} \right] + Constant \qquad (2.6)$$

III. Derivation of Integrals

Derivation of integral (2.1)

$$\int \frac{dy}{\sqrt{[1-(\frac{1+x}{2})\sin^3 y]}} = \int \left[1 - \left(\frac{1+x}{2}\right)\sin^3 y\right]^{-\frac{1}{2}} dy$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \sin^{3m} y\ dy = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \int \sin^{3m} y\ dy$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} (-\cos y)\ \sin^{3m+1} y (\sin^2 y)^{\frac{1-3m}{2}} {}_2F_1 \left[\begin{array}{ccc} \frac{1}{2}, & \frac{1-3m}{2} & ; \\ & & \cos^2 y \\ \frac{3}{2} & & ; \end{array} \right] + Constant$$

$$= -\cos y \sin^{3m+1} y\ (\sin^2 y)^{\frac{-1-3m}{2}}\ F^{1;2}_{0;1} \left[\begin{array}{ccc} \frac{1}{2} & ;\frac{1}{2}, \frac{1-3m}{2} & ; \\ & & \frac{1+x}{2}, \cos^2 y \\ \text{—}; & \frac{3}{2} & ; \end{array} \right] + Constant \qquad (3.1)$$

Derivation of integral (2.2)

$$\int \frac{dy}{\sqrt{[1-(\frac{1+x}{2})\cos^3 y]}} = \int \left[1-\left(\frac{1+x}{2}\right)\cos^3 y\right]^{-\frac{1}{2}} dy$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \cos^{3m} y \; dy = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \int \cos^{3m} y \; dy$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \frac{\sqrt{-\sin^2 y}\cos^{3m+1} y \; \mathrm{cosec}\; y}{(3m+1)} {}_2F_1\left[\begin{array}{c} \frac{1}{2}, \frac{3m+1}{2} \; ; \\ \frac{3m+3}{2} \; ; \end{array} \cos^2 y\right] + Constant$$

$$= \frac{\sqrt{-\sin^2 y}\;\; \mathrm{cosec}\; y \cos^{3m+1} y}{3m+1} F^{1;2}_{0;1}\left[\begin{array}{ccc} \frac{1}{2} & ; \frac{1}{2}, \frac{3m+1}{2} & ; \\ \text{—} & ; \frac{3m+3}{2} & ; \end{array} \frac{1+x}{2}, \cos^2 y\right] + Constant \qquad (3.2)$$

Derivation of integral (2.3)

$$\int \frac{dy}{\sqrt{[1-(\frac{1+x}{2})\tan^3 y]}} = \int \left[1-\left(\frac{1+x}{2}\right)\tan^3 y\right]^{-\frac{1}{2}} dy$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \tan^{3m} y \; dy = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \int \tan^{3m} y \; dy$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \frac{\tan^{3m+1} y}{(3m+1)} {}_2F_1\left[\begin{array}{c} 1, \frac{3m+1}{2} \; ; \\ \frac{3m+3}{2} \; ; \end{array} -\tan^2 y\right] + Constant$$

$$= \frac{\tan^{3m+1} y}{(3m+1)} F^{1;2}_{0;1}\left[\begin{array}{ccc} \frac{1}{2} & ; 1, \frac{3m+1}{2} & ; \\ \text{—} & ; \frac{3m+3}{2} & ; \end{array} \frac{1+x}{2}, -\tan^2 y\right] + Constant \qquad (3.3)$$

Derivation of integral (2.4)

$$\int \frac{dy}{\sqrt{[1-(\frac{1+x}{2})\cot^3 y]}} = \int \left[1-\left(\frac{1+x}{2}\right)\cot^3 y\right]^{-\frac{1}{2}} dy$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \cot^{3m} y \; dy = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \int \cot^{3m} y \; dy$$

$$= -\sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \frac{\cot^{3m+1} y}{(3m+1)} {}_2F_1\left[\begin{array}{c} 1, \frac{3m+1}{2} \; ; \\ \frac{3m+3}{2} \; ; \end{array} -\cot^2 y\right] + Constant$$

$$= -\frac{\cot^{3m+1} y}{(3m+1)} F^{1;2}_{0;1}\left[\begin{array}{ccc} \frac{1}{2} & ; 1, \frac{3m+1}{2} & ; \\ \text{—} & ; \frac{3m+3}{2} & ; \end{array} \frac{1+x}{2}, -\cot^2 y\right] + Constant \qquad (3.4)$$

Notes

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Derivation of integral (2.5)

$$\int \frac{dy}{\sqrt{(1-(\frac{1+x}{2})\ \sec^3\ y)}} = \int \left[1-\left(\frac{1+x}{2}\right)\ \sec^3\ y\right]^{-\frac{1}{2}} dy$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \sec^{3m} y\ dy = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \int \sec^{3m} y\ dy$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!}\ \sin y\ \cos^2(y)^{\frac{3m+1}{2}} \sec^{3m+1}\ y\ {}_2F_1\left[\begin{array}{ccc} \frac{1}{2}, \frac{3m+1}{2} & ; & \sin^2 y \\ \frac{3}{2} & ; & \end{array}\right] + Constant$$

$$= \sin(y) \cos^2(y)^{\frac{3m+1}{2}} \sec^{3m+1}(y)\ F^{1;2}_{0;1}\left[\begin{array}{cccc} \frac{1}{2} & ;\frac{1}{2}, \frac{1+3m}{2} & ; & \\ & & & \frac{1+x}{2}, \sin^2 y \\ -; & \frac{3}{2} & ; & \end{array}\right] + Constant \qquad (3.5)$$

Derivation of integral (2.6)

$$\int \frac{dy}{\sqrt{(1-(\frac{1+x}{2})\ \mathrm{cosec}^3\ y)}} = \int \left[1-\left(\frac{1+x}{2}\right)\ \mathrm{cosec}^3\ y\right]^{-\frac{1}{2}} dy$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \mathrm{cosec}^{3m} y\ dy = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!} \int \mathrm{cosec}^{3m} y\ dy$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!}\ (-\cos y)\ (\sin^2(y))^{\frac{3m+1}{2}} \mathrm{cosec}^{3m+1}\ y\ {}_2F_1\left[\begin{array}{ccc} \frac{1}{2}, \frac{3m+1}{2} & ; & \cos^2 y \\ \frac{3}{2} & ; & \end{array}\right] + Constant$$

$$= -\cos y (\sin^2(y))^{\frac{3m+1}{2}} \mathrm{cosec}^{3m+1} y\ F^{1;2}_{0;1}\left[\begin{array}{cccc} \frac{1}{2} & ;\frac{1}{2}, \frac{1+3m}{2} & ; & \\ & & & \frac{1+x}{2}, \cos^2 y \\ -; & \frac{3}{2} & ; & \end{array}\right] + Constant \qquad (3.6)$$

IV. Conclusion

In our work we have established hypergeometric form of some indefinite integrals . We can only expect that the development presented in this work will stimulate further interest and research in this important area of classical special functions. Just as the mathematical properties of the Gauss hypergeometric function are already of immense and significant utility in mathematical sciences and numerous other areas of pure and applied mathematics, the elucidation and discovery of the formula of hypergeometric functions considered herein should certainly eventually prove useful to further developments in the broad areas alluded to above.

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Notes

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Solution of Kinematic Wave Equation Using Finite Difference Method and Finite Element Method

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Abstract - The Various Numerical Methods are applied to solve the spatially varied unsteady flow equations (Kinematic Wave) in predicting the discharge, depth and velocity in a river. Solutions of Kinematic Wave equations through finite difference method (Crank Nicolson) and finite element method are developed for this study. The computer program is also developed in Lahey ED Developer and for graphical representation Tecplot 7 software is used. Finally some problems are solved to understand the method.

