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## Discovering Thoughts, Inventing Future

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# On the Maximal Ideals in the Banach Space of ${ }^{+}$Ouasicontinuous Functions 

By V. Srinivasa Kumar<br>JNTUH College of Engineering India

Abstract - In this paper some interesting properties of ${ }^{+}$Quasicontinuous functions are presented. The maximal ideals in the Banach space of bounded real valued +Quasicontinuous functions defined on $[0,1]$ are investigated.

Keywords: ${ }^{+}$quasicontinuity, maximal ideal, space of maximal ideals, weak* topology, compact hausdorff space, bounded linear functional, cliquish function.

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# On the Maximal Ideals in the Banach Space of ${ }^{+}$Quasicontinuous Functions 

V. Srinivasa Kumar

Abstract : In this paper some interesting properties of ${ }^{+}$Quasicontinuous functions are presented. The maximal ideals in the Banach space of bounded real valued ${ }^{+}$Quasicontinuous functions defined on $[0,1]$ are investigated. Keywords : +quasicontinuity, maximal ideal, space of maximal ideals, weak* topology, compact hausdorff space, bounded linear functional, cliquish function.
I. Introduction

In this paper, it is shown that the set of all bounded real
${ }^{+}$Quasicontinuous functions defined on [0,1] forms a commutative Banach algebra with identity under the supremum norm. The maximal ideals in this Banach algebra are identified to be of the form $M_{x}=\{f / f(x)=0\}$ or $M_{x}^{+}=\{f / f(x+)=0\}$ for $x \in[0,1]$.

In what follows $\mathbb{R}, I$ and $J$ stand for the real line, the unit closed interval $[0,1]$ and any closed and bounded interval $[a, b]$ respectively.
a) Preliminaries
1.1 Definition: Let $f: J \rightarrow \mathbb{R}$. We define $f(a-)=f(a)$ and $f(b+)=f(b)$.

We say that $f(p+)$ exists at $p \in[a, b)$ and we write $f(p+)=L$, where $L \in \mathbb{R}$ if for every $\varepsilon>0$ there exists a $\delta>0$ such that $|f(x)-L|<\varepsilon \forall x \in(p, p+\delta) \subset J$ Similarly for $p \in(a, b]$ we write $f(p-)=l \in \mathbb{R}$ if for every $\varepsilon>0$ there exists a $\delta>0$ such that $|f(x)-l|<\varepsilon \quad \forall x \in(p-\delta, p) \subset J$
1.2 Definition: A function $f: J \rightarrow \mathbb{R}$ is said to be ${ }^{+}$Quasicontinuous on $J$ if (i) $f(p+)$ exists at every $p \in[a, b)$

[^0](ii) $\quad f(b-)=f(b)$
1.3 Definition: A function $f: J \rightarrow \mathbb{R}$ is said to be cliquish at a point $p \in J$ if for every $\varepsilon>0$ and every neighborhood $U$ of $p$ in $J$ there exists a non-empty open set $W \subset U$ such that $|f(x)-f(y)|<\varepsilon \quad \forall x, y \in W$. We say that $f$ is cliquish on $J$ if it is cliquish at every point of $J$.
1.4 Definition: A mapping $T$ from a linear space $\mathscr{V}$ into a linear space $\mathscr{W}$ is said to be linear if $T(c x+d y)=c T(x)+d T(y)$ for all $x$ and $y$ in $\mathscr{V}$ and constants $c$ and $d$.
1.5 Definition: Let $\mathscr{V}$ and $\mathscr{V}$ be normed linear spaces. A linear map $T: \mathscr{V} \rightarrow \mathscr{W}$ is said to be bounded if there exists a real number $K \geq 0$ such that $\|T(x)\| \leq K\|x\| \quad \forall x \in \mathscr{V}$.
1.6 Definition: A linear functional on a vector space $\mathscr{V}$ over a field $\mathscr{K}$ is a linear mapping from $\mathscr{V}$ to $\mathscr{K}$.
b) Properties of ${ }^{+}$Quasicontinuous functions
2.1 Proposition: Let $c \in \mathbb{R}$. If $f: J \rightarrow \mathbb{R}$ and $g: J \rightarrow \mathbb{R}$ are
${ }^{+}$Quasicontinuous on $J$ then $f+g, c f, f g, f \vee g$ and $f \wedge g$ are
${ }^{+}$Quasicontinuous on $J$, where $(f \vee g)(x)=\max \{f(x), g(x)\}$ and $(f \wedge g)(x)=\min \{f(x), g(x)\}$.

Proof: Let $p \in[a, b)$. (i) Let $\varepsilon>0$ be given. Then there exist $\delta_{1}>0$ and $\delta_{2}>0$ such that $|f(x)-f(p+)|<\frac{\varepsilon}{2} \forall x \in\left(p, p+\delta_{1}\right) \subset J$ and
$|g(x)-g(p+)|<\frac{\varepsilon}{2} \forall x \in\left(p, p+\delta_{2}\right) \subset J$. Put $\delta=\min \left\{\delta_{1}, \delta_{2}\right\}$.
Then $x \in(p, p+\delta) \Rightarrow|(f+g)(x)-(f(p+)+g(p+))| \leq|f(x)-f(p+)|+$

$$
|g(x)-g(p+)|<\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon .
$$

Thus for every $\varepsilon>0$ there exists a $\delta>0$ such that

$$
|(f+g)(x)-(f(p+)+g(p+))|<\varepsilon \quad \forall \quad x \in(p, p+\delta)^{\cdot}
$$

Hence $(f+g)(p+)$ exists and $(f+g)(p+)=f(p+)+g(p+)$. Since $f$ and $g$ are continuous at $b, f+g$ is continuous at $b$.

Hence $f+g$ is ${ }^{+}$Quasicontinuous on $J$.
(ii) If $c=0$ then $c f=O$, where $O: J \rightarrow \mathbb{R}$ is defined by $O(x)=0$.

Then $c f$ is ${ }^{+}$Quasicontinuous on $J$. Now suppose that $c \neq 0$.
Let $\varepsilon>0$ be given. Then there exists a $\delta>0$ such that

$$
\begin{aligned}
& |f(x)-f(p+)|<\frac{\varepsilon}{|c|} \forall x \in(p, p+\boldsymbol{\delta}) \subset J \\
\Rightarrow & |(c f)(x)-(c f)(p+)|<\varepsilon \quad \forall x \in(p, p+\delta)
\end{aligned}
$$

Hence $(c f)(p+)$ exists and $(c f)(p+)=c f(p+)$. Since $f$ is continuous at $b$, $c f$ is continuous at $b$. Hence $c f$ is ${ }^{+}$Quasicontinuous on $J$.
(iii) Since $f$ and $g$ are ${ }^{+}$Quasicontinuous at $p$, for every $\varepsilon>0$ there exists a $\delta>0$ such that $|f(x)-f(p+)|<\varepsilon$ and $|g(x)-g(p+)|<\varepsilon \quad \forall x \in(p, p+\delta) \subset J$ $\Rightarrow|(f g)(x)-f(p+) g(p+)|=|f(x) g(x)-f(x) g(p+)+f(x) g(p+)-f(p+) g(p+)|$ $\leq|f(x)||g(x)-g(p+)|+|g(p+)||f(x)-f(p+)|$

$$
<|f(x)| \varepsilon+|g(p+)| \varepsilon \quad \forall x \in(p, p+\delta)
$$

$$
=|f(x)-f(p+)+f(p+)| \varepsilon+|g(p+)| \varepsilon
$$

$$
<\varepsilon(\varepsilon+|f(p+)|+|g(p+)|) \quad \forall x \in(p, p+\delta)
$$

Hence $(f g)(p+)$ exists and $(f g)(p+)=f(p+) g(p+)$. Since $f$ and $g$ are continuous at $b, f g$ is continuous at $b$.

Hence $f g$ is ${ }^{+}$Quasicontinuous on $J$.
It is easy to verify that $f \vee g$ and $f \wedge g$ are ${ }^{+}$Quasicontinuous on $J$ and we have the following.

$$
(f \vee g)(p+)=\max \{f(p+), g(p+)\} \text { and }(f \wedge g)(p+)=\min \{f(p+), g(p+)\} .
$$

2.2 Proposition: Let $f_{n}: J \rightarrow \mathbb{R}, n=1,2,3, \ldots$, be ${ }^{+}$Quasicontinuous on $J$ and $f_{n} \rightarrow f$ uniformly on $J$. Then $f$ is ${ }^{+}$Quasicontinuous on $J$.

Proof: Let $p \in[a, b)$. Let $\varepsilon>0$ be given. Then there exists an integer $N$ such that $n \geq N \Rightarrow\left|f_{n}(x)-f(x)\right|<\frac{\varepsilon}{2} \quad \forall x \in J$.

Since $f_{N}$ is ${ }^{+}$Quasicontinuous at $p$, there exists a $\delta>0$ such that

$$
\begin{aligned}
&\left|f_{N}(x)-f_{N}(p+)\right|<\mathcal{E} \forall x \in(p, p+\delta) \subset J \\
& x \in(p, p+\delta) \Rightarrow\left|f(x)-f_{N}(p+)\right|=\left|f(x)-f_{N}(x)+f_{N}(x)-f_{N}(p+)\right| \\
& \leq\left|f(x)-f_{N}(x)\right|+\left|f_{N}(x)-f_{N}(p+)\right| \\
&<\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon .
\end{aligned}
$$

Thus for every $\varepsilon>0$ there exists a $\delta>0$ such that

$$
\left|f(x)-f_{N}(p+)\right|<\varepsilon \forall x \in(p, p+\delta) \subset J .
$$

Hence $f(p+)$ exists for every $p \in[a, b)$.
Since each $f_{n}$ is continuous at $b$ and $f_{n} \rightarrow f$ uniformly on $J, f$ is continuous at $b$. Hence $f$ is ${ }^{+}$Quasicontinuous on $J$.
2.3 Remark: It is not necessary that a ${ }^{+}$Quasicontinuous function defined on a compact domain is bounded. It can be seen from the following example.
2.4 Example: Define $f:[-1,1] \rightarrow \mathbb{R}$ by $f(x)=\left\{\begin{array}{lll}1 & \text { if } & 0 \leq x \leq 1 \\ \frac{1}{x} & \text { if } & -1 \leq x<0\end{array}\right.$

This function $f$ is ${ }^{+}$Quasicontinuous on $[-1,1]$ but it is not bounded.
2.5 Remark: We denote the set of all bounded real valued ${ }^{+}$Quasicontinuous functions defined on $I$ by the symbol $\mathscr{C b}^{+}(I)$. By the propositions 2.1 and 2.2 it follows that $\mathscr{C b}^{+}(I)$ forms a commutative Banach algebra with identity under the supremum norm, where the identity $\boldsymbol{e}: I \rightarrow \mathbb{R}$ is defined by $\boldsymbol{e}(x)=1 \forall x \in I$.
2.6 Proposition: Let $f: J \rightarrow \mathbb{R}$ and $p \in J$. If $f(p+)$ exists then $f$ is cliquish at $p$.

Proof: Let $\varepsilon>0$ be given and let $U$ be a neighborhood of $p$ in $J$. Then there exists a $\delta_{1}>0$ such that $\left(p-\delta_{1}, p+\delta_{1}\right) \cap J \subset U$.

Given $f(p+)$ exists. So there exists $\delta_{2}>0$ such that

$$
|f(x)-f(p+)|<\frac{\varepsilon}{2} \quad \forall x \in\left(p, p+\delta_{2}\right) \subset J .
$$

Put $\boldsymbol{\delta}=\min \left\{\boldsymbol{\delta}_{1}, \delta_{2}\right\}$ and $W=(p, p+\boldsymbol{\delta})$.
Then for $x, y \in W$, we have $|f(x)-f(y)|=|f(x)-f(p+)+f(p+)-f(y)|$

$$
\begin{aligned}
& \leq|f(x)-f(p+)|+|f(y)-f(p+)| \\
& <\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon .
\end{aligned}
$$

Thus for every $\varepsilon>0$ and every neighborhood $U$ of $p$, there exists a non-empty open set $W \subset U$ such that $|f(x)-f(y)|<\varepsilon \quad \forall x, y \in W$
$\Rightarrow \quad f$ is cliquish at $p$.
2.7 Remark: From the above proposition it is clear that every ${ }^{+}$Quasicontinuous function is cliquish. The converse is not true as is evident from the following example.
2.8 Example: Define $f:[-1,1] \rightarrow \mathbb{R}$ as follows.

$$
\mathrm{f}(x)=\left\{\begin{array}{lll}
\frac{1}{x} & \text { if } & 0<x \leq 1 \\
0 & \text { if } & -1 \leq x \leq 0
\end{array}\right.
$$

Clearly f is cliquish at $x=0$ but it is not ${ }^{+}$Quasicontinuous.
2.9 Theorem [2]: If $f: J \rightarrow \mathbb{R}$ is ${ }^{+}$Quasicontinuous then the set of points of discontinuity of $f$ is atmost countable.
c) Maximal Ideals in $\mathscr{C b}^{+}(I)$
3.1 Definition: For each $x \in I$, we define the following.
(a) $M_{x}=\left\{f \in \mathscr{O b}^{+}(I) / f(x)=0\right\}$
(b) $M_{x}^{+}=\left\{f \in \mathscr{C}^{+}(I) / f(x+)=0\right\}$.
3.2 Proposition: For each $x \in I$, the sets $M_{x}$ and $M_{x}^{+}$are maximal ideals in the commutative Banach algebra $\mathscr{C b}^{+}(I)$.

Proof: For $x \in I$, define $F_{x}$ and $F_{x}^{+}$on $\mathscr{C b}^{+}(I)$ by $F_{x}(f)=f(x)$ and $F_{x}^{+}(f)=f(x+)$ for $f \in \mathscr{C b}^{+}(I)$.

Clearly $F_{x}$ and $F_{x}^{+}$are multiplicative linear functionals in the dual space $\mathcal{B}_{+}$with kernels $\quad M_{x}$ and $M_{x}^{+}$respectively. Hence $M_{x}$ and $M_{x}^{+}$are ideals. Moreover $M_{x}$ and $M_{x}^{+}$are maximal ideals in $\mathscr{O b}^{+}(I)$.
3.3 Proposition: If $M$ is a maximal ideal in $\mathscr{C b}^{+}(I)$ then either $M=M_{x}$ or
$M=M_{x}^{+}$for some $x \in I$.
Proof: For $x \in I$, define $F_{x}$ and $F_{x}^{+}$on $\mathscr{Q b}^{+}(I)$ by $F_{x}(f)=f(x)$ and $F_{x}^{+}(f)=f(x+)$ for $f \in \mathscr{Q}^{+}(I)$.

Clearly $F_{x}$ and $F_{x}^{+}$are multiplicative linear functionals in the dual space $\mathcal{B}_{+}$with kernels $M_{x}$ and $M_{x}^{+}$respectively. Hence $M_{x}$ and $M_{x}^{+}$are ideals. Moreover $M_{x}$ and $M_{x}^{+}$are maximal ideals in $\mathscr{C b}^{+}(I)$.
3.3 Proposition: If $M$ is a maximal ideal in $\mathscr{C b}^{+}(I)$ then either $M=M_{x}$ or $M=M_{x}^{+}$for some $x \in I$.

Proof: Assume that $M \neq M_{x}$ and $M \neq M_{x}^{+}$for any $x \in I$.
Then there exist $f_{x}$ and $g_{x}$ in $M$ such that $f_{x} \notin M_{x}$ and $g_{x} \notin M_{x}^{+}$.
Define $\varphi_{x}: I \rightarrow \mathbb{R}$ by $\varphi_{x}(t)=f_{x}^{2}(t)+g_{x}^{2}(t+) \quad \forall t \in I$.
Clearly $\varphi_{x} \in \mathscr{C b}^{+}(I)$. Since $\varphi_{x}$ is ${ }^{+}$Quasicontinuous at $x$ and $\varphi_{x}(x)>0$, there exists $\delta_{x}>0$ such that $\varphi_{x}(t)>0 \quad \forall t \in\left[0, \delta_{x}\right)$ and for $x \neq 1$

We have $\varphi_{1}(t)=f_{1}^{2}(t)+g_{1}^{2}(t+) \forall t \in I$.
Since $\varphi_{1}$ is continuous at 1 and $\varphi_{1}(1)>0$ there exists a $\delta>0$ such that $\varphi_{1}(t)>0 \forall t \in(1-\delta, 1]$. Then $[0,1]=\left(\bigcup_{x \neq 1}\left[0, \delta_{x}\right)\right) \cup(1-\delta, 1]$. Since $I$ is compact, there exists $x_{1} \neq 1$ in $I$ such that $[0,1]=\left[0, \delta_{x_{1}}\right) \cup(1-\delta, 1]$.

Put $\varphi=\varphi_{x_{1}}^{2}+\varphi_{1}^{2}$. Then $\varphi \in M$ and $\varphi(t)>0 \forall t \in I \Rightarrow \frac{1}{\varphi} \in M$
Then $e=\varphi \cdot \frac{1}{\varphi} \in M$. This is a contradiction.
Hence it follows that $M=M_{x}$ or $M=M_{x}^{+}$for some $x \in I$.
3.4 Remark: Let $\mathscr{U}_{+}$be the space of all maximal ideals in $\mathscr{Q b}^{+}(I)$. Then $\mathscr{U}_{+}$ is a compact Hausdorff space with the weak ${ }^{*}$ topology on $\mathscr{Q b}^{+}(I)$. Hence $\mathscr{N}_{+}^{2}=\mathscr{N}_{+} \times \mathscr{I}_{+}$is a compact Hausdorff space with the product topology on $\mathbb{Q b}^{+}(I) \times \mathrm{Cb}^{+}(I)$.
3.5 Proposition: Let $\mathscr{A}^{+}=\left\{\left(M_{x}, M_{x}^{+}\right) / x \in I\right\}$. Then there exists a one-to-one correspondence between $I$ and $\mathscr{A}^{+}$.

Proof: Define $\Psi^{+}: I \rightarrow \mathscr{A}^{+}$by $\Psi^{+}(x)=\left(M_{x}, M_{x}^{+}\right)$.
Clearly $\Psi^{+}$is surjective. If $0 \leq s<t \leq 1$, the function

$$
\Psi_{0}^{+}(p)=\left\{\begin{array}{lll}
0 & \text { if } & t \leq p \leq 1 \\
\frac{1}{x-t} & \text { if } & 0 \leq p<t
\end{array}\right.
$$

satisfies $\Psi_{0}^{+} \in M_{t}$ and $\Psi_{0}^{+} \notin M_{s}$.
$\Rightarrow \quad M_{s} \neq M_{t}$
$\Rightarrow \quad\left(M_{s}, M_{s}^{+}\right) \neq\left(M_{t}, M_{t}^{+}\right)$
$\Rightarrow \quad \Psi^{+}(s) \neq \Psi^{+}(t)$
Hence $\quad \Psi^{+}$is $1-1$.
Hence $\Psi^{+}$is a one-to-one correspondence between $I$ and $\mathscr{A}^{+}$.
3.6 Remark: Each maximal ideal in $\mathscr{C b}^{+}(I)$ is the kernel of some multiplicative linear functional on $\mathscr{C b}^{+}(I)$, hence can be identified with a multiplicative linear functional on $\mathscr{C b}^{+}(I)$. Let $M_{x}$ and $M_{x}^{+}$be identified with the multiplicative linear functional $F_{x}$ and $F_{x}^{+}$respectively. So we can write $\mathscr{A}^{+}=\left\{\left(F_{x}, F_{x}^{+}\right) / x \in I\right\}$.
3.7 Proposition: $\mathscr{A}^{+}$is closed in $\mathcal{B}_{+}^{2}=\mathcal{B}_{+} \times \mathcal{B}_{+}$and hence compact.

Proof: We prove that $\mathscr{A}^{+}$is closed. Compactness is an immediate consequence of the Banach - Alaoglu theorem [5]. If $F=\left(F_{1}, F_{2}\right) \in \mathcal{B}_{+}^{2}$, we define $\|F\|=\max \left\{\left\|F_{1}\right\|,\left\|F_{2}\right\|\right\}$. Then $\mathcal{B}_{+}^{2}$ is a Banach space under the above norm.

Let $S=\{F /\|F\| \leq 1\} \subset \mathcal{B}_{+}^{2}$. Put $\mathcal{A}=\mathscr{A}^{+} \bigcup\{O\}$.

The $\mathscr{A}^{+} \subset \mathscr{M}_{+}^{2} \subset \mathcal{A} \subset S \subset \mathcal{B}_{+}^{2}$.
Define $\mathcal{P}^{+}: \mathcal{A} \rightarrow \mathbb{R}$ by

$$
\mathcal{P}^{+}(F)=\left\{\begin{array}{lll}
1 & \text { if } & F \in \mathcal{A} \\
0 & \text { if } & F=O
\end{array} \text { and } F \neq O\right.
$$

Since $\mathcal{P}^{+}$is continuous, $\mathscr{A}^{+}$and $\mathcal{A}$ are closed in $\mathcal{B}_{+}^{2}$.
d) Further Properties
4.1 Proposition: Fix $f \in \mathscr{Q}^{+}(I)$. Define $\psi_{f}: I \rightarrow \mathbb{R}^{2}$ by $\psi_{f}(x)=(f(x), f(x+))$, where $\mathbb{R}^{2}=\mathbb{R} \times \mathbb{R}$ is considered with the norm $\left\|\left(x_{1}, x_{2}\right)\right\|=\max \left\{\left|x_{1}\right|,\left|x_{2}\right|\right\}$. Then $\psi_{f}$ is continuous on $I$ if and only if $f$ is continuous on $I$.

Proof: Assume that $\psi_{f}$ is continuous on $I$. Let $p \in I$ and let $\varepsilon>0$ be given.
Since $\psi_{f}$ is continuous at $p$, there exists a $\delta>0$ such that

$$
\begin{aligned}
& \left\|\psi_{f}(x)-\psi_{f}(p)\right\|<\varepsilon \quad \forall x \in(p-\delta, p+\delta) \cap I . \\
& \Rightarrow\|(f(x), f(x+))-(f(p), f(p+))\|<\varepsilon \quad \forall x \in(p-\delta, p+\delta) \cap I \\
& \Rightarrow\|(f(x)-f(p), f(x+)-f(p+))\|<\varepsilon \quad \forall x \in(p-\delta, p+\delta) \cap I \\
& \Rightarrow \max \{|f(x)-f(p)|,|f(x+)-f(p+)|\}<\varepsilon \quad \forall x \in(p-\delta, p+\delta) \cap I \\
& \Rightarrow|f(x)-f(p)|<\varepsilon \quad \forall x \in(p-\delta, p+\delta) \\
& \Rightarrow \quad f \text { is continuous at } p .
\end{aligned}
$$

Thus if $\psi_{f}$ is continuous at $p$ then $f$ is continuous at $p$.
Conversely suppose that $f$ is continuous on $I$.
Then $\psi_{f}(x)=(f(x), f(x)) \quad \forall x \in I$.
Hence $\psi_{f}$ continuous on $I$.
4.2 Proposition: Let $\mathbf{B}=\left\{\Psi_{f} / f \in \mathscr{L b}^{+}(I)\right\}$. Define $F: \mathscr{Q b}^{+}(I) \rightarrow \mathbf{B}$ by $F(f)=\psi_{f}$. Then $F$ is a one-to-one continuous multiplicative linear mapping from $\mathrm{Qb}^{+}(I)$ onto $\mathbf{B}$.

