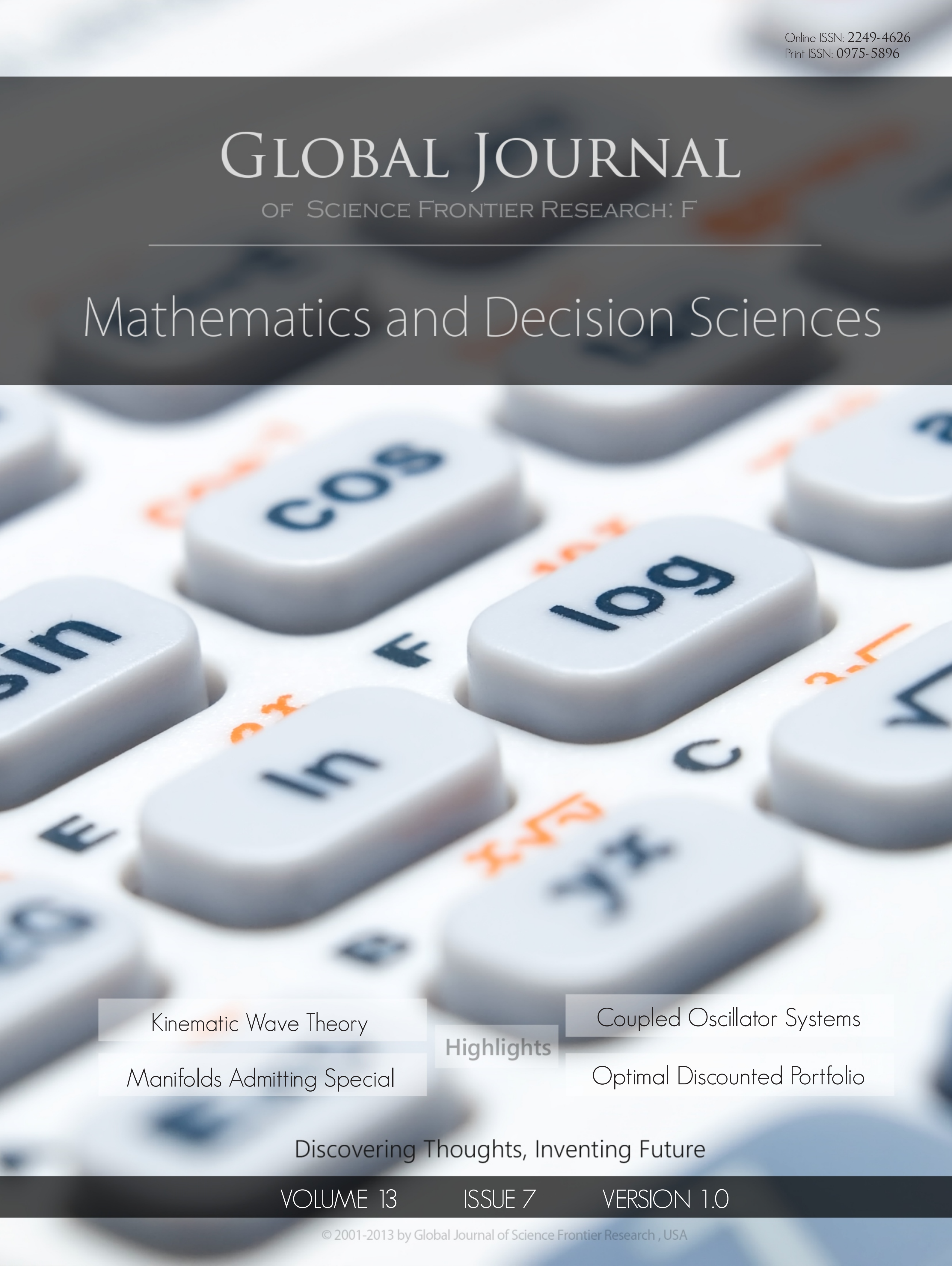

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# Lorentzian Para Sasakian Manifolds Admitting Special Semi Symmetric Recurrent Metric Connection

By Sunil Kumar Srivastava & R. P. Kushwaha
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*Abstract -* Several author as Agashe and Chafle [1], Sengupta, De. U.C, Binh [4] and many other introduced semi symmetric non metric connection in different way. In this paper we have studied LP sasakian manifold with special semi-symmetric recurrent metric connection [2] and discuss it exientance in LP sasakian manifold. In section 3 we establish the relation between the Riemannian connection and special semi-symmetric recurrent metric connection on LP sasakian manifold [4]. The section 4 deals with $\xi$–conformaly flat and $\phi$ concircularly flat of n dimensional LP sasakian manifold and we proved that $\xi$–conformaly flatness with special semi-symmetric recurrent metric connection and Riemannian manifold coincide.

*Keywords :* *lorentzian para saskian manifold, semi symmetric recurrent metric connection, $\xi$–conformaly flat, $\phi$ concircularly flat.*

*GJSFR-F Classification :* *MSC 2000: 53C25*

LORENTZIAN PARA SASAKIAN MANIFOLDS ADMITTING SPECIAL SEMI SYMMETRIC RECURRENT METRIC CONNECTION

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1. Agashe, N. S and Chafle, M. R: A semi symmetric non metric connection on a Riema-nnian Manifold. Indian. J. Pure appl. Math. 399-409, 1992.

# Lorentzian Para Sasakian Manifolds Admitting Special Semi Symmetric Recurrent Metric Connection

Sunil Kumar Srivastava [α] & R. P. Kushwaha [σ]

*Abstract* - Several author as Agashe and Chafle [1], Sengupta, De. U.C, Binh [4] and many other introduced semi symmetric non metric connection in different way. In this paper we have studied LP sasakian manifold with special semi-symmetric recurrent metric connection [2] and discuss it exientance in LP sasakian manifold. In section 3 we establish the relation between the Riemannian connection and special semi-symmetric recurrent metric connection on LP sasakian manifold [4]. The section 4 deals with ξ–conformaly flat and ∅ concircularly flat of n dimensional LP sasakian manifold and we proved that ξ–conformaly flatness with special semi-symmetric recurrent metric connection and Riemannian manifold coincide.

*Keywords : lorentzian para saskian manifold, semi symmetric recurrent metric connection, ξ–conformaly flat, ∅ concircularly flat.*

## I. Introduction

An n-dimensional differentiable manifold $M^n$ is a lorentzian para-sasakian manifold if it admits tensor field of type (1,1), a contravariant vector field $\xi$, a covariant vector field η and a lorentzian metric $g$ satisfying;

$$\emptyset^2 X = X + \eta(x)\xi \tag{1.1}$$

$$\eta(\xi) = -1 \tag{1.2}$$

$$g(\emptyset X, \emptyset Y) = g(X,Y) + \eta(X)\eta(Y) \tag{1.3}$$

$$g(X,\xi) = \eta(X) \tag{1.4}$$

$$(D_X\emptyset)(Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi \tag{1.5}$$

$$D_X\xi = \emptyset X \tag{1.6}$$

For arbitrary vector field *and* $Y$, whrere D denotes the operator of covariant differentiation with respect to lorentzian metric $g$ [5].

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In a LP sasakian manifold with structure $(\emptyset, \xi, \eta, g)$ the following relation hold.

$$\text{(a)} \quad \emptyset(\xi) = 0 \qquad \text{(b)} \quad \eta(\emptyset X) = 0 \qquad \text{rank}\emptyset = n - 1 \tag{1.7}$$

Let us $put$ $F(X, Y) = g(\emptyset X, Y)$ (1.8)

The tensor field $F$ is symmetric $(0,2)$ tensor field ie $F(X, Y) = F(Y, X)$ (1.9)

And
$$(D_X\eta)(Y) = F(X, Y) = g(\emptyset X, Y) \tag{1.10}$$

Also in a LP sasakian manifold the following relation holds;

$$g(R(X, Y)Z, \xi) = \eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y) \tag{1.11}$$

And
$$S(Y, \xi) = (n - 1)\eta(X) \tag{1.12}$$

For any vector field $X, Y, Z$ where $R(X, Y)Z$ is the Riemannian curvarture tensor and $S$ is the Ricci tensor.

Let $(M^n, g)$ be an LP sasakian manifold with Levi-Civita connection D. we define a linear connection $\overline{D}$ on $M^n$ by

$$\overline{D}_X Y = D_X Y - \eta(X)Y \tag{1.13}$$

Where $\eta$ is 1-form associated with vector field $\xi$ on $M^n$, given by

$$g(X, \xi) = \eta(X) \tag{1.14}$$

Using (1.13) the torsoin tensor $\overline{T}$ of $M^n$ with respect to connection $\overline{D}$ is given by

$$\overline{T}(X, Y) = \overline{D}_X Y - \overline{D}_Y X - [X, Y] = \eta(Y)X - \eta(X)Y \tag{1.15}$$

A linear connection satisfying (1.15) is called semi symmetric connection.
Further from (1.13), we have

$$(\overline{D}_X g)(Y, Z) = 2\eta(X)g(Y, Z) \tag{1.16}$$

A linear connection satisfying (1.16) is called semi symmetric recurrent metric connectionThe word special is used to distinguish it from other connection.

## II. Existence of Special Semi-Symmetric Recurrent Metric Connection

Let $\overline{D}$ be a linear connection in $M^n$, given by

$$\overline{D}_X Y = D_X Y + H(X, Y) \tag{2.1}$$

Where H is a tensor of type (1,2).

Now, we determine the tensor field H such that $\overline{D}$ satifies (1.15) and (1.16).

Notes

From (2.1), we have

$$\bar{T}(X,Y) = H(X,Y) - H(Y,X) \tag{2.2}$$

$$\text{Let } G(X,Y,Z) = (\bar{D}_X g)(Y,Z) \qquad \text{then} \tag{2.3}$$

$$g(H(X,Y),Z) + g(H(X,Z),Y) - -G(X,Y,Z) \tag{2.4}$$

From (2.1),(2.2) and (2.4), we have

$$g(\bar{T}(X,Y),Z) + g(\bar{T}(Z,X),Y) + g(\bar{T}(Z,Y),X)$$

$$= g(H(X,Y),Z) - g(H(Y,X),Z) + g(H(Z,X),Y) - g(H(X,Z),Y) + g(H(Z,Y),X) - g(H(Y,Z),X)$$

$$= 2g(H(X,Y),Z) + 2\eta(X)g(Y,Z) - 2\eta(Z)g(X,Y) + 2\eta(Y)g(X,Z$$

Or

$$H(X,Y) = \frac{1}{2}\{\bar{T}(X,Y) + \bar{T}'(X,Y) + \bar{T}'(Y,X)\} - \eta(X)Y - \eta(Y)X + g(X,Y)\xi \tag{2.5}$$

Where
$$g\left(\bar{T}'(X,Y),Z\right) = g(\bar{T}(Z,X),Y) \tag{2.6}$$

Using (1.5), (2.6) we get

$$\bar{T}'(X,Y) = \eta(X)Y - g(X,Y)\xi \tag{2.7}$$

Then in view of (1.15), (2.5) and (2.7),we get

$$H(X,Y) = -\eta(X)Y$$

This implies

$$\bar{D}_X Y = D_X Y - \eta(X)Y$$

Conversely, a connection $\bar{D}$ given by (1.13), satisfies (1.15) and (1.16) show that $\bar{D}$ is a special semi symmetric recurrent metric connection.
So we state the following theorem.

**Theorem 2.1:** let $(M^n, g)$ be an LP sasakian manifold with lorentzian para contact metric structure $(\emptyset, \xi, \eta, g)$ admits a special semi-symmetric connection which is given by

$$\bar{D}_X Y = D_X Y - \eta(X)Y$$

## III. Curvature Tensor of $M^n$ with Respect to Special Semi –Symmetric Recurrent Metric Metric Connection $\bar{D}$

The Curvature tensor of $M^n$ with respect to special semi - symmetric recurrent metric metric connection $\bar{D}$ is given by

$$\bar{R}(X,Y,Z) = \bar{D}_X\bar{D}_Y Z - \bar{D}_X\bar{D}_Y Z - \bar{D}_{[X,Y]}Z$$

Using (1.13) and (1.10) in above we have

$$\bar{R}(X,Y,Z) = R(X,Y,Z) \tag{3.1}$$

Hence we conculude.

**Proposition 3.1:** The Curvature tensor of $M^n$ with respect to special semi-symmetric recurrent metric metric connection $\overline{D}$ coincide with the curvature tensor of connection D of Riemannian manifold.
Taking the inner product of (3.1) with W, we have

$$\bar{R}(X,Y,Z,W) = R(X,Y,Z,W) \tag{3.2}$$

Where $\bar{R}(X,Y,Z,W) = g(\bar{R}(X,Y,Z),$

From (3.2), we have

$$\bar{R}(X,Y,Z,W) = -R(Y,X,Z,W) \tag{3.3}$$

$$\bar{R}(X,Y,Z,W) = -R(X,Y,W,Z) \tag{3.4}$$

Combining above two relation, we have

$$\bar{R}(X,Y,Z,W) = R(Y,X,W,Z) \tag{3.5}$$

We also have,

$$\bar{R}(X,Y,Z) + \bar{R}(Y,Z,X) + \bar{R}(Z,X,Y) = 0 \tag{3.6}$$

This is the Bianchi first identity for $\overline{D}$.

Hence we conclude that the curvature tensor of $M^n$ with respect to special semi - symmetric recurrent metric metric connection $\overline{D}$ satifies the first Bianchi identy.
Contracting (3.2) over X and W, we obtain

$$\bar{S}(Y,Z) = S(Y,YZ) \tag{3.7}$$

Where $\bar{S}$ and S denote the Ricci tensor of the connection $\overline{D}$ and D respectively.

From (3.7) we obtain a relation between the scalar curvature of $M^n$ with respect to the Riemannian connection and special semi-symmetric recurrent metric connection which is given by

$$\bar{r} = r \tag{3.8}$$

So we have following

**Proposition 3.2:** Forn dimensional LP sasakian manifold with special semi symmetric recurrent metric connection $\overline{D}$

**(1)** The curvature tensor $\bar{R}$ is given by (3.1)

**(2)** Ricci tensor $\bar{S}$ is given by (3.7)

**(3)** $\bar{r} = r$

Notes

## IV. Concircular Curvature Tensor of Lp Sasakian Manifold with Respect to Special Semi-Symmtric Recurrent Metric Connection

Analogous to the definition of concircularcurvarure tensor in a Riemannian manifold we define concircular curvature tensor with respect to the special semi symmetric recurrent metric connection $\overline{D}$ as

$$\bar{C}(X,Y,Z) = \bar{R}(X,Y,Z) - \frac{r}{n(n-1)}\{g(Y,Z)X - g(X,Z)Y\} \tag{4.1}$$

Using (3.1) and (3.7) in (4.1), we have

$$\bar{C}(X,Y,Z) = C(X,Y,Z) \tag{4.2}$$

So we have

**Proposition 4.1:** $\bar{C}(X,Y,Z) = C(X,Y,Z)$ that is manifold coincide with Riemannian Manifold.

The notion of an ξ-conformaly flat contact manifold was given by Zhen, Cabrezizo and Fermander [3]. In an analogous we define an ξ-conformaly flat n - dimensional LP sasakian manifold.

**Defination 5.2:** An n-dimensional LP sasakian manifold is called ξ-conformaly flat if the condition $\bar{C}(X,Y)\xi = 0$ holds on $M^n$.

From (4.2) it is clear that $\bar{C}(X,Y)\xi = C(X,Y)\xi$

So we have the following theorem.

**Theorem 4.1:** In an n-dimensional LP Sasakian manifold, an ξ–conformaly flatness with respect to special semi-symmetric recurrent metric connection and Riemannian connection coincide.

**Defination 4.3:** an n dimensional LP Sasakian manifold satisfying the condition

$$\emptyset^2\bar{C}(\emptyset X,\emptyset Y)\emptyset Z = 0 \tag{4.3}$$

Is called $\emptyset -$ concircularly flat.

Letus suppose that $M^n$ be n dimensional $\emptyset -$ concircularly flat LP sasakian manifold with respect to special semi-symmetric recurrent metric connection. It can easily be seen that $\emptyset^2\bar{C}(\emptyset X,\emptyset Y)\emptyset Z = 0$ if and only if

$$g(\bar{C}(\emptyset X,\emptyset Y)\emptyset Z,\emptyset W) = 0 \tag{4.4}$$

For all X,Y, Z, Won T(M).

Using (4.1), $\emptyset -$ concircularly flat means

$$g(\bar{R}(\emptyset X,\emptyset Y)\emptyset Z,\emptyset W) = \frac{r}{n(n-1)}\{g(\emptyset Y,\emptyset Z)g(\emptyset X,\emptyset W) - g(\emptyset X,\emptyset Z)g(\emptyset Y,\emptyset W)\} \tag{4.5}$$

Ref

3. Zhen, G, Cabrezizo J. L, Fernander, L.M and Fernander: on ξ conformally flat contact metric manifold. Indian J. Pure. appl. math 28, 725-734, 1997.

Let $\{e_1, e_2 \ldots., \xi\}$ be a local orthogonal basis of the vector in $M^n$ using the fact that $\emptyset e_1, \emptyset e_2 \ldots. \xi\}$ is also a local orthogonal basis, putting $X = W = e_i$ in (4.5) and summanig with respect to $i$, we have

$$S(\emptyset Y, \emptyset Z) = \frac{r}{n(n-1)}\{g(\emptyset Y, \emptyset Z)\} \tag{4.6}$$

Notes

Putting $Y = \emptyset Y, Z = \emptyset Z$ in (4.6) and using the fact S is symmetric, we have

$$g(\bar{R}(\emptyset X, \emptyset Y)\emptyset Z, \emptyset W) = 0$$

Hence we have

**Theorem 4.2:** an n-dimensional LP sasakian manifold is $\emptyset -$ concircularly flat with respect to special semi-symmetric recurrent metric connectionand manifold coincide with Riemannian Manifold.

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# Global Attractor of the Nonlinearfour Order Wave Equations

By Ling Zhao
*Sichuan Normal University, China*

Abstract - Global attractor of the nonlinear four order wave equations is consideredin this paper. Firstly, it is proved that this system possesses an absorbing set in $L^2(\Omega)\times H^2(\Omega)$. Secondly, it is obtained that the nonlinear four order wave equations have a global attractor in $L^2(\Omega)\times H^2(\Omega)$ by using $C-$condition.

Keywords : *global attractor, $C-$condition, four order wave equations.*

GJSFR-F Classification : *MSC 2010: 35B40, 35B41, 35G30, 35L75*

GLOBAL ATTRACTOR OF THE NONLINEARFOUR ORDER WAVE EQUATIONS

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# Global Attractor of the Nonlinearfour Order Wave Equations

Ling Zhao

*Abstract* - Global attractor of the nonlinear four order wave equations is consideredin this paper. Firstly, it is proved that this system possesses an absorbing set in $L^2(\Omega)\times H^2(\Omega)$. Secondly, it is obtained that the nonlinear four order wave equations have a global attractor in $L^2(\Omega)\times H^2(\Omega)$ by using $C-$ condition.

*Keywords : global attractor, $C-$ condition, four order wave equations.*

## I. Introduction

Nonlinear four order wave equations can be applied in mechanics of elastic constructions, and have a long history([1]-[11]). But few of the studies consider attractor of the nonlinear four order wave equations. In this paper we study attractor of the following nonlinear four order wave equations

$$u_{tt}+\triangle^2 u+\alpha u_t=f(u),\quad x\in\Omega,\quad t>0, \tag{1.1}$$

$$u=\triangle u=0,\quad x\in\partial\Omega,\quad t>0, \tag{1.2}$$

$$u(x,0)=\varphi(x),\quad u_t(x,0)=\psi(x),\quad x\in\Omega, \tag{1.3}$$

where $\alpha>0$, $\Delta$ is the Laplacian operator, $\Omega$ denotes an open bounded set of $R^n(n=1,2,3)$ with smooth boundary $\partial\Omega$, and $f(s)\in C^1(R)$ satisfies the following conditions

$$F(s)=\int_\Omega f(s)dx\le c, \tag{1.4}$$

$$sf(s)-F(s)\le c, \tag{1.5}$$

and there is a $0\le r<\infty$, such that

$$\lim_{s\to\infty}\frac{f'(s)}{s^r}=0. \tag{1.6}$$

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Ref

[1] Eisley JG: Nonlinear vibration of beams and rectangular plates. Z.Angew. Math. Phys. 15, 167-175 (1964)

As far as the theory of infinite-dimensional dynamical system is concerned, we refer to [12]-[22]. In the study of infinite dimensional dynamical system, the long-time behavior of the solution to equations is an important issue. The long-time behavior of the solution to equations can be shown by the global attractor with the finite-dimensional characteristics. Some authors have already studied the existence of the global attractor for some evolution equations. The global attractor strictly defined as $\omega$-limit set of ball, which under additional assumptions is nonempty, compact, and invariant ([14],[16]). We investigate global attractor of the equations (1.1)-(1.3) in this article.