Keywords : kinematic wave, overland flow, channel flow, finite element method, crank-nicolson method.

GJSFR-F Classification : MSC 2010: 51J15 , 81R20, 35R20

SOLUTION OF KINEMATIC WAVE EQUATION USING FINITE DIFFERENCE METHOD AND FINITE ELEMENT METHOD

Strictly as per the compliance and regulations of :

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Dr. M. M. Hossain [α] & J. Ferdous Ema [σ]

Abstract - The Various Numerical Methods are applied to solve the spatially varied unsteady flow equations (Kinematic Wave) in predicting the discharge, depth and velocity in a river. Solutions of Kinematic Wave equations through finite difference method (Crank Nicolson) and finite element method are developed for this study. The computer program is also developed in Lahey ED Developer and for graphical representation Tecplot 7 software is used. Finally some problems are solved to understand the method.

Keywords : kinematic wave, overland flow, channel flow, finite element method, crank-nicolson method.

I. Introduction

Hydrology (from Greek : Yδωρ, hudōr, "water"; and λόγος, logos, "study") is the study of the movement, Distribution, and quality of water throughout the Earth and thus addresses both the hydrologic cycle and water resources. So in the broadest sense it is the study of water in all its phases and includes hydraulics, the physics and chemistry of water, meteorology, geology and biology. But the word as used by the scientists and engineers usually has a considerably narrower connotation. In this more limited sense, "Hydrology can be defined as that branch of physical geography, which is concerned with the origin. distributaries movement and properties of the waters of the Earth". The study of hydrology thus concerns itself with the occurrence and transportation of the waters through air, Over the ground and through the strata of the earth and this includes three important phases of what is known as the hydrological cycle, namely rainfall, runoff and evaporation. Hydrology is therefore, bounded above by meteorology, below by geology and at land's end by oceanology. Engineering hydrology includes those segments of hydrology pertinent to the design and operation of engineering projects for the control and use of water. Hydrology means the science of water. It is a branch of earth science. Basically it is an applied science.

Domains of hydrology include hydrometeorology, surface hydrology, hydrogeology, drainage basin management and water quality, where water plays the central role. In general sense hydrology deals with (i) Water resources estimation (ii) Acquisition of processes such as precipitation, runoff and evapo-transporation.

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II. Model Development

a) Kinemtic Wave Equations From Saint Venant Equations

The St. Venant equations characterizing the dynamic flow can be written as:

Continuity:
$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q+(i-\phi) \tag{1}$$

Momentum:
$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial y_0}{\partial x}=g(s_f-s_0)-q(\frac{u-v}{A}) \tag{2}$$

Notes

The equation (1) may be rewritten in the following form for a ready reference to the various types of wave models that are recognized.

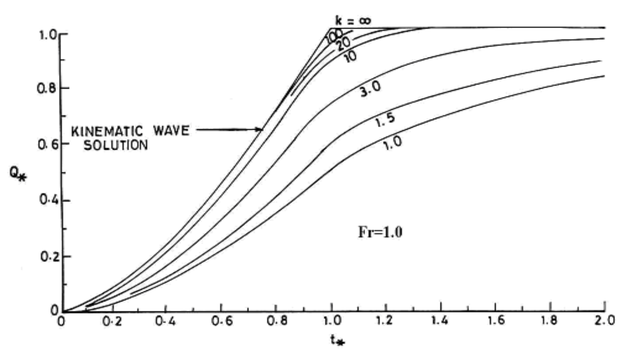
Term: I II III IV Equation of motion: $\frac{1}{g}\frac{\partial u}{\partial t}+\frac{u}{g}\frac{\partial u}{\partial x}+\frac{\partial y_0}{\partial x}+(s_f-s_0)=0$

Local Convective Depth acceleration acceleration slope

Wave model and terms used to describe it are:

Kinematics wave only term	IV= 0
Diffusion wave	III+ IV=0
Steady dynamic wave	II+III+IV=0
Dynamic wave	I+II+III+IV=0
Gravity wave	I+II+III= 0

and other terms are neglected.



b) Hydrodynamic Theory And Kinematic Wave Equations

The hydrodynamic theory for incompressible fluid flows gives the following set of equations (also known as the Navier-Stokes' equations):

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=X-\frac{\partial P}{\partial x}+\mu\nabla^2 u$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=Y-\frac{\partial P}{\partial y}+\mu\nabla^2 v$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=Z-\frac{\partial P}{\partial z}+\mu\nabla^2 w$$

and continuity equation: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

where $$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2};$$

Notes

ρ = the mass density ;
u, v and w are the velocity components in the x, y and z direction respectively;
X, Y, Z are the body forces per unit volume;
P = pressure and μ = viscosity.

c) *Elements Used In Kinematics Wave Models*

In this work, for computational purpose, the following two types of elements have been identified:

(i) Overland flow elements and

(ii) Channel flow elements (Fig. 3.1)

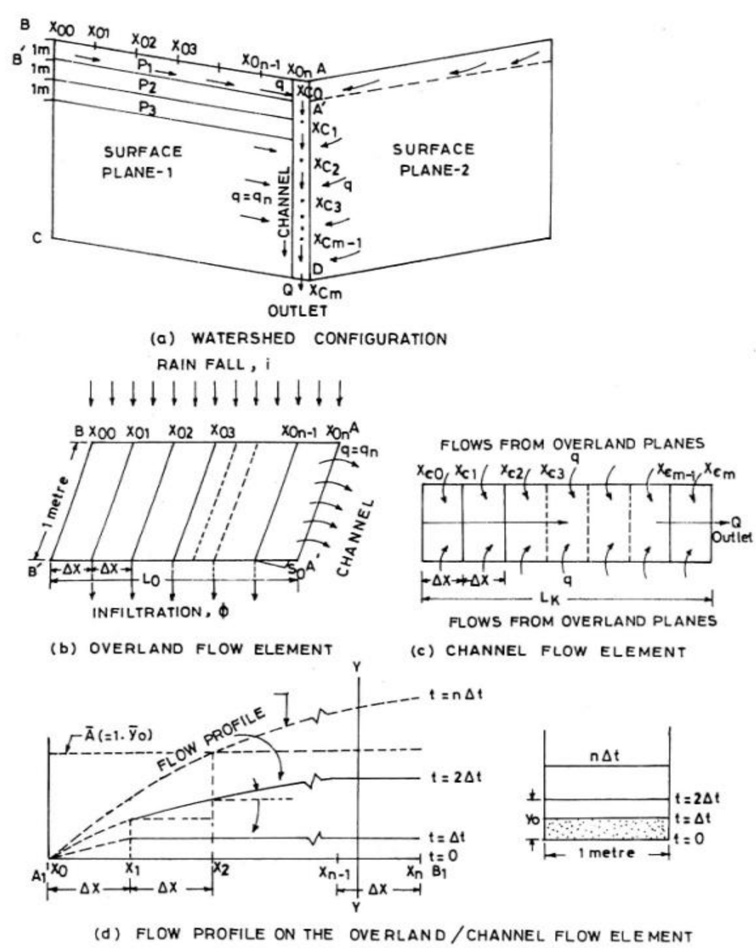


FIG. 3.1 GENERATION OF FLOW PROFILE

d) *Trapezoidal Channel Cross Section*

A trapezoidal cross-section is the most general type of channel cross-section. It is defined by the cannel side slope (Z), and the channel bottom width (B) (Fig.3.2).