Proof: Clearly $F: \mathscr{C b}^{+}(I) \rightarrow \mathbf{B}$ is surjective.
For $f, g \in \mathscr{Q b}^{+}(I), \psi_{f+g}(x)=((f+g)(x),(f+g)(x+))$

$$
\begin{aligned}
& =(f(x), f(x+))+(g(x), g(x+)) \\
& =\psi_{f}(x)+\psi_{g}(x) \quad \forall x \in I
\end{aligned}
$$

Hence $\psi_{f+g}=\psi_{f}+\psi_{g} \quad \forall f, g \in \mathscr{C b}^{+}(I)$
$\Rightarrow \quad F(f+g)=F(f)+F(g) \quad \forall \quad f, g \in \mathscr{C}^{+}(I)$.
Let $c \in \mathbb{R}$.
Then it is easy to see that $F(c f)=\psi_{c f}=c \psi_{f}=c F(f) \forall f \in \mathscr{O} \mathscr{C}^{+}(I)$.
Hence $F$ is linear.
Also we have $\psi_{f g}(x)=((f g)(x),(f g)(x+))$

$$
\begin{aligned}
& =(f(x), f(x+))(g(x), g(x+)) \\
& =\psi_{f}(x) \psi_{g}(x) \forall x \in I
\end{aligned}
$$

Hence $F(f g)=\psi_{f g}=\psi_{f} \psi_{g}=F(f) F(g)$.
$\Rightarrow F$ is multiplicative now we prove that $F$ is $1-1$. For this, suppose that $F(f)=F(g)$
$\Rightarrow \quad \psi_{f}=\psi_{g}$
$\Rightarrow \quad \psi_{f}(x)=\psi_{g}(x) \quad \forall x \in I$
$\Rightarrow \quad(f(x), f(x+))=(g(x), g(x+)) \forall x \in I$
$\Rightarrow \quad f(x)=g(x) \quad \forall x \in I$
$\Rightarrow \quad f=g$.
Hence $F$ is $1-1$.
Suppose that $f_{n} \in \mathscr{C b}^{+}(I), n=1,2,3, \ldots$, and $f \in \mathscr{C b}^{+}(I)$.
Let $f_{n} \rightarrow f$ uniformly on $I$. Then for a given $\varepsilon>0$ there exists an integer $N>0$ such that $\left|f_{n}(x)-f(x)\right|<\frac{\varepsilon}{3}$ for all $n \geq N$ and all $x \in I$.

Fix $x \in I$ and $n \geq N$. Since $f_{n}$ is ${ }^{+}$Quasicontinuous there exists a $\delta_{1}>0$ such that $\left|f_{n}(t)-f_{n}(x+)\right|<\frac{\varepsilon}{3} \quad \forall t \in\left(x, x+\delta_{1}\right)$.

Since $f$ is also ${ }^{+}$Quasicontinuous at $x$, there exists a $\delta_{2}>0$ such that

$$
|f(t)-f(x+)|<\frac{\varepsilon}{3} \quad \forall t \in\left(x, x+\delta_{2}\right) .
$$

Put $\delta=\min \left\{\delta_{1}, \delta_{2}\right\}$. Then for $t \in(x, x+\delta)$ and $n \geq N$,

$$
\begin{aligned}
\left|f_{n}(x+)-f(x+)\right| & =\left|f_{n}(x+)-f_{n}(t)+f_{n}(t)-f(t)+f(t)-f(x+)\right| \\
& \leq\left|f_{n}(x+)-f_{n}(t)\right|+\left|f_{n}(t)-f(t)\right|+|f(t)-f(x+)| \\
& <\frac{\varepsilon}{3}+\frac{\varepsilon}{3}+\frac{\varepsilon}{3}=\varepsilon .
\end{aligned}
$$

Hence $\left|f_{n}(x+)-f(x+)\right|<\varepsilon$ for all $n \geq N$ and all $x \in I$.

$$
\begin{aligned}
n \geq N \Rightarrow\left\|F\left(f_{n}\right)-F(f)\right\| & =\left\|\psi_{f_{n}}-\psi_{f}\right\| \\
& =\sup \left\{\left\|\psi_{f_{n}}(x)-\psi_{f}(x)\right\| / x \in I\right\}<\varepsilon .
\end{aligned}
$$

Hence $F$ is continuous on $\mathscr{C b}^{+}(I)$.
4.3 Proposition: The set $\mathbf{B}=\left\{\psi_{f} / f \in \mathscr{Q} \mathscr{C}^{+}(I)\right\}$ is a commutative Banach algebra with identity $\psi_{e}$ under the norm defined by $\left\|\psi_{f}\right\|=\sup \left\{\left\|\psi_{f}(x)\right\| / x \in I\right\}$, where $\psi_{e}(x)=(1,1) \quad \forall x \in I$.

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## A Scientific Theory of Destiny

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Abstract - Destiny is the most complicated subject in our world and cannot be comprehended without taking a global view in both space and time. Therefore this paper takes a multi disciplinary approach using mathematics, physics, engineering, economics, and philosophy. The paper has two major related objectives (A) Provide a theory for the existence of destiny and (B) Explain away the counter logic.

For part A we take the following approaches. (a1) The idea of destiny is well recognized by Newtonian physics. In our universe everything happens because of simultaneous interactions of everything. Thus destiny is not an individual concept. It is not my destiny or your destiny, it is our destiny. (a2) From this global concept of our destiny, we derive how the destiny plays out locally for an individual; how humans are really part of nature, work in cooperation with nature, and according to its dictates. In particular we show that our first reaction to every natural event is planned. (a3) Reincarnation theory of soul is an integral part of destiny theory. We prove giving real life examples, that reincarnation is a law of nature, and we are all reincarnated souls. Thus destiny is not only global in space; it is also global in time, covering all past lives.

Keywords : uncertainty principle, finite fourier transform, finite laplace transform, dynamical systems, central bank, destiny, reincarnation, eternal recurrence, simultaneity law.

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# A Scientific Theory of Destiny 

Subhendu Das


#### Abstract

Destiny is the most complicated subject in our world and cannot be comprehended without taking a global view in both space and time. Therefore this paper takes a multi disciplinary approach using mathematics, physics, engineering, economics, and philosophy. The paper has two major related objectives (A) Provide a theory for the existence of destiny and (B) Explain away the counter logic.

For part A we take the following approaches. (a1) The idea of destiny is well recognized by Newtonian physics. In our universe everything happens because of simultaneous interactions of everything. Thus destiny is not an individual concept. It is not my destiny or your destiny, it is our destiny. (a2) From this global concept of our destiny, we derive how the destiny plays out locally for an individual; how humans are really part of nature, work in cooperation with nature, and according to its dictates. In particular we show that our first reaction to every natural event is planned. (a3) Reincarnation theory of soul is an integral part of destiny theory. We prove giving real life examples, that reincarnation is a law of nature, and we are all reincarnated souls. Thus destiny is not only global in space; it is also global in time, covering all past lives.

For part B we cover several important opposing views. (b1) The uncertainty principle of quantum mechanics says nothing can be certain, and therefore destiny cannot be true. We point out that the core idea behind Heisenberg's original mathematical proof is based on Fourier Transform theory. Fourier Transform uses infinity, which as we show is not meaningful in nature. Replacing infinity by any finite number changes the uncertainty to certainty, just like the Newtonian physics. (b2) Destiny is not in the mainstream, therefore it must be wrong. We show that it is the opposite. All Mathematical and physical theories have assumptions that are invalid in nature and therefore they cannot describe nature. These theories are not approximate theories, they are outright wrong. As a natural consequence, the real life engineering which uses these incorrect theories is full of kludges and patches. (b3) We must also understand money, money power, and the way it is controlled by the Central Bank. It is the same money that remote controls every activity and the mindset of every individual in our world. If anyone anywhere is allowed to find the truth on any subject, including destiny, then people will eventually discover the truth behind the central bank, which cannot be allowed. The central bank is a very secret private organization, cannot be controlled by any government, as is well known in economics. In the hierarchy of all activities, however, the soul theory and the laws of nature are at the top, above the central bank. Keywords : uncertainty principle, finite fourier transform, finite laplace transform, dynamical systems, central bank, destiny, reincarnation, eternal recurrence, simultaneity law.


## I. Introduction

Everything in the universe is simultaneously and continuously interactive. That is, nothing is isolated in nature. Therefore true knowledge cannot be acquired by isolating objects from nature and then investigating these isolated objects. As an example, we show that Newton's first law of motion assumes isolated environment and therefore fails to characterize the nature. All things must be considered simultaneously and in their natural environment. Any attempt to create controlled and artificial environment will destroy natural environment and subsequently hide the truth also. The truth is always unique and universal and it can come only from nature. Because of limited space in page size a

[^1]complete and full treatment of the subject is not feasible; so a comprehensive overview is presented covering many integrated angles.

Destiny is a very well recognized concept in all of Asian countries, particularly in India. However, it is almost an unbelievable subject and highly debated in the western countries. Somehow people have associated science and technology against all kinds of natural philosophies including the concepts of reincarnation, soul, and destiny. The objective of this paper is to show that both destiny and reincarnation are laws of nature and has nothing to do with god, religion, philosophy, math, science, and engineering (MSE) of any country. They were universal truth billions of years ago and they are still true now.

Take for example computers and internet. Do they invalidate laws of nature, no, they cannot. Whatever we were doing fifty years back using paper and pencil we are doing the same thing using our computers, only at faster rates. Same is true for application of computerized technology in our daily life. We have replaced a maid by a washing machine. We have replaced mechanical controls of automobile by electronic microprocessors. We can go to moon, to a star, but none of these will change anything about nature or about our mind, body, and soul. Our body is created from nature, including its brain. Our soul is part of nature too. No amount of technology can change any of these natural things and their laws.

Assume that we are watching a projected slide on a screen on the wall of a dark room. If we now open all the doors and windows of the room then the daylight will fade away the slide content on the wall. But the content will still be there and a careful observation will still make it visible. Our science, engineering, and math can only distort our views of nature, or will prevent us from observing them, but the natural truth about humans, its soul, and the laws of nature will remain same as they were for billions of years. A careful investigation using yogic power or highly concentrated mind will make them visible. Technology cannot and does not change them.

It must be realized that our present society is completely controlled by money and the money power, which are controlled by the central bank. Money is not an object of nature; it does not follow the laws of nature. Therefore money is artificial. Using an artificial object, real things of nature cannot be investigated and understood. This artificial society cannot change our relations with nature which are invariant and eternal. However, we show that this artificial society is trying hard to hide the truth. We can make our people think and work like mechanical objects and prevent them from listening to their souls. But our souls always know the truth, as we explain.

Many times we talk about statistical analysis. We think as if everything is probabilistic and therefore propose that things must be examined using statistical theory. A philosophy professor [1] says if hundred people jump from empire state building at New York, and if only one person walks out clean and clear then we must realize that there is something that must be correct, and we must do research to find out how he did it. We cannot throw away this data point as a statistically insignificant item or an outlier. The truth does not lie in statistics, truth lies in miracles of nature, miracles of human capabilities, i.e. the yogic power, that our mainstream has never investigated. Yogic power can be acquired through yogic meditation. But to prove the existence of yogic power nature occasionally gives us children born with specific and limited kind of miraculous yogic power as we give examples later.

In this paper we will occasionally refer to scriptures from India. Thus a very important observation should be made here. Hinduism is not a religion. It does not require its followers to regularly gather at local temples. There are no preachers for preaching the
theories at regular intervals. There is no hierarchy of ministers, there are no money transactions. There is no conversion process, nor excommunication method. One of the most revered Hindu philosopher of modern times Vivekanada [2] said there is no god in Vedas; he said - down with gods, down with God himself, I am all the God that ever existed, speak no more of God, I am the infinite. Essentially he says that the soul of god and your soul are same. We are all gods and therefore there is no god. Hinduism is all about - yoga, yoga meditation, yogis, and yogic power; it is all about the laws of nature like - reincarnation, soul theory, destiny, karma, sruti etc. You should not get confused with the mainstream Hinduism. It appears that by introducing God, the mainstream Hinduism has deprived the general population of the universal truth in the scriptures. Vedic theories, Gita, Ramayan, and Mahabharat can all be perfectly explained without god; they represent many scientific theories of nature as we explain in this paper. A text is a scripture if it is based on Sruti, that is, realized using yogic power only.

In addition we may need to have a reference time scale of Indian scriptures. According to latest archaeological discoveries of underwater cities, ocean water level simulation studies, satellite imaging, the time for Mahabharat, the last scripture, is more than 10,000 years old. The referenced video summarizes the above conclusion [3]. There are many such references on the internet, including History channel, BBC news etc.

## Contents

The paper is organized in the following way.

- Mainstream: money power, science power
- Characteristics of nature: global plan, simultaneity law, embedded engineering systems, universal truth, boundedness law, finite time law, engineering experiments.
- Reincarnation law: proof of reincarnation, reincarnation examples, hypnosis, reincarnation and destiny, theory of reincarnation, predicting destiny, eternal recurrence.
- Interconnected souls: yogic power.
- Heisenberg and destiny: Assumptions in quantum mechanics, finite Fourier transform.
- Newton and destiny: sigma law, global destiny, individual destiny, examples.


## iI. Destiny Definitions

There are many definitions of destiny known to us. All of them however indicate the same concept. That means we all understand destiny basically in the same way.

A very common statement in the west is - "As you sow, so you reap". This clearly says what you are doing now is a result of what you did in the past. This is same as: past defines the present or similarly the present defines the future. Thus destiny idea is built into our vocabulary, we do not control the present; the past does. Yet we always insist we have freewill. Freewill and destiny are opposite concepts. If destiny exists then freewill cannot. If the past dictates the present, then we cannot have freewill at the present time.

Merriam Webster dictionary meaning of destiny is: "a predetermined course of events often held to be an irresistible power or agency". Since it is predetermined, that means we do not have any freewill also. Every moment of everything that we do is completely predetermined. It has been written in a physics book [4, p. 46] "Events do not happen; they are just there, and we come across them".

There is a general understanding that destiny is major events in life, which matches in some sense with the definition of Merriam Webster dictionary. And all other things are not predefined and we have freewill for such activities. However it is clear that if a major event happens at time $t$ then at time t -dt, where dt is a small time, something happened that created the major event. Thus activities at t-dt are also destiny. Inductively, this shows that every activity is part of our destiny. The time dt can be few milliseconds like 200 to 500 or may be several years and even decades.

There is another way of thinking, which essentially is same as above. Destiny says everything happens because of some reasons; alternatively, nothing can happen without a cause. Since this reason happens first, then this reason is dictating your action. Thus you are not acting on your own freewill; you are acting based on your own reason, which happened few seconds before. This cause and effect, action and reaction, or reasons etc. are all same thing and they define the destiny. Therefore (a) action-reaction (b) cause and effect (c) reasons (d) past activity (e) irresistible power are all same and can be considered as the result of our past activity. We cannot get out of it and we are helplessly tied by our past. This past can be even less than a second of time, like a moment.

Libet has experimentally found out [5] that we get a ready potential signal (RP) in our brain about 550 milliseconds before we take the action. We also get another awareness of wish to act signal (W) about 200 ms before the actual act. Thus stimulus RP is already present before we know the W signal. This can be interpreted as: before you know it, your soul has talked to you, and your body will naturally follow it, because you are the soul.

Libet's this experiment was not completely conclusive because of veto capability [5], where the subject could abort the action after becoming aware of the action. Under veto condition his experimental system did not have the capability to record the brain signals. However, under a non-veto condition we find that nature or the soul told us to do something well before we became aware of it and did it.

Under this circumstance we can quote the professor "We inescapably live in the objective past so that the objective present and future are completely beyond our awareness and control" $[6, \mathrm{p} 76]$. Note that in Libet's experiment events were internally generated by the subject, they did not come from external nature. Even under this condition destiny will still prevail as we perform a careful analysis. It is hoped that someone in future will repeat the Libet's experiment with proper design and document the results.

Thus our destiny analysis will be based on what will happen in the next moment. We want to show that our reaction in the next moment is planned. The time interval in our analysis will be of the order of 200-500 milliseconds. It is clear though that the process can be continued sequentially to find that all events in our life are planned. We will show that following every event presented to us by nature, our first action within 200 milliseconds is just the opposite of the event and therefore our hands are tied. This is according to the definition (a) listed above, the action reaction law of nature.

Note that destiny does not mean - I did not do anything, destiny did everything for me. All the definitions of destiny are same and show that it is a consequence of action reaction law of nature. I do everything that nature has told me to do. Therefore concepts like morality, immorality, responsibility, and irresponsibility are meaningless, because as we show that destiny is a global plan. The same philosophy has been reflected in Gita also. There is nothing that is good or bad. There is nothing like north pole or south pole of a magnet. They are just two different faces of the same object. So is life and death.

## iil. Destiny Examples

We present several examples to show the action, reaction, and timing of the activities to highlight the facts about nature, and to show that we interact with nature, just like all other nonliving objects of nature. These examples will illustrate how we should look at our behavior and actions to understand that we do not and cannot have freewill.

Our thinking and analysis process must stop at the first reaction. We should not continue thinking about our next actions. That will lead to confusions only. We must always link an action-reaction pair with time gaps of less than one second to understand the non-existence of freewill.

We are reacting according to the events. The events are presented to us from nature, we do not have any control over them, and then we react to resolve the event. In this process we are alone and isolated as individuals. We must then provide equal and opposite reaction to the event.

## Example - 1

Many people have asked - whether I will drink coffee or tea now is dependent on my freewill. A little thinking will show that it is not based on my freewill. Therefore it is destiny. Here we use the definition - "As you sow, so you reap"; that is, past defines the present. Note that here past does not mean karma or ten years past but maybe one hour or even hundred milliseconds in the past.

First of all your choice will depend on what you have taken last time and how long time back. Your choice will also depend on the situation in the coffee room. If there is no coffee in the pot you may be forced to take hot water and a tea bag. When you are there, you may find your boss taking coffee, and so to be in tune with her, you may also want to take coffee instead of tea.

Every single moment you are changing according to the events presented to you by nature. These decisions happen so fast in your brain that you never realize that you are constantly changing your mind in response to events. Thus we are not using our freewill at any time, we are simply interacting, and there is always only one choice available to you. As Newton said: action and reaction are equal and opposite to each other. The Indian ancient scripture Samkhya says cause and effect are similar in nature.

You will make a fundamental mistake, if you think you are your body or your brain. You are your soul and your soul is communicating with all souls constantly and interactively and to you. There is abundance of evidences to prove that. This paper is on this soul theory. Ignoring soul theory is like ignoring central bank in the analysis of economic problems.

Example - 2
The following example shows how the global plan of a corporation emanates events sequentially to us and we just have no freewill but to respond according to the needs of the events. Inside a corporation there cannot have any freewill for anybody. Money power has purchased us to act and behave exactly the way the corporation wants. If you do not obey your boss you will lose the job and face the economic consequences.

You are in your cubicle working intensely on a software programming problem. A system test engineer comes to you and knocks your door. You respond and say hello. So far, we see these acts are just equal and opposite reactions. Test engineer asks which file has voltage limits for the relay number one. You think little bit and try to remember. Again these are all sequential events following exactly the Newton's action reaction law
(27). You cannot use any one of your options; you do not have any choices, but to respond correctly. For, you are working on a corporation under a global schedule in which you have participated. You are constrained by the simultaneity law of all activities of the corporation. You tell the file name relay limits.h in the include directory. The engineer thanks you and leaves the office. This is a straight forward simple case of sequential events emanated, naturally, from the corporation's global activity of test plan and product design.

## Example - 3

Here you are the CEO of a corporation. VP of sales comes to your office and proposes a company labor strike, to prevent the disgrace and financial setback that the planned export may cause, if the defective product lot is shipped this week. Both realize the situation, but the CEO authorizes the VP to secretly carryout the labor strike plan.
This is the movie plot of the Oscar wining director Satyajit Ray in his 1974 film Company Limited (Seema Baddha) [7]. Here we can see although immoral, illegal, but still agreed upon by the CEO for the greater good of the corporation. Again the events happened sequentially, but it was part of a preplanned global activity by the VP, that the CEO had to agree without any options. There are many major world events that have happened because of similar constraints. You can see that money played a very important role in the above example. Without money things could have been different.

You can always say I would have done it in a different way. But remember that another person will also say the same thing after learning about your experience. The events, the environment, your characteristics, and time to react will determine how we react. Since they are all different for every single one of us, we all react differently.

Things happen in real life in a completely different way than what we can imagine sitting in our office. Our imagination can produce many examples to prove that destiny is false. But our imagination will always fail to explain how and why we all behave the way we do. We will also fail to explain the miraculous events that are happening everywhere, all the time in our world. This is true for all real life reincarnation, yogic power, and destiny examples available abundantly for us to examine. Many such examples are discussed in this research paper.

The fact is every example that is happening in our world is part of destiny and can be proven to be so. All we need to show is that there is a reason behind it, which is the definition of destiny.

## IV. The Mainstream

People have a tendency to believe and say that if it is not in the mainstream then it must be wrong and should be considered as suspicious. We want to show that, on the contrary, the mainstream is in general wrong.

The mainstream is the activity where most of the population is involved. There is only about $1 \%$ of the population who thinks differently and performs differently, but still maybe involved in the mainstream activities to earn their livelihood. These are the Galileos of our time. Our analysis in this section will be focused to show that this $1 \%$ only knows the real truth in all aspects of our lives. If you do research, and if you are lucky, you will be able to find researchers in this $1 \%$ group and encounter the real truth. Note that like Galileos, their researches are very thorough and astonishingly convincing.

The reason of this is quite simple and obvious - the money power. We have been purchased by this money. Everybody in this world is controlled by the same money coming from the same source, the central bank. We cannot escape the remote control

## $\mathrm{R}_{\text {ef }}$

capability of this money power. Therefore we must understand the concept, organization, activities, and the objective of this money power to understand why the mainstream is wrong in mathematics, physics, economics, reincarnation, and destiny. Without such an understanding it will not be possible to remove our colored eye glasses and get a rational view.

## a) Money Power

In this subsection we discuss the concept of money power, the owners of this power, its remote control capability, its pervasive characteristics, and how helpless we are under its control. It plays a very important role in our destiny.

## i. Objective of Money Power (MP)

Money is the most important control variable in our society. The same money is used everywhere in the world. It is used in our education, business, research, health care, manufacturing of food, philosophy, and religion. No one can escape this remote control capability of the money power (MP). The fundamental control objective of money is that no one should know or find out the truth of anything - including MP. As Carroll Quigley [8, p. 465], a professor at Georgetown university and a mentor of US President Bill Clinton, says Britain has two groups: classes and masses. In his book he said education for classes has three negatives and one of them is that it must not find the truth. It appears that the same philosophy has now spread all over the world for all groups. We show with examples that, it is indeed the case in our time in history.

## ii. Central Bank (CB)

Most of us do not know that the source of money is the central bank (CB). In USA it is the Federal Reserve Bank (Fed). CB is a privately owned bank and is a very highly secretive organization. In some countries CB may be government owned, however, the money supply and control function of CB is still in private hands. Only CB can print money, and give it to anyone it wants, by any amount, and without any transparency and accountability. In 2008 the Bloomberg News sued the Federal Reserve claiming that the Fed paid secretly $\$ 12.8$ trillion dollars for bailout money [9]. The GDP for USA was $\$ 14.2$ trillion at that time. Thus money supply is unlimited and free for the CB.

Money is not an object of nature. Money is not the paper or the coin, it is the value associated with it. Thus we cannot see, touch, and feel anything about money. So money, not being an object of nature, cannot follow the laws of nature too. Therefore money has to be false, free, and abundant. Unless we understand this real nature of money, it will be difficult to understand economy and the money power; and their influence in mathematics, physics, and destiny.

Everything in nature is given to us as free. How can then something have a price? Thus we have distorted our society, created false money, and then using this false money the CB is controlling every activity of everybody. It has put a price tag for every object. A land, a tree, water, an animal, a human being, our work, everything has been assigned a price tag.

None of us can do anything against the objectives of the CB, and we all know this very well. That is why the Nobel Laureate in economics, Milton Friedman, said: "One unsolved economic problem of the day is how to get rid of the Federal Reserve" [10]. So naturally the goal of CB will be to prevent the discovery of truth. That is, if people find the real truth of any subject, then eventually people will discover the real truth about CB, which must be prevented. However, as mentioned before, $1 \%$ of the population, in any subject area, always knows all the universal truths.