The paper is organized as follows. In Section 2 we recall preliminary results. In Section 3, we obtain global attractor of the equations.

## II. Preliminaries

Let $X$ and $X_1$ be two Banach spaces, $X_1 \subset X$ a compact and dense inclusion. Consider the abstract nonlinear evolution equation defined on $X$, given by

$$\begin{cases} \frac{du}{dt} = Lu + G(u), \\ \\ u(x,0) = u_0. \end{cases} \tag{2.1}$$

where $u(t)$ is an unknown function, $L : X_1 \to X$ a linear operator, and $G : X_1 \to X$ a nonlinear operator.

A family of operators $S(t) : X \to X (t \geq 0)$ is called a semigroup generated by (2.1) if it satisfies the following properties:

(1) $S(t) : X \to X$ is a continuous map for any $t \geq 0$,

(2) $S(0) = id : X \to X$ is the identity,

(3) $S(t+s) = S(t) \cdot S(s), \forall t, s \geq 0$. Then, the solution of (2.1) can be expressed as

$$u(t, u_0) = S(t)u_0.$$

Next, we introduce the concepts and definitions of invariant sets, global attractors, and $\omega$-limit sets for the semigroup $S(t)$.

**Definition 2.1** Let $S(t)$ be a semigroup defined on $X$. A set $\Sigma \subset X$ is called an invariant set of $S(t)$ if $S(t)\Sigma = \Sigma, \forall t \geq 0$. An invariant set $\Sigma$ is an attractor of $S(t)$ if $\Sigma$ is compact, and there exists a neighborhood $U \subset X$ of $\Sigma$ such that for any $u_0 \in U$,

Ref

[12] Babin AV, Vishik MI: Attractors of Evolution Equations, North-Holland, Amsterdam, 1992.

$$\inf_{v\in\Sigma}\|S(t)u_0 - v\|_X \to 0, \quad \text{as} \quad t\to\infty.$$

In this case, we say that $\Sigma$ attracts $U$. Especially, if $\Sigma$ attracts any bounded set of $X$, $\Sigma$ is called a global attractor of $S(t)$ in $X$.

For a set $D \subset X$, we define the $\omega$-limit set of $D$ as follows

$$\omega(D) = \bigcap_{s\geq 0}\overline{\bigcup_{t\geq s} S(t)D},$$

where the closure is taken in the $X$-norm. Lemma 2.2 is the classical existence theorem of global attractor by Temam [16].

**Lemma 2.2** Let $S(t) : X \to X$ be the semigroup generated by (2.1). Assume the following conditions hold

(1) $S(t)$ has a bounded absorbing set $B \subset X$, i.e., for any bounded set $A \subset X$ there exists a time $t_A \geq 0$ such that $S(t)u_0 \in B, \forall u_0 \in A$ and $t > t_A$;

(2) $S(t)$ is uniformly compact, i.e., for any bounded set $U \subset X$ and some $T > 0$ sufficiently large, the set $\overline{\bigcup_{t\geq T} S(t)U}$ is compact in $X$.

Then the $\omega$-limit set $\mathcal{A} = \omega(B)$ of $B$ is a global attractor of (2.1), and $\mathcal{A}$ is connected providing $B$ is connected.

**Definition 2.3**[15] We say that $S(t) : X \to X$ satisfies $C-$condition, if for any bounded set $B \subset X$ and $\varepsilon > 0$, there exist $t_B > 0$ and a finite dimensional subspace $X_1 \subset X$, such that $\{PS(t)B\}$ is bounded, and

$$\|(I-P)S(t)u\|_X < \varepsilon, \quad \forall t \geq t_B \ \text{ and } \ u \in B,$$

where $P : X \to X_1$ is a projection.

**Lemma 2.4**[15] Let $S(t) : X \to X(t \geq 0)$ be a dynamical systems. If the following condition are satisfied

(1) there exists a bounded absorbing set $B \subset X$,

(2) S(t) satisfies $C-$condition,

then $S(t)$ has a global attractor in $X$.

We introduce the spaces

$$H = L^2(\Omega), \quad V = \{u \in H^2 | u|_{\partial\Omega} = \triangle u|_{\partial\Omega} = 0\}, \quad E = H \times V.$$

Ref

[16] Temam R: Infinite-Dimensional Dynamical Systems in Mechanics and physics. Applied Mathematical Science, Vol. 68, Springer-Verlag, New York, 2nd., 1997

The denotations $(\cdot,\cdot), \|\cdot\|$ are the inner product and norm in $H$ respectively, i.e., for any $u,v \in H$, $(u,v) = \int_\Omega uvdx$, $\|u\| = (\int_\Omega |u|^2dx)^{\frac{1}{2}}$. The norm in $V$ is $\|u\|_V = \|\triangle u\|$ and we have $\|\triangle u\|^2 \geq \beta_1\|u\|^2$, where $\beta_1$ is the first eigenvalue of $\Delta u$ with boundary condition (1.2).

**Lemma 2.5** Assume the nonlinear term $f : R \to R$ satisfies (1.6). Then the nonlinear operator $f : V \to H$ is compact.

**Proof.** Let $\{u_n\}$ be a bounded sequence in $V$. By the Soblev embedding theorem, $\{u_n\}$ is bounded in $L^k(\Omega)$ for any $1 \leq k < \infty$, and has a convergent subsequence in $L^2(\Omega)$. Without loss of generality, we assume $\{u_n\}$ converges to $u_0$ in $H$. It is sufficient to prove that $\{f(u_n)\}$ converges to $f(u_0)$ in $H$.

From (1.6), for any $\eta > 0$, we have

$$|f'(s)|^2 \leq \eta|s|^{2r} + c_\eta,$$

where $c_\eta \to 0$ if $\eta \to 0$.

Then, for some $0 \leq \varepsilon \leq 1$, we obtain

$$\int_\Omega |f(u_n) - f(u_0)|^2dx = \int_\Omega |f'(u_0 + (1-\varepsilon)(u_n - u_0))|^2|u_n - u_0|^2dx$$
$$\leq \eta\int_\Omega |u_0 + (1-\varepsilon)(u_n - u_0)|^{2r}|u_n - u_0|^2dx + c_\eta\int_\Omega |u_n - u_0|^2dx$$
$$= \eta\int_\Omega |\varepsilon u_0 + (1-\varepsilon)u_n)|^{2r}|u_n - u_0|^2dx + c_\eta\int_\Omega |u_n - u_0|^2dx$$
$$\leq 4^r\eta\int_\Omega (|u_0|^{2r} + |u_n|^{2r})|u_n - u_0|^2dx + c_\eta\int_\Omega |u_n - u_0|^2dx$$
$$\leq 4^r\eta[(\int_\Omega |u_0|^{2rp}dx)^{\frac{1}{p}} + (\int_\Omega |u_n|^{2rp}dx)^{\frac{1}{p}}](\int_\Omega |u_n - u_0|^{2q}dx)^{\frac{1}{q}} + c_\eta\int_\Omega |u_n - u_0|^2dx$$

where $p > 0, q > 0$, and $\frac{1}{p} + \frac{1}{q} = 1$.

Let $n \to \infty$ and $\eta \to 0$. We have

$$\int_\Omega |f(u_n) - f(u_0)|^2dx = 0.$$

Then the nonlinear operator $f : V \to H$ is compact.

## III. Existence of Global Attractor

**Theorem 3.1** Let $\varphi \in H$, $\psi \in V$, and $f$ satisfy the conditions (1.4) and (1.5). Suppose that the problem (1.1)-(1.3) has a unique weak solution and $S(t), t > 0$, defined by $S(t)(\varphi,\psi) =$

Notes

$(u(t), u_t(t))$, is the semigroup generated by the problem (1.1)-(1.3). Then $S(t)$ has a bounded absorbing ball.

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**Proof.** Take the inner product of (1.1) in $H$ with $v = u_t + \theta u$, $0 < \theta \leq \theta_0$, and $\theta_0$ will be chosen later. we obtain

$$(u_{tt}, v) + (\triangle^2 u, v) + \alpha(u_t, v) = (f, u).$$

Using the condition (1.4), we have

$$\frac{1}{2}\frac{d}{dt}(\|v\|^2 + \|\triangle u\|^2 - 2\int_\Omega F(u)dx) + (\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2$$

$$+(\theta^2 - \alpha\theta)(u, v) - \theta\int_\Omega f(u)udx = 0. \tag{3.1}$$

Using the Hölder inequality, Young inequality and the condition (1.5), we get

$$(\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2 + (\theta^2 - \alpha\theta)(u, v) - \theta\int_\Omega f(u)udx$$

$$\geq (\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2 + (\theta^2 - \alpha\theta)\|u\|\|v\| - \theta\int_\Omega (F(u) + c)dx$$

$$\geq (\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2 - \frac{\alpha\theta}{\beta_1^{\frac{1}{2}}}\|\triangle u\|\|v\| - \theta\int_\Omega (F(u) + c)dx$$

$$\geq (\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2 - \frac{\theta}{2}\|\triangle u\|^2 - \theta\frac{\alpha^2}{2\beta_1}\|v\|^2 - \theta\int_\Omega (F(u) + c)dx$$

$$\geq [\alpha - \theta(1 + \frac{\alpha^2}{2\beta_1})]\|v\|^2 + \frac{\theta}{2}\|\triangle u\|^2 - \theta\int_\Omega (F(u) + c)dx.$$

Choose $\theta_0$, such that $\theta_0(1 + \frac{\alpha^2}{2\beta_1}) = \frac{\alpha}{2}$. Hence, $\alpha - \theta(1 + \frac{\alpha^2}{2\beta_1}) \geq \frac{\alpha}{2} \geq \theta$.

From (3.1), we have

$$\frac{1}{2}\frac{d}{dt}(\|v\|^2 + \|\triangle u\|^2 - 2\int_\Omega F(u)dx) + \frac{\alpha}{2}\|v\|^2 + \frac{\theta}{2}\|\triangle u\|^2 - \theta\int_\Omega (F(u) + c)dx \leq 0.$$

Then, it follows that

$$\frac{d}{dt}[\|v\|^2 + \|\triangle u\|^2 + 2\int_\Omega (c - F(u))dx] + \theta[\|v\|^2 + \|\triangle u\|^2 + 2\int_\Omega (c - F(u))dx]$$

$$\leq 4c\theta|\Omega|, \tag{3.2}$$

where $|\Omega|$ is measure of the $\Omega$.

Let $y(t) = \|v\|^2 + \|\triangle u\|^2 + 2\int_\Omega (c - F(u))dx$. Then $y(t) \geq 0$ and (3.2) can be read as

$$\frac{dy}{dt} + \theta y \leq 4c\theta|\Omega|.$$

Using the Gronwall inequality, we have

$$y(t) \leq y(0)e^{-\theta t} + 4c|\Omega|(1-e^{-\theta t}), \quad t \geq 0.$$

For any bounded set $B$ of $E$, $(\varphi,\psi) \in B$ and $\int_\Omega [c - F(\varphi)]dx$ is bounded. Then

$$R(B) = \sup_{(\varphi,\psi)\in B} y(0) = \sup_{(\varphi,\psi)\in B} \{\|\varphi\|_V^2 + \|\psi + \theta\varphi\| + 2\int_\Omega (c - F(\varphi))dx\} < \infty,$$

and

$$\lim_{t\to\infty} \sup_{(\varphi,\psi)\in B} y(t) \leq 4c|\Omega| = \mu_0^2.$$

Let $\mu_1 > \mu_0$ be fixed, and there is a $t_0 = t_0(R(B),\mu_1) = \frac{1}{\theta}\ln\frac{R(B)}{\mu_1^2-\mu_0^2}$ such that for any $t \geq t_0$, we have $y(t) \leq \mu_1^2$. Then

$$\begin{aligned}
\|u(t)\|_V^2 + \|u_t(t)\|^2 &\leq \|u(t)\|_V^2 + 2\|u_t(t) + \theta u(t)\|^2 + 2\theta^2\|u(t)\|^2 \\
&\leq \|u(t)\|_V^2 + 2\|u_t(t) + \theta u(t)\|^2 + \frac{2\theta^2}{\beta_1}\|u(t)\|_V^2 \\
&\leq 2(1+\frac{\theta^2}{\beta_1})(\|u(t)\|_V^2 + \|u_t(t)+\theta u(t)\|^2) \\
&\leq 2(1+\frac{\theta^2}{\beta_1})y(t) \leq 2(1+\frac{\theta^2}{\beta_1})\mu_1^2.
\end{aligned}$$

Let $\rho_0^2 = 2(1+\frac{\theta^2}{\beta_1})\mu_1^2$. Then For all $t \geq t_0$, we have

$$\|u\|_V^2 + \|u_t\|^2 \leq \rho_0^2, \tag{3.3}$$

which implies that the ball of $E$, $B_0 = B_E(0,\rho_0)$, centered at 0 of radius $\rho_0$, is an absorbing set.

**Theorem 3.2** Assume $f : R \to R$ satisfies (1.4)-(1.6). Then the semigroup $S(t), t \geq 0$ associated with problem (1.1)-(1.3) possesses a global attractor.

**Proof.** The eigenvalue equation

$$\begin{cases} \Delta^2 u = \beta u, \\ u|_{\partial\Omega} = \triangle u|_{\partial\Omega} = 0, \end{cases} \tag{3.4}$$

has eigenvalues $\beta_1, \beta_2, \cdots, \beta_k, \cdots$and eigenvector $\{e_k | k = 1,2,3,\cdots\}$, and $0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_k \geq \cdots$. If $k \to \infty$, $\beta_k \to \infty$. $\{e_k | k = 1,2,3,\cdots\}$ constitutes an orthogonal base of $H$.

For $\forall u \in H$, we have

$$u = \sum_{k=1}^{\infty} u_k e_k, \quad \|u\|_{L^2}^2 = \sum_{k=1}^{\infty} u_k^2,$$

Introduce subspace $E_1 = span\{e_1, e_2, \cdots, e_k\} \subset H$. Let $E_2$ be an orthogonal subspace of $E_1 \subset H$.

For $\forall (u_t, u) \in E$, we find

$$u_t = u_{1t} + u_{2t}, \quad u_{1t} = P_m u_t, \quad u_{2t} = (I - P_m) u_t,$$

and

$$u = u_1 + u_2, \quad u_1 = P_m u \quad u_2 = (I - P_m) u,$$

where $P_m : H \to E_1$ be the orthogonal projection.

Let $P_i : E \to E_i \times E_i$ be the orthogonal projection. Thanks to Definition 2.3, we will prove that for any bounded set $B \subset E$ and $\varepsilon > 0$, there exists $t_* > 0$ such that

$$\|P_1 S(t) B\|_H \le M, \quad \forall t > t_*, \quad M \quad \text{is a constant}, \tag{3.5}$$

$$\|P_2 S(t) B\|_H \le \varepsilon, \quad \forall t > t_*, \quad \varphi \in B. \tag{3.6}$$

From Theorem 3.1, $S(t)$ has an absorbing set $B_M$. Then for any bounded set $B \subset E$, there exists $t_0 > 0$ such that $S(t)B \subset B_M$, $\forall t > t_0$, which imply (3.5).

From Lemma 2.5, $f : V \to H$ is compact, then for any $\varepsilon > 0$, there exists some $m \in N$ such that

$$\|(I - P_m) f\| \le \varepsilon.$$

Multiply (1.1) by $v_2 = u_{2t} + \theta u_2$ $(0 < \theta \le \theta_1)$, and integrate over $\Omega$. We obtain

$$\int_\Omega (u_{tt} + \triangle^2 u + \alpha u_t) v_2 dx = \int_\Omega f v_2 dx.$$

Then, we have

$$\frac{1}{2}\frac{d}{dt}(\|v_2\|^2 + \|\triangle u_2\|^2) + (\alpha - \theta)\|v_2\|^2 + \theta\|\triangle u_2\|^2$$

$$+(\theta^2 - \alpha\theta)(u_2, v_2) \le (f, v_2) \le \|(I - P_m) f\|\|v_2\| \le \varepsilon\|v_2\| \le \frac{\alpha}{4}\|v_2\|^2 + \frac{\varepsilon^2}{\alpha}. \tag{3.7}$$

Using the Hölder inequality and Young inequality, we get

$$(\alpha - \theta)\|v_2\|^2 + \theta\|\triangle u_2\|^2 + (\theta^2 - \alpha\theta)(u_2, v_2)$$

$$\ge (\alpha - \theta)\|v_2\|^2 + \theta\|\triangle u_2\|^2 + (\theta^2 - \alpha\theta)\|u_2\|\|v_2\|$$

$$\ge (\alpha - \theta)\|v_2\|^2 + \theta\|\triangle u_2\|^2 - \frac{\alpha\theta}{\beta_{m+1}^{\frac{1}{2}}}\|\triangle u_2\|\|v\|$$

$$\geq (\alpha-\theta)\|v_2\|^2+\theta\|\triangle u_2\|^2-\frac{\theta}{2}\|\triangle u_2\|^2-\theta\frac{\alpha^2}{2\beta_{m+1}}\|v_2\|^2$$

$$\geq [\alpha-\theta(1+\frac{\alpha^2}{2\beta_{m+1}})]\|v_2\|^2+\frac{\theta}{2}\|\triangle u_2\|^2.$$

Choose $\theta_1$, such that $\theta_1(1+\frac{\alpha^2}{2\beta_1})=\frac{\alpha}{2}$. Hence, $\alpha-\theta(1+\frac{\alpha^2}{2\beta_1})\geq\frac{\alpha}{2}\geq\theta$.

From (3.7), we have

$$\frac{1}{2}\frac{d}{dt}(\|v_2\|^2+\|\triangle u_2\|^2)+\frac{\alpha}{4}\|v_2\|^2+\frac{\theta}{2}\|\triangle u_2\|^2\leq\frac{\varepsilon^2}{\alpha}.$$

Then, it follows that

$$\frac{d}{dt}(\|v_2\|^2+\|\triangle u_2\|^2)+\theta(\|v\|^2+\|\triangle u\|^2)\leq\frac{2\varepsilon^2}{\alpha}. \tag{3.8}$$

Using the Gronwall inequality, we have

$$\|v_2(t)\|^2+\|\triangle u_2(t)\|^2\leq(\|v_2(0)\|^2+\|\triangle u_2(0)\|^2)e^{-\theta t}+\frac{2\varepsilon^2}{\alpha}(1-e^{-\theta(t-t_0)}),\quad t\geq t_0.$$

where $t_0$ is given in the proof of Theorem 3.1. Then

$$\|v_2(0)\|^2+\|\triangle u_2(0)\|^2\leq\rho_0^2.$$

Let $t_1-t_0=\frac{1}{\theta}\ln\frac{\rho_0^2\alpha}{\varepsilon^2}$. Hence

$$\|v_2(t)\|^2+\|\triangle u_2(t)\|^2\leq\frac{3\varepsilon^2}{\alpha},\quad t\geq t_1.$$

Then

$$\|u_2(t)\|_V^2+\|u_{2t}(t)\|^2\leq\|u_2(t)\|_V^2+2\|u_{2t}(t)+\theta u_2(t)\|^2+2\theta^2\|u_2(t)\|^2$$

$$\leq\|u_2(t)\|_V^2+2\|u_{2t}(t)+\theta u_2(t)\|^2+\frac{2\theta^2}{\beta_{m+1}}\|u_2(t)\|_V^2$$

$$\leq 2(1+\frac{\theta^2}{\beta_{m+1}})(\|u_2(t)\|_V^2+\|u_{2t}(t)+\theta u_2(t)\|^2)$$

$$\leq 2(1+\frac{\theta^2}{\beta_{m+1}})(\|v_2(t)\|^2+\|\triangle u_2(t)\|^2)\leq(1+\frac{\theta^2}{\beta_{m+1}})\frac{6\varepsilon^2}{\alpha}.$$

Let $C=\frac{6}{\alpha}(1+\frac{\theta^2}{\beta_{m+1}})$. Then for all $t\geq t_1$, we have

$$\|u\|_V^2+\|u_t\|^2\leq C\varepsilon^2, \tag{3.9}$$

which implies (3.6).

From Lemma 2.4, the equations(1.1)-(1.3) possess a global attractor.

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# On Semi-Open Sets and Semi-Separability

By V. Srinivasa kumar
*JNTU University, India*

*Abstract -* In this paper, we introduce the concepts of semi-limit and semi-separability. We prove that separability and semi-separability are equivalent and also prove a few interesting results in this connection.

*Keywords :* *semi-open, semi-closed, semi-closure, semi-neighborhood, semi-limit, semiseparability, sepa-rability, semi-topology.*

*GJSFR-F Classification :* *AMS Code: 54A10, 54A20, 54C08*

ON SEMI-OPEN SETS AND SEMI-SEPARABILITY

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# On Semi-Open Sets and Semi-Separability

V. Srinivasa kumar

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## I. Introduction

In 1963, Norman Levin introduced the concept of semi-open sets in his paper [2]. It has drawn the attention of various authors including Crossley, Hildebrand and Dorsett and they have probed deeply into this area and developed many interesting concepts like semi-closed sets, semi-compactness etc. In this present paper, we introduce the concepts of semi-limit and semi-separability and prove that semi-separability is equivalent to separability. Also we construct a topology using semi-open sets and we call this topology a semi-topology.