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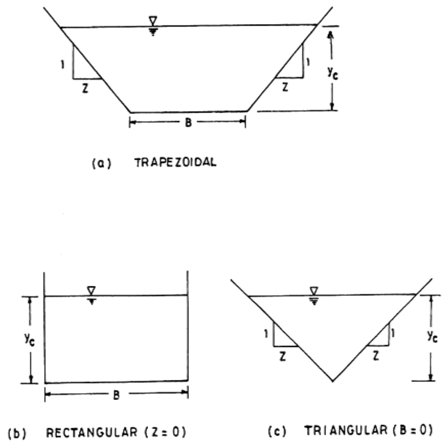

FIG. 3.2 CHANNEL SHAPES FOR KW CHANNEL ROUTING

e) The Final Form of Kinematic Wave Equations For The Channel Flows

The unknown parameters for the channel shapes under consideration i.e. α_k and m_k being the unknown functions. The KW equation for the channel flow can be written by combining equations (3.19) and (3.20) as given below:

$$\frac{\partial A}{\partial t} + \frac{\partial(\alpha_k A^{m_k})}{\partial x} = q$$

If α_k is independent of x, then the equation becomes:

$$\frac{\partial A}{\partial t} + \alpha_k m_k \frac{\partial(A^{m_k - 1})}{\partial x} = q$$

Crank-Nicolson and other methods:

First Order one-way wave equation

The first order wave equation in one-dimensional space is as follows:

$$u_t = cu_x$$

where c is a positive constant, and u(x, t) is subject to the initial condition

$$u(x, 0) = f(x), \qquad -\infty < x < \infty.$$

The solution for $t \geq 0$ and all x is a family of characteristics, which are straight lines shifted to the left in the x, t- plane, inclined to the x-axis at an angle

$$\Theta = \tan^{-1}(\tfrac{1}{c}).$$

The explicit solution is

$$u(x, t) = f(x + ct).$$

Notes

Table 1. Explicit finite difference schemes for first order 1-D wave equation

FD Scheme	Matrix Representation
Forward Euler (FEU) $u_j^{n+1} = u_j^n + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1 & \frac{1}{2}r & & \\ -\frac{1}{2}r & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -\frac{1}{2}r \\ 0 \\ \vdots \\ 0 \\ \frac{1}{2}r \end{bmatrix}$
Upwind (UPW) $u_j^{n+1} = u_j^n + r\left(u_{j+1}^n - u_j^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1-r & r & \\ & \ddots & \ddots \\ & & \ddots & \ddots \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ r \end{bmatrix}$
Lax-Friedrichs (LXF) $u_j^{n+1} = \frac{1}{2}\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 0 & \frac{1+r}{2} & & \\ \frac{1-r}{2} & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} \frac{1-r}{2} \\ 0 \\ \vdots \\ 0 \\ \frac{1+r}{2} \end{bmatrix}$
Lax-Wendroff (LXW) $u_j^{n+1} = u_j^n + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{1}{2}r^2\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1-r^2 & \frac{r(r+1)}{2} & & \\ \frac{r(r-1)}{2} & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} \frac{r(r-1)}{2} \\ 0 \\ \vdots \\ 0 \\ \frac{r(r+1)}{2} \end{bmatrix}$
Leapfrog (LFG) $u_j^{n+1} = u_j^{n-1} + r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \mathbf{u}^{n-1} + \begin{bmatrix} 0 & r & & \\ -r & \ddots & \ddots & \\ & \ddots & \ddots & \ddots \\ & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -r \\ 0 \\ \vdots \\ 0 \\ r \end{bmatrix}$
Fourth-order Leapfrog (LF4) $u_j^{n+1} = u_j^{n-1} + \frac{4}{3}r\left(u_{j+1}^n - u_{j-1}^n\right) - \frac{1}{6}r\left(u_{j+2}^n - u_{j-2}^n\right)$	$\mathbf{u}^{n+1} = \mathbf{u}^{n-1} + \begin{bmatrix} 0 & \frac{4}{3}r & -\frac{1}{6}r & & \\ -\frac{4}{3}r & \ddots & \ddots & \ddots & \\ \frac{1}{6}r & \ddots & \ddots & \ddots & \ddots \\ & \ddots & \ddots & & \\ & & \ddots & & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -\frac{4}{3}r \\ \frac{1}{6} \\ 0 \vdots \\ -\frac{1}{6}r \\ \frac{4}{3}r \end{bmatrix}$

Finite Element Formulation for Solving KW Equation: $\left.\frac{\partial h}{\partial x}\right|_{x=x_j} = \frac{h_{j+1} - h_{j-1}}{2\Delta x}$

$$h(x,t) = \sum_{j=1}^{M} \Phi_j(x)\, h_j(t)$$

Channel Discretization and Selection of Approximations Functions

The flow equations are one-dimensional. The channel is divided into small reaches called elements. Each element will be modeled with the same flow equations but with different channel geometry and hydraulic parameters. The elements equations are later assembled into global matrix equations for solution. By applying the Galerkin's principle to the continuity equation the following equation is obtained:

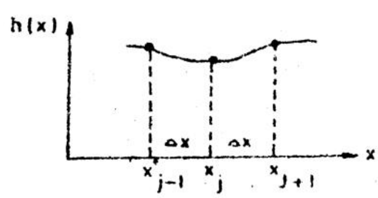

Figure (A) : Finite Difference Computational Mesh

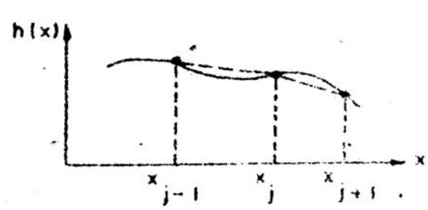

Figure (B) : Finite Element Computational Mesh

$$\sum_{i=I}^{K-1} \int_{x_K}^{x_{K+I}} N^T\left(\frac{\partial y}{\partial t} + y\frac{\partial v}{\partial x} + v\frac{\partial y}{\partial x} - q(x,t)\right) dx = 0$$

Where $\sum_{1}^{K-1}$ is the expression for summary individual element equation from 1 to (k-1) elements; N^T transpose to the shape functions.
Using the shape functions, Equations may be written as

$$\sum_{1}^{K-1} \int_{0}^{1} N^T \frac{\partial y}{\partial t} + Y\frac{\partial v}{\partial x} + v\frac{\partial y}{\partial x} - q(x,t)\, Lds = 0$$

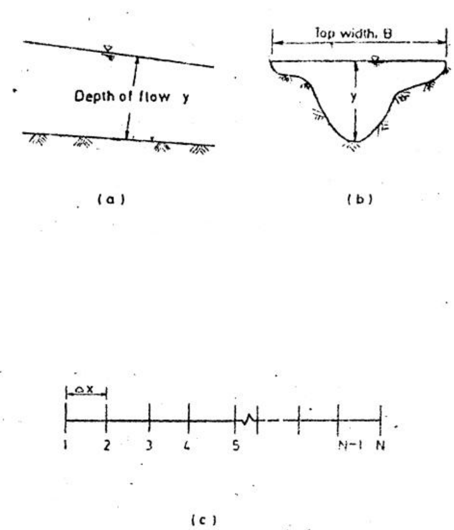

Figure : Natural Idealized Flow Sections (a) Longitudinal Profile (b) vertical Cross sectional Area Flow (c) Longitudinal Channel discretized into finite elements.

Evaluating each term of Equation (5.24) the following elements equation may be written:

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial y_1}{\partial t} \\ \frac{\partial y_2}{\partial t} \end{Bmatrix} + \frac{1}{6}\begin{bmatrix} (2y_1 + y_2)(v_2 - v_1) \\ (y_1 + 2y_2)(v_2 - v_1) \end{bmatrix} + \frac{1}{6}\begin{bmatrix} (2v_1 + v_2)(y_2 - y_1) \\ (v_1 + 2v_2)(y_2 - y_1) \end{bmatrix} - l_q\begin{Bmatrix} 3 \\ 3 \end{Bmatrix} = 0$$

Similar way the momentum Equation for an element can be derived as

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial v_1}{\partial t} \\ \frac{\partial v_2}{\partial t} \end{Bmatrix} + \frac{1}{12}\begin{bmatrix} -2v_1 - v_2 & -v_1 - 2v_2 \\ 2v_1 + v_2 & v_1 + 2v_2 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} + \frac{q}{l}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} + \frac{l_q}{2}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} (\frac{v}{y})_1 \\ (\frac{v}{y})_2 \end{Bmatrix}$$

$$+\frac{q}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} S_{f_1} \\ S_{f_2} \end{Bmatrix} + \frac{gs_0 l}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

Notes

III. Formation of Global Matrix

The element properties originally expressed in local coordinates need to be transformed into global coordinates before a solution algorithm is initated. Based on the node to node relationship, it is possible to generate an overall element property matrix for the entire domain, a process called assembling of element equations.