A past chairman of the Fed, Alan Greenspan, [11] said no government has any power over this bank. CB is very powerful. It controls our government, military, CIA, FBI, Police force. CB is the King of our time. It is the most powerful king that the world has ever seen in its entire history. It rules the whole world using its MP. At one time, during imperialism, the world used to be controlled by the philosophy and the objective of the king. Now it is controlled by the philosophy and objective of the CB. The government cannot control the CB; the CB controls the government by controlling the supply of money to the government. More technical details of these ideas, facts, data, references, and information from publicly available and well authenticated research are collected and provided in the economics paper [12].

## iii. CB and Global Plan

It should be recognized that CB has done an extremely good job all over the world. Whatever good things have happened today in our world has happened because of the CB. All wars were financed by banks and therefore by the CB, including the WW2. Today two billion people of India are free because of WW2. US presidential candidate Patrick Buchanan [13, p.295] of republican party wrote - "As it was, Britain was dragged into an unnecessary war, which cost her nearly 400,000 dead, bankruptcy and the dissolution of the British empire." CB helped to destroy slavery from USA, apartheid from South Africa. Today we do not see any king anywhere. Most of the dictators are also gone. After implementing globalization by USA, national boundaries are vanishing, like in European Union. The law of reincarnation says: human souls do not have nationality, race, and religion. Any soul can take rebirth anywhere in the world. All such credits go to central bank and its money power.

After the 9-11 attack in New York, USA started the Afghanistan and Iraq Wars. These two wars created the immediate opportunity to install the CB in Afghanistan and Iraq. Only two more countries, Iran and North Korea, do not have western style CB. Once they are taken over, CB will have the full control over the world, and will truly become the global king. After that is achieved, and the world is stabilized for some length of time, it will be worth seeing what CB has in its global plan. According to simultaneity law this global plan has been created by many people from many different corners of the society, however, basically under the supervision of the CB. We must take a global view to understand the CB and the destiny of our world.

If money is removed from the world by CB, then that move will bring heaven on earth. Because of the modern networked computerized technology, it will be possible to create a moneyless economy [14], which will run this exact same economy we have now, in the exact same way, without any kind of money, and yet will provide the lifestyle anyone wants. In the meantime however, the world is suffering significantly due to poverty, and will continue to suffer, all because of CB . In the following subsections we track the root causes for this manmade nature of our global destiny.

## iv. Printing Money

Assume that the total material wealth of a nation (GDP) is equivalent to a pot of gold. Similarly assume that the total money available (M3) is a bag of dollar bills. This bag of money has an one to one relationship with this pot of gold. This bag of money was used to create that pot of gold. By the law of conservation (LOC) this pot of gold cannot grow. Therefore this bag of money cannot grow either, because this is equivalent to that pot of gold and is attached to it by the LOC.

As pointed out by the Bloomberg report [9], we know that the Fed can print another bag of money, out of thin air. This extra bag doubles the price of the pot of gold
causing inflation. If this money is allocated only to the top fifth of the population then their share of the pot of gold will increase, changing the wealth distribution. According to the LOC the amount of gold cannot increase, the share of the bottom fifth will then naturally decrease causing transfer of wealth. This transfer of wealth happens not only because the money share decreases for the bottom fifth, the price of gold also increases, and thus reducing their purchasing power.

## v. Transfer of Wealth

Profiting is one of the pillars in the foundation of capitalism headed by the CB. Let us first examine how profiting is against the law of nature and how it helps to transfer your wealth.

Consider a company that manufactures writing pens at a total cost of $\$ 5$ per pen. Now assume that the company sells it at $\$ 15$ per pen making a profit of $\$ 10$ per pen. Thus the cost of the pen for the CEO of the company is $\$ 5$ but for you and me it is $\$ 15$. Therefore every pen is taking away an extra $\$ 10$ from all of us. This process of profiting helps to take money away from the people and to accumulate it in the hands of a very small group of people causing inflation and transfer of wealth. Thus profiting is basically cheating, we all understand that, but cannot do anything to stop it.

The interest charging process is essentially the same activity also. These capabilities of CB violate the law of conservation, which says there is no win-win situation possible. For every win-win situation there will always be a third party who will be the loser. This third party is always the bottom fifth. Thus accumulating paper money is equivalent to transferring material wealth.

Both profiting and interest charging were immoral according to major religions. But now because of MP it is not only legal it is encouraged also [15]. Similarly, creating recessions are major ways to transfer wealth from bottom fifth to upper fifth of the population. Many people will become unemployed in recessions, there will be no jobs, and the unemployed will lose their properties, which will be purchased by rich people at low price, causing large scale transfer of wealth. Thus the activities of CB implemented via capitalism like: profiting, interest charging, giving higher salary to a small group, creating recessions, printing free money, etc. are designed to create classes and masses or rich and poor. Thus money is not only an artificial and free object, it is also used to spread poverty.

## vi. Reincarnation

Capitalism measures people using money. Since money is false, this merit system has to be false also. In business two jobs are never equal and so no two jobs can be compared. Therefore two persons working on those two jobs cannot be compared also. Two persons have entirely different life trajectories based on past life reincarnations. Since past life experiences are playing important role in our present lives, as shown by examples of reincarnations, we cannot compare two persons using only present life. Thus recognition of reincarnation will create fundamental problems in the merit system promoted by money power.

The theory of capitalism requires "you only have one life to live". You will be judged and paid according to your achievements in this one and only one life. So, you make money anyway you can to live better. In Wall Street it is called IBGYBG, I will be gone you will be gone [15]. So remain quiet, obey the business, take money, and help CB. All these principles of capitalism like - I control my life, I have freewill, I am the boss etc. - are against the law of reincarnation and destiny. The discovery of truth behind destiny and reincarnation therefore must be suppressed.

## vii. Nishkam Karma

Karma means work and Nishkam means selfless work. The objective of any human being should be to perform his work in a selfless manner. Only then it will be possible to provide best performance to the humanity. Any amount of selfishness will hinder the search of true knowledge and will prevent proper application of that knowledge for producing high quality work. In most of our activities time is used as a tool to prevent us from performing nishkam karma. In USA it is a very common statement that - we never find time to make things right the first time, but we always have enough time to fix the problems created during the first time. Thus time is an artificial constraint, which prevents discovery of truth using perfect and nishkam karma.

The way of capitalism is to make more and more profit. Central bank will give you more and more money if you make more and more profit. Thus exploit people, product, and services as much as you can to make profit. This is promotion of selfishness. Thus capitalism inherently prevents manufacturing of quality products; a fact which we see abundantly around us. In addition this more and more money makes us loyal to the CB, afraid of losing job, increases insecurity. And we become part of money power and extend its abuse. This also creates a larger and larger group of masses and smaller and smaller group of classes. These activities are against the knowledge of yoga, meditation, nishkam (selfless) karma, reincarnation, and destiny; which are all laws of nature. The laws of nature represent the universal truth. However, they shake the foundation of capitalism, and money power. Both of which are controlled and managed by the CB.

## viii. Suppress the Truth

Keynes [16, p. 235-236], a father of economic theories, said - "By a continuing process of inflation, governments can confiscate, secretly and unobserved, an important part of the wealth of their citizens. By this method they not only confiscate, but they confiscate arbitrarily; and while the process impoverishes many, it actually enriches some. The process engages all the hidden forces of economic law on the side of destruction and it does it in a manner which not one man in a million is able to diagnose". We should recognize that the government is controlled by the CB . The government is not an independent body; it cannot function without money supply, which is managed by the CB. Note also that, the above quotation confirms the idea of - do not find the truth only one in a million may know. For the survival of CB , all truth must remain suppressed and should not be discovered. Universal truths of nature therefore cannot be known from these main stream activities.

If you pick any modern macroeconomics text book published in USA, you will find that it never mentions that

- CB is a private bank and the source of MP
- It cannot be controlled by government
- CB controls the government by controlling the supply of money
- CB is not transparent, not responsible to anybody
- Money is free for CB and its supply is abundant.

Thus no economics student, including professors, will ever discover the true nature of CB, and money power, without ever taking a special research and investigation. This shows why only $1 \%$ of the economic community knows the real truth.

The truth is always unique and universal. That means, something that is true in USA but is false in China cannot be an universal truth. Similarly, the concept that was
true thousand years back all over the world but is not true now also cannot be an universal truth. Only all laws of nature are unique universal truth all over the world and for all time. Anything that violates the laws of nature must be false and should be removed from the design of our society.

## b) Science Power

Science is also a major force against discovery of all real truth including destiny and reincarnation. People believe that science is correct, and think that reincarnation or destiny therefore does not have any scientific basis. However, we show here with some examples and logic that science is wrong, it does not and cannot work, and therefore there is no concept like scientific basis. Money Power (MP) cannot allow science to find any truth. The CB has heavily invested in math and science and has created the science power to keep people away from real truth. As we have mentioned money is false; how can then a false thing be used to find the real truth? You cannot. Science and math have to be false.

As a matter of fact, it is not necessary to understand science, to know that it is wrong. We can easily verify that it has to be wrong, from a philosophical point of view also. You have to accept only one truth: all math and science theories have assumptions. And it is obvious that nature does not and cannot make any assumptions. Therefore math and science are contradictory to nature, and so they are all false. However, we give two examples from physics and mathematics to illustrate the details. A careful observation will show that assumptions do not make science and math approximate; they make them outright wrong as we show here.

Consider the example of Newton's first law [17, p.115], a very well known and popular law that we have studied in our high schools. This law has an assumption that says - "in the absence of any external forces". Let us investigate this assumption; if this assumption is wrong then we can say that the law must be wrong too. Clearly this assumption cannot be true, because there are always gravitational forces due to sun, earth, moon and many other heavenly objects of nature. There is no place in near earth and in deep space where there is no gravitation. Since earth is rotating, there is also a rotational force called Coriolis force. Thus the above assumption is wrong.

Obviously, nature does not and cannot make any assumptions in describing its laws. The nature must be taken as is. Therefore the nature's laws of motion cannot also have any assumptions. Thus the Newton's first law cannot be correct, because it comes with an assumption. Newton started with something that was misleading and confused all of us for centuries. The following statements, confirming our assertion, can be found in a physics textbook, $[18$, p. 8$]$, about the Newton's first law: "We could hardly sustain that this principle (First law) is a strict experimental result. On the one hand it is not evident how to recognize whether a body is free of forces or not. Even if a unique body in the universe were thought, it is undoubted that its movement could not be rectilinear and uniform in every reference system".

It is very easy to verify that if you leave a ball in deep space, it will not remain stationary as predicted by the first law. The ball will start moving and will change its speed as the time varying gravitational force changes. Thus the assumption did not make the law approximate, it made it wrong.

Every theory of physics and mathematics has an assumption about nature. Since nature cannot make any assumptions, all of these theories are wrong. We have explained it in the case of Newton's first law as an example. Many such examples are described in
the paper [19]. This example confirms the theory that: education should not find the truth and is consistent with the objective of money power. Moreover, it is not possible for humans to understand nature. Humans are created by nature. Creation cannot know or understand the creator. For similar reasons, our computers cannot know humans, for example. At some point in the chain of questions on every subject, we have to accept the laws of nature without any explanation.

Now we illustrate the falsity of mathematics with one example. Replacing infinity assumption by a large number does not make math approximate, but makes it completely wrong. Consider the well known mathematical theory of Laplace Transform (LT) which is widely used in engineering and physics. Laplace transform theory is defined by (1):

$$
\begin{equation*}
F(s)=\int_{0}^{\infty} e^{-s t} f(t) d t \tag{1}
\end{equation*}
$$

In the expression (1) the variable $t$ is usually considered as time. It shows that the time must be valid from zero to infinity. The function $\mathrm{F}(\mathrm{s})$ on the left hand side, called infinite Laplace transform (ILT), will be valid only for infinite time system. Now infinity is an assumption in nature, because nature is always a finite time system, as we have discussed.

Let us consider an example with finite time T to bring out the fact [20] that the Laplace transform is based on infinite time assumptions, that is, it cannot be used for finite duration signals. The finite duration step function $f(t)$ is defined by

$$
f(t)= \begin{cases}1 & 0 \leq t \leq T  \tag{2}\\ 0 & \text { otherwise }\end{cases}
$$

Using the definition (2) we get the expression for the finite Laplace transform (FLT):

$$
\begin{align*}
\mathcal{L}_{T}(1) & =\int_{0}^{T} e^{-s t} \cdot 1 \cdot d t \\
& =\frac{1}{s}-\frac{1}{s} e^{-s T}  \tag{3}\\
& =\frac{1-e^{-s T}}{s} \tag{4}
\end{align*}
$$

We can see from (3) that the FLT has the standard ILT term $\frac{1}{s}$ and another expression involving $e^{-s T}$. The second term in (3) is zero only when $T$ is infinity. Thus if we use only the first part of (3), the ILT part, then we will implicitly assume infinite time situation for our finite time problem (2) and the Laplace model will not be correct for real engineering problems.

Observe from expression (4) that the FLT does not have any poles at the origin, but the ILT has. At $s=0$ the expression (4) takes $0 / 0$ form. Thus by using the first part of (3) for finite time engineering problems we artificially inject poles in the models. The entire Laplace Transform theory must be revised and rewritten for applications in finite time engineering [21].

Similarly, data analysis that we have done using infinite time Fourier theory could be wrong also [22]. If you analyze this way then you will find that all mathematical theories will be invalid under all laws of nature discussed in this paper.

The major problem of science (math and physics) is that it is not required to work. It works only on paper and pencil. When it is used in engineering we find that it does not work. This is because engineering, which uses natural components and natural materials, cannot make assumptions also, and must automatically follow natural laws. Engineers add lot of patches, kludges, and redundancies, to make the science work [19]. When you add patches, then the science is not a theory anymore, it is a patch work. Because engineering cannot make assumptions, engineering automatically eliminates all assumptions from science, making science invalid, because these theories work only under their assumptions. Science is used by CB to maintain the power of money. In this sense science indirectly prevents recognition or discovery of destiny law and all associated truths, like reincarnation.

Many of our famous inventors, who changed the world, like Benjamin Franklin, Thomas Edison, Wright Brothers, Bill Gates, Steve Jobs, were not even formally educated. Thus science education is not necessary, that is because they cannot be used in engineering and are all wrong as illustrated with the examples above. We should not throw away the ideas behind destiny and reincarnation by saying that they do not have a scientific basis, because there is no such thing as scientific basis. Science and education are controlled by money power. Therefore science carries the same objectives of the central bank - should not find the truth.

By not disclosing the real truth in the mainstream we are misleading the entire humanity. The society can be completely different, much more trustworthy, and peaceful if we all know the real truth. It is possible to run the exact same economy, in the exact same way we have now, without sacrificing any of our life styles, with complete disclosure of truth in all areas of life. Such a society can only be created by removing money from the world [14].

## V. Characteristics of Nature

To comprehend the problems of science and math we must understand some fundamental characteristics of nature and how they are used in engineering. We will see that the embedded engineering is a miniature replica of nature. This understanding will also allow us to get a feel for the long hand of the money and money power. We are all, in a very subconscious way, working as agents of money power, safeguarding our personal interests, and preventing the discovery of all real and universal truths.

In this section we discuss the following laws: global plan, simultaneity law, boundedness law, finite time law. We also show why testing of scientific theories is not feasible.

## a) Global Plan

There exists a global plan in our planet and our activities are part of that plan. Our galaxy is a giant spiraling structure with its associated motions in space. Our solar system is present in one of its branches and is moving along with this huge galaxy. The sun cannot go anywhere on its own; it must follow the course dictated by the global plan of this galaxy. While staying on earth we do not see the definition of our galaxy. As we go away from earth, in the deep space, the global plan of the galaxy begins to unravel its mystery, structure, and the journey. From this far away point in space we can get the glimpse [23] of the future path or destiny of our earth.

If we look inside a corporation we find a global plan that governs all activities of all employees. When we start a new project in the company, many employees sit together in a conference and create a global plan for the project. They together decide the things to
be done, assign a time schedule for each activity, assign people working on such activities, define how people will interface in creating their individual assignments etc. Once the plan is precisely drawn everyone knows exactly what they will do in the next day for several months, until the project is completed. During this period no one has any freedom to change their work plan, and why they would try to change, they would not, because they jointly created the plan. This corporation is also linked with other corporations in the same industry. Going farther outwards we see that the entire economy is working in a coordinated way under a global plan.

Same is true for our society. This global plan is created by the action-reaction law that started long time back and is still continuing and will continue forever. The reincarnation law says that we were there before. We came to this earth many times in the past and will continue to come again and again. The simultaneity law is also involved in this global plan. Things are happening because of simultaneous action and reaction of many people over global space and global time. These simultaneity, action-reaction, and reincarnation laws together create this global plan. Together they are generating all the activities in this universe. If we move far away, and look at the global history of our political and economical systems for past few centuries, then we will be able to see this global plan and then we can estimate its possible future.

We can see many trends of this global plan in the history of our world for last two three centuries. We have seen all kings have vanished from the world. Systematically one by one they were all removed. British Empire was destroyed by WW2, Mughal Empire from India was removed by British kings. Kings from Russia and China were also removed by communism. On the economic front we see how slowly banking system took over the world. They created huge money power that controls all governments, their militaries, intelligence services, and police force. We see that a monolithic central banking (CB) system has evolved with their owners as the new king of the entire world. This money power emanating from the CB controls entire world now. Our education, religion, medicine, food industries everything is under unified global control of this money power. On the political front we see some kind of parliamentary system has been installed in almost all countries. We can see also that none of these parliamentary systems has any power of their own. They are controlled by lobbyist sent by the money power. No member of these parliaments has any rights for secret ballot system in the management of their governments. Thus globally people lost their freedom to control their governments. We can see that an unified global plan has evolved.

At the individual level money plays an important role in placing us inside the global plan. The supply of money is controlled by the Central Bank (CB) as discussed. There is a huge Money Power (MP) associated with the CB. It is the same money that controls all activities of this world. It is a very coherent and unified force. Using the money supply the CB can remote control each one of us. By terminating us from our jobs, life can be completely changed. Thus we are all forced to participate in the global plan created by the CB and MP. Our destiny therefore is intimately tied with this global plan. It is interesting to observe that CB was evolving over centuries, and incarnation after incarnation new souls entered and left the CB and MP, and yet none in CB and MP did try to change its role. This probably shows that CB and MP did not enter its death process yet, that is the global plan has not reached its final matured form.

Unfortunately, because of the complexity of action-reaction law, lack of knowledge of our past life activities, and the simultaneity law we cannot precisely estimate the future. However, when we come to any point in the future time then we can look back
and see the global plan in the events of the past. Thus the existence of the plan is very definite but the prediction is not realistically definite. This observation is not just for our societies, it can be observed in our personal life also. Any event that appears meaningless or random now will appear meaningful and reasonable at some future time, when taken in global context.

If we look at things in local time, in local region, using local concepts and local philosophy, we will see only local activities and it will appear like noise, random, and with no plan or purpose. The global plan can be understood well if we view the entire world and over long period of time covering 3-4 centuries, just like the galaxy example. A meaningful unified theory can be detected in the global plan only when all major as well as all minor events of the world history can be explained using this unified theory. Sometimes whistleblowers, like [8,24], come out to reveal the inner workings of the power structures (CB and MP) giving us a great deal of the global view. These people were actually hired to reveal the secrets of power structures. There are some news channels like PBS in USA that reveal many secrets. Internet of course is the best resource. Mainstream will never give you the real truth. You have to look in the bushes and branch roads to find the universal truth.

## b) Simultaneity Law

A very important characteristic of nature is its simultaneity. All humans are interacting constantly, simultaneously, and all over the world and for all time. So is true with all physical objects. Nothing is isolated and nothing can be isolated.

A company on precision weight measurement system [25] uses the moon's gravity effect, as it travels over earth, to precisely measure the weight of a mass on earth. Thus simultaneity is global and not just local. This company's products show how complex and sophisticated our modern requirements are. Before we even realize, everything in engineering will be simultaneously integrated together just like our natural world is.

We cannot take earth out of its orbit and analyze it. Then it will not be earth any more. Similarly I am a completely different person when I am outside my home. Thus my characteristics are very strongly dependent on my environment. I will behave completely differently in a controlled environment. The situation will be same for any electron also. An electron out of its orbit is not an electron any more. These facts must be considered carefully when we use mathematics in describing our nature.

Since no two objects can be present at same place in same time, the impact of simultaneity law is different for different objects, including quantum mechanical particles. Thus no two particles can be compared. They have different characteristics. They all come from their own simultaneous environments. For the same reason, a particle taken out of its simultaneous environment is completely different. The simultaneity law implies there is no isolated environment. We show that all algebra, in particular, vector space, operator theory, Schwartz inequality, are designed for isolated environments only. Newton's first law failed because it violated this simultaneity law and assumed instead an isolated environment.

Thus if we want to add two vectors, the addition will be meaningful, only when the vectors belong to same class. But if we assign one physical variable to one vector and another physical variable to another vector then they cannot be added any more. All physical variables are different and have different characteristics. Thus we see that this law has immense implications for mathematics. It indicates that mathematics cannot even comprehend the objects of nature. This law essentially prevents us from using Schwartz inequality to two different quantum mechanical particles. Similarly, it prevents cascading
two operators, because output of one operator cannot be used as input to another operator. Operator inputs must have specific characteristics, once we use an operator, we have changed the characteristics of the output object, and this changed characteristics is invalid as input for the second operator. The simultaneous environments of inputs and outputs are different. An operator takes an object out of its environment, making it a different object.

Our math and science are not yet ready for simultaneity law. Most of the theories that we use now are more than 100 years old. Requirements, concepts, and philosophies of those simpler days are deeply embedded in those theories.

## c) Embedded Engineering System

To understand the limitations of mathematics compared to the characteristics of nature we should understand operating systems. The real time operating system (RTOS) implements the simultaneity law in engineering. It is a multitasking system that interacts with interrupts from external and internal sources. Basically RTOS is a collection of tasks where each one is designed to operate simultaneously. Their job is to monitor the environment and get the required data whenever the environment changes [26]. Many of our embedded systems interact with external computers via serial interfaces. In many cases these interfaces bring user commands also. These interfaces are constantly monitored by several tasks to reconfigure the system according to the changes in the environment. Thus simultaneity is built into all embedded systems just like in nature. Isolated environment or system or object is not meaningful in modern engineering. The output of any engineering experiment is thus a result of many such simultaneous influences.

An interesting result of simultaneity law is humans as an engineering product of nature. A person is described by the social and economic conditions of the place where he was born and raised. The person is also described by the values and culture of his parents. He is similarly defined by all his teachers, schools, and colleges where he got his educations. He is also the product of many simultaneous interactions of all his friends, colleagues, and children. Thus every person has many different characteristics and therefore no two persons can be similar and compared. Similarly the results of two different engineering experiments performed by same embedded engineering system cannot be compared. The results are unique and unequal, because an engineering system senses the environment.

Clearly, RTOS is beyond the scope of mathematics and physics, but it is an integral part of modern engineering as well as nature. Anytime a task switches from one to another, it finds the environment completely changed. When the task was in sleep mode, the simultaneity law worked and changed that environment. But our present mathematics and physics theories rely on the continuity of states from one task to another, but that does not hold under RTOS and all other laws discussed here. It is clear that most of our mathematical and scientific theories do not have any means to accommodate all the laws mentioned in this section. Thus all engineering experiments will remove all isolated environment based assumptions and make the corresponding theories fail.

Mathematics is designed based on the assumption of isolated environment. Nature never obeys that assumption. Thus it is completely unreliable to trust mathematics and think that math can define nature. Mathematics has no comprehension of nature at both galaxy and particle level in describing our nature. The real characteristic of the simultaneity law is truly mind boggling.