In what follows $(X,T)$ stands for a topological space. The symbols $cl(\ )$ and $Int(\ )$ denote the closure and interior in a topological space respectively.

### a) *Preliminaries*

**1.1 Definition:** Let $A$ be a subset of $X$. $A$ is said to be

(i) semi-open in $(X,T)$ if $A \subseteq cl(Int(A))$.

(ii) semi-closed if $X - A$ is semi-open in $(X,T)$.

(iii) semi-neighborhood of a point $x \in X$ if $x \in A$ and $A$ is semi-open in $(X,T)$.

**1.2 Definition:** The semi-closure of a set $A$ in $(X,T)$ denoted by $scl(A)$, is the intersection of all semi-closed supersets of $A$.

**1.3 Definition:** A point $x \in X$ is said to be a semi-limit point of a set $A$ in $(X,T)$, if every semi-neighborhood of $x$ contains a point of $A$ different from $x$ in $X$.

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*b) Semi-Limit and Semi-Topology*

**2.1 Definition:** Let $x \in X$ and let $\{ x_\lambda / \lambda \in \Delta \}$ be a net in $(X, T)$. We say that $x$ is a semi-limit of $\{x_\lambda / \lambda \in \Delta\}$ and we write $x = s.\lim_{\lambda \in \Delta} x_\lambda$ if for every semi-neighborhood $A$ of $x$ in $X$ there exists a $\lambda_A \in \Delta$ such that $x_\lambda \in A \quad \forall \lambda \geq \lambda_A$.

**2.2 Proposition:** For $A \subset X$ and $x \in X$, the following are equivalent.

(i) $x$ is a semi-limit point of $A$

(ii) there exists a net $\{x_\lambda / \lambda \in \Delta\}$ in $A$ such that $x = s.\lim_{\lambda \in \Delta} x_\lambda$

(iii) $x \in scl(A)$

**2.3 Remark:** Let $S(T)$ be the collection of all semi-open sets in $(X, T)$. The set $S(T)$ clearly contains $T$ and is closed under arbitrary unions. However, being not closed under finite intersections, $S(T)$ is not a topology on $X$. However, if $A \in T$ and $B \in S(T)$ then $A \bigcap B \in S(T)$.

**2.4 Definition:** We define $S_0(T) = \{A \in S(T) / A \bigcap B \in S(T) \ \forall B \in S(T)\}$ and $S_{00}(T) = \{A \in S(S_0) / A \bigcap B \in S(S_0) \ \forall B \in S(S_0)\}$ where $S(S_0)$ is the collection of all semi-open sets in the topological space $(X, S_0(T))$.

**2.5 Proposition:**

(a) $S_0(T)$ and $S_{00}(T)$ are topologies on $X$.

(b) $T \subseteq S_0(T) \subseteq S(T)$.

(c) $S(S_0) \subseteq S(T)$.

(d) $S_0(T) = S_{00}(T)$.

**2.6 Remark:** We call the topology $S_0(T)$, a semi-topology on $X$.

**2.7 Notation:** We denote the closure of a subset $A$ of $X$ in the topological space $(X, S_o(T))$ by the symbol $cl_0(A)$ and interior of $A$ by $Int_o(A)$.

**2.8 Proposition:** For $A \subseteq X$, $scl(A) \subseteq cl_0(A) \subseteq cl(A)$.

*c) Semi-Separability*

**3.1 Definition:** $(X, T)$ is said to be separable if there exists a countable subset $A$ of $X$ such that $cl(A) = X$.

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**3.2 Definition:** $(X,T)$ is said to be semi-separable if there exists a countable subset $A$ of $X$ such that $scl(A) = X$ .

**3.3 Proposition:** $(X,T)$ is separable if and only if it is semi-separable.

**Proof:** Suppose that $(X,T)$ is separable.

$\Rightarrow$ there exists a countable subset $A$ of $X$ such that $cl\,(A) = X$ .

Let $x \in X$ and $G$ be a semi-neighborhood of $x$ in $(X,T)$

$\Rightarrow$ $x \in G$ and $G$ is semi-open in $(X,T)$

$\Rightarrow$ there exists $O \in T$ such that $O \subseteq G \subseteq cl\,(O)$ ·

Assume that $G \bigcap A = \phi$ $\Rightarrow A \subseteq X - G \subseteq X - O$

$$\Rightarrow cl\,(A) \subseteq X - O \quad \Rightarrow X = X - O$$

$$\Rightarrow O = \phi \quad \Rightarrow G = \phi \text{ which is a contradiction.}$$

Hence $G \bigcap A \neq \phi$ .

Thus each semi-neighborhood of $x$ in $(X,T)$ intersects $A$

$\Rightarrow$ $x \in s\,cl(A)$ . Hence $s\,cl(A) = X$ $\Rightarrow$ $X$ is semi-separable.

The converse follows from the definitions 3.1, 3.2 and the proposition 2.8.

**3.4 Proposition:** $(X, S_0(T))$ is semi-separable $\Leftrightarrow$ $(X,T)$ is semi-separable.

**Proof:** Suppose that $(X, S_0(T))$ is semi-separable

$\Rightarrow$ there exists a countable subset $A$ of $X$ such that $cl_0(A) = X$

$\Rightarrow cl(A) = X$ $\Rightarrow (X,T)$ is separable and hence it is semi-separable.

Conversely suppose that $(X,T)$ is separable $\Rightarrow (X,T)$ is semi-separable

$\Rightarrow$ there exists a countable subset $A$ of $X$ such that $scl(A) = X$

$\Rightarrow$ $cl_0(A) = X$ $\Rightarrow$ $(X, S_0(T))$ is separable and hence it is semi-separable.

## II. Acknowledgements

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# Lie Algebraic Approach and Complex Invariant Coupled Oscillator Systems

By Jasvinderpal Singh Virdi
*Panjab University, India*

*Abstract* - In classical mechanics, the system of coupled harmonic oscillators is shown to possess the symmetry applicable toa six-dimensional space in complex coordinates, two-dimensional phase space consisting of two position and twomomentum variables. In search into the features of a dynamical system, with the possibility of its complex invariant,we explore this dynamical systems. Dynamical algebraic approach is used to study two-dimensional complex systems(coupled oscillator system) on the extended complex phase plane (ECPS). Scope and importance of invariants in theanalysis of complex trajectories for dynamical systems is discussed.

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# Lie Algebraic Approach and Complex Invariant Coupled Oscillator Systems

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## I. Introduction

The coupled oscillator provides many soluble models in different branches of physics because of its mathematical simplicity. It stays with us in many different forms because it provides the mathematical basis for many soluble models in physics, including the Lee model in quantum field theory [1], the Bogoliubov transformation in superconductivity, relativistic models of elementary particles [2]. In physics coupled harmonic oscillator system in two-dimensions [3], in the Bells inequality experiments employing coupled harmonic oscillators [4]. This also has been used for description of motion of a charged particle in a magnetic field [5, 6] or . They are also studied in context of electrical circuits with time-varying capacitors and inductors, particularly with reference to their memory property, has become of considerable interest in recent years [7]. Blasone *et al* [8] studied momentum-dependent terms in the Hamiltonian structure in the context of the so-called holographic principle and in the treatment of quantum gravity as a dissipative and deterministic system. Hamiltonian for such system is given by

$$H = \frac{1}{2}[\alpha_1 p_x^2 + \alpha_2 p_y^2 + \beta_1 x^2 + \beta_2 y^2] + \alpha_3(p_x y + x p_y). \tag{1}$$

Invariants for above Hamiltonian provided they exist and can be computed, and even the complex invariants [9] exist, are a very useful tool to understand the theoretical structure of this dynamical systems. Since invariants of real Hamiltonian systems have been played a vital role in understanding the underlying dynamics of the systems and so we expect that the complex invariants can also be helpful in exploring some deep insights into features of complex dynamical systems. In the past, complex invariants have been discussed in context of understanding fermion masses and quark mixing, and CP-conserving two-Higgs-doublet model scalar potentials in the Particle physics [10, 11].
In this paper, we construct a complex invariants corresponding coupled oscillators based on the ECPS approach in complex domain [12]. Recently, with a view to explore some role of invariants for complex systems, Kaushal et al. [13] found invariants for some one dimensional systems within the framework of an ECPS. Some quantum mechanical studies within the ECPS are also reported [14]. But most of such studies are restricted in one dimension only. Such studies in higher dimensions is desirable from the intrinsic mathematical interest, to check the validity of various methods/theories and to find solutions of some realistic physical problems. With this motivation, recently we generalized the ECPS in two dimensions and studied the coupled oscillator.

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We make use of the Lie algebraic approach to derive at least one invariant for the TD versions of above coupled oscillator system. In fact, the Lie algebraic approach (ref therein [15]) commands several advantages over the rationalization method, particularly for the TD systems, not only in terms of the closure property of the Poisson bracket algebra of phase space functions but also for its straightforward extension to the corresponding quantum system.

### a) Lie algebraic approach

If we define

$$x = x_1 + ip_3; \quad y = x_2 + ip_4; \quad p_x = p_1 + ix_3; \quad p_y = p_2 + ix_4. \tag{2}$$

then, a two dimensional real phase space $(x, y, p_x, p_y, t)$, may be transformed into the corresponding extended complex phase plane (ECPS) $(x_1, p_3, x_2, p_4, p_1, x_3, p_2, x_4, t)$ The above transformations add four additional degrees of freedom, $(x_3, x_4, p_3, p_4, t)$, which can make mathematical analysis of a problem a bit more involved. But nevertheless, these type of transformations are used in many studies [12, 13, 14, 13]. From eq.(2) one can easily obtain one can easily obtain

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x_1} - i\frac{\partial}{\partial p_3}; \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial x_2} - i\frac{\partial}{\partial p_4}; \quad \frac{\partial}{\partial p_x} = \frac{\partial}{\partial p_1} - i\frac{\partial}{\partial x_3}; \quad \frac{\partial}{\partial p_y} = \frac{\partial}{\partial p_2} - i\frac{\partial}{\partial x_4}. \tag{3}$$

Now consider a complex phase space function $I(x, y, p_x, p_y, t)$ as

$$I = I_1(x_1, p_3, x_2, p_4, p_1, x_3, p_2, x_4, t) + iI_2(x_1, p_3, x_2, p_4, p_1, x_3, p_2, x_4, t). \tag{4}$$

Further, the invariance of I implying

$$\frac{dI}{dt} = \frac{\partial I}{\partial t} + [I, H] = 0, \tag{5}$$

where [., .] is the Poisson bracket, which in view of the definition, eq.(2), turns out to be

$$\begin{aligned}[I, H]_{(x,p)} = {} & [I, H]_{(x_1,p_1)} - i[I, H]_{(x_1,x_3)} - i[I, H]_{(p_3,p_1)} - [I, H]_{(p_3,x_3)} \\ & + [I, H]_{(x_2,p_2)} - i[I, H]_{(x_2,x_4)} - i[I, H]_{(p_4,p_2)} - [I, H]_{(p_4,x_4)}.\end{aligned} \tag{6}$$

### b) Example

Consider a coupled harmonic oscillator systems in two-dimensions, whose Hamiltonian is given by (1). Using (2), the above Hamiltonian (see [12] for detail method) can be expressed as

$$\begin{aligned} H = {} & \frac{\alpha_1}{2}p_1^2 - \frac{\alpha_1}{2}x_3^2 + i\alpha_1 p_1 x_3 + \frac{\alpha_1}{2}p_2^2 - \frac{\alpha_1}{2}x_4^2 + i\alpha_1 p_2 x_4 + \frac{\alpha_2}{2}x_1^2 - \frac{\alpha_2}{2}p_3^2 + i\alpha_2 p_3 x_1 p_4 \\ & + \frac{\alpha_2}{2}x_2^2 - \frac{\alpha_2}{2}p_4^2 + i\alpha_2 x_2 + \alpha_3(p_1x_2 + ip_1p_4 + ix_3x_2 - x_3p_4 + x_1p_2 + ix_1x_4 + ip_2p_3 - p_3x_4) \\ = {} & \sum_{m=1}^{20} h_m(t)\Gamma_m(x_1, p_3, x_2, p_4, p_1, x_3, p_2, x_4), \end{aligned} \tag{7}$$

and the various Γ's and h(t)'s for the above complex $H$ are given as

$$\begin{aligned} & \Gamma_1 = \frac{p_1^2}{2};\ \Gamma_2 = \frac{x_3^2}{2};\ \Gamma_3 = p_1x_3;\ \Gamma_4 = \frac{p_2^2}{2};\ \Gamma_5 = \frac{x_4^2}{2};\ \Gamma_6 = p_2x_4;\ \Gamma_7 = \frac{x_1^2}{2};\ \Gamma_8 = \frac{p_3^2}{2}; \\ & \Gamma_9 = x_1p_3;\ \Gamma_{10} = \frac{x_2^2}{2};\ \Gamma_{11} = x_2p_4;\ \Gamma_{12} = \frac{p_4^2}{2};\ \Gamma_{13} = p_1x_2;\ \Gamma_{14} = p_1p_4;\ \Gamma_{15} = x_3x_2; \\ & \Gamma_{16} = x_3p_4,\ \ \Gamma_{17} = x_1p_2;\ \ \Gamma_{18} = x_1x_4;\ \ \Gamma_{19} = p_3p_2;\ \ \Gamma_{20} = p_3x_4. \end{aligned} \tag{8}$$

With

$$\begin{aligned} & h_1 = h_2 = \alpha_1;\ h_3 = i\alpha_1;\ h_4 = h_5 = \alpha_2;\ h_6 = i\alpha_2;\ h_7 = h_8 = \beta_1;\ h_9 = i\beta_1;\ h_{10} = h_{11} = \beta_2 \\ & h_{12} = i\beta_2;\ h_{14} = h_{15} = i\alpha_3;\ h_{13} = \alpha_3;\ h_{16} = -\alpha_3;\ h_{17} = \alpha_3;\ h_{18} = h_{19} = i\alpha_3;\ h_{20} = -\alpha_3. \end{aligned} \tag{9}$$

Ref

14. R. S. Kaushal, On the quantum mechanics of complex Hamiltonian systems in one dimension, J. Phys. A: Math. Gen. **34** (2001) L709-721; R. S. Kaushal and Parthasarathi, Quantum mechanics of complex Hamiltonian systems in one dimension, J. Phys. A **35**, (2002) 8743-8754.

The dynamical algebra in this case is not closed. To find closure property for the above system, we have to add sixteen more phase space functions $(\Gamma_l)$'s. The additional $(\Gamma_l)$'s are as follow

$$\begin{aligned} &\Gamma_{21} = p_1p_3;\ \Gamma_{22} = p_1x_1;\ \Gamma_{23} = x_2p_1;\ \Gamma_{24} = p_1p_4,\ \Gamma_{25} = p_3x_3;\ \Gamma_{26} = x_1x_3;\ \Gamma_{27} = x_2x_3;\ \Gamma_{28} = x_3p_4; \\ &\Gamma_{29} = x_2p_2;\ \Gamma_{30} = p_4p_2;\ \Gamma_{31} = p_2p_3;\ \Gamma_{32} = p_2x_1,\ \Gamma_{33} = x_2x_4;\ \Gamma_{34} = x_4p_4;\ \Gamma_{35} = x_1x_4;\ \Gamma_{36} = p_3x_4, \end{aligned} \tag{10}$$

with corresponding $h_l(t) = 0$. Now in the light of Poisson bracket (6) for complex systems, we get large number (288 no. of) nonvanishing Poisson brackets (for more detail see [12]). Therefore, their use (5) yields the following set of PDEs in $\lambda$'s as described in (**??**) section two:

$$\dot{\lambda}_1 = 4\alpha_1(i\lambda_{21} - \lambda_{22}) - 4\alpha_3(\lambda_{23} - i\lambda_{24}), \tag{11}$$

$$\dot{\lambda}_2 = -4\alpha_1(\lambda_{25} + i\lambda_{26}) - 4\alpha_3(i\lambda_{27} + \lambda_{28}), \tag{12}$$

$$\dot{\lambda}_3 = -2\alpha_1(\lambda_{21} + i\lambda_{22} + i\lambda_{25} - \lambda_{26}) - 2\alpha_3(i\lambda_{23} + \lambda_{24} + \lambda_{27} - i\lambda_{28}), \tag{13}$$

$$\dot{\lambda}_4 = -4\alpha_2(\lambda_{29} + i\lambda_{30}) + 4\alpha_3(i\lambda_{31} - \lambda_{32}), \tag{14}$$

$$\dot{\lambda}_5 = -4\alpha_2(\lambda_{34} + i\lambda_{33}) - 4\alpha_3(\lambda_{36} + i\lambda_{35}), \tag{15}$$

$$\dot{\lambda}_6 = -2\alpha_2(i\lambda_{29} + \lambda_{30} - i\lambda_{34} + \lambda_{33}) - 2\alpha_3(\lambda_{31} + i\lambda_{32} + \lambda_{35} - i\lambda_{36}), \tag{16}$$

$$\dot{\lambda}_7 = -4\beta_1(-\lambda_{22} + i\lambda_{26}) + 4\alpha_3(\lambda_{32} - i\lambda_{35}), \tag{17}$$

$$\dot{\lambda}_8 = 4\beta_1(i\lambda_{21} + \lambda_{25}) + 4\alpha_3(\lambda_{36} + i\lambda_{31}), \tag{18}$$

$$\dot{\lambda}_9 = 2\beta_1(\lambda_{21} + i\lambda_{22} - i\lambda_{25} + \lambda_{26}) + 2\alpha_3(\lambda_{31} + i\lambda_{32} + \lambda_{35} - i\lambda_{36}), \tag{19}$$

$$\dot{\lambda}_{10} = 4\alpha_3(\lambda_{23} - i\lambda_{27}) - 4\beta_2(\lambda_{29} + \lambda_{33}), \tag{20}$$

$$\dot{\lambda}_{11} = 2\alpha_3(i\lambda_{23} + \lambda_{24} + \lambda_{27} - i\lambda_{28}) - 2\alpha_3(\lambda_{29} + i\lambda_{30} + \lambda_{34} - i\lambda_{33}), \tag{21}$$

$$\dot{\lambda}_{12} = 4\alpha_3(i\lambda_{24} + \lambda_{28}) - 4\beta_2(\lambda_{30} + \lambda_{34}), \tag{22}$$

$$\dot{\lambda}_{13} = 2\alpha_3(\lambda_{22} - i\lambda_{33} - i\lambda_{26} + \lambda_{29}) + 2\beta_1(i\lambda_{23} - i\lambda_{27}) + 2\beta_2(\lambda_{32} - i\lambda_{35}), \tag{23}$$

$$\dot{\lambda}_{14} = 2\alpha_3(\lambda_{26} + \lambda_{30} - i\lambda_{34} + i\lambda_{22}) + 2\beta_1(\lambda_{24} - i\lambda_{28}) + \beta_2(-\lambda_{32} + i\lambda_{35}), \tag{24}$$

$$\dot{\lambda}_{15} = 2\alpha_3(\lambda_{21} - i\lambda_{25} + \lambda_{33} + i\lambda_{29}) + 2\beta_1(i\lambda_{23} + \lambda_{27}) + \beta_2(\lambda_{31} - \lambda_{36}), \tag{25}$$

$$\dot{\lambda}_{16} = 2\beta_2(i\lambda_{21} + \lambda_{25} + i\lambda_{30} + i\lambda_{34}) + 2\beta_1(i\lambda_{24} + \lambda_{28}) - \beta_2(\lambda_{31} + i\lambda_{36}), \tag{26}$$

$$\dot{\lambda}_{17} = 2\alpha_3(i\lambda_{21} - \lambda_{22} - \lambda_{29} + i\lambda_{30}) + 2\alpha_1(i\lambda_{31} - \lambda_{32}) - 2\alpha_2(i\lambda_{23} + \lambda_{24}), \tag{27}$$

$$\dot{\lambda}_{18} = -2\alpha_3(\lambda_{21} + i\lambda_{22} + i\lambda_{29} + \lambda_{30}) + 2\alpha_1(-\lambda_{31} + i\lambda_{32}) + \alpha_2(-i\lambda_{23} + \lambda_{24}), \tag{28}$$

Ref

12. J.S. Virdi, and S.C. Mishra, Exact complex integrals in two dimensions for shifted harmonic oscillators, Pramana J. Phys. **79** (2012) 19-40; J.S. Virdi, F. Chand, C.N. Kumar, and S.C. Mishra, Complex dynamical invariants for two-dimensional complex potentials, Pramana J. Phys. **79**, (2012) 173-183.