Notes

The concept of discreatization employed earlier is based on the fact that a domain with varying geometric and hydraulic properties can be treated independently as subdomains but systematically from one subdomain to another. Considering N number elements of varying lengths the assembled global matrix equations for continuity and momentum equations become:

$$\begin{bmatrix} 2l_1 & l_1 & \cdots & & & \bigcirc \\ l_1 & 2(l_1+2) & l_2 & & & \\ & l_2 & 2(l_2+l_3) & l_3 & & \\ & .. & .. \; . & .. & & \\ & \bigcirc & & l_i & 2(l_i+l_{i+1}) & l_{i+1} \\ & & & \cdots & \cdots & .. \\ & & & & & l_{N-1}\;(l_{N-1}) \end{bmatrix} \begin{Bmatrix} \frac{\partial y_1}{\partial x} \\ \frac{\partial y_2}{\partial x} \\ \frac{\partial y_3}{\partial x} \\ \cdots \\ \frac{\partial y_N}{\partial x} \end{Bmatrix} + \begin{bmatrix} v_2 \; -4v_1 & 2v_2+v_1 & & & \\ -v_2-2v_1 & v_3-v_1 & 2v_3-v_z & & \\ & v_3-2v_2 & v_4-v_2 & 2v_4-v_3 & \\ & .. & .. \; . & .. & \\ & & v_{1+1}-2v_i \;\; v_{i+2}-v_1 & 2v_{1+2}-v_{i+1} & \\ & & & -v_N-2v_{N-1} & v_N-v_{N-1} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_i \\ y_N \end{Bmatrix}$$

$$-3\begin{Bmatrix} l_1 q_1 \\ q_1 l_1 + q_2 l_2 \\ q_2 l_2 + l_3 q_3 \\ \ldots \\ l_i q_i + l_{i+1} q_{i+1} \\ \ldots \\ l_{N-1} q_{N-1} \end{Bmatrix} = 0$$

In matrix form the global continuity equation can be written as

$$[A]\begin{Bmatrix} \frac{dy}{dt} \end{Bmatrix} + [B]\{y\} - \{c\} = 0$$

Where A, B are the matrices and C is the column vector, Y is the dependent variable. The global momentum equation can be formed similarly.

The Solution of time dependent global matrix Equation is sought through a semi discrete approach, This approach requires the time derivative of the dependant variable at each node to be replaced by finite difference scheme (in time domain). Such as the forward, backward, and central differences and are given below with time level k as:

Forward difference, $\frac{dy}{dt} = \frac{y^{k+1} - y^k}{\Delta t}$

Backward difference, $\frac{dy}{dt} = \frac{y^k - y^{k-1}}{\Delta t}$

Central difference, $\frac{dy}{dt} = \frac{y^{k+1} - y^{k-1}}{2\Delta t}$

Substitution of Equation (5.29a) in Equation (5.28) yields

$$[A]\left\{\frac{y^{k+1}-y^{k}}{\Delta t}\right\}+[B]\left\{y^{k}\right\}-\{c\}=0$$

An implicit equation will be generated from this Equation with the aid of the time weighting factor in the next section.

Development of the Numerical models

The deterministic stream flow models are investigated with three distinct options: (1) the kinematic flow models comprises (a) the simplified version of momentum equation that neglects pressure and inertia terms are compared to friction and gravity terms and (b) the complete form of continuity equation; (2) the diffusion flow models comprises (a) the simplified momentum equation that accounts only for pressure, friction, and gravity terms and (b) the complete form of continuity equation; and (3) the complete flow model comprises (a) the complete form of momentum equation and (b) the complete continuity equation.

The kinematic flow model is investigated in both explicit and implicit sense. The explicit kinematic flow model leads to linear equations. They are solved using a direct method similar to the tridiagonal matrix algorithm set up by varga (1962). The solution proceeds by matrix reduction similar to Gaussian elimination. In contrast the explicit model, the implicit kinematic model yields a set of non-linear tridiagonal matrix equations which are solved by the functional Newton-Raphson iterative method.

The diffusion model as well as the complete flow model each results in a non-linear bitridiagonal matrix equation. The functional Newton-Raphson's method, along with the direct solution algorithm, triangular decomposition technique that yields a recursion algorithm (Douglas, et al, 1959, Von Rosenberg, 1969), is utilized to predict depth and velocity of flow for each option.

Finite Element Kinematic Wave Model

Explicit Model:

The non-linear continuity equation is easily converted to linear form by use of geometric and flow relations:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}-q(x,t)=0$$

Where, A = Area of flow, L^2 ;

Q = volumetric flow rate, $\frac{L^3}{T}$

The appropriate simplified momentum equation for coupling with the continuity equation has been obtained and is presented below

$$S_f = S_0 = \frac{n_1^2\, v^2}{R^{4/3}} = \frac{v^2 R^{4/3}}{M^2}$$

$$\text{Or}\quad Q = \frac{AR^{2/3}S_0^{1/2}}{n_1} \quad = \quad MAR^{2/3}\, S_0^{1/2}$$

These equations are written in matrix units. For fps units first equation to be divided by 2.216 and the second equation to be multiplied by 1.486.

Notes

Applying the Galerkin's weighted residual method results in the following liner first order ordinary differential equation.

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial A_j}{\partial t} \\ \frac{\partial A_2}{\partial t} \end{Bmatrix} + \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} - \frac{ql}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 0$$

For total length of the stream reach the assembled matrix equation becomes:

$$\begin{bmatrix} 2l_1 & l_1 & \dots & & & \circ \\ l_1 & 2(l_1+l_2) & l_2 & & & \\ & l_2 & 2(l_2+l_3) & l_3 & & \\ & .. & .. \ . & .. & & \\ & \circ & -l_i & 2(l_i+l_{i+1}) & l_{i+1} & \\ & & & \dots & \dots & .. \\ & & & l_{N-1} & 2l_{N-1} & \end{bmatrix}\begin{Bmatrix} \frac{\partial A_1}{\partial t} \\ \frac{\partial A_2}{\partial t} \\ \frac{\partial A_3}{\partial t} \\ \dots \\ \frac{\partial A_i}{\partial t} \\ \dots\dots \\ \frac{\partial AN}{\partial t} \end{Bmatrix} + \begin{Bmatrix} Q_2 - Q_1 \\ Q_3 - Q_1 \\ Q_4 - Q_2 \\ \dots\dots\dots\dots \\ Q_{i+1} - Q_{i-1} \\ \dots \\ Q_N - Q_{N-1} \end{Bmatrix} - \frac{l}{2}\begin{Bmatrix} l_1 q_1 \\ l_1 q_1 + l_2 q_2 \\ l_2 q_2 + l_3 q_3 \\ \dots \\ l_i q_i + l_{i+1} q_{i+1} \\ \dots \\ l_{N-1} q_{N-1} \end{Bmatrix} = 0$$

The above Equation is expressed in a matrix form:

$$[K]\left\{\frac{dy}{dt}\right\} + [D]\{F\} = 0$$

The solution of this Equation is possible upon implementation of the forward differencing in time derivative.

$$[K]\{A\}^{N+1} = [K]\ \{A\}^N + \Delta t\ \{F\}^n - \Delta t\ \ \{D\}^N$$

The solution of the area of flow at various nodes proceeds forward in time with the right hand side evaluated at a previous time level, n. Thus, the Equation can be expressed in more compact form:

$$[K]\ \{A\}^{N+1} = \{X\}^N$$

Where X is the known column vector at previous time level. The matrix, K is a linear and tridiagonal type that easily leads to direct solution algorithm. The computer program solving Equation is facilitated by the use of the compact tridiagonal algorithms proposed by Varga (1962). The computed area of flow at current time level, n+1, is used to update cycle is repeated as new time level is reached. The coded explicit finite element scheme exhibits dynamic instability to restriction on the step. This drawback is inherent in explicit numerical schemes, is expected regardless of the finite element approach.