We cannot do anything in this world all by ourselves. Everything happens because of simultaneous interactions of all people over all past and present times. We can see it clearly as a result of globalization of our economy. We are all simultaneously influenced by the environment of the earth. Earthquake, tsunami, environmental pollution, financial recessions, all simultaneously affect us over all time including future time. We call the result of this law as the global space time (GST) effect. In a later section we show that all souls are also constantly talking to each other using some method unknown to us, but known to yogis of India.

It may appear that this global plan is controlled by some kind of supervisor. But that is not necessary, and may not be true. If you look at the global plan of a corporation, you will find not the founder, nor is the CEO controlling or guiding everything. CEO is getting ideas from his VPs, who in turn are getting feedbacks from his people. All activities are controlled by the simultaneous action and reaction of everybody. Nobody is making a single decision that is changing everything. That will violate the action reaction or cause and effect law. In the same way if you look at the global plan of our solar system, you will find that everything is created by the simultaneity law. RTOS actually functions in the above way. A VP is very much like a task in a RTOS.

A best example will be an electronic circuit board. Every component is doing its precisely defined work, all components are working simultaneously, and none of them are CEOs or VPs in the circuit board. Although, the components have different sizes, shapes, colors etc. but each one is important; if any one fails, the entire circuit board will fail or begin to malfunction. The embedded engineering system is a simpler replica of nature. Whenever we need to think about nature, we can imagine about an embedded engineering circuit board for analysis.

## d) Universal Truth

The truth is always universal and unique in nature. There cannot be multiple truths on any subject, in the sense that the truth is constant over time and over space. Thus it cannot be true that reincarnation only happens in the east and is not happening in the west. It must be happening everywhere and for all time. There are ample documentations on truth of reincarnations for all countries in the world. Reincarnation law is not based on religion, race, or nationality. Thus the reincarnation law cannot be invalid for Christian and Muslim religions and valid only for Buddhists. This is because reincarnation is a law of nature and therefore is an unique universal truth.

But humans will never be able to find the universal truth. Because of the simultaneity law and the reincarnation law, every event is a result of actions of many people from many different places and over many incarnations. It is not possible for an ordinary human being to know all the information given by expression (33). This information space is a multidimensional space, and every individual lives only in one of its very small subspaces, as is highlighted for some variables in (33). A part cannot know the whole. The global plan cannot be visible from local space time state. Universal truth and the global plan are thus intimately related.

Only yogis can know the universal truth. Only with the help of yogic power a human can directly communicate with the soul of anybody and acquire all information it wants. Only such meditation and yogic power allows one to discover the mystery of soul and other characteristic of the universe. It is quite a miracle that yogis around the globe discovered the reincarnation law more than ten thousand years before. This fact alone should convince us about the claims of yogic power. There are many examples of communicating with the soul by such yogis, even in our modern times [27, 28, 29]. The
yogic power cannot be understood by any other means. These experiences of humans show that ordinary person cannot find the universal truth. We have shown that math and science cannot also find the unique universal truth.

Observe that in one engineering sense we have captured a very important universal truth. Consider a digital microprocessor. It is an electronic integrated circuit that can perform miracles. It can solve all problems of the world. You give any problem, or any complex algorithm, a person will be able to convert it into C-programming language or native assembly language, and execute it on the microprocessor. This microprocessor does
know what it is solving, it is only executing its own CPU logic one by one in sequence, but you are seeing that it is solving your problem. Thus the design of a microprocessor has captured the universal truth behind all our engineering and other kinds of problems of our modern society. A microprocessor proves the existence of unique universal truth behind all our analytical thoughts.

The universal truth is always eternal. It always appears in all occasions. It cannot be suppressed. In our context the universal truths are: the reincarnation law, simultaneity law, the action-reaction law, the conservation law, and birth and death processes etc. Any process that tries to suppress the universal truth will die sooner than later. One universal truth we always know: that is the death process for all objects. We all know one day we all will die. This is true for our economic and political systems also.

In the old days we used to draw front view, side view, and top view of many mechanical objects in our engineering drawings. We then had to imagine from them the true picture of the object in the mind's eye. Today technology has advanced; we can draw the real 3-D object directly on our computer and then rotate the object to see it from many different angles, all on the computer screen. Thus we now have the real truth, the unique universal truth of the object. In old days we had only views, none of them were correct or complete. The same can be true for all objects and subjects of our life. The reason is that truth is multidimensional as represented by (33). If two persons differ in their views, then only one of us must be true or both may be wrong, because all of us are missing other dimensions. This is particularly true for non-physical things like, soul, reincarnation, and destiny.

## e) Boundedness Law

Let x denote any engineering or physical variable, like voltage, current, water pressure, etc. In engineering x always has a lowest and a highest possible value. Or in other words they cannot take any arbitrary value from minus infinity to plus infinity. We call this feature of a variable as the boundedness law of nature. We will also call it limiter, nonlinearity or saturation law of engineering. Using mathematical notations we can express this law in the following way:

$$
\begin{equation*}
\mathrm{L} \leq \mathrm{x} \leq \mathrm{U} \tag{5}
\end{equation*}
$$



Figure 1: Saturation Non-Linearity

Here $L$ can be positive, zero, or any negative number but cannot be minus infinity. Similarly U can be negative, zero, or any positive number, but cannot be positive infinity. However U must be greater than L.

Figure 1 describes graphically the logic to implement (5). The horizontal axis is the input axis for the variable x . Along the horizontal axis, x can take any value from minus infinity to plus infinity. We show the boundedness of $x$ in the vertical or output axis of the graph. The graph shows when x is between a and b , whatever way x changes, similar changes happen in the output axis. However, when $x$ goes beyond $b$ you can see that on the output axis it stays fixed at U . That is the output is limited to U . The same is true in the lower direction of $x$, and is limited by L. The box in Figure 1 represents a non-linear system.

Wherever there is an engineering variable in a system, an embedded engineer adds this box at that location of the system, to protect the components there, otherwise the system will fail or burnout. This box is implemented in analog hardware circuits in the form of automatic gain control. It is also implemented in both digital hardware and embedded microprocessor based software. All hands-on experienced engineers will automatically place these boxes in their design. This is a very common embedded engineering practice.

Since the box is a nonlinear box representing the natural law, and implemented by design, all engineering systems are nonlinear systems by nature. Or in other words there are no linear systems in engineering. Note that linear systems are not approximations to this kind of boundedness law. It can be shown that local linearized solutions do not satisfy modern complex engineering requirements.

The examples of such systems with boundedness law are abundant in engineering. All we have to look for is to see the schematic of any embedded hardware and the source code of real time embedded software or firmware. It is not that engineering has completely ignored it, theory has been developed, as an example see [30]. But it is still not practical for the complex problems of our time. However mathematics and science have largely ignored the boundedness law. Observe that this is not a philosophical issue, it is a real engineering problem, and creates conflict with existing theories that we use. Because of this law all linear mathematics will automatically fail in engineering. And engineers will be forced to fix the failures by adding kludges and patches. Note again that the boundedness law is a law of nature.

## f) Finite Time Law

The finite time law is essentially an extension of the boundedness law. These days most of the complex engineering systems are controlled by one or more digital microprocessors and software. All activities that these systems perform are done in small interval of time duration, of the order of several micro or milliseconds. And such activities are repeated continuously [22].

Consider the example of a robotic arm, picking up an item from one place and dropping it in another place and repeating the process in, say, less than a second of time. Similarly a digital communication receiver system, like GPS, receives an electrical signal of milliseconds duration, for example, representing the data, extracts the data from the signal, sends it to the output, and then goes back to repeat the process.

Our software runs under real time multitasking operating systems (RTOS) which are also nothing but finite state machines. A finite state machine is a collection of finite number of activities of finite durations, repeated asynchronously and/or synchronously based on the external as well as internal events. Every time a task returns, it finds a
different environment. The previous tasks have operated on the system and created a new environment. Thus the same finite duration task or activity is always performed on different signal and under different environment.

Although our systems run continuously, like GPS transmitters and receivers, traffic light systems at street corners, but if you look at the internals you will always find that the building blocks are based on finite duration processes.

It is quite surprising that the embedded system evolved to perform things as continuous repetition of finite time activities. If we observe carefully we will see that the nature is also composed of finite time activities. Everything in nature has a birth process, maturity process, and death process. Each one of the processes is also a finite duration process. We also see that earth rotates over finite time around its axis and around the sun. Thus repeating finite time process is a law of nature.

Yet, most of the mathematical theories that we commonly use, assume infinite time. It can be shown that if we replace the infinite time by finite number, however large, the theory will completely fail [21]. Thus again, these are not approximations of large numbers, these are collapse of theories, as explained here. In particular we show that the uncertainty principle will not work if we eliminate infinity by any finite value. Thus in real life and in uncertainty principle cannot be correct.

## g) Engineering Experiments

In many cases we see that experimental results seem to suggest that the theory is correct. But that is not a formal verification of a theory. All experiments are engineering experiments. Engineering is part of nature and therefore obeys the laws of nature. Since nature does not and cannot make any assumption, all experiments automatically eliminate all assumptions of all theories. Therefore experiments do not and cannot prove theories.

To test a theory, which claims to be a law of nature, we must first establish the environment, to verify the assumptions in nature, and not in an artificial environment. Then we must test the results or conclusions under those assumptions. As an example, Newton's First law can never be tested, because it has an assumption - "In the absence of any interaction with something else". This assumption is never valid in nature. Gravitational force is always there in space and near earth. Such an assumption cannot be implemented by any engineering experiment. Thus Newton's first law has never been tested and cannot be tested also.

Same is true for uncertainty principle which, as we show, uses Fourier Transform (FT). The FT uses infinite time assumption. Since we can never perform an experiment for infinite time, we can never verify uncertainty principle. If we replace infinite time by finite time, we will not get an approximate result, as we show there will be no uncertainty, a dramatic change [31]. Thus in reality, we have never tested the uncertainty principle.

All we want is to make the readers understand, that assumptions are invalid in engineering, and therefore in nature also; and thus assumptions cannot be used to describe a law of nature. We claim that all mathematics and therefore sciences have assumptions. Therefore no theory can be tested using any experiment, because all experiments will eliminate all assumptions, thus making the theories not testable and therefore invalid.

The elimination of assumptions naturally leads to the conclusion that engineering never used a true theory, and it always had to correct all applications of all theories by adding kludges and patches in both hardware and software. This is really true in practice;
if you ask any knowledgeable and hands-on test engineer in embedded engineering software, he will always confirm the above conclusions affirmatively [19].

## Vi. Reincarnation Law

In this section we briefly discuss the reincarnation phenomenon for humans [32]. We show, by giving real life examples, that reincarnation is a law of nature. Thus our life is a repeating process of many life cycles, in each cycle we make some contribution towards the objective of our global plan. These examples will show that reincarnation is not based on religion, faith, or any kind of belief system. It is happening all over the world for all time.

Destiny is a consequence of action-reaction law and is generated by the global plan described over global time. Past life events and actions continue to affect us for all time in future. Thus without reincarnation destiny cannot be completely explained and understood.

## a) Proof of Reincarnation

Our normal understanding of proof comes from the science and engineering concepts. Let us say that we want to prove that hydrogen and oxygen when combined properly will produce water. We prove this inside a chemistry laboratory. We take precisely known amounts of hydrogen and oxygen, mix them together in a test tube and then burn or give an electric spark [33] to ignite the gases to produce water. In our context we want to say that this is not really a proof; instead this is a demonstration of a theory.

We may want to say that the proof is the long process that required lot of research by many people over many years to find out that we needed hydrogen and oxygen. Then we did lot of trial and error to find out the exact amounts needed of them. Again after long investigation researchers have found that the items must be ignited by some means to combine them. We call this entire process as the proof. Once we have come to a definite answer to all our questions, then we repeated the test and documented the final steps. Now anyone can repeat those final steps to demonstrate the theory.

Reincarnation also can be proven in the same way. In this case the laboratory will be the whole world. Similar amount of research needs to be done over long time and by many people to identify and short list very convincing cases of Jatismaras, the children who remember past life, which cannot be explained by any other known processes. Then any one can take these final selected cases, carefully analyze them, document the results, and then publish them. This is basically what has been done by Professor Ian Stevenson [34] and his colleagues for over 40 years for more than 3000 cases all over the world.

Dr. Ian Stevenson (1918-2007) was the head of the Department of Psychiatric Medicine, at the University of Virginia, School of Medicine, at Charlottesville, Virginia, USA. His research was funded by one of the founders of Xerox Corporation. People say Stevenson will be considered as Galileo of next century.

But long before, Stevenson's proof, the reincarnation theory has been proven by local people and by their own methods of verification [35], which were similar to what Stevenson did. Each family examined their own cases more thoroughly, than Stevenson did, over the entire 4-5 years of the child's early ages. Their effort was more rigorous and thorough than possibly anyone else can do, because the parents lived with the child. They have examined the details of every moment in the child's daily activities. This is how reincarnation was proven over and over again for more than several thousand years and
all over the world. The parents believed, local community believed, and eventually the entire society, civilization, and the world believed.

Many say - those who believe in reincarnation will not require any proof and those who do not believe no amount of proof will convince them. But we are not professing on belief or faith in this paper. Faith and belief are not at all scientific. When we talk about a physics or chemistry problem we do not mention belief or faith. We just ask you to go to the lab and do the experiment on your own or read the details of the experiment from the literature.

For reincarnation the idea is same. You can go and perform the experiment on any of the established recent cases. That is, go to the town and talk to the boy and their parents, stay there for two three days, and convince yourself. Alternatively, read the details of any one of the cases investigated by Professor Stevenson. Dr. Stevenson has documented [34] most thoroughly each of the cases he has investigated.

Has science answered any question? No it has not. Take for example - solar eclipse. Do we know why it happens? Some will say because all heavenly bodies rotate around each other. But then we would ask why do they rotate? Do we have answers for that? Yes, because they attract. But we do not know why they attract. We used the term gravity instead of the word attract and fooled all of us; as if gravity is science and attraction is not. Thus we see that at some point in the sequence of questions we failed to answer the original question and had to accept things as law of nature without any proof. We have gone down from one level to another level, ultimately down to quantum mechanical level. This quantum mechanical level does not have any experimental foundation, is based on theories, which are full of assumptions, as we have shown in this paper, and therefore cannot be correct.

Then why not accept all natural phenomena right away as they are. Engineers never try to explain things. They observe how things happen, and then they build the product using that observation. They keep on patching until they are satisfied. The fact is we can never explain how things happen in nature, because nature evolved over billions of years and our scientific knowledge is only 200 years old. Moreover nature created us; therefore the creator can never be understood by the ordinary people. Just like our computers can never understand humans. However, by acquiring yogic power we can understand our soul, which is the ultimate creator.

Note that these reincarnation examples are not case studies. Nature is teaching us to learn, by creating examples or experiments, to demonstrate to humanity the real truth of reincarnation. Here every experiment is unique and is happening throughout the centuries over all countries. Physics experiments are also little different when performed in different laboratories. Thus there are no differences between the two types of experiments. One is done by men and the other one is done by nature. One done by nature definitely represents the ultimate truth than the one done by men. Nature does not use money power to fool people.

## b) Reincarnation Examples

A generalized overview of one important class of real life reincarnation examples can be described using the following scheme.

- A baby is born with two gunshot wounds: G1 on the chest and G2 at the back.
- Around age two, when he learns to speak, the boy says: in his past life his name was N , he lived in town T , and on date D he was killed by a gunshot at chest.
- Researchers go to the hospital of town T. They find that on date D a person named N indeed died there due to a gunshot.
- They ask the hospital to show the autopsy photos of the dead body of N.
- From the photos they verify that the wounds at chest and back are exactly like G1 and G2. Not only that, they are at same locations also.

In the above generalized example we see that physical marks from the previous body appeared on a newly born baby. This experience and observation cannot be denied by any means as a case of reincarnation. There are thousands of such well documented birth-defect type cases. This is as straightforward as - someone dropped a ball and it fell on the ground. It is not possible to deny such a trivial phenomenon. Thus again, it has nothing to do with religion, faith, or belief system. This phenomenon is happening all over the world in all countries and to people of all faiths. Reincarnation is an unique and universal law of nature.

## i. Birth Defect Type

The following case is taken from [36]. Here we see that birth defects extend deep inside the body. This is the case of an American boy named DG, who was born in 1997. When he learned to speak he mentioned to his mother that DG was her father LS. DG made a number of statements that indicated knowledge about his maternal grandfather's life that his mother thought he could not have obtained through normal means. DG's parents were Christian but his mother was open to reincarnation ideas. DG discussed the death of his grandfather. He said how LS died in an incident in a store where several people were shooting. He also described many aspects of the life of LS. He talked about the two cats LS had and even mentioned the nickname that LS had given to one of them.

DG had a narrowing of the pulmonary artery at the site of the valves. His mother reported no infections during her pregnancy that could cause such problems for her baby. There was no family history of congenital heart defects. DG's birth defect was very similar to the fatal wounds suffered by his grandfather in a shooting. The autopsy report of LS said something like the following: The gunshot wound went through the skin, soft tissue, and left ribs. Then it cut through the left lung, and main pulmonary artery. There was a 4 cm lacerated wound of the main pulmonary artery.

The paper [36] says DG's birth defects had a pulmonary valve artesia with intact ventricular septum. This means narrowing of pulmonary artery. He had poor development of the right side of the heart. DG underwent a shunt, the first of several procedures, and has done quite well with no lifestyle restrictions. The narrowing of DG's pulmonary artery was a close match with the wound to LS's pulmonary artery.

## Note

In this example a very important concept must be observed. When the baby was in mother's womb, the soul constructed the body of the baby exactly the way it was near the end of the previous life. The entire wound of the past life has been clearly and precisely reproduced in the present body of the new baby. This proves that a soul can do anything it wants during the growth of the baby.

It is then quite possible that everything else in our normal body is also constructed by our soul. Gita says you are born in the same state as you died. Theory of differential equations also says the same thing. Statistical data, in a small town in Sweden, has been found to link the health conditions of grandfather generation to grandson generation [37], which can be explained using the reincarnations of same soul. The soul is the ultimate
creator of the universe and life. Thus our features and characteristics will entirely depend on what souls want from the next life. Details of which are not available probably, but some descriptions of soul theory is given in both Gita [38] and Samkhya [39]. Again these can only be revealed by Yogis or huge computerized reincarnation tracking software and database.

## ii. Scholastic Type

In this type the child not only demonstrates and proves that he is a reincarnation of a previous life, that is, Jatismara, but also demonstrates extraordinary human talent that one cannot attain by any means at that age. These talents are like exceptional musical talents or philosophical understanding of a subject, with an ability to express such knowledge better than any adult. We want to say that these people are born with some yogic power, exceptional but narrow in scope. All of us have yogic power to some extent.

Our first example is of a boy named Maung Tun Kyaing (MTK), from Burma (now Myanmar) during the British period, who preached Buddhism from very early age of 4, with a profound understanding of the subject. The case has been authenticated by Lama Govinda, and also by the then British governor Sir Harcourt Butler of Burma. Both of them personally talked to the boy. Govinda has interviewed the boy, boy's father, and investigated many details related to MTK [27, pp. 190-197].

Lama Anagarika Govinda (1898-1985) was a German man, later became a Buddhist. Robert Thurman, a Columbia University professor describes Govinda as the greatest philosopher of the west, although little known. Note that the middle name "Anagarika", is a Sanskrit word and it means a person with no nationality.

MTK was born in a very poor and illiterate mat-weavers family. Suddenly at age four he started preaching Buddhism, better than any religious teacher could have done. He visited a Buddhist Monastery in the local village, where he lived in his previous life as an abbot named U Pandesia. He recognized the present abbot and correctly mentioned his name; MTK showed the room where he meditated, and identified the things that he used. At the monastery, the child read the scriptures written in ancient Pali language and explained its meaning, which MTK could not have learnt in his home.

People were so impressed by the sermons of MTK that they came by thousands to listen and see him. Eventually his fame reached the ears of the governor. The governor invited MTK to convince himself of MTK's extraordinary gifts and remembrance of his previous birth. Governor was very pleased with the masterful exposition of Buddhist tenets by MTK and gave him a hundred rupee note, which the boy could not accept, as MTK said he cannot sell Dharma. Governor was greatly touched by the little boy when MTK wanted to give him a gift. MTK opened his rosary from his wrist and gave it to the governor. Smilingly accepting the gift the governor said - "now you must tell me how to use this rosary". MTK explained in details that this is to meditate on three marks of existence - impermanence, suffering, and egolessness.

The boy spoke not like the one who was taught, to repeat words which he himself could not understand, on the contrary he spoke with such conviction and sincerity that the governor was profoundly impressed by this child. He asked MTK to go from one end of the country to other and preach to high and low and even to prisoners in the jail. Governor believed even the hardest criminals will melt in hearing his genuine faith and sincere goodwill. The boy did so and the prison doors were opened for him.

## $\mathrm{Ref}_{\text {ef }}$

 1994The case of the American boy Jay Greenberg (1991-) is similar to MTK. Jay was born in a family which did not have any musical background. However, Jay was not a Jatismara. But from age two he started showing talent in music. Jay says he hears the music as a streaming audio from nature. He does not need to edit it. Some say he is the greatest musician to come in last 200 years. He wrote 5 symphonies by the time he was 13. At age 14 London Symphony Orchestra has recorded his music for Sony Company. His case was first aired on 60 minutes CBS TV program, in 2006 in USA [40]. Mozart was also a famous child prodigy, but he was born in a musical family. Those who speak fluently a foreign language, between ages $2-4$, should also be considered under this category, although it has a special name - Xenoglossia.

## iii. Common Type

In this category, a child between the ages of 2-6, starts talking about his previous life. He is a Jatismara. He names the person of his previous life, identifies the city or town where he lived, recognizes people, photos, houses, and other objects that he used in his past life. The parents of the child explore the details; local people cooperate, and help to solve the mysteries. Researchers visit the place, explore the details systematically, document the discussions, take videos etc., and finally publish the results to authenticate the case.

The following example is taken from the TV news from the ABC channel of USA. The transcript [41] says that a six year old boy named James Leininger talked about his previous life as James Huston, who died at age 21 as a navy pilot, in a military plane accident, 60 years back, during World War II. Leininger's parents were an educated Christian family in USA. They never expected such things to happen to them. But over time, covering many incidents, and conversations with their child about extraordinary details of the pilot's life, they have become convinced that this is a case of reincarnation.

Here are some excerpts from the video [42]. From age two, Leininger used to get nightmares. Once he woke up from his dream and said "Airplane crash on fire, little man can't get out." Another time he said that the toy plane has a drop tank. The words "drop tank" were completely unknown to the family. Leininger gave many detailed accounts of the plane, that no one could have told him, or even could have known also. He said the plane used to get flat tires, which was verified later. The parents came to know about many such unusual and detailed information from their son. They investigated, many details that their son told about the air plane, pilot's friends who were still living, the name of the aircraft carrier, shooting down incident of the plane etc. Pilot's sister was still living and became convinced that the boy was her brother.

The proof of reincarnation does not come from only the few statements reported here and in the original references. Many of the above statements can be countered using standard logics. Professor Stevenson has discussed one by one all the powerful counter logics in his book [34]. But the totality of information covering the exact statements, the body languages, the circumstances, the trigger moments for each incidents, the objects that helped to trigger, many question and answer sessions with many people, and many such details, that spanned over 4-5 years in such cases of Jatismaras, cannot be disproved completely using any consistent logic. This is not just one case; there are hundreds of such different but unique cases that happen all over the world in every year. Most importantly there are no motive behind these cases; there are also not any financial gains, or fames, to be made by anyone involved. Reincarnation is the only single unified theory that can explain all of these cases.

## c) Hypnosis and Reincarnation

Past life regression (PLR) therapy is now a well established method of treatment in Psychiatric branch of medical science. In this approach a patient is usually placed in a hypnosis environment and regressed backward in time slowly to past life. Some doctors do not use hypnotic method, yet take the patient consciously to past life memories. A good literature survey on the methodologies of hypnosis is presented in the Master's thesis [43]. Some specialists claim, based on the real life experiences with their patients, that this method proves the validity of reincarnation law.