$$\dot{\lambda}_{19} = 2\alpha_3(i\lambda_{25} - \lambda_{26} - i\lambda_{29} - \lambda_{30}) - 2\alpha_1(\lambda_{31} + i\lambda_{32}) + 2\alpha_2(-\lambda_{27} + i\lambda_{28}), \tag{29}$$

$$\dot{\lambda}_{20} = -2\alpha_3(\lambda_{25} + i\lambda_{26} + i\lambda_{29} + \lambda_{30}) - 2\alpha_1(\lambda_{31} + i\lambda_{32}) + 2\alpha_2(-i\lambda_{27} - \lambda_{28},) \tag{30}$$

$$\dot{\lambda}_{21} = 2\beta_1(i\lambda_1 + \lambda_3) + 2\alpha_1(-\lambda_9 + i\lambda_8) + 2\alpha_3(-i\lambda_{16} - \lambda_{15}) + 2\alpha_3(i\lambda_{17} + \lambda_{18}), \tag{31}$$

$$\dot{\lambda}_{22} = 2\beta_1(\lambda_1 - i\lambda_3) + 2\alpha_1(i\lambda_9 + \lambda_7) + 2\alpha_3(i\lambda_{14} - \lambda_{13}) + 2\alpha_3(\lambda_{17} - i\lambda_{18}), \tag{32}$$

$$\dot{\lambda}_{23} = 2\beta_2(i\lambda_{17} - i\lambda_{18}) + 2\alpha_1(-\lambda_{13} + i\lambda_{15}) + 2\alpha_3(-i\lambda_3 + \lambda_1) + 2\alpha_3(i\lambda_{11} - \lambda_{10}), \tag{33}$$

$$\dot{\lambda}_{24} = 2\alpha_3(i\lambda_1 + \lambda_3) + 2\alpha_3(-\lambda_{11} + i\lambda_{12}) + 2\alpha_1(-i\lambda_{16} - \lambda_{14}) + 2\beta_2(\lambda_{18} - \lambda_{17}), \tag{34}$$

$$\dot{\lambda}_{25} = 2\beta_1(-\lambda_2 - i\lambda_3) - 2\alpha_1(i\lambda_9 + \lambda_8) + 2\alpha_3(-\lambda_{16} - i\lambda_{15}) + 2\alpha_3(i\lambda_{19} + \lambda_{20}), \tag{35}$$

$$\dot{\lambda}_{26} = 2\beta_1(-i\lambda_2 + \lambda_3) + 2\alpha_1(i\lambda_7 - \lambda_9) + 2\alpha_3(-i\lambda_{13} - \lambda_{14}) + 2\alpha_3(\lambda_{19} - i\lambda_{20}), \tag{36}$$

$$\dot{\lambda}_{27} = 2\alpha_3(-i\lambda_2 + \lambda_3) + 2\alpha_3(-i\lambda_{10} - \lambda_{11}) - 2\alpha_1(i\lambda_{13} + \lambda_{15}) + 2\beta_2(\lambda_{19} + i\lambda_{20}), \tag{37}$$

$$\dot{\lambda}_{28} = 2\alpha_3(\lambda_2 + i\lambda_3) + 2\alpha_3(-i\lambda_{11} - i\lambda_{12}) - 2\alpha_1(\lambda_{14} - i\lambda_{16}) + 2\beta_2(-\lambda_{19} + i\lambda_{20}), \tag{38}$$

$$\dot{\lambda}_{29} = 2\beta_2(\lambda_4 + \lambda_6) + 2\alpha_2(-\lambda_{10} + i\lambda_{11}) + 2\alpha_3(i\lambda_{15} - \lambda_{13}) + 2\alpha_3(\lambda_{17} - i\lambda_{19}), \tag{39}$$

$$\dot{\lambda}_{30} = -2\alpha_3(\lambda_4 + i\lambda_6) + 2\alpha_2(-\lambda_{11} - i\lambda_{12}) + 2\alpha_3(-i\lambda_{16} - i\lambda_{14}) + 2\alpha_3(i\lambda_{17} + \lambda_{19}), \tag{40}$$

$$\dot{\lambda}_{31} = 2\alpha_3(i\lambda_4 + \lambda_6) + 2\alpha_2(i\lambda_8 + \lambda_9) + 2\alpha_2(-i\lambda_{16} - \lambda_{15}) + 2\beta_1(i\lambda_{17} + \lambda_{19}), \tag{41}$$

$$\dot{\lambda}_{32} = 2\alpha_3(\lambda_4 - i\lambda_6) + 2\alpha_3(\lambda_7 + i\lambda_9) + 2\alpha_2(i\lambda_{14} - \lambda_{13}) + 2\beta_1(\lambda_{17} - i\lambda_{19}), \tag{42}$$

$$\dot{\lambda}_{33} = 2\beta_2(\lambda_5 + \lambda_6) + 2\alpha_2(-\lambda_{10} - i\lambda_{11}) + 2\alpha_3(-i\lambda_{13} - \lambda_{15}) + 2\alpha_3(\lambda_{18} - i\lambda_{20}), \tag{43}$$

$$\dot{\lambda}_{34} = 2\beta_2(-\lambda_5 - \lambda_6) + 2\alpha_2(-i\lambda_{11} + \lambda_{12}) + 2\alpha_3(-\lambda_{14} - i\lambda_{16}) + 2\alpha_3(i\lambda_{18} + \lambda_{20}), \tag{44}$$

$$\dot{\lambda}_{35} = 2\beta_1(-\lambda_{18} - \lambda_{20}) + 2\alpha_2(-i\lambda_{13} - \lambda_{14}) + 2\alpha_3(-i\lambda_5 + \lambda_6) + 2\alpha_3(i\lambda_7 - \lambda_9), \tag{45}$$

$$\dot{\lambda}_{36} = 2\alpha_3(\lambda_5 + i\lambda_6) - 2\alpha_3(i\lambda_9 + \lambda_8) + 2\alpha_2(-\lambda_{16} + i\lambda_{15}) + 2\beta_1(i\lambda_{18} + \lambda_{20}). \tag{46}$$

As such the solution to these 36 coupled equation turns out to be difficult. Therefore, we make the following choices about $\lambda$'s which facilitate to find solutions of above equations.
From eqs.(11), (12) and (13), we get $2\dot{\lambda}_3 = i\dot{\lambda}_1 - i\dot{\lambda}_2$. If we consider $\lambda_3 = c_3$ (a constant), and by taking $\lambda_1 = \lambda_2 = \eta_1(t)$; which immediately gives

$$\lambda_1 = \eta_1(t) + c_1; \qquad \lambda_2 = \eta_1(t) + c_2. \tag{47}$$

From eqs.(14), (15) and (16), we get $2\dot{\lambda}_6 = i\dot{\lambda}_4 - i\dot{\lambda}_5$. If we consider $\lambda_6 = c_6$ (a constant), and by taking $\lambda_4 = \lambda_5 = \eta_2(t)$; which immediately gives

$$\lambda_4 = \eta_2(t) + c_4; \qquad \lambda_5 = \eta_2(t) + c_5. \tag{48}$$

Notes

Notes

Again From eqs.(17), (18) and (19), we get $2\dot{\lambda}_9 = i\dot{\lambda}_7 - i\dot{\lambda}_8$. If we set $\lambda_9 = c_9$ (a constant), and consider $\lambda_7 = \lambda_8 = \eta_3(t)$; which immediately gives

$$\lambda_7 = \eta_3(t) + c_7; \qquad \lambda_8 = \eta_3(t) + c_8. \tag{49}$$

From eqs.(20), (21) and (22), we get $2\dot{\lambda}_{11} = i\dot{\lambda}_{10} - i\dot{\lambda}_{12}$. If we set $\lambda_{11} = c_{11}$ (a constant), and consider $\lambda_{12} = \lambda_{10} = \eta_4(t)$; which immediately gives

$$\lambda_{12} = \eta_4(t) + c_{12}; \qquad \lambda_{10} = \eta_4(t) + c_{10}. \tag{50}$$

Now, in order to find solutions for $\lambda_{13}$, $\lambda_{14}$, $\lambda_{15}$ and $\lambda_{16}$ we have to make simplification for complications of above set of 24 eqs (23-46). (i.e. $\alpha_1 = \alpha_2 = \alpha_3$, and $\beta_1 = \beta_2 = \alpha_3$). From eqs.(17), (20) and (23), we get $2\dot{\lambda}_{13} = \dot{\lambda}_7 + i\dot{\lambda}_{10}$. If we consider $\lambda_{13} = c_{13}$ (a constant), and considering the eqn from above relation (with $\lambda_{10} = \eta_4(t) + c_{10},\ \lambda_7 = \eta_3(t) + c_7;$) gives

$$\lambda_{13} = \eta(t) + c_{13}. \tag{51}$$

where $\eta(t) = \frac{1}{2i}[\eta_4(t) + \eta_3(t);]$ is an another function of time, and ($c_{13} = c_7 + c_{10}$; a constant.)
From eqs.(18), (20) and (25), we get $2i\dot{\lambda}_{15} = \dot{\lambda}_{10} + i\dot{\lambda}_8$, If we consider $\lambda_{15} = c_{15}$ (a constant), and considering the eqn from above relation (with $\lambda_{10} = \eta_4(t) + c_{10},\ \lambda_8 = \eta_3(t) + c_8;$) gives

$$\lambda_{15} = \eta(t) + c_{15}, \tag{52}$$

where $\eta(t) = \frac{1}{2i}[\eta_4(t) + \eta_3(t)]$ is an another function of time and $c_{15} = c_{10} + c_8$; a constant.
In order to find solutions for $\lambda_{16}$, from eqs.(18), (22) and (26), we get $2i\dot{\lambda}_{16} = \dot{\lambda}_8 + \dot{\lambda}_{12}$. If we consider $\lambda_{16} = c_{16}$; (a constant), and considering the relation (with $\lambda_8 = \eta_3(t) + c_8,\ \lambda_{12} = \eta_4(t) + c_{12};$) gives

$$\lambda_{16} = \eta(t) + c_{16}, \tag{53}$$

where $\eta(t) = \frac{1}{2i}[\eta_4(t) + \eta_3(t)]$; and $c_{16} = c_8 + c_{12}$; a constant.
From eqs.(17), (22) and (24), we get $2i\dot{\lambda}_{14} = \dot{\lambda}_7 + \dot{\lambda}_{12}$, If we consider $\lambda_{14} = c_{14}$; (a constant), and considering relation (with $\lambda_7 = \eta_3(t) + c_7,\ \lambda_{12} = \eta_4(t) + c_{12};$) gives

$$\lambda_{14} = \eta(t) + c_{14}, \tag{54}$$

where $\eta(t) = \frac{1}{2i}[\eta_4(t) + \eta_3(t)]$; and $c_{16} = c_7 + c_{12}$ a constant.
Now, to find solutions for $\lambda_{17}$, $\lambda_{18}$, $\lambda_{19}$ and $\lambda_{20}$, refer from eqs.(11), (14) and (27), we get $2\dot{\lambda}_{17} = \dot{\lambda}_1 + \dot{\lambda}_4$; and considering the relation $\lambda_1 = \eta_1(t) + c_1,\ \ \lambda_4 = \eta_2(t) + c_4$; gives

$$\lambda_{17} = \phi(t) + c_{17}, \tag{55}$$

where $\phi(t) = \frac{1}{2}\int[\dot{\eta}_1(t) + \dot{\eta}_2(t)]dt$; and $c_{17} = c_1 + c_4$, a constant. From eqs.(11), (15) and (28), we get $2\dot{\lambda}_{18} = i\dot{\lambda}_1 - i\dot{\lambda}_5$; and considering the relation $\lambda_1 = \eta_1(t) + c_1,\ \lambda_5 = \eta_2(t) + c_5$; will results

$$\lambda_{18} = \varphi(t) + c_{18}, \tag{56}$$

where $\varphi(t) = \frac{1}{2}\int[i\dot{\eta}_1(t) - i\dot{\eta}_2(t)]dt$; and $c_{18} = c_1 + c_5$, a constant. Similarly to find solutions of $\lambda_{19}$, from eqs.(12), (14) and (29), we get $2\dot{\lambda}_{19} = i\dot{\lambda}_2 - i\dot{\lambda}_4$; and considering the relation $\lambda_2 = \eta_1(t) + c_2,\ \ \lambda_4 = \eta_2(t) + c_4$; gives

$$\lambda_{19} = \chi(t) + c_{19}, \tag{57}$$

where $\chi(t) = \frac{1}{2}\int[i\dot{\eta}_1(t) - i\dot{\eta}_2(t)]dt$ and $c_{19} = c_2 + c_4$, a constant.
Again from eqs.(12), (15) and (30), we get $2\dot{\lambda}_{20} = \dot{\lambda}_2 + \dot{\lambda}_4$; and considering the relation $\lambda_2 = \eta_1(t) + c_1,\ \ \lambda_4 = \eta_2(t) + c_4$; gives

$$\lambda_{20} = \quad (t) + c_{20}, \tag{58}$$

where $\quad (t) = \frac{1}{2}\int[\dot{\eta}_1(t) + \dot{\eta}_2(t)]dt$; and $c_{20} = c_2 + c_4$, a constant.
Solutions for $(\lambda_{21} - \lambda_{28})$ can be obtained respectively as from eqs.(31-38), we obtain following equations

$$i\dot{\lambda}_{21} + \dot{\lambda}_{22} = 2(\alpha_1\lambda_7 - \alpha_1\lambda_8 - \alpha_3\lambda_{13} + i\alpha_3\lambda_{14} - i\alpha_3\lambda_{15} + \alpha_3\lambda_{16}) = 0. \tag{59}$$

$$\dot{\lambda}_{23} + i\dot{\lambda}_{24} = 2(-\alpha_3\lambda_{10} + \alpha_3\lambda_{12} - \alpha_1\lambda_{13} + i\alpha_1\lambda_{15} - i\alpha_1\lambda_{14} + \alpha_1\lambda_{16}) = 0. \tag{60}$$

$$\dot{\lambda}_{25} - i\dot{\lambda}_{26} = 2(\alpha_1\lambda_7 - \alpha_1\lambda_8 - \alpha_3\lambda_{13} + i\alpha_3\lambda_{14} - i\alpha_3\lambda_{15} + \alpha_3\lambda_{16}) = 0. \tag{61}$$

$$\dot{\lambda}_{27} + i\dot{\lambda}_{28} = 2i(-\alpha_3\lambda_{10} + \alpha_3\lambda_{12} - \alpha_1\lambda_{13} + i\alpha_1\lambda_{15} - i\alpha_1\lambda_{14} + \alpha_1\lambda_{16}) = 0. \tag{62}$$

Notes

Since

$$\lambda_7 = \lambda_8, \lambda_{10} = \lambda_{12}; \qquad \lambda_{13} = \lambda_{14} = \lambda_{15} = \lambda_{16} = \eta(t), \tag{63}$$

or if we set

$$\begin{aligned} &\dot{\lambda}_{21} = -i\dot{\lambda}_{22} = \dot{\xi}(t); \quad \dot{\lambda}_{23} = -i\dot{\lambda}_{24} = \dot{\theta}(t); \\ &\dot{\lambda}_{25} = i\dot{\lambda}_{26} = \dot{\delta}(t); \quad \dot{\lambda}_{27} = -i\dot{\lambda}_{28} = \dot{\zeta}(t). \end{aligned} \tag{64}$$

which immediately gives

$$\begin{aligned} &\lambda_{21} = \xi(t) + c_{21}; \quad \lambda_{22} = -i\xi(t) + c_{22}; \quad \lambda_{23} = \theta(t) + c_{23}; \quad \lambda_{24} = -i\theta(t) + c_{24}; \\ &\lambda_{25} = \delta(t) + c_{25}; \quad \lambda_{26} = -i\delta(t) + c_{26}; \quad \lambda_{27} = \zeta(t) + c_{27}; \quad \lambda_{28} = i\zeta(t) + c_{28}. \end{aligned} \tag{65}$$

Solutions for $(\lambda_{29} - \lambda_{36})$ can be obtained respectively as, from eqs.(39-46), we obtain following equations

$$\dot{\lambda}_{31} - i\dot{\lambda}_{32} = -2(-i\alpha_3\lambda_7 + i\alpha_3\lambda_8 + i\alpha_2\lambda_{13} + \alpha_2\lambda_{14} - \alpha_2\lambda_{15} - i\alpha_2\lambda_{16}) = 0. \tag{66}$$

$$\dot{\lambda}_{29} + i\dot{\lambda}_{30} = 2(-\alpha_2\lambda_{10} + \alpha_2\lambda_{12} - \alpha_3\lambda_{13} + i\alpha_3\lambda_{15} - i\alpha_3\lambda_{14} + \alpha_3\lambda_{16}) = 0. \tag{67}$$

$$\dot{\lambda}_{33} + i\dot{\lambda}_{34} = 2i(-\alpha_2\lambda_{10} + \alpha_2\lambda_{12} - \alpha_3\lambda_{13} + i\alpha_3\lambda_{15} - i\alpha_3\lambda_{14} + \alpha_3\lambda_{16}) = 0. \tag{68}$$

$$\dot{\lambda}_{35} + i\dot{\lambda}_{36} = -2(-i\alpha_3\lambda_7 + i\alpha_3\lambda_8 + i\alpha_2\lambda_{13} + \alpha_2\lambda_{14} - \alpha_2\lambda_{15} - i\alpha_2\lambda_{16}) = 0. \tag{69}$$

Since

$$\lambda_7 = \lambda_8, \lambda_{10} = \lambda_{12}; \qquad \lambda_{13} = \lambda_{14} = \lambda_{15} = \lambda_{16} = \eta(t), \tag{70}$$

or if we set

$$\dot{\lambda}_{29} = -i\dot{\lambda}_{30} = \dot{\gamma}(t); \quad \dot{\lambda}_{31} = i\dot{\lambda}_{32} = \dot{\mu}(t); \quad \dot{\lambda}_{33} = -i\dot{\lambda}_{34} = \dot{\rho}(t); \quad \dot{\lambda}_{35} = -i\dot{\lambda}_{36} = \dot{\sigma}(t). \tag{71}$$

Which immediately gives

$$\begin{aligned} &\lambda_{29} = \gamma(t) + c_{29}; \quad \lambda_{30} = i\gamma(t) + c_{30}; \quad \lambda_{31} = \mu(t) + c_{31}; \quad \lambda_{32} = -i\mu(t) + c_{32}; \\ &\lambda_{33} = \rho(t) + c_{33}; \quad \lambda_{34} = i\rho(t) + c_{34}; \quad \lambda_{35} = \sigma(t) + c_{35}; \quad \lambda_{36} = i\sigma(t) + c_{36}. \end{aligned} \tag{72}$$

where the arbitrary function $\eta$'s, $\phi, \varphi, \chi, \psi, \xi, \theta, \delta, \zeta, \gamma, \mu, \rho$ and $\sigma$'s and integration constants, $c_i$'s, $(i = 1, ....36)$. are obtained through the equations, when we have solved eqs.[(11)-(46)]. Therefore, substitution of the solutions for $\lambda_i$'s yield the, complex invariant for a two dimensional complex oscillator systems as

$$\begin{aligned} I = &\frac{1}{2}\eta_1(p_1^2 + x_3^2) + \frac{1}{2}\eta_2(p_2^2 + x_4^2) + \frac{1}{2}\eta_4(x_1^2 + p_3^2) + \frac{1}{2}\eta_3(x_2^2 + p_4^2) + \eta(x_1x_3 + p_3p_4 + x_2p_3 + x_1p_4) \\ &+\phi p_1p_2 + \varphi p_1x_4 + \chi x_3p_2 + \quad x_3x_4 + \xi(p_1p_3 - ip_1x_1) + \theta(p_1x_2 - ip_1p_4) + \delta(p_3x_3 - ix_1x_3) \\ &+\zeta(ip_4x_3 + x_2x_3) + \gamma(ip_2p_4 + p_2x_2) + \mu(p_3p_2 - ix_1p_2) + \rho(ip_4x_4 + x_2x_4) + \sigma(ip_3x_4 + x_1x_4). \end{aligned} \tag{73}$$

is our desired invariant.

## II. Conclusion

We have shown in this paper, the coupled harmonic oscillator system with two space and one time variable share the same mathematical framework as the coupled harmonic oscillators in one dimension ECPS. The role of a linear invariant designed, however, for a rotating TD harmonic oscillator in $N$-dimensions is investigated by Malkin and Man'ko [16] in the context of coherent states. While the use of the quantum analogue of such TD systems in one dimension has been known [17, 18] for more than three decades.

Notes

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# Sensitivity Analysis for the Physiographic Parameters of Overland as well as Cannel Flow Elements Based on Kinematic Wave (Kw) Theory

By Dr. M. M. Hossain, J. Ferdous & M. Motateb Hossain
*University of Dhaka, Bangladesh*

*Abstract* - In the recent past, in many developing countries a good deal of research has been carried to solve the problems of 'large basins' whereas not much has been done with regard to the hydrologic problems of small watersheds. Small watersheds to play important roles e.g. a village pond catered by its own small watershed; in hilly watersheds, the generated runoff causes flash flood, resulting into disruptions of communication lines etc. Therefore, it is necessary to look into these aspects of the hydrologic problems with greater attention. The hydrologic responses of a small watershed depend upon the mechanics of surface runoff, which is primarily a nonlinear process. We have discussed the sensitivity of physiographic parameters through lumped kinematics wave models and found that the overland roughness and overland slope are more sensitive than other physiographic parameters of overland and channel flows. Appropriate discussions of results and conclusions as arrived at in different parts have been summarized.