To solve the KW model through the above finite element method one can study the flow problem of overland flow as well as channel flow by using practical data collecting from any river in Bangladesh.

IV. Conclusion

A hydrological model is an important tool for estimating and organizing quantitative hydrologic information. The main objectives of this thesis is to develop a suitable surface hydrological model for study the movement of overland, (i.e. through its surface runoff) as well as stream flow components of the hydrologic cycle. To achieve these objectives, various techniques and available models were studied. It was concluded that the dynamic approached are the best to account for the physical processes associated with the runoff mechanics of the watersheds. Among these approaches, the kinematic wave theory is the best suited to the prevailing condition.

A further work can be done by developing computer program using these methods to solve KW equation for channel and overland flows for various practical data set collecting from any small river in Bangladesh

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Classes of Fuzzy Real-Valued Double Sequences Related to the Space ℓ^p

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Abstract - In this article, corresponding to certain general sequence $\phi = \{\phi_{nk}\}$, we introduce the fuzzy real valued double sequence spaces ${}_2m(\phi, p)$ space where $p = (p_{nk})$ is a double sequence of bounded strictly positive numbers, closely related to the space ℓ^p. We study their different properties. We study some algebraic and topological properties of the space ${}_2m(\phi, p)$. Also we obtained the necessary and sufficient conditions for inclusion and equality of ${}_2m(\phi, p)$ and ${}_2m(\psi, p)$.

Keywords : *fuzzy real valued double sequence, normal, symmetric etc.*

GJSFR-F Classification : *MSC 2010: 03E72, 26E50*

CLASSES OF FUZZY REAL-VALUED DOUBLE SEQUENCES RELATED TO THE SPACE LP

Strictly as per the compliance and regulations of :

Ref

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Classes of Fuzzy Real-Valued Double Sequences Related to the Space ℓ^p

Bijan Nath [α] & Santanu Roy [σ]

Abstract - In this article, corresponding to certain general sequence $\phi = \{\phi_{nk}\}$, we introduce the fuzzy real valued double sequence spaces ${}_2m(\phi, p)$ space where $p = (p_{nk})$ is a double sequence of bounded strictly positive numbers, closely related to the space ℓ^p. We study their different properties. We study some algebraic and topological properties of the space ${}_2m(\phi, p)$. Also we obtained the necessary and sufficient conditions for inclusion and equality of ${}_2m(\phi, p)$ and ${}_2m(\psi, p)$.

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I. Introduction

To overcome limitations induced by vagueness and uncertainty of real life data, neoclassical analysis [6] has been developed. It extends the scope and results of classical mathematical analysis by applying fuzzy logic to conventional mathematical objects, such as functions, sequences and series etc. Since the introduction of the concept of fuzzy sets by Zadeh [28] in 1965, fuzzy set theory has become an active area of research in science and engineering. The ideas of fuzzy set theory have been used widely not only in many engineering applications, such as, in the computer programming [10], in quantum physics [15], in population dynamics [2], in the control of chaos [9], in bifurcation of non-linear dynamical system [12], but also in various branches of mathematics, such as, theory of metric and topological spaces [8], in the theory of linear systems [17], studies of convergence of sequences of functions [5,13,27] and in approximation theory [1].

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Using the notion of fuzzy real numbers, different types of fuzzy real-valued sequence spaces have been introduced and studied by several mathematicians. The initial works on double sequences of real or complex terms are found in Bromwich [4]. Hardy [11] introduced the notion of regular convergence for double sequences of real or complex terms. Moričz [16], Basarir and Solancan [3], Choudhary and Tripathy [7], Tripathy and Dutta [22,23], Tripathy and Sarma [24,25] are a few to be named those who studied different aspect of double sequences of fuzzy real numbers

The space $m(\phi)$ was introduced by Sargent [19]. He studied some properties of the space $m(\phi)$. Later on it was studied from sequence space point of view and some matrix classes with one member as $m(\phi)$ were characterized by Rath and Tripathy [18], Tripathy and Sen [26] and many others.

II. Definitions and Background

Throughout the thesis *N*, *R* and *C* denote the sets of **natural**, **real** and **complex** numbers respectively and *w*, ℓ_∞ denote the spaces of all and bounded sequences of complex terms respectively.

A fuzzy real number *X* is a fuzzy set on *R*, ie. a mapping $X : R \to L(=[0,1])$

associating each real number *t* with its grade of membership $X(t)$.

A fuzzy real number *X* is said to be **convex** if

$$X(t) \geq X(s) \wedge X(r) = \min [X(s) \wedge X(r)], \text{ where } s < t < r.$$

If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number *X* is said to be **normal**.

A fuzzy real number *X* is said to be **upper semi-continuous** if for each $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$, for all $a \in L$ is open in the usual topology of *R*. The set of all upper semi continuous, normal, convex fuzzy number is denoted by $R(L)$. Throughout the thesis, by a fuzzy real number we mean that the number belongs to $R(L)$.

The **α-level** set of a fuzzy real number *X*, $0 < \alpha \leq 1$, is denoted and defined as

$$[X]^\alpha = \{ t \in R : X(t) \geq \alpha \}.$$

Ref.

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Notes

Every real number r can be expressed as a fuzzy real number $\bar{r}$ as follows :

$$\bar{r}(t) = \begin{cases} 1 & \text{if} \quad t = r \\ 0 & \text{otherwise} \end{cases}$$

The additive identity and multiplicative identity in $R(L)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

The **absolute value** $|X|$ of $X \in R(L)$, is defined as (one may refer to Kaleva and Seikkla [42]):

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if} \quad t \geq 0 \\ 0 & \text{if} \quad t < 0. \end{cases}$$

Let D be the set of all closed bounded intervals $X = [X^L, X^R]$ on the real line R.

If $Y = [Y^L, Y^R]$, then $X \leq Y$ if and only if $X^L \leq Y^L$ and $X^R \leq Y^R$.

Also let $d(X,Y) = \max(|X^L - X^R|, |Y^L - Y^R|)$. Then (D, d) is a complete metric space.

Let $\bar{d} : R(L) \times R(L) \to R$ be defined by

$$\bar{d}(X,Y) = \sup_{0 \leq \alpha \leq 1} d([X]^\alpha, [Y]^\alpha), \text{ for } X, Y \in R(L).$$

Then $\bar{d}$ defines a metric on $R(L)$ and $(R(L), \bar{d})$ is a complete metric space.

A fuzzy real-valued double sequence is a double infinite array of fuzzy real numbers X_{nk} for all $n, k \in N$ and is denoted by (X_{nk}), where $X_{nk} \in R(L)$.

A fuzzy real-valued double sequence (X_{nk}) is said to be **convergent** in Pringsheim's sense to the fuzzy real number X_0, if for every $\varepsilon > 0$, there exists $n_0 = n_0(\varepsilon)$, $k_0 = k_0(\varepsilon) \in N$ such that $\bar{d}(X_{nk}, X_0) < \varepsilon$, for all $n \geq n_0$ and $k \geq k_0$.