It has been found that many chronic migraine headache cases for many adults are related to past life problems. In one such case [44] all normal medical treatments, over long period of time, failed to cure the patient. Accidentally the patient discovered the PLR technique and made an appointment for hypnosis. It was found out that, in her past life she was raped, had a baby, and then died in an accident where she got wounded in her head. Once she discovered all the details of her past, via PLR, she was completely cured automatically. This is a common experience with Jatismaras also, once you learn about your past life, the problems do not interfere with present life any more.

In a similar incident, as described in a BBC video, a child started growing a large cyst in his throat [45]. Doctors performed the operation but it grew again. One day the boy told his mother how he died in his past life due to a gunshot wound on his neck in a military battle field. Once the details were found out and discussed with the boy, the cyst and the growth in the throat vanished completely. The normal medical doctors could not explain but had to accept it as a miracle. This case does not come under birth mark type; however it is similar to migraine headache problem. It is quite possible that a detail medical examination of the physical body would reveal an internal birth mark in both cases. Here again we see how soul interferes with the physical activities of the human body.

Brian L Weiss (1944-) graduated from Yale university school of medicine, was a head of psychiatry at Mount Sinai Medical center, Miami, Florida, USA. He is famous for his research in past life regression therapy and reincarnation. He is a preeminent authority in the west on reincarnation law. He has published many books on the subject.

In 1980 in a hypnosis session, one of his patients Catherine revealed past life memories. In another session Catharine provided many details about the deceased father and son of Dr. Weiss. This revelation has convinced Dr. Weiss about the reincarnation as a natural law. The PLR process on Catherine completely relived her from her paralyzing phobias; and put Dr. Weiss in a life changing path. The orthodox medical school has tried to censor Brian, and victimize as another Galileo, after he published his first book on the subject [82]. Dr. Weiss has appeared in all major TV channels and shows world over.

One interesting observation revealed by many incarnations of Catherine was that many of her associates in many of her past lives are here with her in her present life. She could recognize them in her PLR sessions. This means that our physical appearances and our physical characteristics do not change much over incarnations on life after life, over centuries, and we remain recognizable. That means again, this same I with this same body, was there many times in past lives in past years. The books by Dr. Weiss reveal many important feature of our past lives, in particular he points out that group of souls live together in many lives.
 lives of many thousand people. This vision, this ability to hear music, and ability to acquire Buddhist knowledge etc are part of shruti capability described in Vedic literature, which can be acquired by yogic power only.

## d) Reincarnation and Destiny

All humans are tightly integrated in our world with our interactive action and reaction forces. This integration with nature is not just in present life, because of reincarnation law we were connected in our past lives also. The past life actions are working in present life too. Whatever we started in our past life, we are continuing with that activity in our present life. Thus this action reaction law or cause and effect law maintains a coordination of activities of past, present, and future.

Consider the stone throwing example. We throw a stone and watch its trajectory in the space as it flies, as shown in Figure 2. We can clearly see and understand where it will go. We have studied in high school physics class, the formulas that describe the motion of this moving stone. We also know that if we see only a portion of the trajectory, then we can find out where it came from and where it will go. This means only a small portion of the trajectory, like in the window A in Figure 2, has all the information about the entire trajectory. That is same thing as saying that, any portion of the trajectory has the memory of the complete trajectory. In fact the theory of differential equations says that any point in the trajectory defines the entire trajectory. This is because the differential equation is a memory law. This memory concept has been used in sniper detection technologies at the Iraq war battlefield by military [47].

A soul also has a similar trajectory, but it is distributed over space and time. We cannot see or feel about its entire trajectory. The reincarnation examples show that this concept of trajectory is valid in real life. Our present life is only a small part of this trajectory, just like the window A of figure 2. Thus all of us are in a long journey and for eternity with different trajectories. We inherit this trajectory; our present life is defined by experiences gained from this trajectory. If we know any event in any window like in A then we will be able to identify all details of other past and future lives, as pointed out in the predicting destiny section. Thus our life follows many laws of nature just like nonliving objects.

However, predicting destiny is not possible, unless we find a yogi or that special person born with special yogic skill to predict future. The world has experienced many such special persons in every decade. You just have to find them to learn about your destiny. It is a fact though that destiny is predictable and with very precise details, as we described in this paper.

## e) Theory of Reincarnation

Samkhya Karika [39] provides the most brilliant scientific theory in the shruti collection of Vedic system. It is credited to Kapila muni. In Gita [38] verse (Gita 10:26) Krishna has been defined as - "... among perfected beings I am the sage Kapila". Mahabharat says [48, Vol. 10, p.8] -"There is no knowledge that is equal to this (Sankhyas). Do not yield to any kind of doubt. The knowledge which is described in the system of the Sankhyas is regarded as the highest." Vivekananda [49], a famous Indian philosopher of modern time, says the following - "This wonderful man, the most ancient of philosophers, is mentioned even in the shruti - O lord, thou who produced the sage Kapila in the Beginning".

According to Samkhya Karika, the human body is associated with three kinds of objects (a) the soul (b) the subtle body and (c) the gross body. We can see and feel only the gross body. We cannot see the other two. Only through extensive yoga meditation one can acquire the knowledge of the other two objects. The soul never reincarnates, the subtle body is everlasting and incarnates, and the gross body is perishable. The subtle body consists of Intelligence, I-Principle, eleven sense organs, and five primary elements. However the soul is intimately tied with other two objects. The soul is like perfume on a garment; and garment is the subtle body [39].

It is the subtle body that reincarnates after death, as we can read in Samkhya verse (SK:40) - "The mergent subtle body, produced primordially, unconfined, constant, composed of the tattavas beginning with Mahat and ending with tanmatras, transmigrates, free from experience, and tinged with dispositions". The translator of Samkhya uses the word transmigration instead of reincarnation. But Samkhya does not say that humans can take birth as non-humans.

In the beginning there was soul and primordial nature or primal matter according to (SK:03). Soul is eternal, it is not part of nature, it does not die, it does not follow the laws of nature. Creation happens, according to Samkhya Karika, when the soul interacts with nature. At this time nature creates (SK:22) the subtle body, consisting of components (tattavas), in the following order: Intelligence (Mahat), then I-Principle (Ahankar), then 11 sense organs, and finally five primary elements (tanmatras). The book says that the subtle body is so subtle that it can pass through the mountains [39]. The soul never reincarnates, as stated in verse (SK: 62) - "Thus, verily, Purusha (soul) is never bound, nor is he released, nor does he migrate. It is the Prakriti (nature), being the support of manifold creation that migrates, is bound and is released". Thus subtle body is considered as part of nature, however, the soul remains identified with the subtle body for eternity.

The Samkhya also describes the cause and effect law of nature. In verse (SK:9) it says a nonexistent cannot become existent. It qualifies a cause as efficient cause, which means a cause must be appropriate to make the effect happen, that is, not just any cause can make any effect happen, it must be meaningful and compatible. The verse also points out that the effect is of same nature as of cause. Thus we see the origin of action reaction law and the law of conservations is very ancient.

The well known Hindu scripture Gita basically explains the Samkhya Karika in its own simpler way, as a dialogue between two persons Krishna and Arjun, so that the common people can understand the concept of reincarnation. Presenting theories in the form of dialogue is an important method for presenting complicated idea. Use of FAQ

## Ref

(frequently asked questions) in modern websites essentially is a similar concept. The Sanskrit word Samkhya has been used in verse (Gita 5:4)- "Only the ignorant speak of devotional service (karma yoga) as being different from analytical study of the material world (Sankhya). Those who are actually learned say that he who applies himself well to one of these paths achieves the results of both" [38].

The verse (Gita 2:20) reconfirms, as stated in Samkhya, that soul does not reincarnate - "For the soul there is neither birth nor death at any time. He has not into being, does not come into being, and will not come into being. He is unborn, eternal, ever existing and primeval. He is not slain when the body is slain".

Gita says there is no discontinuity in characteristics of the subtle body; although from reincarnation examples we can see that there can be large break in time and space between two births. Thus the life continues from the end of one birth, to the next birth, without interruption of continuity of the subtle body. The verse (Gita 8:6) confirms "Whatever state of being one remembers when he quits his body, O son of Kunti, that state he will attain without fail". There are many real life examples of reincarnation which reflect concept of continuity in the above verse. Therefore it seems that Indian Scriptures do not say that human subtle body improves during the gap between death and rebirth. All improvements must happen during physical life time.

The ideas behind subtle body, like the concepts of intelligence, I-principle, and sense organs are buried in the verse (Gita 5:23)- "Before giving up this present body, if one is able to tolerate the urges of the material senses and check the force of desire and anger, he is well situated and is happy in this world". If the reverse happens, then the imprint of the gross body will be placed on the subtle body, creating the possibility of birth marks. This explains that although 20,000 people die every year in USA due to gunshot wounds, but only a very small fraction of them talk [36] about their past life wounds in the present body.

Both Ramayan and Mahabharat give at least one example of reincarnation in their stories. In Ramayana we see [50, pp.1607-1610] that Sita, the main female character, is the reincarnation of Vedavati; and in Mahabharat [48, Vol.5, p. 266] we find Shikhandin reincarnated as a man from her previous life as woman named Amba.

It should be recognized that besides reincarnation law, Ramayan and Mahabharat also talk about the other laws of nature like, cause and effect, destiny, theory of karma, law of conservation, simultaneity law over multiple life spans etc.

## f) Predicting Destiny

In many places in India there are secret books on astrological predictions of all humans of the world. These books were written by ancient yogis of India. Most famous of them are attributed to Vrigu and Agasthya. It is said that Vrigu lived during Dakkha period, which was several thousand years before Ramayan period [51]. Agasthya lived during Ramayan period [52]. Vrigu invented the astrology and used his yogic power to write these predictions.

These books are commonly called Vrigu Samhita and Agasthya Nadi Samhita. These books are written in Sanskrit (for Vrigu) and ancient Tamil language (for Agasthya), on palm leaf pages, and are in several volumes. Many original volumes are lost during Muslim period. The remaining volumes are scattered all over India. Some do not have the original volumes and the readers give false predictions. People from all over the world go there to learn about their destiny.

Each horoscope contains several pages. The pages give details of names of the visitor, his parents, and family members. It describes past, present, and future on many subjects of the visitor. Some of them contain medical histories. They also include past and future incarnations of the visitor. Some predictions are written in question and answer form.

For every visitor, a particular page has to be found out. If that page is not found then the horoscope predictions cannot be given. The page is found from one of many ways: (a) date and time of arrival of the visitor (b) birth place, date, and time (c) horoscope chart etc. For Agasthya Samhita thumb prints are used [53] to locate the pages. The readers ignore the page if it does not contain your name and date of birth correctly.

Destiny defines everything in our life. So whether a visitor will go to such a Samhita place or his page can be found or not is also part of destiny. In fact everything can be and should be interpreted using destiny concept only. It will be clear from the concept of destiny, eternal recurrence, mathematical logic etc. that predicting the future precisely is quite meaningful, although we cannot tell exactly how this happens. This is because we do not know science behind this yogic vision. As we have shown the science that we study is full of assumptions, is misleading, and erroneous. And the money power has prevented us from discovering the real science.

The following are some well known predictions from Samhita. A four star army general JN Choudhury, visited one of these centers [54] where people claim to have some volumes of Vrigu. Mr. Choudhury was planning to retire from service. But the pundit's readings of his pages said he will become chief of army. Eventually things turned that way and he was selected as chief and was appointed for the war against Pakistan.

Another political leader, KM Munshi visited the place [54] in 1945. The pundit started reading his horoscope and described his position as a political leader. But the next sentence said he will have to leave now immediately without completing the rest of the predictions. Mr. Munshi's secretary came and reminded him of his appointment that he must honor now and leave the place. This shows how destiny is confirmed, so many thousand years later at some precise moments, and at such level of details in our life.

In another incident in 2007 Pratibha Patil visited, for Vrigu Samhita [55] and it was predicted that she will become president of India. Six months later she, an unlikely candidate, became the first woman president to hold that office.

The following prediction will show the level of details included in the Agasthya Samhita [53]; Vrigu Samhita also has similar capability. They mention the visitor's name, and names of wife, mother, and father, the age of the visitor, number of sons and daughters. Then they describe the birth horoscope signs, position of stars, all written on that ancient page. They read from the manuscript the education of the visitor, children's schooling, and the place of work of the visitor. It talked about undue delay in job promotion, break in children's education, wife's skin disease, car accident, court case. It described past life sins and good deeds and their effects on the present life. It recommends some pilgrimage or prayers to certain places for correcting sins of past life. It should be understood, that no matter what you do or do not do, the destiny cannot be changed. Thus these remedies are stated as part of destiny also. The destiny is not defined using any kind of if-then-else type conditions.

In the western world there was a very famous astrologer and palmist named Cheiro (1866-1936), who predicated many great things about many great historical figures of his time. His predictions appeared in many news papers including New York Times.

This is what Mark Twain said about Cheiro and can be found in [56]: "Cheiro has exposed my character to me with humiliating accuracy. I ought not to confess this accuracy; still I am moved to do so". Cheiro predicted the date of Queen Victoria's death, the year and month when King Edward VII would pass away, the grim destiny that awaited the late Czar of Russia, the assassination of King Humbert of Italy, the attempt on the Shah's life in Paris, and for many others he foretold with equal accuracy the outstanding events of their careers [57].

We do not want to discuss how people do these predictions, whether there is some kind of meaningful methodology or not, all we see is that future can be predicted and not only that, how it will happen is also predictable. We should recognize that there is nothing like "meaningful" in our nature. We cannot understand nature and the yogic power.

## g) Eternal Recurrence

The German man, Fredric Wilhelm Nietzsche (1844-1900), one of the most influential philosophers of the west in past 150 years, was greatly influenced by Indian studies in his philosophy and works. He is credited with the philosophy of eternal recurrence [58], which says that everything that is happening now has happened many times in the past and will happen many times in the future. That is, destiny is recurrent or history repeats itself. He also believed in exactness of such events. Life is precisely defined and repeated infinitely many times, i.e. eternally recurrent.

There is a very interesting example of eternal recurrence of a mother and daughter pair whose palm lines were identical, as Chiero was showing them to Mark Twain [56]. In this case every action of the girl's life repeated exactly, even to dates, the action of mother's life, although twenty years separated them in time. Both had same illnesses at same age, married at same age, had five children, and were a widow at same age. As we show later, the example of Mahabharat as a repetition of Ramayan, is a recognition of the eternal recurrence phenomenon by ancient yogis like Byasdev, the author of Mahabharat and Gita.

Vrigu Samhita’s prediction for Mr. Munshi [54], where Mr. Munshi had to leave the place without completing the prediction readings, seems to confirm such an eternal recurrence theory. How can such detailed level prediction happen so many thousand years later? Not only that, Vrigu also says who you were in previous life and where you will be born in next life. This means, it is not just an event, it is also the same soul that is coming back and repeating the process. Interestingly, PLR sessions of with Caroline by Dr. Brian Weiss [46] show that she was able to recognize many past life persons who are physically present in her present life; confirming a repeating process. In this paper it is shown that theoretically Nietzsche is correct, but it will never happen exactly in reality for all people at same time. Although at very high level things will remain very similar for all of us.

Nietzsche also believed in the simultaneity law. That is everything in this universe is simultaneously linked and working together [59]. He points out that if any one changes anything anywhere that change will be propagated everywhere in this universe, which is a natural consequence of the simultaneous interconnection law for all objects including humans.

According to Nietzsche this hidden eternal recurrence cannot be seen by ordinary people. Because he says the crowd can only see what exists. Thus he creates an overman [60], who is like a very powerful yogi, like an avatar, who can show people the true nature
of eternal recurrence. He also thought that his overman can liberate the humanity from the distress of recurrence. We show that his concept and philosophy is fairly well embedded in the destiny theory of the universe. Using Indian scriptures Ramayan and Mahabharat we show how our life is truly following the eternal recurrence, simultaneity, and overman.

At very high level the two stories of Ramayan and Mahabharat are exactly identical, even though the gap between them is over ten thousand years.

- We see that Ram in Ramayan and Krishna in Mhabharat are two highly powerful yogic persons; creating major wars against evil forces; kills millions of people; and does this for greater good of ordinary people. Thus we can see that the two wars were same events, i.e. recurrent.
- At some detail level we see that Ram had to live in forest for fourteen years and in Mahabharat we see that Judhistir had to live in forest for thirteen years.
- We see that in Ramayan there was a woman character Manthara who was responsible for sending Ram to forest. In Mahabharat we see Sakuni a similar male character who was responsible for sending Judhistir to forest.
- Manthara changed the world of Ram in one night and Sakuni in Mahabharat also did the same to Judhistir in one night. The two characters, Sakuni and Manthara, are almost identical in nature. One used to walk with a stick and the other with the dice in hands. Both persons limp when they walk.
- In both stories two great women, Sita and Draupadi, were insulted and humiliated. Society mistreated them very much, beyond expectation of any civilized kingdoms.

Are these similarities intentional? Yes of course, both stories were written by yogis of highest order. Mahabharat was written after Ramayan by Byasdev. Byasdev knows the concept of eternal recurrence and therefore created the stories along the same line. In fact in one place Krishna tells Arjun that they both came to this world many times. Krishna knew each life of Arjun, but Arjun does not remember (Gita:4.5).

## i. Science of Eternal Recurrence

Consider two sinusoidal functions of slightly different frequencies. Assume that they start at same time at exact zero phase. After the first period the two waves will have slightly different positions, they will not coincide in amplitude. But after several cycles, depending on their frequency difference, the two waves will meet again at same phase. This way the two events will recur exactly and periodically after several distances apart.

If we now take a third signal with slightly different frequencies, the situation will be similar again, but this time the gaps between exact recurrences of all three will be longer. We see such similarities happening in Ramayan and Mahabharat stories over several thousand years. The core reason is the periodicity of lives and events in our universe. Since there are millions of periodic events there will be almost zero probability that exact same events will repeat exactly in the same way. At any point in time we will however see many similarities and much dissimilarity. Moreover there are periodic events that are completely destructive in nature, like tsunami, ice age, which may be quite catastrophic. These events will superimpose over other periodic events to destroy many similarities. Or in other words, there will be quiet periods, in the series of recurrence.

Everything in nature is precisely deterministic and therefore can be expressed by a set of simultaneous equations as in (33). This equation is written in matrix form by (34). This coefficient matrix will also have eigenvalues, some of which can be complex
conjugate numbers, which will produce sinusoidal waveforms. Thus equation (33) embeds periodicity and hence eternal recurrence. The equation (33) also says that any point in time the vector x will have many components very close to each other and many will be widely different, just like the sinusoidal function example. Thus observing the events of nature at any time we will always be able to see the similarities, because they were all created by an underlying equation representing a unified theory. The similarity is maintained by the equations of nature.

## ii. Poincare's Recurrence Theorem (PRT)

This theorem [61, p.82] is very much like the mathematical equivalent of Nietzsche's eternal recurrence theory of nature. PRT says that a dynamical system under fairly general conditions will come close to any point in its state space infinitely many times over infinite time interval. If we apply this to nature, it amounts to saying history repeats itself. We see in Mahabharat, things can be fairly close to Ramayan, but many things can be different too. Thus theoretically things can be exactly similar but in real life there will be many differences. Moreover we should remember infinity is nonexistent in nature. Everything has a finite time death process. A star will die eventually.

The PRT theory can be explained using the following hypothetical physical simulation. Consider a transparent sealed cubic box of one inch side. Assume that we have one microscopic colored particle inside it that is moving randomly everywhere in all directions hitting the walls and coming back to everywhere. Assume that you have one observation probe at any location inside the box. You will find that the particle will hit the probe every now and then, infinitely many times, over infinite time intervals.

For completeness we include the PRT statement below [61, p.82]. In PRT, T represents a dynamical system. T takes any point x and moves it to another new point $\mathrm{T}(\mathrm{x})$. The operator T can be used repeatedly to define the entire trajectory: $T^{n}=T \circ T \circ$ $\ldots \circ T$, (n-times). The idea of measure preserving means T operates on a constant volume space. PRT says if we observe any bounded region A, then the trajectory will come back to A many times, very much like the box experiment described above. The measure is considered a probability measure with $\mu(X)=1$. The space X can be any n-dimensional space and $\mathcal{B}$ is a $\sigma$-algebra on X . We say that $\mu$ is T invariant measure or equivalently T is a measure preserving transformation if $\mu\left(T^{-1} A\right)=\mu(A)$, for all $A \in \mathcal{B}$.

Theorem (PRT): Let $T: X \rightarrow X$ be a measure preserving transformation of $(X, \mathcal{B}, \mu)$ and let $A \in \mathcal{B}$ have $\mu(A)>0$. Then for $\mu$ almost everywhere, $x \in A$, the orbit $\left\{T^{n}(x)\right\}_{n=0}^{\infty}$ returns to A infinitely often.

## Vil. Interconnected Souls

Soul to soul communication is the most amazing feature of nature. It appears that this communication system gives us

- complete source of knowledge from nature
- creates the global plan for the destiny and
- allows individuals to learn about the features of personal destiny.

This feature, the transparent connectedness of all souls, is the foundation of the entire universe. Because of this reason some say soul is one and unique. However, Samkhya does not confirm that theory.

Our universe provides abundant information and indications to learn about our souls. Consciousness or soul is everywhere, in physical objects, plants, animals, and humans. They are all continuously connected and talking to each other. This soul to soul direct communication is much stronger and precise than our oral communication. The ancient and modern yogis of India communicated using this method. Ordinary people like you and I often sense this ability as our inherent power. Our literature is full of such information, formal and informal, experimental and non-experimental, and as major news reports on many TV channels. We discuss briefly some examples of such soul to soul communications.

We do not know anything about the nature and we may never be able to know how and why things happen. For example we do not know why moon attracts earth, we call it gravitation, but gravitation is just another word for attraction. We have to accept things as they happen without any questions just like the engineers do. All we have to do is to learn how to use nature to live in tune with nature. We cannot test anything in our laboratories related to consciousness. All our past physics experiences are gross, artificial, non-conscious, and with many kinds of unnatural assumptions. In our laboratories we have completely ignored real nature of things. All theories have assumptions; they must be removed and then tested. Or we must first implement the assumptions to test the theories. We have not done that, as we have discussed in this paper. We also must consider the existence of consciousness and their interactions before testing such complex experiments on human subjects.

## a) With Material Objects

In Princeton university laboratory and in many other laboratories a simple experiment has demonstrated that the human soul can communicate with physical experimental setup and can cause its outcome to change. A random number generator with computerized recorder was used to graph its average values over several trials. Any operator could make the graph go upwards the normal level when intended to do so. Similarly, he could make the graph of the average values to go down the normal level [62].

In another experiment a Japanese researcher, Dr. Emoto, found that human mind can control the characteristic shapes of water crystals. Different states of mind, when interact with water, create crystals of different shapes. A film mentions his ideas and says if we can do this to water, then imagine what we can do to humans [63]. Emoto's experiment which relies on consciousness is very sensitive and requires natural environment. A controlled environment inside a laboratory may destroy the effect of consciousness.

A book reviewer, of the book "Hidden messages from water" by Dr. Emoto, which was a New York Times bestseller, says "While most of the scientific community refers to his studies as unorthodox, uncorroborated and not reliable, audiences in Switzerland, Germany, Australia, the Netherlands, Canada and the United States, wept when they saw the photographs of beautifully formed orange, red and blue crystals that were told 'you're cute' or 'I am sorry;' or when the music of Mozart Symphony No. 40 or Bach's famous arrangement 'Air on the G String,' played to the crystals" [64]. Japanese people are indeed very sensitive to nature. Only they could produce people like Emoto.

## b) With Animals

A cat, named Oscar, in a nursing home in Rhode Island, USA, goes to a bed two to four hours before and stays there until the patient dies [65]. This has happened consistently and accurately for nearly 50 times, up to the year 2010, in that hospital. The
doctors have carefully recorded these cases and documented in the New England Journal of Medicine in 2007.