*Keywords : kinematic wave, overland flow, channel flow, lumped physiographic models, watershed, sensitivity analysis.*

GJSFR-F Classification : *MSC 2010: 53A17*

SENSITIVITY ANALYSIS FOR THE PHYSIOGRAPHIC PARAMETERS OF OVERLAND AS WELL AS CANNEL FLOW ELEMENTS BASED ON KINEMATIC WAVE KWTHEORY

*Strictly as per the compliance and regulations of :*

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# Sensitivity Analysis for the Physiographic Parameters of Overland as well as Cannel Flow Elements Based on Kinematic Wave (Kw) Theory

Dr. M. M. Hossain[α], J. Ferdous [σ] & M. Motateb Hossain [ρ]

*Abstract -* In the recent past, in many developing countries a good deal of research has been carried to solve the problems of ‘large basins’ whereas not much has been done with regard to the hydrologic problems of small watersheds. Small watersheds to play important roles e.g. a village pond catered by its own small watershed; in hilly watersheds, the generated runoff causes flash flood, resulting into disruptions of communication lines etc. Therefore, it is necessary to look into these aspects of the hydrologic problems with greater attention. The hydrologic responses of a small watershed depend upon the mechanics of surface runoff, which is primarily a nonlinear process. We have discussed the sensitivity of physiographic parameters through lumped kinematics wave models and found that the overland roughness and overland slope are more sensitive than other physiographic parameters of overland and channel flows. Appropriate discussions of results and conclusions as arrived at in different parts have been summarized.

*Keywords : kinematic wave, overland flow, channel flow, lumped physiographic models,watershed, sensitivity analysis.*

## I. Introduction

Water is one of the most important natural resources vitally needed for sustenance of life. The use of water is continuously increasing with the increase in population. This increased demand for water has forced the planners, scientists and hydrologist not only to concentrate on major ‘basin planning’ but also to pay sufficient attention to small watersheds. The watershed planning may help in micro-level water budgeting to meet the localized demands of water for small population concentrations scattered over the entire basin. In Bangladesh, in majority cases, each village has its own pond (or tank) from which the water is drawn for domestic and other needs. These ponds have their own watersheds from which the rainwater is fed to them. Prior to onset of monsoon, after the sustained dry period most of the ponds either get dries up or are left with insignificant quantities of water. The fine silt or clay, which is accumulated at the bottom of the tank, is removed by the local people for plastering of huts and other general repairs of the houses. Thus, the lost capacity of the pond due to accumulation of silt is recouped. Though is may appear quite insignificant, yet the role of small watersheds is very important is most of the developing tropical counties is general and in the Indian subcontinent is particular. The micro-climatic influences onto the small watersheds are of

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particular significance. Response of a watershed to small, medium heavy and very heavy rainfall leasing to near cloud burst conditions is of importance not only for meeting the water demands but also for the protection of life and property. In hilly and mountainous catchments, the situations may get worse. Heavy intensity rains and resulting runoffs may cause flash floods and landslides and thus disrupt the communication lines of vital importance. These disruptions may result into major economic problems as one region gets completely cutoff from rest of the country.

To find suitable solutions to these problems, it is necessary that micro-level in-depth studies be carried out for computations of watershed responses to vastly varying meteorological conditions. There are different types of watershed models, which have been developed in the past to estimate the peak flows and runoff hydrographs for small watersheds. With the advent of powerful high-speed digital computers, it is now possible to study through mathematical models the basic physical processes viz. transformation of rain into the runoff land also the movement of water in the surface as well as in channels. These models rely on mathematical statements to represent the system.

In a small watershed, the generation of runoff can better be studied through the 'overland phase' and 'channel phase'. The conversion of rainfall excess into surface runoff and then the movement of flood flows in the channel is recognized as a nonlinear process. These days this process is generally modeled through two approaches, viz. the hydrodynamic approaches and the system's approach. For small watersheds the hydrodynamic approaches have been found more useful for the transformation of rainfall excess into the surface runoff (viz. in the overland phase) and the movement of flood flows in the channel (viz. in the channel phase).

### *a) Objective of the Study*

The objectives of the present work may thus be summarized as below:

(i) To develop suitable mathematical models based on KW theory, best suited to the typical problems of some small watersheds.

(ii) To confirm the applicability of finite difference schemes which are most suitable for small watersheds under study.

(iii) To carry out the sensitivity analysis for the physiographic parameters of overland as well as cannel flow elements and to identify the parameters which are most sensitive. Further, to find out the role of effective overland roughness in the context of natural high slope watersheds.

Computer can solve only a finite number of digits to represent each number at each step of a calculation. Hence, round-off errors are incorporated. The computation is stable if the growth of these errors is within reasonable limits or controlled.

A numerical model with consistent equations, convergent solutions and stable error propagation forms a computationally stable scheme and gives results, which are quite close to the exact solution.

### *b) Nature of Hydrologic System*

The nature of the hydrologic system is completely defined, through one property each, mentioned in the following three sets of possible system behaviors.

(1) Linear or nonlinear

(2) Lumped or distributed

(3) Time-invariant or time-variant.

A brief description of these three characteristics is given in the following subsections.

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i. *The Linear and Nonlinear Systems*

A hydrologic system is said to be linear, if a step input fed to the system produces an output, which is directly proportional to their input. A linear system may be represented mathematically by a linear equation. The principle of superposition applies to it may be defined mathematically as under:

$$f(\Psi_1)_t + f(\Psi_2)_t + --- + f(\Psi_n)_t = f(\Psi_1 + \Psi_2 + --- + \Psi_n)_t$$

where $\Psi_i$ is any response function and t is the time of its applicability.

The principle of homogeneity, which is a particular case of principle of superposition, thus may be described as below $f(k\Psi)_t = kf(\Psi)_t$ where k is a constant function.

A nonlinear system is represented by a nonlinear function. The extent of nonlinearity depends upon the system itself.

The hydrologic system may be defined by a general type of differential equation and can be written as follows:

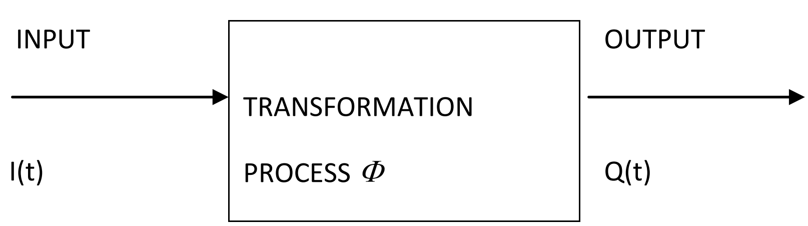
$$f(\Psi) = a_n \frac{d^n\Psi}{dt^n} + a_{n-1}\frac{d^{n-1}\Psi}{dt^{n-1}} + --- + a_1\frac{d^1\Psi}{dt^1} + a_0\Psi \,.$$

The system would be nonlinear if any on these coefficients $a_0, a_1, a_2$, - - -, $a_n$ etc. are the function of $\Psi$ or the function $\Psi$ carries an exponent other than unity.

ii. *The Lumped and Distributed System*

The hydrologic system is said to be lumped if its input functions or parameters do not vary with respect to spatial coordinates (Fig.2.3 (a)). For the lumped systems, average conditions or values of input and parameters are applicable. Thus, lumped systems are represented by ordinary differential equations.

The system is defined as distributed if the input or the transfer function and other parameters do vary with the spatial coordinates (Fig.2.3 (b)). Such systems are mathematically represented by the partial differential equations. The theoretical solution of such systems (a differential equation) this requires completes knowledge of the boundary conditions.




(a) LUMPED SYSTEM

$\Phi(x)$

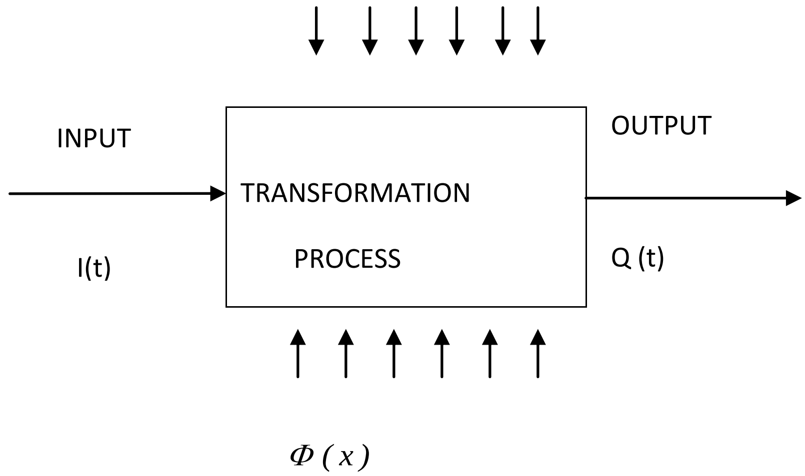



*Figure :* The Lumped and Distributed Systems

iii. *Time-Invariant and Time-Variant Systems*

A time-variant system is one in which the input-output relationship is not dependent upon the time at which the input is applied to the system. The concept of time-invariant makes the analysis simpler.

A time-variant system is one in which the input-output relationship is a dependent function of time. It may be concluded that a lumped, linear and time-invariant system is easiest to work with. But in surface hydrology, the hydrologic system happens to be distributed, nonlinear and time-variant in its behavior. This behavior of the system is the most complex one and quite difficult not only to formulate mathematically but also to solve. Therefore, a compromise has to be reached so that the complicated natural hydrologic system may be solved satisfactorily by making suitable assumptions with respect to the aspects stated above.

c) *Numerical Technique (Finite Difference Methods) for Solving the Kinematic Wave Equations*

The computational schemed will require the initial values for the entire domain $(X_i\,'s)$ and the upstream boundary conditions for all $(t_j$'s). Solutions of the governing equations through the finite difference scheme will be obtained at each of this grid point. Computations advance along the downstream direction for a time step $\Delta t$, until all the discharges as well as water areas are computed at all the grid points in the entire longitudinal length L.

Next, the computations are advance ahead in time by another time step $\Delta$t and the computations proceed likewise.

The rainfall excess intensity $(i_e)$ is assumed constant within a time step $\Delta t$. But it may change from one time step to the next time step, to account for the variation in rates of rainfall excess intensities occurring within a storm event.

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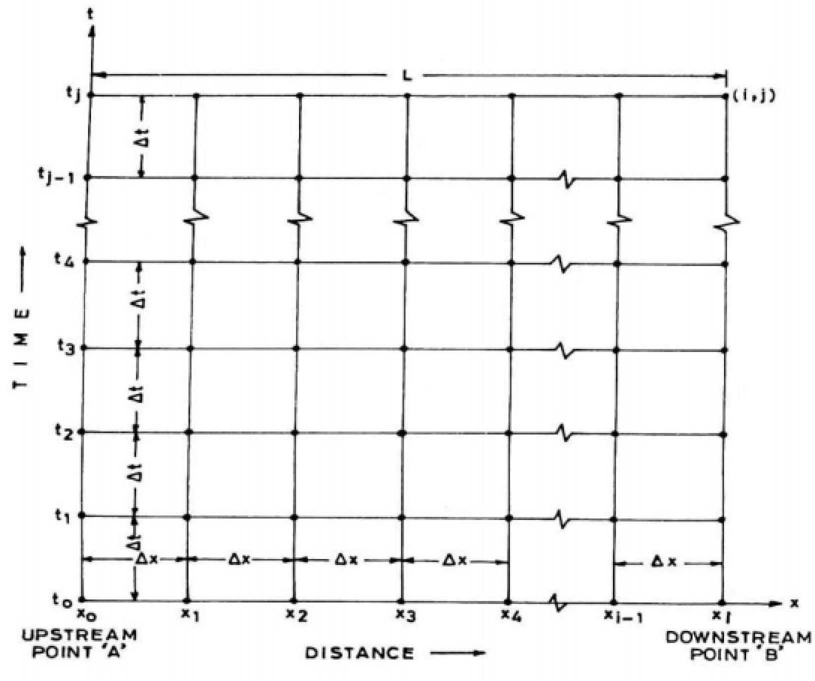


*Figure 3.3 :* Flow Domain Representation Through a Fixed X – t Grid

## *d) Physiographic Models Employed*

### i. *Lumped Physiographic Model*

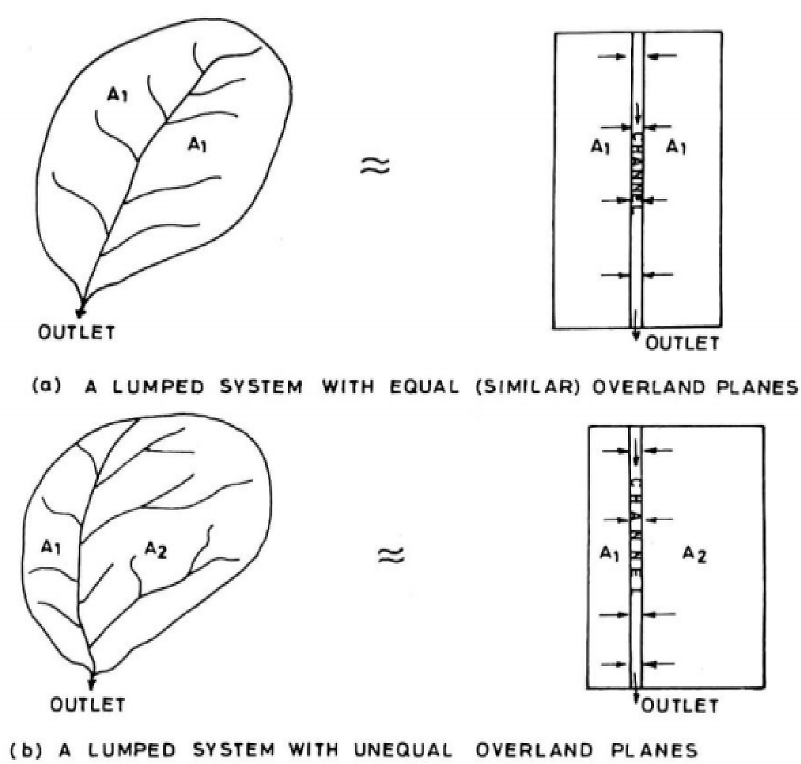


*Figure 3.5 :* Lumped Physiographic Models

*a. Initial Conditions*

$$\left.\begin{array}{l} Q(x,0)=0 \\ A(x,0)=0 \end{array}\right\} \text{for all x}$$

For the overland flows,

$$\left.\begin{array}{l} q(x,0)=0 \\ y_0(x,0)=0 \end{array}\right\} \text{for all x}$$

*b. Boundary Conditions*

For channel flow:

$$\left.\begin{array}{l} Q(0,t)=0 \\ A(0,t)=0 \end{array}\right\} \text{for all t}$$

And for the overland flow:

$$\left.\begin{array}{l} q(0,t)=0 \\ y_0(0,t)=0 \end{array}\right\} \text{for all t}$$

*e) Physiographic Details of Watershed of Railway Bridge No. 319.*

The small watershed of Railway Bridge No. 319 selected for this study is situated in Bangalore district of Karnataka of India. This bridge is located on Arsikere Bangalore Section of the Indian Railways. The index maps giving details of the watershed is given in Fig. Some important features, general information and the physiographic data as extracted from the available records and the topographic map of this bridge is given in Table.

*Table 1 :* General Information and Salient Features of Watershed of Railway Bridge No. 319

| Sl. No. | Particulars | Units |
|---|---|---|
| 1. | a. Name of the zone<br>b. Name of the sub zone | Deccan Plateau 3i |
| 2. | Geographical location | i. Longitude(app.) $77^0 10^{'}$ East<br>ii. Latitude (app.) $13^0 18^{'}$ North |
| 3. | Terrain | Hilly |
| 4. | Shape of the basin | Oblong |
| 5. | Climate | Humid |
| 6. | Type of soil | i. Rocky<br>ii. Red Earth |

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| | | |
|---|---|---|
| 7. | Land use | Partially Dry cultivation |
| 8. | No. of rain gauge station | 1 |
| 9. | No. of discharge location | 1 |
| 10. | Altitude (average elevation from MSL) | 833 meter |
| 11. | Watershed area | 82.0 hectare |
| 12. | Length of the main channel(longest) | 1650.0metre |
| 13. | Total average annual rainfall | 500-1000(mm) |

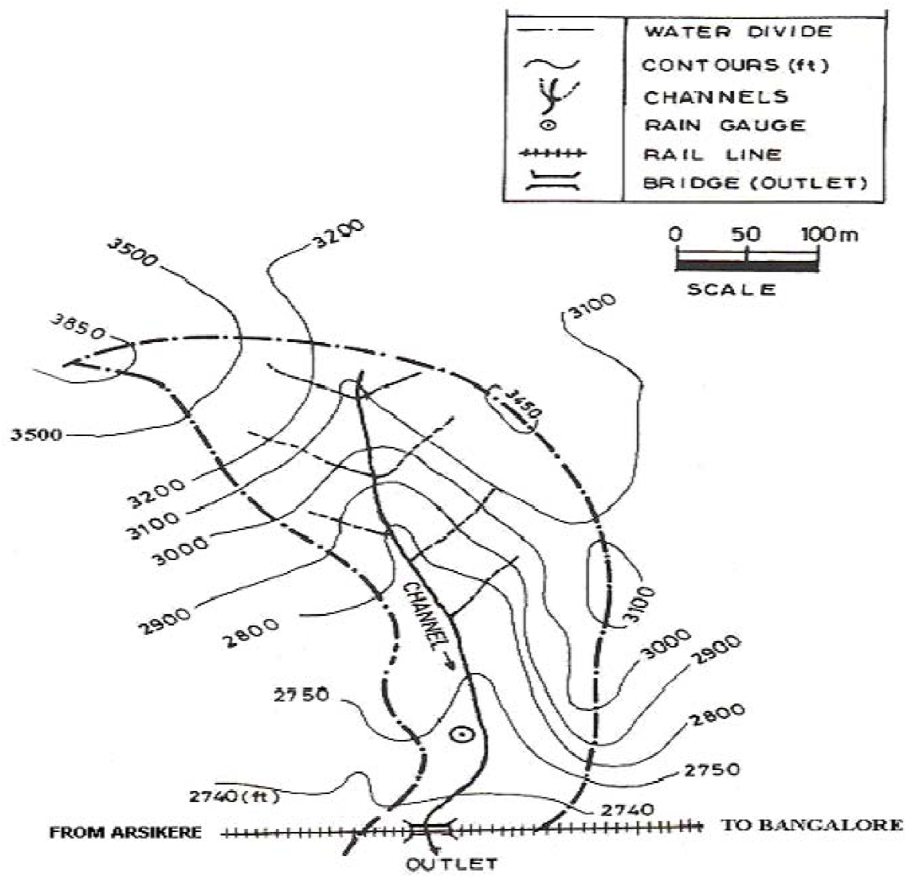


*Figure 4.1 :* The Index Map of Bridge no. 319

*f) Estimation of Physiographic Parameters for Bridge No. 319*

The topographic details of this watershed are shown in Fig. 4.1. In this natural watershed, only one main drainage channel exists that too in the central part of the watershed. As discussed the lumped physiographic model is used to compute the model parameters for the application of KW theory. For this purpose, a lumped model of the type given in Fig. (a) is adopted for the estimation of parameters. The equivalent watershed has been obtained by dividing equally the total drainage area into the two sides 1650 meters long main channel. The schematic representation of this model is shown in Fig.

As discussed in Section, the physiographic parameters were computed. The values of physiographic parameters are given in Table.