A fuzzy real-valued double sequence (X_{nk}) is said to be bounded if $\sup_{n,k} \bar{d}(X_{nk}, \bar{0}) < \infty$.

A fuzzy real valued double sequence space E^F is said to be **solid** (or **normal**) if $(Y_{nk}) \in E^F$, whenever $\bar{d}(Y_{nk}, \bar{0}) \leq \bar{d}(X_{nk}, \bar{0})$ for all $n, k \in N$ and $(X_{nk}) \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **monotone** if E^F contains the canonical pre-image of all its step spaces.

Throughout π denotes a permutation over $N \times N$. For $X = (X_{nk})$ a given sequence, $S(X)$ denotes the set of all permutation of the elements of (X_{nk}), that is $S(X) = \{(X_{\pi(n,k)})\}$.

A fuzzy real valued double sequence space E^F is said to be **symmetric** if $S(X) \subset E^F$, for all $X \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **sequence algebra** if $(X_{nk} \otimes Y_{nk}) \in E^F$, whenever $(X_{nk}), (Y_{nk}) \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **convergence free** if $(Y_{nk}) \in E^F$, whenever $(X_{nk}) \in E^F$ and $X_{nk} = \bar{0}$ implies $Y_{nk} = \bar{0}$.

Let $\wp$ denote the set of all subsets of N. For any $s \in N$, $\wp_s$ denote the class of all $\sigma \in \wp$ such that σ does not contain more than s elements. $\phi = (\phi_{nk})$ is a non-decreasing sequence such that

$$(n+1)(k+1)\phi_{nk} \geq nk\phi_{n+1,k+1} \text{ for all } n,k \in N.$$

A BK-space is a Banach space of complex double sequences $x = (x_{nk})$ in which the co-ordinate maps are continuous, that is,

$$\left|x_{nk}^{(i)} - x_{nk}\right| \to 0, \text{ whenever } \left\|x^{(i)} - x\right\| \to 0 \text{ as } n,k \to \infty,$$

where $x^{(i)} = \left(x_{nk}^{(i)}\right)$, for all $i \in N$ and $x = (x_{nk})$.

The space $m(\phi)$ introduced by Sargent [19] is defined as

$$m(\phi) = \left\{(x_k) \in w : \|x\|_{m(\phi)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \sum_{n \in \sigma} |x_k| < \infty\right\}.$$

Tripathy and Sen [26] introduce the sequence spaces $m(\phi, p)$ as follows :

For $1 \leq p < \infty$,

$$m(\phi, p) = \left\{(x_n) \in w : \|x\|_{m(\phi,p)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \left\{\sum_{n \in \sigma} |x_n|^p\right\}^{\frac{1}{p}} < \infty\right\},$$

For $0 < p < 1$,

$$m(\phi, p) = \left\{(x_n) \in w : \|x\|_{m(\phi,p)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \sum_{n \in \sigma} |x_n|^p < \infty\right\}.$$

Notes

Generalizing the above sequence spaces, we now introduce the spaces ${}_2m(\phi, p)$ as follows:

$$m(\phi, p) = \left\{ X = (X_{nk}) \in {}_2w^F : \|X\|_{{}_2m(\phi,p)} = \sup_{s\geq 1, \sigma\in\wp_s} \sup_{r\geq 1, \sigma'\in\wp_r} \frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}, \bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}} < \infty \right\};$$

where $p = (p_{nk})$ is a double sequence of bounded strictly positive real numbers.

III. Main Results

Theorem 1. *The class of sequences* ${}_2m(\phi, p)$ *is a linear space.*

Proof. With standard techniques, we can easily prove the result.

Theorem 2. *The class of sequences* ${}_2m(\phi, p)$ *is complete.*

Proof. Let $(X^{(i)})$ be a Cauchy sequence in ${}_2m(\phi, p)$ where $X^{(i)} = (X_{nk}^{(i)})$.

$$\sup_{s\geq 1, \sigma\in\wp_s} \sup_{r\geq 1, \sigma'\in\wp_r} \left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}, \bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \infty, \quad \text{for all } n, k \in N.$$

Then for a given $\varepsilon > 0$, there exists $n_0 \in N$ such that

$$\|X^{(i)} - X^{(j)}\| < \varepsilon, \text{ for all } i, j \geq n_0.$$

$$\Rightarrow \sup_{s\geq 1, \sigma\in\wp_s} \sup_{r\geq 1, \sigma'\in\wp_r} \left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk}^{(j)})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \varepsilon, \text{ for all } n, k \in N \quad \ldots(1)$$

$$\Rightarrow \bar{d}(X_{nk}^{(i)}, X_{nk}^{(j)}) < \varepsilon\phi_1, \text{ for all } i, j \geq n_0, \text{ for all } n, k \in N.$$

Hence for each fixed $n, k \in N$, the sequence $(X_{nk}^{(i)})$ is a Cauchy sequence in $R(L)$.

Since $R(L)$ is complete, the sequence $(X_{nk}^{(i)})$ converges in $R(L)$, for each $n, k \in N$.

Let $\lim_{i\to\infty} X_{nk}^{(i)} = X_{nk}$, for all $n, k \in N$, where $X = (X_{nk})$.

We now show that (i) $X \in {}_2m(\phi, p)$ and

and (ii) $X^{(i)} \to X$.

From equation (1), we get for each fixed s and r

$$\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk}^{(j)})]^{p_{nk}} < \varepsilon^{p_{nk}}(\phi_s\phi_r)^{p_{nk}}, \text{ for all } i, j \geq n_0, \ \sigma \in \wp_s.$$

Letting $j \to \infty$, we get

$$\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}{}^{(i)}, X_{nk})]^{p_{nk}} < \varepsilon^{p_{nk}}(\phi_s\phi_r)^{p_{nk}}, \text{ for all } i \ge n_0,\ \sigma \in \wp_s.$$

$$\Rightarrow \sup_{s\ge1,\sigma\in\wp_s}\ \sup_{r\ge1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}{}^{(i)}, X_{nk})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \varepsilon, \text{ for all } i \ge n_0 \quad \ldots(2)$$

$$\Rightarrow X^{(i)} - X \in {}_2m(\phi, p), \text{ for all } i \ge n_0.$$

Hence $X = X^{(i)} + (X - X^{(i)}) \in {}_2m(\phi, p)$, since ${}_2m(\phi, p)$ *is a linear space.*

Also (2) $\Rightarrow \left\|X^{(i)} - X\right\|_{{}_2m(\phi,p)} < \varepsilon$, for all $i \ge n_0$.

$$\Rightarrow X^{(i)} \to X \in {}_2m(\phi, p).$$

Hence ${}_2m(\phi, p)$ is complete.

Theorem 3. *The class of sequences* ${}_2m(\phi, p)$ *is a B K-space.*

Proof. By Theorem 2, ${}_2m(\phi, p)$ is a Banach space.

Let $\left\|X^{(i)} - X\right\|_{{}_2m(\phi,p)} \to 0$, as $i \to \infty$.

Then for a given $\varepsilon > 0$, there exists $n_0 \in N$ such that

$$\left\|X^{(i)} - X\right\|_{{}_2m(\phi,p)} < \varepsilon, \text{ for all } i \ge n_0.$$

$$\Rightarrow \sup_{s\ge1,\sigma\in\wp_s}\ \sup_{r\ge1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}{}^{(i)}, X_{nk})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \varepsilon, \text{ for all } i \ge n_0.$$

$$\Rightarrow \bar{d}(X_{nk}{}^{(i)}, X_{nk}) < \varepsilon\phi_1, \text{ for all } i \ge n_0, \text{ for all } n, k \in N.$$

$$\Rightarrow \bar{d}(X_{nk}{}^{(i)}, X_{nk}) \to 0, \text{ as } i \to \infty.$$

Hence ${}_2m(\phi, p)$ *is a B K-space.*

This completes the proof of the theorem.