The nursing home has many other cats, but none of them has this power of Oscar. Many times doctors and their modern instruments failed to predict the death cases accurately, but Oscar never did.

Many people seem to suggest that dying patients release some kind of smell that cats can detect. Some even suggested that the body makes some sound, or the body does not make any movements etc. But Oscar seems to detect even the day of death, not just few hours before death; he goes away for his round 10 hours before and comes back four hours before. All sense signals are analyzed by brain, which in turn is controlled by soul. Thus it is the soul that detects the communication and the time of death. Oscar has the special power to sense, communicate, and link with souls. The hospital administration gets some time to inform the immediate relatives of the patients to take care of them during their last hours. The cat that way helps to physically connect the souls of relatives.

This example shows that the human souls do communicate in some way to the world that they will be leaving soon and their time has come to an end. This shows the predictability of life. These cannot be computed by Newton's action reaction law or using the concept of differential equations. However, these laws do say in a very convincing way that the nature is predictable. Most of us cannot hear, see, or feel it that way, but some of us surely can. Many animals have this well known characteristic of recognizing this communication signals. We have heard these cases for many household dogs. Such amazing events appear in the TV and magazines every now and then.

## c) Mind Reading

A 9-year old autistic girl from India can read her mother's mind completely. When mother sees a six digit number the girl writes that number on the computer. When mother reads a poem, the girl types it out exactly on her computer [66]. This is an example of how accurately and precisely one human can read the mind of another human. In this case we see a normal child with autism as disorder. Indian yogis can achieve this same capability by intense practice of yogic meditation. In this specific case we find a girl is born with such a yogic power. There are many such examples where we find people are born with such yogic power. The case of the musician Jay Greenberg is one such example, where he is acquiring music directly from nature as an audio stream; which is same as the sruti power of yogis of yore.

The yogic meditation procedure is still practiced in India. In his book Lama Govinda [27] pointed out how his Tibetan guru read his mind completely even before he spoke to him. The guru told Govinda, as Govinda was planning to talk, that before guru leaves, the guru will give him a gift as a souvenir. Govinda was a German man and his guru was a Tibetan. Thus we can realize memory information is stored in some universal format and is independent of our formal language, very much like in our computers. Soul to soul direct communication is much more precise than language based oral communication which is based on some kind of translations from the universal natural format.

In his book Yogananda described [29, pp. 297-302] how he found the newly conceived baby, still in mother's womb, who was his student in his past life, in the Kolkata city of India. Yogananda was able to connect to the call of the soul in mother's womb and track him along a street, and finally to the house where the expectant parents were living. This process of finding the newborn Lamas of Tibet is very well known and
well established. It seems yogis know exactly how to communicate with a soul and get all desired information. Our society should learn this method and teach our students the soul communication method.

Compared to this kind of mind reading power, the simple telepathic experiments in the lab are almost meaningless. We should try to discover more such people and document the results to let people know what is out there in nature. Indian and Tibetan yogis have demonstrated such power since ancient days, thousands of years back. All Indian scriptures talk about such powers. Many books written by modern yogis have also documented such yogic power.

## d) Patent Law

There is a significant controversy about the patent law. We have mentioned that the law of conservation says we cannot invent anything, because that will violate the law of existence. This law says a nonexistent object cannot become existent. It has been found [67] that hundreds of new technologies were invented almost simultaneously from multiple different places by people working independently. This happens because all souls are simultaneously interconnected, and they are continuously communicating among themselves. One physics book says things do not happen they are already there we just pass through them [4, p. 46].

## e) Identical Twins

Two identical twin sisters, Adriana and Tamara, were separated from their birth place in Mexico. They were united 20 years later in New York, where they were living all those years only 25 miles apart [68]. Adriana always wanted and talked about having a sister. Her adopted mother knew, wanted to adopt both of them, but for many reasons that did not happen. Justin met Adriana, went out couple of days, but there was no spark between them. Justin's buddy Rueben decided to introduce Justin to Tamara who Rueben thought looked like Adriana. Justin was shocked to see Tamara, and immediately realized they must be sisters. After some inquiry Justin learnt about their birth place, date of birth, etc. and then decided to introduce them to each other. Eventually the sisters met and found each other again. Two weeks before their meeting, Tamara's father died. Adriana's father died long time back. Both families have come to believe that their fathers brought them together. Adriana said - "They like to think they have control in every single thing in their life, but it's not, to a certain extent, things are always planned out." It is interesting see how direct experience changes the understanding of life. It is the details that convince us. On the surface of this description it may appear an absurd idea, but the details will always give solid foundation of our life events with a profoundly natural touch.

In the above story many souls, including the sisters, from many different places worked together to make the union happen. It should be realized that these are not rare or unique cases. These are happening all over the world and have been well documented in many ways.

## f) Yogic Power

Indian yogis have demonstrated, thousands of years back and even today, what humans can achieve in terms of its mental capabilities, by following a rigorous discipline of yogic meditations. The main idea behind these meditation procedures is to achieve a level of concentration to make direct connection to your soul. It is called self realization or Swaroop Darshan in Sanskrit. Once you connect to your soul you achieve the immense power of nature to observe things exactly the way nature does things. Many books [27, 28,

29] have documented some of these capabilities by the modern time yogis living in Himalayan and Tibetan caves. Many western devotees have gone to India and Tibet and learned these powers of mind and body control.

The book [69] on Yoga and Kriya, most exhaustive book of its kind, describes in details a methodology to achieve such a yogic power. From reading the book it can be realized that it requires tremendous amount of dedication, sacrifice, and concentration. It is probably a ten year long process depending on the characteristics of the student. This book only gives one of the many processes available today. An expert guru may be able to lead to another specific process.

Some of the capabilities of these ancient Indian yogis are listed below. Another such set of capabilities are also listed in [6]:

- Extend life to over 1000 years
- see past, present, and future
- see anything at any place from anywhere
- simultaneously exist at more than one place
- heal anyone from any kind of sickness
- curse and bless anyone with results
- transplant divine knowledge, power, and vision directly into someone's brain
- create material objects out of nothing
- die by spontaneous burning
- spontaneously mind read anyone
- acquire the ability to hear Sruti
- die at will
- live without eating any food
- remain invisible to camera photography
- switch to a dead younger body and continue to live, etc.

A person who has acquired at least one or more of the above skills using yogic meditation will be called a yogi. Although the above skills can be attained by anyone by following a yogic meditation, occasionally children are born with one of the skills mentioned, with some limited scope. We have already mentioned the case of 9-year old girl who can read exactly everything in her mother's mind. We have also discussed the musician boy Jay Greenberg who hears the music like a sruti message from nature. MTK, the philosopher boy of Burma, also can be considered as hearing Sruti. There are some documented examples of persons who have survived without eating any food for entire life time. One such case is described in the book [29, pp. 264-271] about an woman named Giribala.

A photo of Giribala taken in 1936 with Swami Yogananda says she did not eat since 1880. Her non-eating state has been rigorously investigated by the Maharaja of Burdwan. She employs certain yoga technique to recharge her body with cosmic energy from the ether, sun, and air. Maharaja invited her to his palace. She agreed to a test and lived for two months locked up in a small section of his home. Later she returned for a palace visit of twenty days; and then for a third test of fifteen days. The Maharaja himself told that these three rigorous scrutinies had convinced him beyond doubt of her non-eating state. Another similar case has been documented in the British news paper [70].

No amount of astrological theories can predict the destiny the way Nostradamus, Cheiro, and Indian ancient Yogis have done. These people are examples of how yogic power can help to visualize the future and the past. The example of hypnosis sessions with Catherine [46] shows how much yogic power are hidden in our mind and how easily it can be experienced and demonstrated.

## g) Soul to Soul

Telepathy or ESP is only a small part of soul to soul communication. We are talking about complete knowledge of all souls about everything that is happening now and is going to happen in future. Since all souls are working together using the simultaneity law, all souls know what the global plan is. As a result, a person will physically come and express the idea or about the possible future event to you. You may not even notice the interaction but you will remember the details of interaction only after the incident which was predicted to you happens. This happens to everybody. Sometimes your own soul predicts to you about the possible future events. Note that it is not just your soul telling you; it is actually triggered by some other events or souls. Reactions happen only due to some actions. That is the destiny law. Sometimes it may take, days, weeks, months or even decades to understand the destiny prediction at past time by a person.

What is important is to realize that souls will come to interact with you directly or indirectly to inform the future events in your destiny. It is also important to realize that your soul is constantly talking to you, listen carefully and you will be able to hear it. This happens because of the simultaneity law of nature, you are not alone. This fortune telling is constantly happening in our life.

Interestingly, life stories of all of us are just like the above. Nature always comes to us and tells us about our destiny. Great literary authors who wrote many great novels in the past and in modern times know this feature of nature very well. If you read any one of them you will find how in a very subtle way the authors always tell about the future events that are going to happen to their actors and actresses. But none of them can understand these predictions. You as a reader will notice always these predictions. This is the story telling art that authors have learnt from nature. When we read the book first time we may not catch these points, but if you read it second time, you will always recognize these predictions. It happens because now you know the global plan of the story, but the novel characters are familiar with the local events only, and they don't see the destiny. Destiny can be understood only when we get the skills to view the global plan, which is a very rare skill, but not impossible to acquire it.

All human souls are continuously and interactively interconnected. Our own soul is continuously responding to such communications. Whatever we are doing now is in response to our souls' desire. As pointed out by professor, "If you look back over your life you will see that people you never knew existed suddenly entered your life and brought with them enormous life changes. They might have been lovers, teachers, enemies or gurus" [6]. We have discussed in this paper how Dr. Weiss realized Catherine changed his life via hypnosis sessions.

A very good example of this concept has been captured in the American film (1946) "It's a wonderful life" [71]. The movie is considered one of the greatest movies made in USA, and is aired in many TV channels every year during Christmas time. The movie shows how we interact with many people and do very little and almost unknowingly and instinctively, to other people, and yet bring profound changes in their lives.

## Vili. Heisenberg and Destiny

Heisenberg has created the uncertainty principle. He said there are many things in nature that are related by some lower bound number. If you try to measure one of them precisely then the other variable will be very much in error. That is, the product of their measurement accuracies must be greater than a lower bound number. That means there is some uncertainty in measuring any variable. Nothing can be measured precisely.

This uncertainty prevents the concepts of destiny defined by Newtonian physics as derived here. In this section we show [31] that uncertainty principle violates the laws of nature and therefore cannot be correct. This conclusion will then reestablish the destiny concept and the strength of the Newtonian physics.

## a) Assumptions in Physics

In a previous section, called characteristics of nature, we have discussed that nature has the following features that are not obeyed by our mathematical and physical theories: boundedness law, finite time law, simultaneity law. Most important assumption we make is the linearity, which violates the boundedness law. In Laplace and Fourier transform we use infinity violating the finite time law. Many theories assume isolated environments like Newton's laws of motions. We have shown in that section how these theories fail. They are not approximate theories they are outright wrong. In this section we go deeper into the uncertainty principle to show how this theory ignores the fundamentals of nature.

No two objects of nature can be compared. It is a very fundamental characteristic of nature. The simple algebraic expression $\mathrm{x}=\mathrm{y}$ cannot be used for any object in nature. All objects are unique and they all are like apples and oranges. Even two apples are not equal. Thus the operator theory used for proving uncertainty principle cannot be true also. Every object has a soul, without a soul an object cannot be created, humans are not exceptions. The souls have eternal existence. Comparing things is an idea propagated by the central bank, money, and money power. Money is used to measure people, which is completely impossible and irrational thing to do. It ignores the fundamental reality of soul, reincarnation, past life knowledge and experiences. Yet no scientists, economists dare to object this comparison system, because they all have been purchased by the money power. Therefore you can imagine the pervasive nature of money power, and how it is used to suppress the truth. The objective of this research is to show that truth is unique, universal, and eternal.

## b) Assumptions in $Q M$

In his book [72] of 1930 Heisenberg gives details about the proof of his uncertainty principle. More information of this theory can be found in [31]. In his proof he made two important assumptions:

- He assumes that momentum and position are related by Fourier Transform (FT) pair, and
- He ignores the infinity assumption of FT theory.

Both assumptions violate the laws of nature.
The following proof is taken from the book: Heisenberg [72] pages 15-19. In all integrals, Heisenberg assumes, the lower limit is $-\infty$ and the upper limit is $+\infty$. The average value of the position $q$ of an electron can be given by the probability amplitude S( $q^{\prime}$ ) as:

$$
\bar{q}=\int q^{\prime}\left|S\left(q^{\prime}\right)\right|^{2} d q^{\prime}
$$

Then $\Delta q$ is defined by

$$
\begin{equation*}
(\Delta q)^{2}=2 \int\left(q^{\prime}-\bar{q}\right)^{2}\left|S\left(q^{\prime}\right)\right|^{2} d q^{\prime} \tag{6}
\end{equation*}
$$

This expression (6) can be called the uncertainty in the knowledge of the electron's position. In the same way the momentum $p$ and its uncertainty may be defined as

Here $T\left(p^{\prime}\right)$ is the probability amplitude of the electron's momentum. After these definitions, Heisenberg makes his very crucial assumption. He claims that the two probability amplitudes are related by the equations

$$
\begin{align*}
& T\left(p^{\prime}\right)=\int S\left(q^{\prime}\right) R\left(q^{\prime} p^{\prime}\right) d q^{\prime}  \tag{8}\\
& S\left(q^{\prime}\right)=\int T\left(p^{\prime}\right) R^{*}\left(q^{\prime} p^{\prime}\right) d p^{\prime} \tag{9}
\end{align*}
$$

Observe that (8-9) are equivalent to an abstract Fourier Transform pair. He reveals the details within next few steps given below. This is a very unrealistic assumption on nature. There is no reason to believe, that nature will know our mathematics, and then obey the relation (8) and (9), connecting position and momentum. Position and momentum can be two completely independent variables in nature.

He defines $R\left(q^{\prime} p^{\prime}\right)$ as the matrix of the transformation from a Hilbert space, in which $q$ is a diagonal matrix, to one in which $p$ is diagonal. The matrix $R\left(q^{\prime} p^{\prime}\right)$ satisfies the following equation

$$
\begin{equation*}
\int p\left(q^{\prime} q^{"}\right) \mathrm{R}\left(\mathrm{q}^{\prime \prime} p^{\prime}\right) d q^{\prime \prime}=\int R\left(q^{\prime} p^{\prime \prime}\right) \mathrm{p}\left(\mathrm{p} " p^{\prime}\right) d p^{\prime \prime} \tag{10}
\end{equation*}
$$

which is equivalent to the differential equation

$$
\begin{equation*}
\frac{h}{2 \pi i} \frac{\partial}{\partial q}, R\left(q^{\prime} p^{\prime}\right)=p^{\prime} R\left(q^{\prime} p^{\prime}\right) \tag{11}
\end{equation*}
$$

and whose solution is given by

$$
\begin{equation*}
R=c e^{\frac{2 \pi i}{h} p^{\prime} q^{\prime}} \tag{12}
\end{equation*}
$$

The details of (10-11) are given in the Appendix of the same book [72]. Observe that (10-12) are equivalent statements, i.e., one can be derived from the other. Thus the form of R in (12) could have been assumed directly.

Normalizing gives c the value of $1 / \sqrt{h}$. He claims, quite naturally that, the values of $\Delta \mathrm{p}$ and $\Delta \mathrm{q}$ are thus not independent. This is also obvious from his assumption of relating p and q via (8-9). To simplify further calculations, he introduces the following abbreviations:

$$
\begin{gathered}
x=q^{\prime}-\bar{q}, \quad y=p^{\prime}-\bar{p} \\
s(x)=S\left(q^{\prime}\right) e^{\frac{2 \pi i}{h} \bar{p} q^{\prime}} \\
t(y)=T\left(p^{\prime}\right) e^{-\frac{2 \pi i}{h} \bar{q}\left(p^{\prime}-\bar{p}\right)}
\end{gathered}
$$

$$
\begin{align*}
& (\Delta q)^{2}=2 \int x^{2}|s(x)|^{2} d x  \tag{13}\\
& (\Delta p)^{2}=2 \int y^{2}|t(y)|^{2} d y \tag{14}
\end{align*}
$$

While equations (8-9) become

$$
\begin{align*}
& t(y)=\frac{1}{\sqrt{h}} \int s(x) e^{\frac{2 \pi i}{h} x y} d x  \tag{15}\\
& s(x)=\frac{1}{\sqrt{h}} \int t(y) e^{-\frac{2 \pi i}{h} x y} d y \tag{16}
\end{align*}
$$

The expressions (15) and (16) can be verified by direct substitution of definitions of $x, y, t(y), s(x)$, and $R$ in (8) and (9). Observe that (15-16) could have been directly assumed instead of (8-9), since they all are merely assumptions. There was no need to introduce the intermediate steps related to (8-12). In fact there are proofs of uncertainty principle that jumps from (7) directly to (15) [31].

Now we can clearly see that (15) and (16) are classical infinite Fourier Transform pair, (14) represents the variations in time and (13) represents variations in spectrum. Thus the uncertainty principle is nothing but the product of time and bandwidth. Combining (13-15), the expression for $(\Delta p)^{2}$ may be transformed, giving

$$
\begin{aligned}
& \frac{1}{2}(\Delta p)^{2}=\frac{1}{\sqrt{h}} \int y^{2} t^{*}(y) d y \int s(x) e^{\frac{2 \pi i}{h} x y} d x \\
& =\frac{1}{\sqrt{h}} \int t^{*}(y) d y \int s(x)\left(\frac{h}{2 \pi i} \frac{d}{d x}\right)^{2} e^{\frac{2 \pi i}{h} x y} d x \\
& =\frac{1}{\sqrt{h}}\left(\frac{h}{2 \pi i}\right)^{2} \int t^{*}(y) d y \int \frac{d^{2} s}{d x^{2}} e^{\frac{2 \pi i}{h} x y} d x \\
& =\left(\frac{h}{2 \pi i}\right)^{2} \int s^{*}(x) \frac{d^{2} s}{d x^{2}} d x
\end{aligned}
$$

Thus he writes, by using integration by parts, and noting that $S\left(q^{\prime}\right)$ is related to probability density function vanishing at two ends:

$$
\begin{equation*}
\frac{1}{2}(\Delta p)^{2}=\frac{h^{2}}{4 \pi^{2}} \int\left|\frac{d s}{d x}\right|^{2} d x \tag{17}
\end{equation*}
$$

Now the following expression

Hence it follows from equations (13) and (17) that

$$
\frac{1}{2}(\Delta p)^{2} \geq \frac{1}{2} \frac{h^{2}}{4 \pi^{2}} \frac{1}{(\Delta q)^{2}}
$$

or

$$
\begin{equation*}
\Delta p \Delta q \geq \frac{h}{2 \pi} \tag{19}
\end{equation*}
$$

which was to be proved. The equality can be true in (19) only when the left side of (18) vanishes, i.e., when

$$
s(x)=c e^{-\frac{x^{2}}{2(\Delta q)^{2}}}
$$

or

$$
S\left(q^{\prime}\right)=c e^{-\frac{\left(q^{\prime}-\bar{q}\right)^{2}}{2(\Delta q)^{2}}-\frac{2 \pi i}{h} \vec{p} q^{\prime}}
$$

where c is an arbitrary constant. Thus Gaussian probability distribution causes the product $\Delta p \Delta q$ to assume its minimum value.

In summary, Hesienberg assumed that momentum and wave functions are related by the Fourier transform pair (15) and (16). Then he defined the variances of the time and spectrum functions using (13) and (14). Then a simple algebraic manipulation proved the uncertainty relation (19). From (19) we see that uncertainty is the product of time and spectrum variances (bandwidth) of FT pair. It is clear from the above proof that there is no physical or experimental support behind the result (19), the uncertainty principle. It is a consequence of Fourier Transform which has its own assumptions as we will examine later. In particular, we will show that by removing infinity assumption we can remove the uncertainty.


Thus to create the uncertainty principle Heisenberg made

- one vital assumption - the momentum and position are related by FT pair and
- overlooked the hidden assumption of infinity in the definition of FT pair.

Of course he also made the global assumption - whatever happens in mathematics will happen in nature too - a tremendous ego of humanity over nature. We must be very humble to nature, which has created us over a period of billions of years. It is impossible for humanity to understand nature; just like our computers cannot know its creator, the humans. Next we show that uncertainty principle cannot be a law of nature by changing the infinity assumption of FT.

## c) Infinite Fourier Transform

The Fourier Transform (FT) has become a fundamental tool for many branches of science and engineering; quantum mechanics, as we have just seen, is no exception. The FT pair is defined as:

$$
\begin{gather*}
X(w)=\int_{-\infty}^{\infty} x(t) e^{-i w t} d t  \tag{20}\\
x(t)=\frac{1}{2 \pi} \int_{-\infty}^{\infty} X(w) e^{i w t} d w \tag{21}
\end{gather*}
$$

Equation (20) gives the FT $\mathrm{X}(\mathrm{w})$ of the time function $\mathrm{x}(t)$. Expression (21) gives the Inverse FT from the spectrum function $\mathrm{X}(w)$ and produces the time function $\mathrm{x}(\mathrm{t})$. In the previous section we have given the proof that shows that uncertainty principle is derived from the FT theory. In this section we analyze the FT in more details to find the root cause of this uncertainty.

Observe that both integrals have infinity as limits. One way to examine this infinity requirement of FT is to visualize the example of the delta function. Its FT is 1 for all w . That means all cosine functions that create the delta function have unit amplitude and zero phase. If you draw some of these cosine functions, see Priemer [73] in pages 178179, you will find that the functions are adding up to create the pulse and becoming zero at all other places. This example shows that all cosine functions must be defined over all time, and the same must be true for the delta function also. That is, the delta function must exist as zero for the entire real line except the place where it is non-zero.

Consider the time function shown in Figure 3 and the corresponding Fourier transformed spectrum function shown in Figure 4. The graph in Figure 4 was obtained using expression (20). Both functions must be defined and must exist for the entire x -axis as required by the FT theory. The width of the distribution in spectrum $\Delta \mathrm{w}$ is $4 \pi / \Delta \mathrm{T}$ and the width $\Delta t=\Delta T$ of the distribution in time function can be regarded as uncertainties. The product of these two uncertainties show that

$$
\begin{equation*}
\Delta w \Delta t=4 \pi \tag{22}
\end{equation*}
$$

The quantum mechanics textbook by Ohanian [74] page 35, says (22) "is an instance of Heisenberg's uncertainty relation". The same idea is described in the engineering book, Soliman [75] pages 214-216, and has the section heading "The Uncertainty Principle". Note that (22) is not derived from any assumption. This is the property of Fourier Transform pair where time and spectrum functions are related by the Fourier integral.

The expression (22) is known as Time-Bandwidth product, and also as Dimensionality theorem in the field of digital communication engineering, see [76] page 93. Since the spectrum function is derived from the time function, the two functions are related, and (22) represents one such relation.

## d) Finite Fourier Transform

First we present a sampling theorem for finite duration continuous functions [77] without proof. We will use this result for our theory on finite Fourier transform.

Let $f(t)$ be a continuous time function defined over $L_{2}[a, b]$, the space of square integrable continuous functions over [a,b]. For engineering we do not need measurability and Lebesgue integrability. Assume that we divide the finite time interval [a,b] into $\mathrm{n} \geq 1$ equal parts using equally spaced points $\left\{t_{1}, t_{2}, \ldots t_{n}, t_{n+1}\right\}$. Where $t_{1}=a$ and $t_{n+1}=b$. Use the following notations to represent the t -subintervals

$$
\Delta t_{i}=\left[t_{i}, t_{i+1}\right), i=1 \ldots n-1 \text { and } \Delta t_{n}=\left[t_{n}, t_{n+1}\right]
$$

then define the characteristic functions:

$$
X_{i}(t)=\left\{\begin{array}{l}
1 \text { when } t \in \Delta t_{i} \\
0 \text { otherwise }
\end{array} \text { for } i=1 \ldots n\right.
$$

and the simple functions:

$$
f_{n}(t)=\sum_{i=1}^{n} f\left(t_{i}\right) X_{i}(t) \quad \forall t \in[a, b]
$$

## Theorem 1

$$
f_{n}(t) \rightarrow f(t) \text { as } n \rightarrow \infty, \text { on }[a, b] \text { in } L_{2}[a, b] \text { norm. }
$$

The above theorem, [77], essentially says that the sequence of step functions, with step height defined by the sample values, converges to the original function. Thus given any accuracy limit, a step function can be generated that will represent the function with that accuracy. The theorem says that this conclusion is valid for any finite interval.