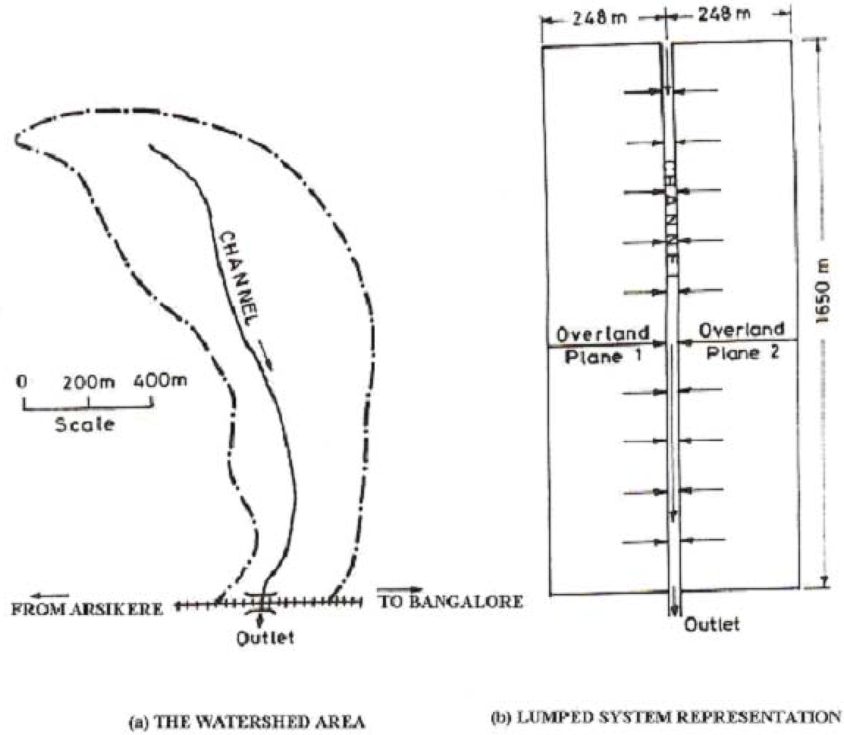


*Figure 4.2 :* Schmatic Representation Through a Lumped Model (Bridge No. 319)

*Table 2 :* The Lumped Physiographic Parameter Values of Bridge No. 319

| Sl.No. | Particular | Unit |
|---|---|---|
| 1. | Area | 82.0 hectares |
| 2. | Overland (Plane):<br>(a). Average length (each side)<br>(b). Average slope (each side)<br>(c). Overland roughness | <br>248.0 meters<br>0.092<br>0.052 |
| 3. | Channel:<br>(a). Average length<br>(b). Average slope<br>(c). Average roughness<br>(d). Average bed width<br>(e). Average side slope | <br>1650.0 meters<br>0.072<br>0.035<br>3.0 meters<br>2.5 H: IV |

*g) Overland Flow of Watershed of Bridge No. 319 for Different Values of Overland Roughness (An)*

In overland there are two physiographic parameters land roughness and slope. The effect of overland roughness on overland flow of watershed of bridge no. 319 is shown in the following Fig.



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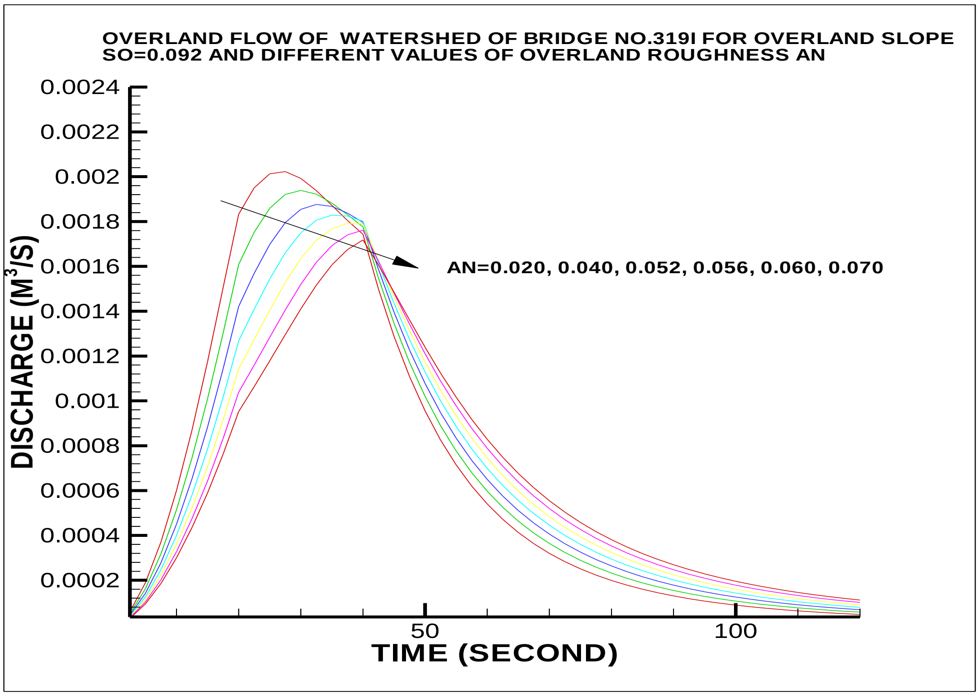


*Figure :* Overland Flow for Different Values of Overland Rougheness

From the above figure, we observed that if the overland roughness is increasing then peak flow is decreasing. Therefore, the overland roughness (AN) is sensitive.

*h) Overland Flow of Watershed of Bridge No. 319 for Different Values of Overland Slop (So)*

Different overland has different overland slope. It has an effect on overland flow. The effect of overland slope on overland flow of watershed of bridge no. 319 is shown in the following Fig. 4.4

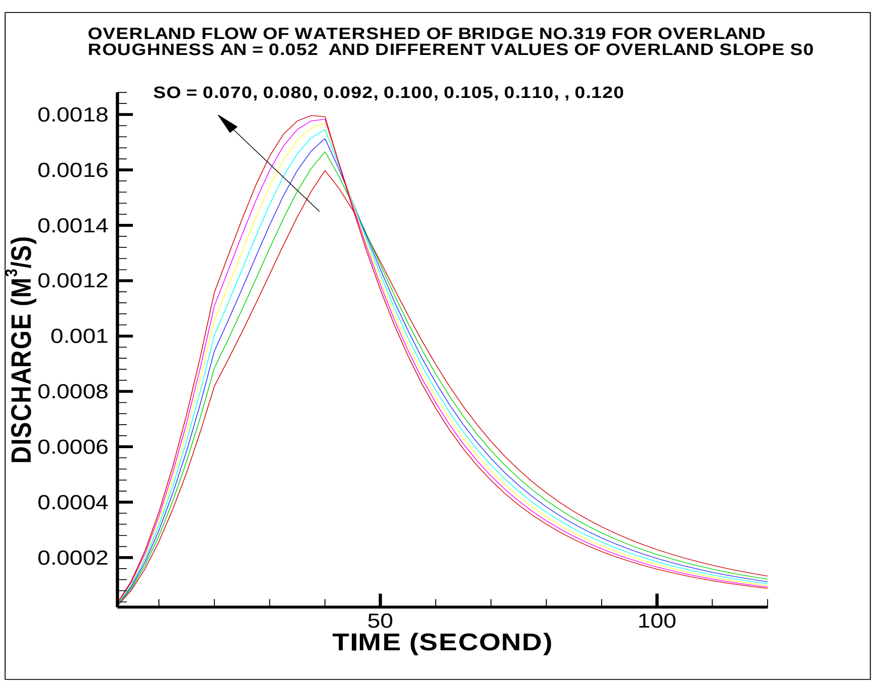


*Figure :* Overland Flow for Different Values of Overland Slope

From the above figure, we observe that if the overland slope is increasing then the peak flow is increasing for high slope. Therefore, the overland slope is sensitive.

*i) Channel Flow Routing*

A channel is a conduit in which water flows with a free surface. Classified according to its origin a cannel may be either natural or artificial. Natural channels include all watercourse that exist naturally on the earth, varying is size from tiny hillside rivulets through brooks, streams, small and large rivers, to tidal estuaries. Underground streams carrying water with a free surface are also considered natural channels.

*j) Channel Flow Routing of Watershed of Bridge No. 319 for Different Value of Overland Roughness (An)*

Flow of watershed is made of overland and channel flow. Overland roughness is a important factor for channel flow. The effect of overland roughness on channel flow of watershed of bridge no. 319 is shown in the Fig. 4.5

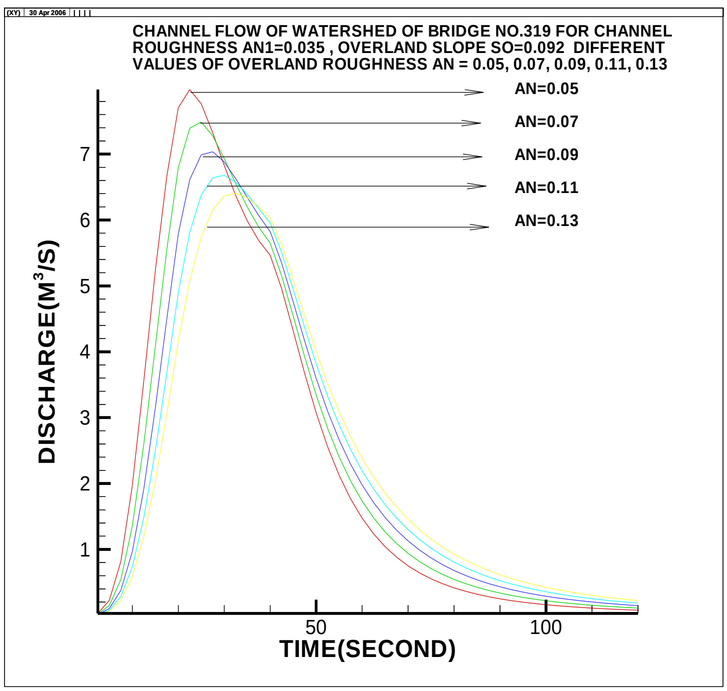


*Figure :* Channel Flow Routing for Different Value of Overland Roughness (An)

From above Fig, we observed that if overland roughness is increasing then the peak flow is decreasing and the flow routing changes is shapes quickly. So the overland roughness on channel flow is sensitive.

*k) Channel Flow Routing of Watershed of Bridge No. 319 for Different Value of Channel Roughness (An1)*

Different channel has different channel roughness. It is a factor of channel flow. The effect of channel roughness on channel flow of watershed of bridge no.319 is shown in the following Fig.4.6.

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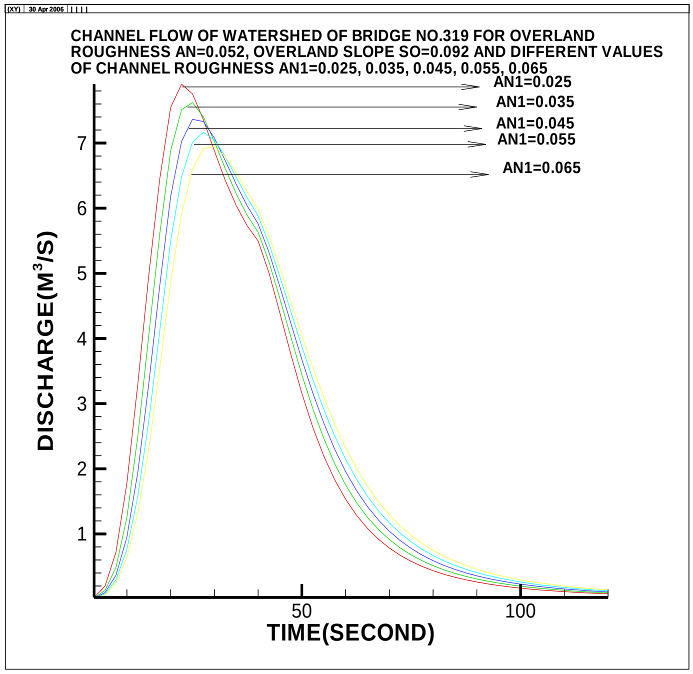


*Figure :* Channel Flow Routing for Different Value of Channel Roughness (An1)

From the above figure we easily observed that the effect of channel roughness on channel flow peak is sensitive but is not like overland roughness. So channel roughness is very sensitive but not like as overland roughness on channel flow.

1) *Channel Flow Routing of Watershed of Bridge No.319 for Different Value of Side Slope (Bz)*

Channel flow depends on different physiographic parameters of overland and channel. Here the effect of side slope (BZ) on channel flow of watershed of bridge no. 319 is shown in Fig.

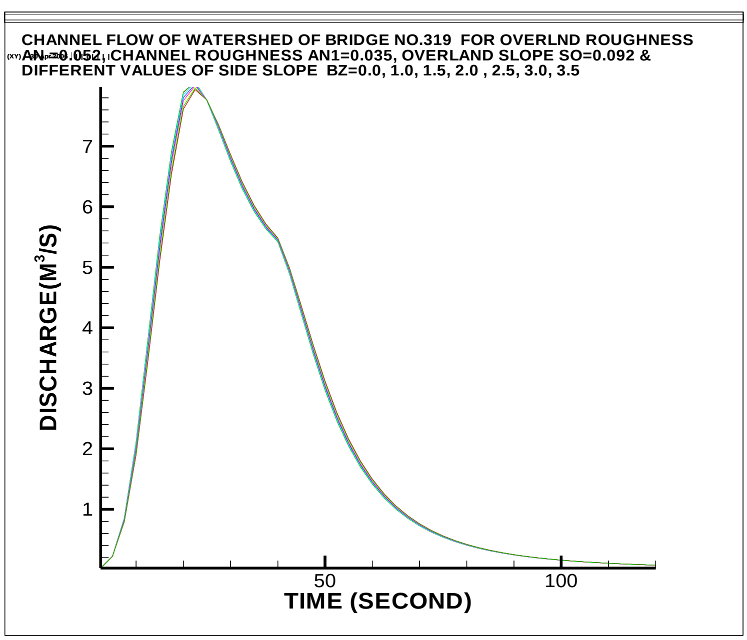


*Figure :* Channel Flow Routing for Different Value of Side Slope (Bz)

From the above figure we observed that if side slope is increasing then peak flow is decreasing very slowly. Therefore, the effect of side slope on the flow routing of channel flow is not important factor and hence side slope is not sensitive so much.

*m) Channel Flow Routing of Watershed of Bridge No. 319 for Different Value of Overland Slope (So)*

Channel flow is constituted of direct rainfall and overland flow. Channel flows depend on different physiographic parameters of overland and channel. Here the effect of overland slope (SO) on channel flow of watershed of bridge no. 319 is shown in the following Fig.

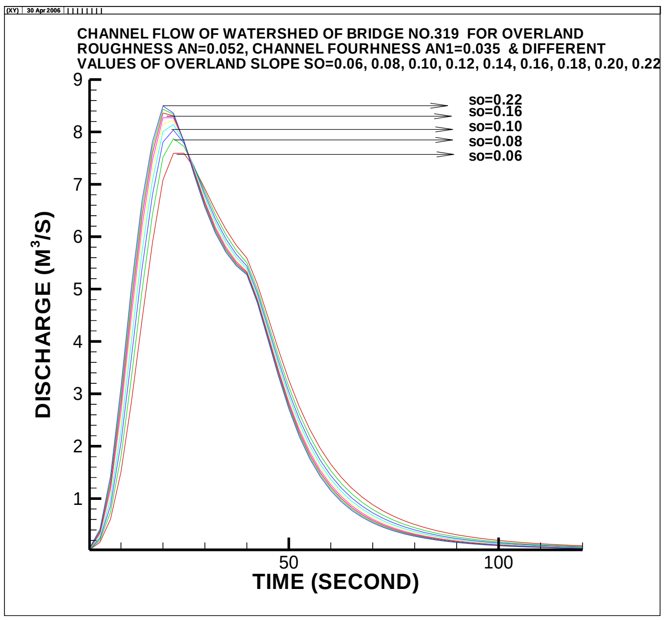


*Figure :* Channel Flow Routing for Different Value of Overland Slope

From the above figure, we observe that the effect of overland slope (SO) on the peak flow routing of channel flow if SO is increasing then flow routing is increasing, but after a period of flow, the flow is not increasing. So the overland slope is not so sensitive for channel flow.

## II. Conclusion

From the above discussion we come in conclusion that the overland roughness and overland slope and channel roughness are more sensitive than the other parameters on overland and channel flow.

## III. Scope of Furthur Study

The present study can be extended as follows:

(i) Distributed method can be used for channel flow and overland flow routing.

(ii) Three fully off-center first order explicit finite difference methods have been used for the solution of proposed KW models. A single step, second explicit method can also be used for above-mentioned models. Also, implicit finite difference methods and the finite element methods may also be used for this purpose.

(iii) In the proposed models, the contributing surface plane elements have assumed rectangular and perpendicular to the elements. The shape and the alignment of

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these planes can be oriented according to the prevailing overland slopes. An altogether, different approach will have to be developed for such cases of modeling.

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# Optimal Discounted Portfolio, Expected Wealth and Strategic Consumption for a Defined Contribution Pension Scheme

By C. I. Nkeki
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*Abstract -* This paper deals with optimal discounted portfolio, expected wealth and strategic consumption process for a defined contribution (DC) pension scheme. The aims of this paper are to find: the optimal discounted portfolio and optimal discounted consumption choice to be adopted by the pension plan member (PPM) up to retirement period; the expected discounted wealth and variance of the expected wealth for the plan member. The financial market is composed by a riskless and a risky assets, and the effective salary of the plan member is assume to be stochastic. The expected discounted wealth and its variance are obtained. The discounted portfolio and consumption processes of the plan member are obtained. It is find that part of the discounted portfolio value is proportional to the ratio of the present value of the discounted expected future contributions to the optimal discounted wealth value overtime. It is also find that there is the need for gradual transfer of part of the portfolio value in risky asset to the riskless one against unforeseen shocks.

*Keywords : Strategic discounted consumption, optimal discounted portfolio, pension plan member, unfore-seen shocks, stochastic salary, defined contribution, pension scheme.*

GJSFR-F Classification : *MSC 2010: 91G10*

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# Optimal Discounted Portfolio, Expected Wealth and Strategic Consumption for a Defined Contribution Pension Scheme

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*Abstract* - This paper deals with optimal discounted portfolio, expected wealth and strategic consumption process for a defined contribution (DC) pension scheme. The aims of this paper are to find: the optimal discounted portfolio and optimal discounted consumption choice to be adopted by the pension plan member (PPM) up to retirement period; the expected discounted wealth and variance of the expected wealth for the plan member. The financial market is composed by a riskless and a risky assets, and the effective salary of the plan member is assume to be stochastic. The expected discounted wealth and its variance are obtained. The discounted portfolio and consumption processes of the plan member are obtained. It is find that part of the discounted portfolio value is proportional to the ratio of the present value of the discounted expected future contributions to the optimal discounted wealth value overtime. It is also find that there is the need for gradual transfer of part of the portfolio value in risky asset to the riskless one against unforeseen shocks.

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## I. Introduction

We consider optimal discounted portfolio and discounted consumption problem in a defined contributory pension scheme. This paper follows the work of [14] and [16]. In their paper, they considered the optimal portfolio and strategic consumption in the lifecycle of a PPM in pension scheme. But, the area of discounted portfolio, discounted consumption and the variance of expected discounted wealth of a PPM are yet to be considered to the best of my knowledge in the literature of financial mathematics and actuarial science. In this paper, we assume that the PPM consume continuously throughout his or her life time and consumption terminates when the individual dies.

In the literature, [9] considered a numerical solution as well as analytical results to the intertemporal consumption problem for portfolio management. [17] examined a tractable model of precautionary savings in continuous time and assumed that the uncertainty was about the timing of the income loss in addition to the assumption of non-stochastic asset return. [3] considered labour supply flexibility and portfolio choice of individual life cycle. They considered the objective of maximizing the expected discounted lifetime utility by assumed that the utility function has two argument (consumption and labour/leiture). [2] adopted the quadratic utility function that characterized a linear marginal utility function. They asserted that the utility function was not attractive in describing the behaviour of individual towards risk as it implies increasing absolute risk

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aversion. [3] concluded that labour income induced the individual to invest an additional amount of wealth to the risky asset. They established that labour income and investment choices are related, but they failed to analyzed the optimal consumption process of the investor. [4] studied the generally the optimal management of a defined contribution pension plan where the guarantee depends on the level of interest rates at a fixed retirement date. [5] considered optimal dynamic asset allocation strategy for a DC pension plan by taking into account the stochastic features of the plan member's lifetime salary progression and the stochastic properties of the assets held in accumulating pension fund. They emphasised that salary risk (the fluctuation in the plan member's earning in response to economic shocks) is not fully hedgeable using existing financial assets. They further emphasized that wage-indexed bonds could be used to hedge productivity and inflation shocks. They further asserted that such bonds are not widely traded. They referred the optimal dynamic asset allocation strategy stochastic lifestyling. They compared it against various static and deterministic lifestyle strategies in order to calculate the costs of adopting suboptimal strategies. Their solution technique made use of the present value of future contribution premiums into the plan. This technique can be found in [1], [4], [8]. Deterministic lifestyling which is the gradual switch from equities to bonds according to present rules is a popular asset alloction strategy during the accumulation phase of DC pension plans and is designed to protect the pension fund from a catastrophic fall in the stock market just prior to retirement (see [1], [5], [6]). [5] and [9] analysed extensively the occupational DC pension funds, where the contribution rate is a fixed percentage of salary.

The classical dynamic lifetime portfolio selection in a continuous time model was developed by [10], [11]. [9] used a dynamic programming approach to derived a formula for optimal investment allocation in a DC scheme and compared three risk measures to analyzed the terminal net replacement ratio achieved by members. They suggested that when the choice of investment strategy is determined, risk profiles of individual and different risk measures are both important factors which should be taken into consideration.