Notes

Notes

Theorem 4. (i) *The class of sequences* ${}_2m(\phi, p)$ *is a symmetric space. If* $X \in {}_2m(\phi, p)$ and $U \in S(X)$, where $S(X)$ denotes the set of all permutation of the elements of (X_{nk}), then $\|U\|_{{}_2m(\phi,p)} = \|X\|_{{}_2m(\phi,p)}$.

(ii) *The class of sequences* ${}_2m(\phi, p)$ *is a normal space.*

Proof. The proof of the result follows from definition.

Theorem 5. ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$ if and only if $\sup_{s\ge1}\left(\frac{\phi_s}{\psi_s}\right) < \infty$.

Proof. Let $\sup_{s\ge1,}\left(\frac{\phi_s}{\psi_s}\right) = K(<\infty)$.

Then $\phi_s \le K\psi_s$.

Let $(X_{nk}) \in {}_2m(\phi, p)$. Then

$$\sup_{s\ge1,\sigma\in\wp_s}\ \sup_{r\ge1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \infty$$

$$\Rightarrow \sup_{s\ge1,\sigma\in\wp_s}\ \sup_{r\ge1,\sigma'\in\wp_r}\left[\frac{1}{K\psi_s}\frac{1}{K\psi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \infty$$

$$\Rightarrow \|X\|_{{}_2m(\psi,p)} < \infty.$$

Hence ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$.

Conversely let ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$. we have to show that $\sup_{s\ge1}\left(\frac{\phi_s}{\psi_s}\right) = \sup_{s\ge1}(\eta_s) < \infty$.

If possible let $\sup_{s\ge1,}(\eta_s) = \infty$. Then there exists a subsequence (η_{s_i}) of (η_s) such that $\lim_{i\to\infty}(\eta_{s_i}) = \infty$.

Then for $(X_{nk}) \in {}_2m(\phi, p)$, we have

$$\sup_{s\ge1,\sigma\in\wp_s}\ \sup_{r\ge1,\sigma'\in\wp_r}\left[\frac{1}{\psi_s}\frac{1}{\psi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] \ge \sup_{s\ge1,\sigma\in\wp_s}\ \sup_{r\ge1,\sigma'\in\wp_r}\left[\eta_{s_i}\frac{1}{\phi_{s_i}}\eta_{r_i}\frac{1}{\phi_{r_i}}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] = \infty.$$

$\Rightarrow (X_{nk}) \in {}_2m(\psi, p)$, a contradiction.

This step concludes the proof of the theorem.

In view of the above theorem, we formulate the following result.

Corollary 1. ${}_2m(\phi, p) = {}_2m(\psi, p)$ if and only if $\sup_{s\geq 1,}(\eta_s) < \infty$ and $\sup_{s\geq 1,}(\eta_s^{-1}) < \infty$, where

$\eta_s = \dfrac{\phi_s}{\psi_s}$.

Theorem 7. ${}_2m(\psi, p) = \ell^p$ if and only if $\sup_{s\geq 1,}(\phi_s) < \infty$ and $\sup_{s\geq 1,}(\phi_s^{-1}) < \infty$.

Proof. Putting $\psi_{nk} = 1$, for all $n, k \in N$, in Corollary 1, we get the result easily.

Theorem 8. *If* $0 < p_{nk} < q_{nk} \leq \sup_{n,k} q_{nk}$, then ${}_2m(\phi, p) \subset {}_2m(\phi, q)$.

Proof. Using the properties of ℓ^p spaces, we get the result easily.

Theorem 9. ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$ *if* $0 < p_{nk} < q_{nk} \leq \sup_{n,k} q_{nk}$ and $\sup_{s\geq 1,}\left(\dfrac{\phi_s}{\psi_s}\right) < \infty$.

Proof. Using the properties of ℓ^p spaces and Theorem 5, we get the result.

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Coefficient Problem for Certain Subclass of Analytic Functions Using Quasi-Subordination

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Abstract - An analytic function f is quasi-subordinate to an analytic function g, in the open unit disk if there exist analytic function φ and w, with $|\varphi(z)| \leq 1$, w (0) = 0 and $|w(z)| <$ 1 such that $f(z) = \varphi(z)g(w(z))$. Certain subclasses of analytic univalent functions associated with quasi-subordination are defined and the bounds for the Fekete-Szego coefficient functional $|a_3 - \mu a_2^2|$ for functions belonging to these subclasses are derived.

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COEFFICIENT PROBLEM FOR CERTAIN SUBCLASS OF ANALYTIC FUNCTIONS USING QUASI-SUBORDINATION

Strictly as per the compliance and regulations of :

Coefficient Problem for Certain Subclass of Analytic Functions Using Quasi-Subordination

B. Srutha Keerthi [α] & S. Prema [σ]

Abstract - An analytic function f is quasi-subordinate to an analytic function g, in the open unit disk if there exist analytic function φ and w, with $|\varphi(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = \varphi(z)g(w(z))$. Certain subclasses of analytic univalent functions associated with quasi-subordination are defined and the bounds for the Fekete-Szego coefficient functional $|a_3 - \mu a_2^2|$ for functions belonging to these subclasses are derived.

I. Introduction and Motivation

Let A be the class of analytic function f in the open unit disk $D = \{z : |z| < 1\}$ normalized by $f(0) = 0$ and $f'(0) = 1$ of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. For two analytic functions f and g, the function f is subordinate to g, written as follows:

$$f(z) \prec g(z), \tag{1.1}$$

if there exists an analytic function w, with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$. In particular, if the function g is univalent in D, then $f(z) \prec g(z)$ is equivalent to $f(0) = g(0)$ and $f(D) \subset g(D)$. For brief survey on the concept of subordination, see [1].

Ma and Minda [2] introduced the following class

$$S^*(\phi) = \left\{ f \in A : \frac{zf'(z)}{[f(z)]} \prec \phi(z) \right\}, \tag{1.2}$$

where ϕ is an analytic function with positive real part in D, $\phi(D)$ is symmetric with respect to the real axis and starlike with respect to $\phi(0) = 1$ and $\phi'(0) > 0$. A function $f \in S^*(\phi)$ is called Ma-Minda starlike (with respect to ϕ). The class $C(\phi)$ is the class of functions $f \in A$ for which

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$1 + zf''(z)/f'(z) \prec \phi(z)$. The class $S^*(\phi)$ and $C(\phi)$ include several well-known subclasses of starlike functions as special case.

In the year 1970, Robertson [3] introduced the concept of quasi-subordination. For two analytic functions f and g, the function f is quasi-subordinate to g, written as follows:

$$f(z) \prec_q g(z), \tag{1.3}$$

if there exist analytic function φ and w, with $|\varphi(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = \varphi(z)g(w(z))$. Observe that when $\varphi(z) = 1$, then $f(z) = g(w(z))$, so that $f(z) \prec g(z)$ in D. Also notice that if $w(z) = z$, then $f(z) = \varphi(z)g(z)$ and it is said that f is majorized by g and written $f(z) \ll g(z)$ in D. Hence it is obvious that quasi-subordination is a generalization of subordination as well as majorization. See [4,5,6] for works related to quasi-subordination.

Throughout this paper it is assumed that ϕ is analytic in D with $\phi(0) = 1$. Motivated by [2,3], we define the following classes.

Definition 1.1. *Let the class $R_q^*(\alpha, \phi)$ consists of functions $f \in A$ satisfying the quasi-subordination*

$$\frac{z^{1-\alpha} f'(z)}{[f(z)]^{1-\alpha}} - 1 \prec_q \phi(z) - 1, \quad \alpha \geq 0 \tag{1.4}$$

Example 1.2. *The function $f : D \to C$ defined by the following*

$$\frac{z^{1-\alpha} f'(z)}{[f(z)]^{1-\alpha}} - 1 = z[\phi(z) - 1], \quad \alpha \geq 0 \tag{1.5}$$

belongs to the class $R_q^*(\alpha, \phi)$.