We now show that if we eliminate the infinity assumptions from Fourier transform expressions in (20) and (21) then we can overcome this lower bound error limit (22) from the uncertainty principle. In case of a band limited waveform, that is, a waveform whose spectrum is zero outside a finite bandwidth $[-B,+B]$, the expression for inverse Fourier transform (21) can be rewritten as

$$
\begin{equation*}
x(t)=\frac{1}{2 \pi} \int_{-2 \pi B}^{2 \pi B} X(w) e^{i w t} d w \tag{23}
\end{equation*}
$$

We use the principles behind the numerical inversion of Laplace transform method as described in [78]. Let the measurement window for the function $\mathrm{x}(\mathrm{t})$ be $[0, \mathrm{~T}]$, where T is finite and not necessarily a large number. Divide the frequency interval 2 B into K smaller equal sub-intervals of width $\Delta \mathrm{w}$ with equally spaced points $\left\{w_{j}\right\}$ and assume that $\left\{X\left(w_{j}\right)\right\}$ is constant but unknown over that j -th interval. Then we can express the integration in (23) approximately as:

$$
\begin{equation*}
x(t) \approx \frac{1}{2 \pi}(\Delta w) \sum_{j=1}^{K} e^{i t w_{j}} X\left(w_{j}\right) \tag{24}
\end{equation*}
$$

The right hand side of (24) is a linear equation in $\left\{X\left(w_{j}\right)\right\}$, which are unknown. Now we can also divide the interval $[0, T]$ into $K$ equal parts with equally spaced points $\left\{\mathrm{t}_{\mathrm{j}}\right\}$ and let the corresponding known sample values be $\left\{\mathrm{x}\left(\mathrm{t}_{\mathrm{j}}\right)\right\}$. Then if we repeat the expression (24) for each sample point $\mathrm{t}_{\mathrm{j}}$ we get K simultaneous equations in the K unknown variables $\left\{X\left(w_{j}\right)\right\}$ as given by the equivalent expression (25).

$$
\begin{gather*}
x=\frac{\Delta w}{2 \pi} E(t, w) X, \quad \text { Where }  \tag{25}\\
E=\left\{E_{m n}\right\}=e^{i t_{m} w_{n}}, m, n=1, \ldots K
\end{gather*}
$$

The equations in the matrix vector expression (25) are independent because exponential functions in (25) are independent. Therefore we can solve (25) for $\left\{\mathrm{X}\left(\mathrm{w}_{\mathrm{j}}\right)\right\}$. Theorem 1 ensures that the sets $\left\{\mathrm{X}\left(\mathrm{w}_{\mathrm{j}}\right)\right\}$ and $\left\{\mathrm{x}\left(\mathrm{t}_{\mathrm{j}}\right)\right\}$ can be selected to achieve any level of accuracy requirements in (23) for both $\mathrm{x}(\mathrm{t})$ and $\mathrm{X}(\mathrm{w})$. Note that the accuracy for both $\mathrm{x}(\mathrm{t})$ and $\mathrm{X}(\mathrm{w})$ increase simultaneously, as we increase the sample rate. For convenience we assume that the number of terms $K$ in (25) is equal to $K=T k f_{s}=2 k B T$. Here $f_{s}$ is the Nyquist sample rate and $\mathrm{k}>1$. We state the following theorem from [77], which essentially is a modification of the Nyquist's sampling theorem.

## Theorem 2

Let $\mathrm{x}(\mathrm{t})$ be a band limited function with bandwidth restricted to $[-\mathrm{B},+\mathrm{B}]$ and available over the finite measurement window $[0, T]$. Then given any accuracy estimate $\varepsilon$ there exists $k>1$ such that 2 kBT equally spaced samples of $\mathrm{x}(\mathrm{t})$ over $[0, \mathrm{~T}]$ will completely specify the Fourier transform $\mathrm{X}(\mathrm{w})$ at the given accuracy $\varepsilon$.

Theorem 2 says that by increasing the sample rate we can achieve any accuracy in describing the spectrum function and the time function. The sampling factor k is a multiple of the Nyquist rate. In industry it is quite common to use 4 or 5 times the

Nyquist rate to get correct results. In [77] a numerical example is given to show how k affects the accuracy in recovering the time functions.

In Figure 5 we show, using a numerical example, how the unit pulse of Figure 3 with $\Delta \mathrm{T}=2$ can be reconstructed using (25) for various values of bandwidth. The red graph is derived for a bandwidth of $\pi$, and $\mathrm{K}=20$ as number of samples. The black graph represents the classical FT of the pulse in Figure 3. Figure 5 shows how a narrow bandwidth red spectrum can generate the same time function when we eliminate infinity assumption. A larger bandwidth spectrum could have been used also, but with a different sample rate to create the same pulse. We call method (25) as numerical inverse finite Fourier transform, NIFFT.

It is clear from Figure 5 that the uncertainty relation (22) does not hold if we use (25). Expression (25) gives many solutions for spectrum for the same time function and depends on values of K and bandwidth. By eliminating infinity, the formula (25) brings two additional degrees of freedom, a choice of bandwidth and a sample rate. These two degrees of freedom make (25) an independent method of constructing waveforms. The rectangular pulse generated from the red spectrum is same as shown in Figure 3.


Figure 5 : Different spectrums for same pulse

## e) Operator Theory

There is another proof of uncertainty principle based on operator theory of functional analysis. The details of this approach are provided in the paper [31]. This theory is entirely based on algebra and vector space, which essentially is based on algebra

There is a very basic assumption in algebra that all physics and engineering people ignore. The assumption is that in algebra all numbers must be real numbers. That is even the basic equation like $\mathrm{a}-\mathrm{b}=0$ assumes that both variables a and b are real numbers. But in nature no two objects are equal. Therefore no two objects can be compared. No objects in nature can be converted to real numbers. A cat, a dog, or a human being cannot be converted to a real number and therefore cannot be compared. Similarly two electrons cannot be compared also. Thus this operator theory is fundamentally wrong in its application to science and engineering.

But our society is so accustomed to measuring everything using numbers that we often times forget its applicability to nature. We measure people using their salaries. This is a completely wrong process. Money power is controlling us in our subconscious mind. Human merits cannot be compared and cannot be evaluated also. Thus this merit system
of our economy is completely baseless and cannot be used for nature. We cannot extend this artificial economic theory to electrons and compare them like in Schwartz inequality theorem. Even two characteristics of same object, like position and momentum, cannot be compared using numbers. Nature is very complex; everything has lot of details. Nature has evolved over billions of years, and has many microscopic features that will prevent comparisons of two objects or two features of same object. Thus algebra is completely unusable for nature, more so for quantum mechanical particles.

An electron taken out of its orbit has completely different characteristics. I am a completely different person when I am outside my home. Even two electrons are different, because they have different positions in their orbits. The resultant simultaneous effects on these different positions are different. Thus the basis of the operator theory is completely false for nature.

## IX. Newton and Destiny

Classical mechanics or Newtonian theory is considered perfectly deterministic, that is, there are no probability or chance events in nature. Using this approach we derive a conceptual expression for global destiny and then show how individual destiny can be interpreted. This approach will thus show absence of any freewill. The idea of destiny based on Newtonian theory is pretty well established in science community; however, we derive very systematically an analytical model to illustrate the global and local views of destiny.

In our universe there are only two types of objects: some material objects and some force type objects. Material objects are those that we can experience with our sense organs. These are objects that we can touch, feel, or see. They include all non-living and living objects, including humans. The force type objects are like heat energy, gravitational force, electromagnetic force, light energy etc. It should be recognized that all force type objects are always associated with some material type objects. Gravitational force is always from a material object like sun, moon etc. Similarly light energy is from sun or a light bulb. Some of the forces are also created by human objects using hands, feet, mouth etc. Thus essentially we can think of everything that is happening in our world is due to interactions of either some material objects or force objects.

## a) Sigma Law

The sigma law can be stated using the following sigma notation:

$$
\begin{equation*}
\sum_{i=1}^{N} x_{i}=0 \tag{26}
\end{equation*}
$$

Here the set $\left\{\mathrm{x}_{\mathrm{i}}\right\}$ represents any physical variable. If you use any consistent set of physical variables, with proper units and dimensions, then you will always find that the above sum in the left hand side of (26) will be zero. The right hand side can become a given constant, but the constant can be absorbed in the left hand side. The variables, for example, can be forces, energies, or masses etc. Thus (26) is a balancing equation; things are balanced in nature.

The law of conservation (LOC) of mass and energy are two important laws of nature. It has been shown [79] that this is the only meaningful law of nature; all other laws of physics are either equivalent to or can be derived from this law. The law says that mass and energy cannot be created or destroyed. Thus the sigma of all masses is a constant; the same is true for energy. They can only be transformed or transferred. Thus
to produce a car we must use materials and energy from nature. It cannot be created out of nothing. We call LOC as the sigma law.

## b) Action Reaction Law

The action-reaction law or the Newton's third law is another important law of nature. It can be found in [17, p.120] and has been explained in the following way. In an isolated environment, the forces always occur in pairs or that a single isolated force cannot exist. Any one of these two forces can be called the action force, and the other one then can be called the reaction force. The reaction force is equal in magnitude of the action force and of opposite in direction and can be written as in (27):

$$
\begin{equation*}
F_{2}=-F_{1} \quad \text { or } \quad F_{1}+F_{2}=0 \tag{27}
\end{equation*}
$$

Since everything is interacting simultaneously over the entire global space and time, for every action, $\mathrm{F}_{1}$, there will always be more than one reaction $\left\{\mathrm{F}_{2}, \mathrm{~F}_{3}, \ldots \mathrm{~F}_{\mathrm{N}}\right\}$. It is thus not possible to create an isolated system and produce a single reaction as in (27). However, the summation of all reactions must still be equal to the original action that produced all the reactions. Therefore in real life we should have (28):

$$
\begin{align*}
& F_{1}=-\left(F_{2}+F_{3}+\cdots+F_{N}\right) \quad \text { or } \\
& \sum_{i=1}^{N} F_{i}=0 \tag{28}
\end{align*}
$$

Expression (28) shows that the Newton's third law is a sigma law.
We should recognize that all human actions satisfy this law also. Our hands create physical forces, our ears receive physical sound wave actions; we see energy of natural lights through our eyes. Thus humans are tightly integrated with this physical world. Inside our bodies also this same action-reaction law works. We can think and imagine, but all our actions are guided by this action-reaction law of nature.

## c) Time and Action-Reaction

In this subsection we show how laws of nature can be used to prove that the present is defined by the past. That is, the destiny idea follows clearly and in a straight forward way from the action reaction law.

As we have mentioned, this law says that for every action there is an equal and opposite reaction. This reaction, according to the law, is supposed to happen instantaneously. But in nature nothing happens like that, everything takes some time. Thus if an action F1 happens at time t, then its reaction F2 will happen at dt time later, where dt is a very small time, can be called observational delay, measurement delay, or reaction time etc. Thus F2 will happen at time $t+d t$. We know that this F 2 will be equal and opposite of F1, thus we can write F2 = -F1. Since we know F1, we thus know F2 also. Thus future is clearly known; this is future because $t+d t$ is future time. Thus the action reaction law clearly states that the destiny is precisely defined and can be found if we know the present. This analysis essentially leads to expression (27).

The logic can be applied in the same way to describe the past, of any action that happened now, at time t. Suppose an action F1 happened at present time t. Then according to action reaction law an action F2 must have happened at t-dt time, and F2 must be equal to -F1. Thus the past F2 is completely known from the present F1. If that was not the case then the action F1 could not have happened. Thus the entire trajectory of all actions is completely planned. In this case also, the logic leads to the equation (27).

The action reaction forces that are happening within the dt time interval will be called the first reaction events. If we know the first reaction events then we can sequentially and repeatedly apply this law to determine all future and past events. In the case of individual destiny analysis we will show how to predict this first reaction event. Which of course will be equal and opposite to the force that caused the event.

However, as we have mentioned, the action reaction law has an assumption of isolated environment, which cannot be true in nature. There exists a simultaneity law in nature, which says that: everything in nature happens because of simultaneous interactions of every object in nature. We have explained all humans are tightly integrated in our world with our actions and reactions. This integration is not just in present life, because of reincarnation law we were connected in our past lives also. The past life actions are working in present life too. Therefore according to the simultaneity law for any action there will be more than one reaction at any future time $t+d t$. So the number of reactions, N in (28), will be very large, and the share of any action among all the reactions will be very difficult to find out. But theoretically we understand that, it can be found out, that is, the action reaction law is valid. Thus we must extend our logic from expression (27) to expression (28) to incorporate the simultaneity law of nature.

Note that at time $t+d t$ many other new reactions will also interfere with the object, beside the reactions originated by $\mathrm{F}_{1}$. Thus all objects must be considered simultaneously to find the total reaction on any object. It must be understood clearly, that because of simultaneity law I do not create my destiny. My destiny is created by the actions of many people working together, including past life activities with many people. Thus destiny is tied with the global plan of the society. The notion that I create my destiny, I create my karma, I am independent are not correct and goes against the simultaneity law and reincarnation law of nature. We are not isolated individuals, we are together, and are highly integrated through our activities. Thus it is not my destiny or your destiny, it is our destiny.

## d) Global Destiny

We extend further the action reaction law to create a dynamic model of the global destiny. The sigma law essentially gives us the global plan. We can write:

$$
\begin{align*}
& \sum_{i=1}^{N} x_{i} \neq 0 \quad \text { о } \\
& x_{1}+x_{2}+x_{3}+\cdots x_{N}=0 \tag{29}
\end{align*}
$$

In (29) we can assume that all variables are representing action-reaction forces including that of all humans. Since all the variables represent different parts of the same physical quantities, they may require to be normalized based on scale factors, units, or dimensions. Thus we can add coefficients to each term of (29) to describe the last equation as in (30).

$$
\begin{equation*}
a_{1} x_{1}+a_{2} x_{2}+a_{3} x_{3}+\cdots+a_{N} x_{N}=0 \tag{30}
\end{equation*}
$$

We can now separate the first variable from all other variables and write:

$$
x_{1}=\left(1+a_{1}\right) x_{1}+a_{2} x_{2}+a_{3} x_{3}+\cdots+a_{N} x_{N}
$$

Redefining the coefficients we can rewrite the above expression in the following way:

$$
\begin{equation*}
x_{1}=a_{1} x_{1}+a_{2} x_{2}+a_{3} x_{3}+\cdots+a_{N} x_{N} \tag{31}
\end{equation*}
$$

The equation (31) shows how the action $\mathrm{x}_{1}$ depends on all action variables. It also means that every action is dependent simultaneously on all actions. Since all actions are continuously changing with time, we can represent the left hand side of (31) as the time derive of the variable, represented by a dot as superscript. Thus (31) reduces to:

$$
\begin{equation*}
\dot{x}_{1}=a_{1} x_{1}+a_{2} x_{2}+a_{3} x_{3}+\cdots+a_{N} x_{N} \tag{32}
\end{equation*}
$$

Now the above must also hold true for all the variables, like $\mathrm{x}_{2}, \mathrm{x}_{3}$ etc, that is, we should repeat the same equation, for all variables placed in the left hand side. To accommodate we need to change the subscripts on the coefficients. Using matrix notation then we can write as in (33). Here we have extended (33) to a very large set of equations, almost infinite number of equations in infinite number of variables. This equation represents the action-reaction model of the entire universe. Therefore it cannot have any control variable or independent action force from outside, since there is no outside. Using matrix vector notation we express (33) as in (34).

$$
\begin{align*}
& \dot{x}=A x, \quad x\left(t_{0}\right)=x_{0} \tag{34}
\end{align*}
$$

Since we have derived (33) from the sigma law, every equation in (33) represents a balance equation for all actions and reaction in the global space and global time frame. One highlighted area in (33) may represent all actions of one person. Similarly the other highlighted area may represent the actions of another person. Equation (33) is a mathematical representation of the simultaneity law, which says we are all simultaneously working together in this world. The equation reflects that none of us are identical and cannot function as an isolated individual.

Figure 6 describes equation (33) in an intuitive graphical form. Here the rectangular boxes represent objects of nature. They can be material objects and/or human objects. Each object is connected to all other objects with spring like interfaces as shown by the zigzag and dashed lines. These interfaces generate the coefficients in (31), converting objects into actions. If any object moves then we can see how all objects of the universe will move and react. Philosopher Nietzsche thought it that way also [80, p.10]. Thus the universe is continuously moving as a dynamic system. Figure 6 thus represents the global plan created by the simultaneous action reaction of all objects, including all humans in nature.

The expression (33) is an equation, therefore it has a solution. The simple model (34) has a simple closed form solution also and is given, for a finite but very large dimensional system, by (35):

$$
\begin{equation*}
x\left(t-t_{0}\right)=e^{A\left(t-t_{0}\right)} x\left(t_{0}\right) \tag{35}
\end{equation*}
$$

Here $t_{0}$ is the present time, $A$ is the matrix of coefficients in (33), and $t$ may represent both past and future time. The mathematical theory of differential equations ensures the existence of solution even for more complicated versions of (34) and under fairly realistic conditions [81, p.40]. It has also been shown that the solution is unique under a given set of initial conditions. The theory says that the solution can be extended from minus infinity to plus infinity on time scale.

The above solution has an important property worth noting. If we produce two solutions starting from two different initial conditions then their trajectories will never intersect. If they intersect then it will mean two events will be exactly identical. But that cannot happen according to the theory of differential equation. Thus two events are never equal in both space and time in the entire universe. We discuss this idea again in the local destiny subsection.

We must point out that it is not that the nature is following our model, we are trying to explain how nature may be working using our quantitative model of simultaneity law. We cannot do anything on our own in any place at any time. If we try, it will disturb all the equations and all variables in (33). We must follow what these equations are telling us to do. And in fact, as an example, we are just doing that every day in our corporate world. We know what exactly we have to do tomorrow in our office. We have our work cut out exactly. The life is very easy that way; we do not have to take the trouble of making decisions every day every moment. Following nature is much better than going against it and exerting our influences on the world. We actually cannot, as we will see the situation, in our local view described later. Thus simultaneity law represents the global destiny law. It represents our destiny.

## e) Individual Destiny



Figure 7 : Sequential interactions with

There is a law of physics or nature that says no two objects can occupy the same space at same time. We can call it the space-time problem. That is, you cannot bring two glasses at same location at same time. If you force them, then one of the glasses will break to make room for the other, that is, there will be a conflict.

This law has an important contribution for all conflict resolution problems. Whenever there is a conflict, you will always be able to find that two objects are trying to occupy same space at same time. For example, one person is trying to do a job and at the same time another person is also trying to do the same job and thus creating a conflict of interests. All conflicts can be prevented from happening if we can design our management system so that such occupation of same place does not happen at same time. This method can be applied to even persons, family, kids, work place, and in all business problems also. For example, when mother asks the kid to do something, at that same time the father should not also ask the kid to do something else. That is, two tasks are coming to the same place at same time to be performed by one child. Thus space-time problem is an important philosophy, based on laws of nature, for conflict resolution.

Figure 7 shows how a global plan becomes a sequence of events from the local point of view of an individual object or human being like A. Here we consider A as a single point in space, the personal space of one person. In his personal space he is isolated and alone. The right hand side represents progress of time. The global plan generates events sequentially, as represented by the sequence of dots, for every single person. This is sequential, because two events cannot happen at same place and at same time. The person A is also evolving with time as he gains experiences in life. This progress of characteristics of A is also represented by a sequence of dots as shown by bottom array. Thus both events and characteristics are emanating from the global plan. Although we have isolated A from the global plan in the figure, but in reality they are all inside the global plan. We have isolated for demonstrating how the global plan becomes a local plan for an individual. The entire environment where this individual event will happen is also dependent on space, time, and the individual. The environment can be home, work place, shopping mall, or highways etc.

Nature is continuously changing and time is running. Every moment the environment around A would be changing due to continuous evolution of events and personal characteristics. From the diagram we can see that the person cannot react independently. He is reacting only in response to events which are actions. Note that not acting is also a reaction. At a single point, the person is isolated and working alone in response to actions presented to him sequentially, that is, only one action at a time. He has to follow the Newton's action-reaction law (27), which is based on only two forces. He has therefore only one choice, apply the equal and opposite force. We can see that the situation will not give us any possibility of exercising our options. Or in other words the environment will itself eliminate all but one option. The idea is illustrated using the following example.

## Example

Assume that you are driving on a busy highway in a city like Los Angeles and then realize that you have a flat tire. This flat tire is one of the sequential events that appeared to you from nature's global plan. All events appear to us from nature. You sense the event, and understand the event. This sensing the event is your first reaction. You slow down your car on the highway. You do not have any choice, but to slow down. Thus the very first reaction, within 200-500 milliseconds, to the event was not under your control. We reacted just like a physical non-living object of nature, we produced the equal and
opposite reaction. If we analyze every moment carefully in this way, in this short time interval, and as response to an action, you will find there is no possibility of freewill anywhere.

The flat tire event or action happened at time $t$ and your reaction happened at time $\mathrm{t}+\mathrm{dt}$. Your reaction at dt time later is your first reaction. Clearly in this case then my reaction will be equal and opposite to the action force. I have to stop my driving force. There is no other choice for my first reaction at $\mathrm{t}+\mathrm{dt}$ time.

As we have mentioned before, it is the first reaction that is very important for analysis to understand that we do not have any freewill. Nature presented us an event, and we must react. This first reaction will obey the action reaction law given by (27). Our reaction will be equal and opposite of the event presented to us by nature. The car is automatically slowing down and you stop accelerating. Things are happening very fast at this level of time scale. The time dt is very small, less than one second, of the order of 200 to 500 milliseconds. During this time we can only react, there is no time to draw a decision tree diagram and analyze the tree for optimal choice.

These events are happening so fast in milliseconds rate that the person does not even know that he is playing in the hands of nature. Soul to soul communication plays the most important role here. The RP, ready potential signal, and the W , the wish to act signal, of Libet's experiment as discussed before [5], can give an alternative explanation of such communication facts. The yogic power can allow us to see such events. People born with yogic power can also see things in the same way but with a limited scope.

For every human being, if you notice carefully, you will find all interactions with all persons happen sequentially. The time factor is always there in (33) whenever we are focusing on a single point in space around any one of us. We have also discussed in the context of (35) that the solutions are unique. This observation allows us to take care of things sequentially. Human brain also cannot do things simultaneously. It performs all actions one after another in sequence. Therefore the way you will act will be sequential also. Thus the guiding factors for our actions are:

- events are continuously coming, sequentially, from nature, that is, from the global plan to you
- you are continuously changing because of the continuously changing environment created by the global plan
- the time is of essence, is continuously progressing, and you must act
- your reaction will be equal and opposite to the single and unique action presented by the event.

Thus what you will do within the first reaction time dt is fixed, and preplanned by nature. You are doing equal and opposite to what nature is dictating by presenting the event to you.

## f) Destiny and Responsibility

Destiny is not same for everybody. Every individual will encounter different series of events and his characteristics will also evolve in an unique way according to his past life and present life experiences. The entire incident will happen in a specific environment. Thus reactions, and therefore destiny, of every individual will be unique and different.