In this paper, we aim at finding the optimal discounted portfolio and optimal discounted consumption choice to be adopted by the pesnion plan member (PPM) up to retirement period and the entire lifetime; and the expected discounted wealth and variance of the expected discounted wealth for the plan member.

The structure of the remainder parts of the paper is as follows. Section 2 presents the formulation of the problem which include the financial models, wealth process and stochastic salary of a PPM. In section 3, we present the discounted wealth and discounted consumption process of a PPM. Section 4 presents the present value of discounted future contribution of a PPM. In section 5, we presents the valuation of the discounted wealth process of a PPM. Section 6 present the optimal discounted portfolio and consumption process of a PPM. In section 7, we present the optimal expected discounted wealth valuation and discounted consumption process of a PPM. Section 8 presents the accumulated expected discounted consumption process of a PPM. Finally, section 9 concludes the paper.

## II. Problem Formulation

We consider a continuous-time financial market where there are two investment instruments: a riskless and a risky assets. The price dynamics of the two assets are given, respectively, by

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$$dB(t) = rB(t)dt,\ B(0) = 1, \tag{1}$$

$$dS(t) = S(t)\big(\mu dt + \sigma dW(t)\big),\ S(0) = s_0, \tag{2}$$

$t \in \Re$. We allow $r > 0, \mu > r$, to be constants. $\mu$ is the predictable 1-dimensional process of excess appreciation rate in relation to the stock, $\sigma = (\sigma_1, \sigma_2)$ is a predictable process of volatility vector of stock, $\sigma_1$ is volatility of stock arising from inflation and $\sigma_2$ is the volatility stock arising from the stock market. $\big(W(t) = (W_1(t), W_2(t))'; t \in \Re_+\big)$ is a standard 2- dimensional Brownian motion on a filtered probability space $\left(\Omega, \Im, \{\Im_t\}_{t\in[0,T]}, P\right)$ with $W(0)$ a null vector almost surely. $W_1(t)$ is the source of risk of inflation and $W_2(t)$ is the source of risk of the stock market. We assume that the filtration $\{\Im_t\}_{t\in[0,T]}$ is generated by the Brownian motion and is right continuous, and that each $\Im_t$ contains all the $P-$ null sets of $\Im$ We denote by $L^2_{\Im}$ the set of square integrable $\{\Im_t\}_{t\in[0,T]}-$ adapted processes,

$$L^2_{\Im} = \left\{ \begin{array}{l} \Delta \,|\, The\ \ process\ \ \ \Delta = \{\Delta(t)\}_{t\in[0,T]} \\ adapted\ \ \ process\ \ such\ \ that \\ \int_0^T E\left[\Delta^2(t)\right]dt < \infty \end{array} \right\}$$

and by $L^2_{\Im_T}$ the set of square integrable $\Im_T -$ measurable random variables,

$$L^2_{\Im_T} = \left\{ \Delta \left| \begin{array}{l} \Delta\ \ is\ \ an\ \ F_T - measurable\ \ \ random \\ \text{var}\,iable\ \ such\ \ that\ \ E\left[\Delta^2\right] < \infty \end{array} \right. \right\}$$

.From (1), we have

$$B(t) = \exp(rt\,),\ B(0) = 1; \tag{3}$$

We assume that the financial market is arbitrage-free, complete and continuously open between time 0 and $T$, i.e., there is only one process $\theta$ satisfying

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix},$$

where, $\theta_1$ is the market price of inflation risk and and

$$\theta_2 = \frac{\mu - r - \sigma_1\theta_1}{\sigma_2},$$

is the market price of stock risk.

The exponential process

$$Z(t) = \exp\left[-\theta' W(t) - \frac{1}{2}\|\theta\|^2 t\right], \tag{4}$$

is assumed to be a martingale

From (1) and (4), we have the discouted factor to be

$$\Lambda(t) = B(t)^{-1} Z(t) \tag{5}$$

Taking the differential (5), we have

$$d\Lambda(t) = -\Lambda(t)\big(rdt + \theta' dW(t)\big) \tag{6}$$

The dynamics of the PPM effective salary is given by

$$dY(t) = Y(t)\big(\omega dt + \sigma^{I} dW(t)\big), \quad Y(0) = y > 0, \tag{7}$$

where $Y(t)$ is the salary of the PPM at time $t$, $\omega$ is the expected growth rate of salary of PPM and $\sigma^{I} = (\sigma_{Y1}, \sigma_{Y2})$ is the volatility vector of salary which is driven by the source of uncertainty of inflation, $W_1(t)$ and the stock market, $W_2(t)$. The two sources of risk are partial correlated. We assume that $\omega > 0$ is a constant and and $\sigma^{I}$ is a constant vector.

## III. Discounted Wealth and Discounted Consumption Process of a Ppm

In this section, we consider the discounted wealth process of the PPM in pension scheme.

Let $\Delta(t)$ portfolio process invested in risky asset at time $t$ and $C(t)$ the consumption process at time $t$, then the pair $(\Delta, C)$ is said to be self-financing if the corresponding wealth process $X^{\Delta,C}(t), t \in [0,T]$, satisfies

$$dX^{\Delta,C}(t) = \Delta(t) X^{\Delta,C}(t)\frac{dS(t)}{S(t)} + \big(1-\Delta(t)\big)X^{\Delta,C}(t)\frac{dB(t)}{B(t)} + \big(cY(t) - C(t)\big)dt, \tag{8}$$

where $\Delta_0 = 1 - \Delta(t)$ represents the proportion of the portfolio invested in cash account at time $t$.

Substituting the assets returns in (1) and (2) into (8), we obtain the following

$$dX^{\Delta,C}(t) = (X^{\Delta,C}(t)\big(r + \Delta(t)(\mu - r)\big) + cY(t) - C(t))dt + \sigma\Delta(t)X^{\Delta,C}(t)dW(t) \tag{9}$$

This is our stochastic differential equation which represents the wealth process of the PPM at time $t$.

Using (6) and (9), we have the discounted wealth process to be

$$d\big(\Lambda(t)X^{\Delta,C}(t)\big) = \big(c\Lambda(t)Y(t) - \Lambda(t)C(t)\big)dt + \Lambda(t)X^{\Delta,C}(t)(\Delta(t)\sigma' - \theta)' dW(t) \tag{10}$$

## IV. Present Value of Discounted Future Contribution

In this section, we present the present value of pension plan member future contribution in pension scheme.

Definition 1: The present value of the expected discounted future contributions process is defined as

$$\tilde{\Phi}(t) = E_t\left[\int_t^T \frac{\Lambda(s)}{\Lambda(t)} cY(s)ds\right], \tag{11}$$

Notes

where $c > 0$ is the proportion of the salary contributed into the pension funds and $E_t$ is the conditional expectation with respect to the Brownian filtration $\{\Im(t)\}_{t\geq 0}$

$$\Lambda(t) \equiv Z(t)\exp[-rt], 0 \leq t \leq T. \tag{12}$$

is the stochastic discount factor which adjusts for nominal interest rate and market price of risk.

Notes

Definition 2: The expected discounted future consumption process is defined by

$$\Psi(t) = E\left[\int_t^\infty \frac{\Lambda(u)}{\Lambda(t)} C(u)du\right], t \in [T, \infty). \tag{13}$$

We take limits of integration in Eq. (13) from $t = T$ to $t = \infty$ because, we assume that consumption starts when the PPM retired and consume his/her investment (benefits) till he/she is dead.

Theorem 1: Let $\widetilde{\Phi}(t)$ be the present value of expected future contributions into the pension funds, then

$$\widetilde{\Phi}(t) = \frac{cY(t)}{\varphi}\left(\exp[\varphi(T-t)] - 1\right) \tag{14}$$

where $\varphi = \omega - r - \sigma^I \theta$

Proof: (see Nwozo and Nkeki (2011)).

Find the differential of bothsides of Eq. (14), we obtain

$$d\widetilde{\Phi}(t) = \widetilde{\Phi}(t)\left[\left(r + \sigma^I \theta\right)dt + \sigma^I dW(t)\right] - cY(t)dt \tag{15}$$

Using (6) and (15), we have

$$d\left(\Lambda(t)\widetilde{\Phi}(t)\right) = -c\Lambda(t)Y(t)dt + \Lambda(t)\widetilde{\Phi}(t)(\sigma^I{}' - \theta)' dW(t) \tag{16}$$

## V. Valuation of the Discounted Wealth Process of a PPM

Definition 3: The value of wealth of the PPM is define as

$$V(t) = X(t) + \widetilde{\Phi}(t). \tag{17}$$

So that the discounted value of wealth becomes

$$\Lambda(t)V(t) = \Lambda(t)X(t) + \Lambda(t)\widetilde{\Phi}(t).$$

Then, dynamics of the value of the discounted wealth of the PPM is obtain as

$$d\left(\Lambda(t)V(t)\right) = d\left(\Lambda(t)X(t)\right) + d\left(\Lambda(t)\widetilde{\Phi}(t)\right). \tag{18}$$

Substituting in (10) and (16) into (18), we obtain the change in wealth of the PPM as follows:

$$d\left(\Lambda(t)V(t)\right) = -\Lambda(t)C(t)dt + (\Lambda(t)X^{\Delta,C}(t)\left(\Delta(t)\sigma' - \theta\right) + \Lambda(t)\widetilde{\Phi}(t)\left(\sigma^I{}' - \theta\right))' dW(t) \tag{19}$$

## VI. Optimal Portfolio and Consumption Process of a Ppm

In this section, we derived the optimal discounted portfolio and discounted consumption process of a PPM under dynamic programming principle. We assume that the PPM chooses power utility function in maximizing the expected utility of the terminal discounted wealth and consumption process. The choice of the power utility (which is a linear combination of two power utility functions. The first term is with respect to the wealth process while the second is with respect the consumption process) is motivated by the fact that pension scheme are in general large investment conpanies whose strategic plans are with respect to the size of funds they are managing.

We now give the optimal discounted portfolio and discounted consumption process for pension funds planners at time $t$.

The PPM's problem is to choose an admissible strategy so as to maximize the expected utility of accumulative consumptions and terminal discounted wealth,

$$U(t,V) = \sup_{\substack{\Delta\in\prod( \ ,\Phi) \\ C\in\Re}} E_t^{x,\Phi} J(t,x,\Phi,\widetilde{C})$$

where

$$J(t, \ ,\Phi,\widetilde{C}) = \int_t^T e^{-\rho t} U(u,\widetilde{C}(u))du + U(V(T)),$$

We assume that $U$ is concave and

$U((V,t) \in C^{1,2}(\Re\times[0,T])$, $\rho$ is the PPM's preference discount factor, $\Pi(x,\Phi)$ set of admissible portfolio strategy that are $F_V$ - progressively measureable, that satisfy the integrability condition

$$E\left[\int_0^T \Delta(s)^2 ds\right] < \infty,\ E\left[\int_0^T \widetilde{C}(s)^2 ds\right] < \infty,$$

$E_0^{x,\Phi}$ denotes the conditional expectation at time $t$ given the initial endowment,

$$\Lambda(t)X^{\Delta,C}(t) = x,\ \Lambda(t)\widetilde{\Phi}(t) = \Phi,$$

$$\Lambda(t)C(t) = \widetilde{C}(t),$$

and the utility function is taken as

$$U(V) = \frac{V^\gamma}{\gamma}, 0<\gamma<1.$$

It turns out that $U(t,V)$ satisfies the following Hamilton–Jacobi–Bellman equation

$$\begin{aligned} &U_t + \frac{1}{2}\ ^2(\Delta(t)\sigma'-\theta)'(\Delta(t)\sigma'-\theta)U_{xx} + \frac{1}{2}\Phi^2(\sigma^I{}'-\theta)'(\sigma^I{}'-\theta)U_{\Phi\Phi} \\ &\Phi(\sigma^I{}'-\theta)'(\Delta(t)\sigma'-\theta)U_{x\Phi} - \widetilde{C}(t)U_x + U(\widetilde{C}(t))\exp(-\rho t) = 0. \end{aligned} \tag{20}$$

Notes

This yields the HJB equation for the value function

$$\left(U_t V\right),t \;+\mathrm{H}(t,\Delta,C)=0,$$

Subject to:

$$U(V)=\frac{(x+\Phi)^{\gamma}}{\gamma}-\frac{\widetilde{C}(t)^{\gamma}}{\gamma},0<\gamma<1. \tag{21}$$

where

$$\mathrm{H}(t,\Delta,\widetilde{C})=\frac{1}{2}x^2\left(\Delta(t)\sigma'-\theta\right)'\left(\Delta(t)\sigma'-\theta\right)U_{xx}$$

$$+\frac{1}{2}\Phi^2\left(\sigma^{I}{}'-\theta\right)'\left(\sigma^{I}{}'-\theta\right)U_{\Phi\Phi}$$

$$x\Phi\left(\sigma^{I}{}'-\theta\right)'\left(\Delta(t)\sigma'-\theta\right)U_{x\Phi}$$

$$-\widetilde{C}(t)U_x+U(\widetilde{C}(t))\exp(-\rho t)$$

Hence, we have the following optimal discounted portfolio and optimal discounted consumption, respectively as follows:

$$\Delta^*(t)=\frac{\sigma^{-1}\theta}{x}-\frac{\Phi\sigma^{-1}\left(\sigma^{I}{}'-\theta\right)U_{x\Phi}}{xU_{xx}}, \tag{22}$$

$$\widetilde{C}*(t)=I\left(U_x\,\exp[\rho t]\right) \tag{29}$$

$$\text{where, } I=\left(\frac{dU(\widetilde{C})}{d\widetilde{C}}\right)^{-1}.$$

Substituting Eq. (22), and (29) into Eq.(20), we have

$$U_t+\frac{1}{2}\theta'\theta(1-x^2)U_{xx}-$$

$$\frac{1}{2}\Phi^2(\sigma^{I}{}'-\theta)'(\sigma^{I}{}'-\theta)\frac{U_{x\Phi}^2}{U_{xx}}$$

$$+\frac{1}{2}\Phi^2(\sigma^{I'}-\theta)'(\sigma^{I'}-\theta)U_{\Phi\Phi}$$

$$-I\left(U_x\,\exp(\rho t)\right)U_x+$$

$$\exp(-\rho t)U\left(I\left(U_x\,\exp(\rho t)\right)\right)=0, \tag{36}$$

We assume that in this paper that

$$U(t,V)=\frac{(x+\Phi)^{\gamma}Q(t)^{\gamma}}{\gamma} \qquad (23)$$

and

$$U(t,\widetilde{C})=\frac{\left(\widetilde{C}(t)Q(t)\right)^{\gamma}}{\gamma}.$$

Finding the partial derivative of (23) with respect to $t, x, xx, x\Phi, \Phi\Phi$, we have the following:

$$U_t=(x+\Phi)^{\gamma}Q(t)^{\gamma-1}Q'(t)$$

$$U_x=(x+\Phi)^{\gamma-1}Q(t)^{\gamma}$$

$$U_{\Phi}=(x+\Phi)^{\gamma-1}Q(t)^{\gamma}$$

$$U_{xx}=(\gamma-1)(x+\Phi)^{\gamma-2}Q(t)^{\gamma}$$

$$U_{x\Phi}=(\gamma-1)(x+\Phi)^{\gamma-2}Q(t)^{\gamma}$$

$$U_{\Phi\Phi}=(\gamma-1)(x+\Phi)^{\gamma-2}Q(t)^{\gamma}$$

We observe that the assumption of concavity of $U$ turns out to be true, as

$$U_{xx}=U_{\Phi\Phi}=U_{x\Phi}=(\gamma-1)V^{\gamma-2}<0 \text{ since,}$$
$\gamma<1$.

Substituting the partial derivatives into (36) we have

$$Q'(t)+\frac{1}{2V^2}\theta'\theta(1-x^2)(\gamma-1)Q(t)+\frac{1-\gamma}{\gamma V}\left[\left(Q(t)^{\frac{\gamma^2}{\gamma-1}}\exp\left[\frac{\rho t}{\gamma-1}\right]\right)\right]=0 \qquad (24)$$

(24) can be solve numerically.

Suppose that

$$U(t,v_0)=\frac{(x_0+\Phi_0)^{\gamma}Q(t)^{\gamma}}{\gamma} \qquad (25)$$

Then (25) becomes

$$Q'(t)+\frac{1}{2v_0^2}\theta'\theta(1-x_0^2)(\gamma-1)Q(t)+\frac{1-\gamma}{\gamma v_0}\left[\left(Q(t)^{\frac{\gamma^2}{\gamma-1}}\exp\left[\frac{\rho t}{\gamma-1}\right]\right)\right]=0 \qquad (26)$$

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Notes

Solving (26), we have

$$Q(t)=\tilde{Q}(t)^{\frac{1-\gamma}{1-\gamma+\gamma^2}},$$

where,

$$\tilde{Q}(t)=\frac{2v_0e^{\frac{\rho t}{\gamma-1}}((\gamma-1)-2\gamma(\gamma-1))}{\psi\gamma(\gamma-1)}+$$

$$\frac{2v_0e^{\frac{\rho t}{\gamma-1}}(\gamma^2(\gamma-1)+\gamma^2+2\gamma^3+\gamma^4)}{\psi\gamma(\gamma-1)}+$$

$$\frac{2v_0e^{\frac{\rho t}{\gamma-1}+\frac{r(T-t)}{\gamma-1}}((\gamma-1)(1+(\gamma-1)\gamma))}{\psi\gamma}\times$$

$$e^{\frac{1}{1-\gamma}\left(r-\rho+\frac{(\ _0^2-1)(1+(\gamma-1)\gamma)(1-\gamma)\theta'\theta}{2v_0^2}\right)(T-t)}$$

and

$$\psi=(x_0^2-1)(\gamma-1)(1+(\gamma-1)\gamma)\theta'\theta+2v_0^2\rho.$$

Therefore, the optimal discounted protfolio for the PPM at time $t$ is obtained as

$$\Delta^*(t)=\frac{\sigma^{-1}\theta\big(\Lambda(t)X^*(t)+\Lambda(t)\Phi(t)\big)}{\Lambda(t)X^*(t)}-\frac{\sigma^{-1}\sigma^{I}{}'\Phi(t)}{X^*(t)},$$

$$\Delta_0=1+\frac{\sigma^{-1}\sigma^{I}{}'\Phi(t)}{X^*(t)}-\frac{\sigma^{-1}\theta\big(\Lambda(t)X^*(t)+\Lambda(t)\Phi(t)\big)}{\Lambda(t)X^*(t)}, \qquad (27)$$

the optimal discounted consumption process $C^*(t)$ of the PPM at time $t$, given that (25) holds, is obtained

$$\tilde{C}^*(t)=(x_0+\Phi_0)(g(t)+h(t)\times$$

$$e^{\frac{1}{1-\gamma}\left(r-\rho+\frac{(x_0^2-1)(1+(\gamma-1)\gamma)(1-\gamma)\theta'\theta}{2v_0^2}\right)(T-t)}) \qquad (28)$$

where

$$g(t)=\frac{2v_0e^{\frac{\rho t}{\gamma-1}}((\gamma-1)-2\gamma(\gamma-1))}{\psi\gamma(\gamma-1)}+\frac{2v_0e^{\frac{\rho t}{\gamma-1}}(\gamma^2(\gamma-1)+\gamma^2+2\gamma^3+\gamma^4)}{\psi\gamma(\gamma-1)}$$

and

$$h(t) = \frac{2v_0 e^{\frac{\rho t}{\gamma-1}+\frac{r(T-t)}{\gamma-1}}\big((\gamma-1)(1+(\gamma-1)\gamma)\big)}{\psi\gamma}.$$

At time $t = 0$, (28) becomes

$$\widetilde{C}^*(0) = (x_0 + \Phi_0)(g(0) + h(0) \times e^{\frac{1}{1-\gamma}\left(r-\rho+\frac{({}_0^2-1)(1+(\gamma-1)\gamma)(1-\gamma)\theta'\theta}{2v_0^2}\right)T}) \tag{29}$$

where $$g(0) = \frac{2v_0\big((\gamma-1)-2\gamma(\gamma-1)\big)}{\psi\gamma(\gamma-1)} + \frac{2v_0(\gamma^2(\gamma-1)+\gamma^2+2\gamma^3+\gamma^4)}{\psi\gamma(\gamma-1)},$$

and $$h(0) = \frac{2v_0 e^{\frac{rT}{\gamma-1}}\big((\gamma-1)(1+(\gamma-1)\gamma)\big)}{\psi\gamma}$$

At time $t = T$, (28) becomes

$$\widetilde{C}^*(T) = (x_0 + \Phi_0)(g(T) + h(T)), \tag{29}$$

where

$$g(T) = \frac{2v_0 e^{\frac{\rho T}{\gamma-1}}\big((\gamma-1)-2\gamma(\gamma-1)\big)}{\psi\gamma(\gamma-1)} +$$

$$\frac{2v_0 e^{\frac{\rho T}{\gamma-1}}(\gamma^2(\gamma-1)+\gamma^2+2\gamma^3+\gamma^4)}{\psi\gamma(\gamma-1)}$$

and $$h(T) = \frac{2v_0 e^{\frac{\rho T}{\gamma-1}}\big((\gamma-1)(1+(\gamma-1)\gamma)\big)}{\psi\gamma}.$$