It is well known (see [10]) that the n^{th} coefficient of a univalent function $f \in A$ is bounded by n. The bounds for coefficient give information about various geometric properties of the function. Many authors have also investigated the bounds for the Fekete-Szego coefficient for various classes [11, 12, 13,14, 15,16,17,18,19, 20,21, 22,23,24,25]. In this paper, we obtain coefficient esti–mates for the functions in the above defined classes.

Let Ω be the class of analytic functions w, normalized by $w(0) = 0$, and satisfying the condition $|w(z)| < 1$. We need the following lemma to prove our results.

Lemma 1.3 (see [26]). *If $w \in \Omega$, then for any complex number f*

$$|w_2 - tw_1^2| \leq \max\{1; |t|\}. \tag{1.6}$$

The result is sharp for the functions $w(z) = z^2$ or $w(z) = z$.

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II. Main Results

Throughout, let $f(z) = z+a_2z^2+a_3z^3+\cdots$, $\phi(z) = 1+B_1z+B_2z^2+B_3z^3+\cdots$, $\varphi(z) = c_0 + c_1z + c_2z^2 + c_3z^3 + \cdots$, $B_1 \in R$ and $B_1 > 0$.

Theorem 2.1. *If $f \in A$ belongs to $R_q^*(\alpha, \phi)$, then*

$$|a_2| \leq \frac{B_1}{1+\alpha},$$
$$|a_3| \leq \frac{B_1}{2}\left(1+\max\left\{1, B_1\left|\frac{1-\alpha}{1+\alpha}+\frac{\alpha}{2B_1}\right|+\left|\frac{B_2}{B_1}\right|\right\}\right) \tag{2.1}$$

and for any complex number μ,

$$|a_3-\mu a_2^2| \leq \frac{B_1}{2}\left(1+\max\left\{1, B_1\left|\frac{1-\alpha}{1+\alpha}-\frac{2\mu}{(1+\alpha)^2}+\frac{\alpha}{2B_1}\right|+\left|\frac{B_2}{B_1}\right|\right\}\right). \tag{2.2}$$

Proof. If $f \in R_q^*(\alpha, \phi)$, then there exist analytic functions φ and w, with $|\varphi(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$ such that

$$\frac{z^{1-\alpha}f'(z)}{[f(z)]^{1-\alpha}} - 1 = \varphi(z)(\phi(w(z))-1). \tag{2.3}$$

Since

$$\phi(w(z)) - 1 = B_1w_1z + (B_1w_2 + B_2w_1^2)z^2 + \cdots$$

$$\varphi(z)(\phi(w(z))-1) = B_1c_0w_1z + (B_1c_1w_1 + c_0(B_1w_2 + B_2w_1^2))z^2 + \cdots \tag{2.4}$$

it follows from (2,3) that

$$a_2 = \frac{B_1c_0w_1}{(1+\alpha)}$$

$$a_3 = \frac{1}{2+\alpha}\left[\frac{\alpha}{2}B_1c_0w_1 + B_1c_1w_1 + B_1c_0w_2 + c_0\left(\left(\frac{1-\alpha}{1+\alpha}\right)B_1^2c_0 + B_2\right)w_1^2\right] \tag{2.5}$$

Since $\varphi(z)$ is analytic and bounded in D, we have [27, page 172]

$$|c_n| \leq 1-|c_0|^2 \leq 1 \quad (n > 0). \tag{2.6}$$

By using this fact and the well-known inequality, $|w_1| \leq 1$, we get

$$|a_2| \leq \frac{B_1}{1+\alpha}. \tag{2.7}$$

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Further,

$$a_3 - \mu a_2^2 = \frac{1}{2+\alpha}\Big[B_1 c_1 w_1 + c_0(B_1 w_2 + \frac{\alpha}{2} B_1 w_1 + \left(B_2 + \left(\frac{1-\alpha}{1+\alpha}\right) B_1^2 c_0 - \frac{2\mu}{(1+\alpha)^2} B_1^2 c_0\right) w_1^2\Big]. \quad (2.8)$$

Then

$$|a_3 - \mu a_2^2| \leq \frac{1}{2+\alpha}\Bigg(|B_1 c_1 w_1| + \left|B_1 c_0 \left(w_2 - \left(\frac{2\mu}{(1+\alpha)^2} B_1 c_0 - \left(\frac{1-\alpha}{1+\alpha}\right) B_1 c_0 + \frac{\alpha}{2}\frac{w_1}{c_0} - \frac{B_2}{B_1}\right) w_1^2\right)\right|\Bigg). \quad (2.9)$$

Again applying $|c_n| \leq 1$ and $|w_1| \leq 1$, we have

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2+\alpha}\left(1 + \left|w_2 - \left(\frac{\alpha}{2} - \left(\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2}\right) B_1 c_0 - \frac{B_2}{B_1}\right) w_1^2\right|\right). \quad (2.10)$$

Applying Lemma 1.3 to

$$\left|w_2 - \left(\frac{\alpha}{2} - \left(\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2}\right) B_1 c_0 - \frac{B_2}{B_1}\right) w_1^2\right| \quad (2.11)$$

yields

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2+\alpha}\left(1 + \max\left\{1, \left|\frac{\alpha}{2} - \left(\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2}\right) B_1 c_0 - \frac{B_2}{B_1}\right|\right\}\right). \quad (2.12)$$

Observe that

$$\left|\frac{\alpha}{2} - \left(\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2}\right) B_1 c_0 - \frac{B_2}{B_1}\right| \leq B_1 |c_0| \left|\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2} + \frac{\alpha}{2B_1}\right| + \left|\frac{B_2}{B_1}\right|, \quad (2.13)$$

and hence we can conclude that

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2}\left(1 + \max\left\{1, B_1 \left|\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2} + \frac{\alpha}{2B_1}\right| + \left|\frac{B_2}{B_1}\right|\right\}\right). \quad (2.14)$$

For $\mu = 0$, the above will reduce to estimate of $|a_3|$.

Theorem 2.2. *If $f \in A$ satisfies*

$$\frac{z^{1-\alpha} f'(z)}{[f(z)]^{1-\alpha}} - 1 \ll \phi(z) - 1, \quad (2.15)$$

Notes

then the following inequalities hold:

$$|a_2| \le \frac{B_1}{1+\alpha},$$
$$|a_3| \le \frac{1}{2+\alpha}(B_1 + B_1^2 + |B_2|), \tag{2.16}$$

and, for any complex number μ,

$$|a_3 - \mu a_2^2| \le \frac{1}{(2+\alpha)(1+\alpha)^2}((1+\alpha)^2 B_1 + |(1+\alpha)^2 - (2+\alpha)\mu|B_1^2 + (1+\alpha)^2|B_2|). \tag{2.17}$$

Proof. The result follows by taking $w(z) = z$ in the proof of Theorem 2.1.

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Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

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The Global Journals Inc. (US) are enclosed by our publishing's Early View service. Early View articles are complete full-text articles sent in advance of their publication. Early View articles are absolute and final. They have been completely reviewed, revised and edited for publication, and the authors' final corrections have been incorporated. Because they are in final form, no changes can be made after sending them. The nature of Early View articles means that they do not yet have volume, issue or page numbers, so Early View articles cannot be cited in the conventional way.

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Before start writing a good quality Computer Science Research Paper, let us first understand what is Computer Science Research Paper? So, Computer Science Research Paper is the paper which is written by professionals or scientists who are associated to Computer Science and Information Technology, or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

Index

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