Now, for $0 < \gamma < 1$, we have intuitively that the growth rate (GRC) of the optimal expected discounted consumption is obtain as

$$GRC = \frac{1}{1-\gamma}\left(r - \rho + \frac{(x_0^2-1)(1+(\gamma-1)\gamma)(1-\gamma)\theta'\theta}{2v_0^2}\right). \tag{30}$$

This is referred to as the Euler equation for the intertemporal maximization of discounted consumption under uncertainty. The positive term $\theta'\theta$ captures the uncer-

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tainty of the financial martet. When the financial market is risky, it will induce investors to shift consumption over time. Suppose there is no initial wealth, then (30) becomes

$$GRC = \frac{1}{1-\gamma}(r-\rho+\frac{(1+(\gamma-1)\gamma)(\gamma-1)\theta'\theta}{2\Phi_0^2}).$$

Notes

From (27), the first term is the variational form of the classical portfolio strategy while the last term is an intertemporal hedging strategy that offset any shock to the stochastic salary of the PPM. At $t = 0$, we have

$$\Delta^*(0) = \frac{\sigma^{-1}\theta(x_0+\Phi_0)}{x_0} - \frac{\sigma^{-1}\sigma^{I}{}'\Phi_0}{x_0},$$

$$\Delta_0 = 1 + \frac{\sigma^{-1}\sigma^{I}{}'\Phi_0}{x_0} - \frac{\sigma^{-1}\theta(x_0+\Phi_0)}{x_0}.$$

## VII. Optimal Expected Discounted Wealth Valuation and Discounted Consumption Process of a PPM

In this section, we present the optimal expected value of wealth of the PPM in pension plan at time t. From (19), we have that

$$\begin{aligned} d(\Lambda(t)V(t)) &= -\Lambda(t)C(t)dt + \theta(1-\Lambda(t)V(t) \\ &+ \Lambda(t)\Phi(t))dW(t) \end{aligned} \tag{31}$$

$$\begin{aligned} d(\Lambda(t)V(t))^2 &= (-2\Lambda(t)V(t)\Lambda(t)C(t)dt + \\ &\theta'\theta(\Lambda(t)V(t))^2 - \theta'\theta\Lambda(t)V(t) - \\ &\theta'\theta\Lambda(t)V(t)\Lambda(t)\Phi(t) + \\ &\theta'\theta(1+(\Lambda(t)\Phi(t))^2 + (\Lambda(t)\Phi(t))dt \\ &+ 2\theta'\Lambda(t)V(t)(1-\Lambda(t)V(t)+\Lambda(t)\Phi(t))dW(t) \end{aligned} \tag{32}$$

$$dE(\Lambda(t)V(t)) = -E(\Lambda(t)C(t))dt \tag{33}$$

$$\begin{aligned} dE(\Lambda(t)V(t))^2 &= E(-2\Lambda(t)V(t)\Lambda(t)C(t) + \\ &\theta'\theta(\Lambda(t)V(t))^2 - \theta'\theta\Lambda(t)V(t) - \\ &\theta'\theta\Lambda(t)V(t)\Lambda(t)\Phi(t) + \\ &\theta'\theta(1+(\Lambda(t)\Phi(t))^2 + (\Lambda(t)\Phi(t))dt \end{aligned} \tag{34}$$

Solving, (33) and (34), we have

$$E(\Lambda(t)V(t)) = v_0 - E\left(\int_0^t (\Lambda(s)C(s))ds\right) \tag{35}$$

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$$E(\Lambda(t)V(t))^2 = v_0^2 e^{\theta'\theta t} + \int_0^t \mathrm{K}(s) e^{\theta'\theta(t-s)} ds \tag{36}$$

where,

$$\mathrm{K}(t) = 2E\left(\Lambda(t)C(t)(v_0 - \int_0^t \Lambda(s)C(s)ds)\right) +$$

$$\theta'\theta E\left(v_0 - \int_0^t \Lambda(s)C(s)ds\right) +$$

$$\frac{c\theta'\theta e^{\omega t}(e^{\varphi(T-t)}-1)}{\varphi} E\left(v_0 - \int_0^t \Lambda(s)C(s)ds\right) +$$

$$\theta'\theta(1 + \frac{c^2 e^{2\omega t}(e^{\varphi(T-t)}-1)^2}{\varphi^2} + \frac{c e^{\omega t}(e^{\varphi(T-t)}-1)}{\varphi}).$$

$$Var(\Lambda(t)V(t)) = E(\Lambda(t)V(t))^2 - (E(\Lambda(t)V(t)))^2$$

$$= v_0^2 e^{\theta'\theta t} + \int_0^t \mathrm{K}(s) e^{\theta'\theta(t-s)} ds - \tag{37}$$

$$\left(v_0 - E\left(\int_0^t (\Lambda(s)C(s))ds\right)\right)^2$$

At $t = T$, we have

$$E(\Lambda(T)V(T)) = v_0 - E\left(\int_0^T (\Lambda(s)C(s))ds\right) \tag{38}$$

$$Var(\Lambda(T)V(T)) = E(\Lambda(T)V(T))^2 - (E(\Lambda(T)V(T)))^2 =$$

$$v_0^2 e^{\theta'\theta T} + \int_0^T \mathrm{K}(s) e^{\theta'\theta(T-s)} ds$$

$$-\left(v_0 - E\left(\int_0^T (\Lambda(s)C(s))ds\right)\right)^2.$$

## VIII. Accumulated Expected Discounted Consumption Process of a PPM

The accumulated expected discounted consumption process of the PPM up to retirement period is given by

$$\tilde{\Psi}(T) = E\left[\int_0^T \tilde{C}(u)du\right] = \int_0^T (x_0 + \Phi_0)(g(t) + h(t)e^{GRC(T-t)})dt$$

$$= (x_0 + \Phi_0)\int_0^T g(t)dt + (x_0 + \Phi_0)\int_0^T h(t)e^{GRC(T-t)})dt \tag{39}$$

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Solving (39), we have

$$\widetilde{\Psi}(T)=\frac{2v^2{}_0\left((\gamma-1)-2\gamma(\gamma-1)\right)}{\psi\gamma\rho}(1-e^{\frac{\rho T}{\gamma-1}})$$

$$+\frac{2v^2{}_0(\gamma^2(\gamma-1)+\gamma^2+2\gamma^3+\gamma^4)}{\psi\gamma\rho}(1-e^{\frac{\rho T}{\gamma-1}})$$

$$+\frac{2v^2{}_0\left((1-\gamma)(1+(\gamma-1)\gamma)\right)e^{\frac{(\rho+\alpha)T}{\gamma-1}}}{\alpha\psi\gamma}(1-e^{\frac{\alpha T}{1-\gamma}})$$

where $$\alpha=\frac{(x_0^2-1)(1+(\gamma-1)\gamma)(1-\gamma)\theta'\theta}{2v_0^2}.$$

If we allow $T$ to tend to infinity, we have

$$\lim_{T\to\infty}E_0\left[\int_0^T\widetilde{C}(u)du\right]=E_0\left[\int_0^\infty\widetilde{C}(u)du\right]=$$

$$\lim_{T\to\infty}\widetilde{\Psi}(T)=\widetilde{\Psi}_\infty=\frac{2v^2{}_0\left((\gamma-1)-2\gamma(\gamma-1)\right)}{\psi\gamma\rho}$$

$$+\frac{2v^2{}_0(\gamma^2(\gamma-1)+\gamma^2+2\gamma^3+\gamma^4)}{\psi\gamma\rho}.$$

This shows that throughout the life-cycle of the PPM, his or her accumulated expected discounted consumption will remain nonempty. If $\widetilde{\Psi}_\infty$ is nonnegative, it implies that the PPM is unable to finished his or her consumption before he or she dies. If $\widetilde{\Psi}_\infty$ is negative, it implies that the PPM is finished his or her consumption why still alive. If $\widetilde{\Psi}_\infty$ is zero, it implies that the period the PPM finishes his or her consumption was the period he or she dies. Interestingly, the nature of $\widetilde{\Psi}_\infty$ absolutely depend on the value of $\gamma,\rho$. This implies that the level of risk the PPM takes will depend on how large or how small the value of the accumulated discounted consumption will be and consumption rate. We observe that at $\rho=0$ or

$$\rho=\frac{(x_0^2-1)(\gamma-1)(1+(\gamma-1)\gamma)\theta'\theta}{2v_0^2},$$

optimal expected discounted consumption will be unbounded. The optimal expected discounted consumption will be negative if

$$\rho<\frac{(x_0^2-1)(\gamma-1)(1+(\gamma-1)\gamma)\theta'\theta}{2v_0^2},$$

and non negative if

$$\rho > \frac{(x_0^2 - 1)(\gamma - 1)(1 + (\gamma - 1)\gamma)\theta'\theta}{2v_0^2}.$$

## IX. Conclusion

Notes

This paper dealt with optimal discounted portfolio, expected wealth and strategic life-time consumption process for a defined contributory pension scheme. The expected discounted wealth and its variance are obtained. The discounted variational classical portfolio and consumption process of the plan member are established. The accumulated discounted consumption process of the PPM throughout his or her life-cycle was established. It was found that part of the discounted portfolio value is proportional to the ratio of the present value of the discounted expected future contributions to the optimal discounted portfolio value overtime. It was found that there is the need for gradual transfer of part of the portfolio value in risky asset to the riskless one against unforeseen shocks.

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# On the Right Perfect Ring

By R.H. Sallam
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*Abstract -* We prove that a right self right perfect algebra which is at most countable dimensional modulo their Jacobson radical is right artinian.

*Keywords :* *self injective rings, right perfect rings, quasi-frobenius rings.*

*GJSFR-F Classification :* *MSC 2010: 16D50*

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# On the Right Perfect Ring

R.H. Sallam

*Abstract* - We prove that a right self right perfect algebra which is at most countable dimensional modulo their Jacobson radical is right artinian.

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## Section 1

## I. Introduction

In this note we present a proof for algebra R over a field K which is at most countable imensional modulo their Jacobson radical i.e. R/Rad(R) is at most $N_o$

This includes for example the important situation when R/Rad R is not only semisimple but also finite dimensional' Let R be a ring with identity and RadR is its Jacobson radical. If R is right perfect (i.e. R/RadR is semisimple and RadR is left T-nilpotent) then R/RadR is semisimple and hence $(R/RadR)_R$ and $_R(R/RadR)$ are semisimple right and left R-module.

Note that since R/RadR is semisimple then an R-module M is semisimple iff it is cancelled by RadR. Indeed every simple is cancelled by Rad R and if RadM=0 then M has R/RadR –module structure which is semisimple, therefore M is semisimple as the lattice of R-submodules and R/RadR sub-modules coincide in this case.

$(R/RadR)_R$ is semisimple and finitely generated, so it has a composition series i.e.

$$J/J^2 \subset J^2/J^3 \subset \ldots \subset J^{n-1}/J^n$$

is of length n-1 where each $J^k/J^{k+1}$ is semisimple for all k

Note that if R is right self injective then each decomposable (eR) has simple socle. Indeed if we have a nontrivial decomposition of the socle $socle(eR) = M \oplus N$

Then we can find E(M) and E(N)injective hulls of M, N contained in eR and we obtain eR= E(M) + E(N) nontrivial decomposition that is a contradiction.

Note also that if R is right self injective right perfect then for each simple Right R-module, the left module Hom (S, R) is simple.

First Hom (S, R) is non zero. Looking for the isomorphism types of indecomposable modules eR, these are projective, local and the cover of some simple R-module.

The number of isomorphism types of such modules equals the number of isomorphism types of simple modules equal t, say.

Moreover since the indecomposable eR's also injective with simple socle we see that they are isomorphic if and only if their socle isomorphic. This shows that the distinct

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types of isomorphism of simples occurring as socle of some eR is also t, and so each simple S must appears as a socle of some l eR (i.e. it embeds in R) this shows that

$Hom(S,R) \neq 0$ , for each simple R-modules S. If $f$ , $g \in Hom(S,R)$ and $f \neq 0$, *then* $f: S \to R$

Is a monomorphism and since $R_R$ is injective then there is some $h: R \to R$

Such that $h \circ f = g$. If $f(x) = xc \ \ \forall x \in R$ we get $f(x).c = g(x)$ i.e. $f.c = g$ in *Hom* (*S*, *R*), this shows that *Hom*(*S*, *R*) is generated by $f \neq 0$ so it is simple.

In particular since each simple module embeds in R which is right self injective, it follows that R is injective cogenerator in the category of right R-module, i.e. it is a right PF (Pseudo- Frobonius) ring.

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## Section 2

Let S be a set of repesenatives for simple right R-modules and let t= S and $R/RadR = \oplus S^{K_s}$. Let W= socle ($R_R$) be the right socle of R so W is an R-sub-bimodule of R.

Since each indecomposable module (Re) has simple socle, we have Length (W) equals the number of terms in the indecomposable decomposition $R = \underset{S}{\oplus}$ Re which equals length $(R/RadR)_R$, since each Indecomposable (Re) is local

Let $W = \oplus S^{P_s}$ we have $\sum_{s \in S} P_s = \sum_{s \in S} K_s$

*Proposition 1*

Let R be a right self injective right perfect ring then the set $\{Hom(S, R); S \in S\}$ is a set of representatives for simple left R-modules.

In particular *Hom*(*S*, *R*) and *Hom*(*T*, *R*) are nonisomorphic for nonisomorphic, $S, T \in R$

*Proof*

Since R is right self injective the monomorphism

$0 \to W \to R$, gives rise to the epimorphism

$$Hom(R, R) \to Hom(W.R) \to 0$$

Note that

$$Hom(W, R) = \oplus Hom(S, R)$$

Since $Hom(S, R) \neq 0 \ \forall \ S \in S$, we have

$$Hom(W, R) = \oplus Hom(S, R)^{P_s},$$

has length equal to the length $W = \sum_{s \in S} P^s = length(R/RadR)_R$

By classical Wedderburn –Artin Theorem:

$$length(R/RadR)_R = length_R(R/RadR)$$

Since $Hom(W,R)$ is semisimple, the kernel of $R \to Hom(W,R)$

Contains *RadR* and therefore since

$$length_R(R/RadR) = length \ Hom_R(W, R),$$

we obtain $R/RadR \cong Hom(W, R)$ as left R-module.

This shows that all types of isomorphism of simple left R-modules are found among components of $Hom(W,R)$ and the statement is proved

Note: we note that the above proof further shows that there is an exact sequence of left R-modules

Notes

$$0 \to RadR \to R \to Hom(W, R) \to 0$$

This means that

$$\{r \in R, \text{suchthat}, rW = 0\} = RadR. \text{ That is } ann({}_R W) = RadR$$

This shows that W is also semisimple as a left R-module, i.e. the right socle of R is contained in the left socle, and hence the left and the right socle or the right PF ring coincide.

For a right R-module M, denote $M^* = Hom(M,R)$ this is a left R-module.

*Proposition 2*

Let R be a right injective ring and let M be a right R-module such that there is an exact sequence

$$0 \to S_R \to M_R \to L_R^{(\alpha)} \to 0$$

With S, L simple modules and assume S= socle (M), $L_R^{(\alpha)}$ denote the coproduct of α copies of $L_R$. Then $M^*$ is local left R-module with unique maximal ideal

$$S^{\perp} = \{f \in Hom(M, R); f|_S \neq 0\}$$

Which is semisimple isomorphic to $(L_R^*)^{(\alpha)}$

*Proof*

Since

$$0 \to S_R \to M_R \to L_R^{(\alpha)} \to 0$$

is an exact sequence, so we get the exact sequence,

$$0 \to Hom(L^{(\alpha)}, R) \to Hom(M, R) \to Hom(S, R) \to 0$$

That is

$$0 \to (L^*)^{\alpha} \to M^* \to S^* \to 0$$

The kernel of the morphism

$$M^* = Hom(M, R) \xrightarrow{\gamma^*} S^* = Hom(S, R)$$

$$\text{Is} \quad \{f \in Hom(M, R); f|_S \neq 0\} = S^{\perp}$$

Hence $S^{\perp} \cong (L^*)^{\alpha}$ which is left semisimple module since it is cancelled by $RadR$.

Now since M has simple socle, and its socle embeds in R which is injective it follows that M embeds in R. We note that $M^*$ is generated by any $f \notin S^{\perp}$ which will show that $M$ is cyclic. Indeed such $f$ must be a monomorphism and given any other $h: M \to R$.

By injectivity of $R_R$ there is $g \in Hom(R,R)$ such that $g \circ f = h$. If $g(x) = cx$ for $c \in R$ then we have $h = c.f$ in $M^*$.This shows that $f..R = M$ which shows that $S^{\perp}$ is the only maximal submodules of the cyclic left Rmodule $M^*$, that ends of the proof.

Note that the fact that $M^*$is local can also be proved by embedding M in some indecomposable (eR) for an indecomposable idempotent e, and then, by applying the exact functor $Hom(\text{-},R)$ one obtain an epimorphism $Hom(eR, R) = eR \to M$, and so $M^*$ is local because $eR$ is.

Let α be the largest cardinality for which there is a right R-module M with simple socle and such that $M/(socle(M)) \cong L^{\alpha}$ for some simple module L.

Such cardinality exists, since any such module is contained in R because R is injective. In fact if $W_1$ is the second socle of R, then $\alpha \le length(W_2 / W)$ we note that if α is infinite this is an equality.

Indeed if for each simple modules S, L we denote by $\alpha_{S,L} = [(E(S)/S) : L]$ - the multiplicity of L in the second socle of the injective hull E(S) of S, then $\alpha = \max_{S,L}(\alpha_{S,L})$.

Therefore $\alpha \le \sum_{S,L \in S} \alpha_{S,L} \le n\alpha = \alpha$ if α is infinite.

We note also that if $\sum_k$ is the k'$^{th}$ socle then the $length(W_k / W_{k-1}) \le \alpha$ ; this follows by induction on k.
If this is true for k, then there is an embedding

$$W_k / W_{k-1} \hookrightarrow R^{(\alpha)}$$

And therefore we have

$$length\ (W_{k+1} / W_k) \le length\ (W_1 / W_0)^{(\alpha)} = \alpha \times \alpha = \alpha\ ,$$

since α is infinite cardinal. So we have the following theorem:

*Theorem*

Let R be a right self injective right perfect algebra such that the dimension of each simple R-module is at most countable (equivalently the dimension of $R / RadR$ is at most countable) hence R is countable, and hence right artinian.

*Proof*

With the above notation, assume α is infinite. The length of each $W_k / W_{k\text{-}1}$ is at most α, so since the dimension of each simple is at most $\aleph_0$, its dimension is at most $\aleph_0$, × α = α (since α is infinite).

Thus the dimension of R is at most α, and so it equals α (since $length(W_1 / W_0) = \alpha$).

On the other hand, by proposition (2.2) there is a local left R-module $M^*$, with socle $L^{\alpha}$ for some simple $L$.

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Note that $\dim(L^{\alpha}) \geq 2^{\alpha}$ and that there is an epimorphism $R \rightarrow M^{*}$
So $\dim(R) \geq 2^{\alpha}$, this is a contradiction.

By the result of Lawrence [3] R is right self injective countably generated algebra over a field so it is quasi -frobenius and hence right artinian.

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2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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**33. Report concluded results:** Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

**34. After conclusion:** Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

## Informal Guidelines of Research Paper Writing

**Key points to remember:**

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

**Final Points:**

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

**General style:**

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

**Title Page:**

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

**Abstract:**

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

**Introduction:**

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

**Procedures (Methods and Materials):**

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

**Results:**

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

**Discussion:**

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

## Administration Rules Listed Before Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

**Segment Draft and Final Research Paper:** You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.

- Do not give permission to anyone else to "PROOFREAD" your manuscript.

- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

## Criterion for Grading a Research Paper (Compilation) by Global Journals Inc. (US)

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| Topics | Grades | | |
|---|---|---|---|
| | **A-B** | **C-D** | **E-F** |
| ***Abstract*** | Clear and concise with appropriate content, Correct format. 200 words or below | Unclear summary and no specific data, Incorrect form<br><br>Above 200 words | No specific data with ambiguous information<br><br>Above 250 words |
| ***Introduction*** | Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited | Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter | Out of place depth and content, hazy format |
| ***Methods and Procedures*** | Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads | Difficult to comprehend with embarrassed text, too much explanation but completed | Incorrect and unorganized structure with hazy meaning |
| ***Result*** | Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake | Complete and embarrassed text, difficult to comprehend | Irregular format with wrong facts and figures |
| ***Discussion*** | Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited | Wordy, unclear conclusion, spurious | Conclusion is not cited, unorganized, difficult to comprehend |
| ***References*** | Complete and correct format, well organized | Beside the point, Incomplete | Wrong format and structuring |

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