The time and the moment dictate the destiny. There are many such small and large scale real life examples all over the world, in political, religious, and governmental scenarios. We can thus see that morality and immorality, good or bad, are not meaningful. Since everything is part of destiny. The differences in interpretations of
results are due to global versus local views of the incident. Only global space time (GST) view can give the universal truth. All local views, that ignore the global view, will always be wrong.

Since everything is a result of a global plan no one can be responsible for anything. No one is responsible for wars, poverty, murderers, killings, diseases, money, and money power. Death is also a part of global plan. Before we die everything deteriorates including societies, their values. Erath is spinning around its axis is a result of simultaneous law of nature. Earth has life is a also a law of nature. Events are happening and we are all responding according to the laws of nature. The global plan is also a law of nature. Local plan is derived from the global plan is also a law of nature. Eternal recurrence is a law of nature too. The history shows we had better societies at one time and we will have better systems again. Everything goes in a cycle of birth, maturity, and death process. Societies and environments cannot be exceptions.

No two events are same. They happened at two different times, so they are different. They can also happen at two different places and therefore they are different. Thus you cannot say you would have taken a different action for the same situation. This is because someone else would say the same thing for the actions you have taken in your life events. Since all persons are different, all events are different, all positions are different, and time of occurrences are different then everything must be different.

Everything in nature is like apples and oranges, and therefore they cannot be compared. Even two apples are different, they have different colors, different shapes and thus they are different. Two electrons are not same. They came from different orbits, or had different quantum numbers. Moreover, as soon as you take an electron out of its orbit it will act differently. Just like, I am completely a different person when I am outside of my home. Thus nobody can be blamed for his actions, because these things are happening according to nature's spontaneous evolution, and we are part of that. No one or nothing is good or bad; they are just different as mentioned in Gita (G-2:50) also.

You may say, I will sit idle and destiny will take care of it. No, that will not happen also. If it is not written for you then you will not be able to sit idle. What actions you will take is defined by events in nature and your characteristics. In a corporation you cannot go and sit idle. For, you have made your own plan about your all future activities of the project on which you are working. Thus sitting idle is not in your plan. This schedule of activities was created by all of your project team members in a planning and design level meeting. You cannot sit idle now. Why would you change your own plan without any reasons?

We are always in sync with nature, and working whatever nature is telling us to do. These things happen so quickly and so naturally that we never feel that we are forced to obey nature. Billion years of human evolution never makes us feel frustrated with nature. Thus our actions appear quite natural and almost like freewill. However, the analysis in this subsection has shown that everything is guided by the force of nature, and we are integrated with it, and not free at all. Our souls receive information well before we realize it. We always live in the past.

## X. Conclusions

Human destiny is created by the global plan of the universe. Simultaneous interactions of all objects, including all humans, help to generate this global plan. The global destiny spans the entire space and time history via the reincarnation law of all living objects. Individual destiny is derived from this global plan as a sequence of events.

These events present action and within 200-500 milliseconds we must act. This first reaction is always equal and opposite to the action presented to us. We react the way the nature tells us to do by offering the actions. We have shown that there are no uncertainties in nature because the concepts and mathematics used in the derivation of the uncertainty principle are inconsistent with the laws of nature. It is the soul that controls our physical body. The soul receives the information for what actions to perform. We are souls and not our body or brain.

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# On Some Problems in Fuzzy Sets Theory 

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Abstract - The aim of this paper is to sketch some connections between concepts and results of the classical mathematics and, respectively the mathematics of fuzzy sets.

GJSFR-F Classification : For Code: 54H05, 03D60

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## On Some Problems in Fuzzy Sets Theory

I. Tofan

## Abstract - The aim of this paper is to sketch some connections between concepts and results of the classical mathematics and, respectively the mathematics of fuzzy sets.

## I. Introduction ([3,6])

Let us remember some known definitions and results useful for the next sections.

Let $U$ be a nonempty set. A pair $(U, \mu)$, where $\mu: U \rightarrow[0,1]$ is a mapping is called a fuzzy set. We shall also call $\mu: U \rightarrow[0,1]$ a fuzzy subset of $U$ and denote

$$
\mathcal{F}(U)=[0,1]^{U}=\{\mu \mid \mu: U \rightarrow[0,1]\} .
$$

For $(U, \mu)$ and $\alpha \in[0,1]$ the set ${ }_{\mu} U_{\alpha}=\{x \in U \mid \mu(x) \geq \alpha\}$ (also denoted $\mu_{\alpha}$ ) is called the $\alpha$-level set of $(U, \mu)$.

In this context we have:
1.1 For any $x \in U, \mu(x)=\sup \left\{k \in[0,1] \mid x \in{ }_{\mu} U_{k}\right\}$.
1.2 Let $\left(U_{\alpha}\right)_{\alpha \in[0,1]} \subseteq \mathcal{P}(U)$ be a family of subsets of $U$. Then $\left(U_{\alpha}\right)_{\alpha \in[0,1]}$ is the family of level sets of a fuzzy subset $\mu: U \rightarrow[0,1]$ if and only if it satisfies the conditions:
i) $U_{0}=U$;
ii) $\alpha \leq \beta \Rightarrow U_{\beta} \subseteq U_{\alpha}(\alpha, \beta \in[0,1])$;
iii) for any increasing sequence $\left(\alpha_{i}\right)_{i \in \mathbb{N}}, \alpha_{i} \in[0,1], \forall i \in \mathbb{N}$, having limit $\alpha$ we have $U_{\alpha}=\bigcap_{i \in \mathbb{N}} U \alpha_{i}$.

It is clear that a fuzzy set is completely determined by the family of its level sets.
1.3 In relation with $\operatorname{Im} \mu(\mu: U \rightarrow[0,1])$ it is sayed that $\mu$ satisfies the sup property if any nonempty subset of $\operatorname{Im} \mu$ has the greatest element. In other words, $\mu$ has the sup property if and only if for every nonempty subset $A$ of $\operatorname{Im} \mu$, these exists $x \in\{y \in U \mid \mu(y) \in A\}$ such that $\mu(x)=\sup A$.
1.4 Using the couples $\left(t, t^{*}\right)$ where $t$ is a $t$-norm, and $t^{*}$ is its dual $t$ conorm the operations with fuzzy subsets are defined: for $\mu, \eta \in \mathcal{F}(U)$ we define $\mu \cap_{t} \eta$ and $\mu \cup_{t} \eta$ by

$$
\begin{aligned}
& \mu \cap_{t} \eta: U \rightarrow[0,1], \mu \cap_{t} \eta(x)=t(\mu(x), \eta(x)) \\
& \mu \cup_{t} \eta: U \rightarrow[0,1], \mu \cup_{t} \eta(x)=t^{*}(\mu(x), \eta(x)) .
\end{aligned}
$$

The most useful couples $\left(t, t^{*}\right)$ are given by:
i) $t_{1}(, y)=\min \{x, y\}, \quad t_{1}^{*}(x, y)=\max \{x, y\}$;
ii) $t_{2}(x, y)=x y, t_{2}^{*}(x, y)=x+y-x y$;
iii) $t_{3}(x, y)=\max \{x+y-1,0\}, t_{3}^{*}(x, y)=\min \{x+y, 1\}$.

The empty fuzzy set is given by $\widetilde{\emptyset}: U \rightarrow[0,1], \widetilde{\emptyset}(x)=0, \forall x \in U$. By $\widetilde{U}$ one intend the application $\widetilde{U}: U \rightarrow[0,1] \widetilde{U}(x)=1, \forall x \in U$. We shall denote $\mu \subseteq \eta$, if $\mu(x) \leq \eta(x), \forall x \in U$. The complement $\bar{\mu}$ of $\mu$ is given by $\bar{\mu}: U \rightarrow[0,1], \bar{\mu}(x)=1-\mu(x), \forall x \in U$.
1.5 A fuzzy subset $\mu: \mathbb{R} \rightarrow[0,1]$ (where $\mathbb{R}$ is the field of real numbers is called a fuzzy number if satisfies the following conditions:
i) there exists $x_{\mu} \in \mathbb{R}$ with $\mu\left(x_{\mu}\right)=1$;
ii) the set $\{x \mid \mu(x) \neq 0\}$ is bounded;
iii) the level sets ${ }_{\mu} \mathbb{R}_{\alpha}, \alpha \in[0,1]$, are closed intervals.

One usually takes the fuzzy numbers of the following type:

$$
\mu(x)= \begin{cases}0, & x<a \\ \pi_{1}(x), & x \in[a, b) \\ 1, & x \in[b, c] \\ \pi_{2}(x), & x \in[c, d] \\ 0, & x>d,\end{cases}
$$

where $a \leq b \leq c \leq d$ are reals and $\pi_{1}$ and $\pi_{2}$ satisfy the conditions that turn $\mu$ in a fuzzy number.

For $\pi_{1}(x)=\frac{x-a}{b-a}, \pi_{2}(x)=\frac{d-x}{d-c}(a \neq b, c \neq d)$ one gets trapezoidal fuzzy numbers. If also $b=c$, triangular fuzzy numbers are obtained. It is clear that a triangular fuzzy number is perfectly determined be a triple $(x, y, z)$, where $x, y, z$ are reals such that $x \leq y \leq z$ (for $x=y=z$, practically one obtains the reals).
1.6 Finaly, we recall that by fuzzy relation on $U$ one intends a fuzzy subset of $U \times U$.

## II. Level Sets ([1,7,8,9])

Using the notion of level set of a fuzzy set one can define:
2.1 Let $(G, \cdot)$ be a group and let $\mu: G \rightarrow[0,1]$ be a fuzzy subset of $G$. We say that $\mu$ is a (normal) fuzzy subgroup of $G$ if the level sets of $G,{ }_{\mu} G_{\alpha}$, are (normal) subgroups of $G$, for all $\alpha \in \operatorname{Im} \mu$.

One obtains that $\mu: G \rightarrow[0,1]$ is a fuzzy subgroup of $G$ if and only if:
i) $\mu(x \cdot y) \geq \min \{\mu(x), \mu(y)\}$, for any $x, y \in G$;
ii) $\mu\left(x^{-1}\right) \geq \mu(x), \forall x \in G$.

In the case of normal fuzzy subgroups we must add the condition iii) $\mu(x$. $y)=\mu(y \cdot x)$, for any $x, y \in G$.
2.2 In a similar way as above it is possible to define the notions of fuzzy subring or fuzzy ideal of a ring. In the following, we shall construct the fuzzy ring of quotients (a detailed study is given in [8]).

Let $(R,+, \cdot)$ be a unitary commutative ring and let $\sigma: R \rightarrow[0,1]$ be a fuzzy subset of $R$. We say that $\sigma$ is a fuzzy multiplicative subset of $R$ if every level set $\sigma_{t}, t \geq \sigma(0)$ is a multiplicative system (in the classical sense).

One obtain that $\sigma: R \rightarrow[0,1]$ is a fuzzy multiplicative subset of $R$ if and only if the following conditions are satisfied:
i) $\sigma(x \cdot y) \geq \min \{\sigma(x), \sigma(y)\}$, for any $x, y \in R$;
ii) $\sigma(0)=\min \{\sigma(x) \mid x \in R\}$;
iii) $\sigma(1)=\max \{\sigma(x) \mid x \in R\}$.

If $\sigma$ is a fuzzy multiplicative subset of $R$, then for every $t>\sigma(0)$, we may construct the classical ring of fractions $\sigma_{t}^{-1} R=S_{t}$ with respect to the multiplicative system $\sigma_{t}$. Let $\varphi_{t}$ denote the canonical ring homomorphism $R \rightarrow S_{t}$. If $\sigma(0) \leq s \leq t$, since $\sigma_{t} \leq \sigma_{s}$, the universality property of the ring of fractions yields to the existence of a unique ring homomorphism $\varphi_{t s}: S_{t} \rightarrow S_{s}$ such that $\varphi_{t s} \circ \varphi_{t}=\varphi_{s}$.

The system of rings and homomorphisms $\left(S_{t}, \varphi_{t s}\right)$ is an inductive system ( $[\sigma(0), 1]$ being endowed with the reverse of the usual order).

Let $\sigma^{-1} R$ denote the inductive limit of this system. It is natural to call $\sigma^{-1} R$ the ring of quotients relative to the fuzzy multiplicative system $\sigma$ (a universality property is satisfied $[8,9]$ ).
2.3 In the case of field extensions let $F / k$ be a field extension and $\mu$ : $F \rightarrow[0,1]$ be a fuzzy subset of $F . \mu$ is called a fuzzy intermediate field of $F / k$ if $\forall \alpha \in \operatorname{Im} \mu$, the level set $\mu_{\alpha}$ is an intermediate field of $F / k$.

We obtain that $\mu$ is a fuzzy intermediate field of $F / k$ if and only if the following conditions are satisfied:
i) $\mu(x-y) \geq \min \{\mu(x), \mu y\}$, for any $x, y \in F$;
ii) $\mu\left(x \cdot y^{-1}\right) \geq \min \{\mu(x), \mu(y)\}$, if $y \neq 0$;
iii) for every $x \in F \mu(x) \leq \mu(z), \forall z \in K$.
2.4 Let $X$ be a nonempty set and $\rho: X \times X \rightarrow[0,1]$ be a fuzzy relation on $X . \rho$ will be called a similarity relation on $X$ if $\forall \alpha \in[0,1], \rho_{\alpha}$ is an equivalence relation on $X$.

We obtain that $\rho$ is a similarity relation on $X$ if and only if the following conditions are satisfied:
i) $\rho(x, x)=1, \forall x \in X$;
ii) $\rho(x, y)=\rho(y, x)$, for any $x, y \in X$;
iii) $\rho(x, z) \geq \sup _{y \in X} \min \{\rho(x, y), \rho(y, z)\}$, for any $x, z \in X$.
2.5 The above described context suggests the following open problem: what kind of properties from classical mathematics have a fuzzy counterpart?

## III. Sup Property ( $[1,8,9]$ )

In relation with sup property we have the following remarks:
3.1 An unitary commutative ring $R$ is artinian if and only if every fuzzy ideal of $R$ has the sup property.
3.2 If $F / k$ is a field extension then every fuzzy intermediate field of $F / k$ has the property if and only if there are no infinite strictly decreasing sequences of intermediate fields of $F / k$.
3.3 Similarly, let $G$ be a group. Then every fuzzy subgroups of $G$ has the sup property if and only if there are no infinite strictly decreasing sequences of subgroups of $G$.
3.4 It is clear that this kind of results can be obtained for any algebraic structure for which is defined a notion of fuzzy substructure.

The investigation of the cosequences of the sup property in, for example, lattice theory can give interesting results.

## IV. Fuzzy Numbers ([5,10])

In relation with fuzzy numbers at least two problems are of interest: firstly, the investigation of some possible arithmetic operations with fuzzy numbers as well as of the derived algebraic structures based on the proposed operations - and - secondly, the building of the sets of fuzzy numbers using, for example, 2.2.

The standard operations on the set of fuzzy numbers are tipically defined using Zadeh's extension principle or using the level subsets of the operands, but they do not have, for example, the property of distributivity among other lacking desirable properties.

Some new operations, in the case of triangular fuzzy numbers, are proposed in [10].
4.1 Usually a triangular fuzzy number $(x, y, z) \in \mathbb{R}, x \leq y \leq z$ can be uniquely represented by a triple $(\lambda, y, \rho)$, where $\lambda=y-x, \rho=z-y$ are positive reals and are called left, respectively, right tolerance.

We will use the notation $(y, \lambda, \rho)$ instead of $(\lambda, y, \rho)$, with $y \in \mathbb{R}, \lambda, \rho \in \mathbb{R}$, $\lambda, \rho \geq 0$.
4.2 We consider the operations

$$
\begin{aligned}
& (a, \lambda, \rho) \oplus\left(b, \lambda^{\prime}, \rho^{\prime}\right)=\left(a+b, \max \left\{\lambda, \lambda^{\prime}\right\}, \max \left\{\rho, \rho^{\prime}\right\}\right) ; \\
& (a, \lambda, \rho) \odot\left(b, \lambda^{\prime}, \rho^{\prime}\right)=\left(a \cdot b, \max \left\{\lambda, \lambda^{\prime}\right\}, \max \left\{\rho, \rho^{\prime}\right\}\right),
\end{aligned}
$$

and the relation " $\sim$ " (on the set of triangular fuzzy numbers) given by

$$
(a, \lambda, \rho) \sim\left(b, \lambda^{\prime}, \rho^{\prime}\right) \text { if }\left\{\begin{array}{l}
a=b \\
\lambda-\lambda^{\prime}=\rho-\rho^{\prime}
\end{array}\right.
$$

4.3 One obtains:
i) " $\sim$ " is an equivalence relation;
ii) $\oplus, \odot$ are commutative and associative;
iii) " $\odot$ " is distributive with respect to $\oplus$;
iv) $(0,0,0$,$) is neutral element for \oplus ;(1,0,0)$ is neutral element for $\odot$;
v) $(a, \lambda, \rho) \oplus(-a, \rho, \lambda) \sim(0,0,0), \forall a \in \mathbb{R}$;
for $a \neq 0(a, \lambda, \rho) \odot\left(a^{-1}, \rho, \lambda\right) \sim(1,0,0)$.
More than that, these operations are unique, such that the tolerance (left and right) of the result is not less than the tolerance of any of operands and satisfy a property of monotonicity.
4.4 As an open problem one can propose the investigation of the case in which the left and the right tolerance of the result can be obtained as functions of the all tolerances of the operands and as having proper form for any of the operations $\oplus, \odot$.

## V. Fuzzy Probability ([2,4,11])

Finally, another type of transfer between classical and, respectively, fuzzy mathematics consist in the substitution of the set $\mathcal{P}(U)$ with $\mathcal{F}(U)$ (with keeping fit of the properties).
5.1 Let $\Omega \neq \Phi$ and $\mathcal{F}(\Omega)$. By fuzzy field of events we intend $K \subseteq \mathcal{F}(\Omega)$ such that:
i) $\widetilde{\Omega} \in K$;
ii) $\mu, \eta \in K \Rightarrow \mu \bigcup_{t} \eta \in K$;
iii) $\mu \in K \Rightarrow \bar{\mu} \in K$.

We obtain:
iv) $\widetilde{\emptyset} \in K$;
v) $\mu \bigcap_{t} \eta \in K$;
5.2 Let $K$ be a fuzzy field of events. By probability on $K$ one intend $P: K \rightarrow[0,1]$ such that
i) $P(\widetilde{\Omega})=1$;
ii) $\mu \bigcap_{t} \eta=\widetilde{\phi} \Rightarrow P\left(\mu \bigcup_{t} \eta\right)=P(\mu)+P(\eta)$.

We obtain:
iii) $P(\widetilde{\emptyset})=0$;
iv) $P(\bar{\mu})=1-P(\mu)$;
v) $P\left(\mu \bigcup_{t} \eta\right)+P\left(\mu \bigcap_{t} \eta\right)=P(\mu)+P(\eta)$;
5.3 In the case $t=t_{3}\left(t^{*}=t_{3}^{*}\right)$ supposing in 2.1 that $\mu, \eta \in K \Rightarrow$ $\mu \odot \eta \in K$, where $\mu \otimes \eta: \Omega \rightarrow[0,1], \mu \otimes \eta(x)=\mu(x) \cdot \eta(x)$ we shall denote $P(\mu / \eta)=P(\mu \otimes \eta) / P(\eta),(P(\eta) \neq 0)$. In the above condition we have

$$
P(\mu / \eta)=\frac{P(\mu) \cdot P(\eta / \mu)}{P(\mu) \cdot P(n / \eta)+P(\eta) \cdot P(\mu / n)}
$$

and
if $\mu_{1}, \ldots, \mu_{n} \in K$ are such that $\mu_{i} \bigcap_{t} \mu_{j}=\widetilde{\emptyset}$ for $i \neq j$, then $P\left(\mu, \bigcup_{t} \mu_{2} \bigcup_{t} \ldots \bigcup_{t} \mu_{n}\right)=P\left(\mu_{1}\right)+P\left(\mu_{2}\right)+\ldots+P\left(\mu_{n}\right)$.
5.4 In this area many open problems can appear:

- the connection with other operations with subsets (difference, implication);
- the next step can be to substitute $[0,1]$ with the, for example, $I_{t}=$ $\{(a, \lambda, \rho) \mid a \in[0,1], 0 \leq \lambda \leq a, 0 \leq \rho \leq 1-a\}$ (some new operations are needed).


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# Prediction of Stock Price using Autoregressive Integrated Moving Average Filter ((Arima (P,D, Q)) 

By Olayiwola Olaniyi Mathew, Apantaku Fadeke Sola, Bisira Hammed Oladiran \& Adewara Adedayo Amos<br>Federal University of Agriculture, Nigeria


#### Abstract

The financial system of any economy is seen to be divided between the financial intermediaries (banks, insurance companies and pension funds) and the markets (bond and stock markets). This study was designed to look at the behavior of stock price of Nigerian Breweries Plc with passage of time and to fit Autoregressive Integrated Moving Average Filter for the prediction of stock price of the Nigerian Breweries Plc.

The data were collected from Nigerian Stock exchange and Central Securities Clearing System (CSCS). Time plot was used to detect the presence of time series components in the daily stock prices of Nigerian breweries from 2008 to 2012 and to check if the series is stationary. The structure of dependency was measured by using autoovariance, the auto-correlation and partial autocorrelation. An autoregressive model and moving average model were fitted to stationary series to predict the future stock prices. Alkaike Information Criteria (AIC) was used to determine the order of the fitted autoregressive model.


Keywords : stationary series, autocorrelation, partial autocorrelation, AIC, arima.
GJSFR-F Classification : MSC 2010: 00A05

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## Prediction of Stock Price using

 Autoregressive Integrated Moving Average Filter ((Arima (P,D,Q))Olayiwola Olaniyi Mathew ${ }^{\alpha}$, Apantaku Fadeke Sola ${ }^{\sigma}$, Bisira Hammed Oladiran ${ }^{\rho}$ \& Adewara Adedayo Amos ${ }^{\omega}$

Abstract - The financial system of any economy is seen to be divided between the financial intermediaries (banks, insurance companies and pension funds) and the markets (bond and stock markets). This study was designed to look at the behavior of stock price of Nigerian Breweries Plc with passage of time and to fit Autoregressive Integrated Moving Average Filter for the prediction of stock price of the Nigerian Breweries Plc.

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The time plot showed an irregular upward trend. A first difference of the non stationary series made the series stationary. The plots of the Autocorrelation and Partial Autocorrelation showed that stationary has been introduced into the original non-stationary series in which most of the Plotted points decaying to zero sharply. The plot of Akaike Information Criterion showed that the order of the fitted autoregressive model was 8. The ARIMA model diagnostic check showed that the fitted ARIMA model had a reasonable fit for the original series. Predicted stock price ranges from 138.66 to 141.49.

Keywords : stationary series, autocorrelation, partial autocorrelation, AIC, arima.

## I. Introduction

The Nigerian Stock Exchange was established in 1960 as the Lagos Stock Exchange. In December 1977, it became The Nigerian Stock Exchange, with branches established in some of the major commercial cities of the country. At present, there are six branches of The Nigerian Stock Exchange. Each branch has a trading floor. The branch in Lagos was opened in 1961; Kaduna, 1978; Port Harcourt, 1980; Kano, 1989; Onitsha, February 1990; and Ibadan August 1990; Abuja, October 1999 and Yola, April 2002 and has Lagos as the Head Office of The Exchange. The Exchange started operations in 1961 with 19 securities listed for trading. Today, there are more than 262 securities listed on The Exchange, which includes 11 government Stocks, 49 Industrial

[^2]loan (Debenture/Preference) Stocks and more than 194 Equity/Ordinary Shares of Companies.

## a) Objective of the Study

This study was designed to look at the behavior of stock price of Nigerian Breweries Plc with passage of time and to fit Autoregressive Integrated Moving Average Filter for the prediction of stock price of the Nigerian Breweries Plc.

## b) Source of Data

The data are secondary data obtained from Nigerian Stock exchange and Central Securities Clearing System (CSCS).

## iI. Literature Review

The financial system of any economy is seen to be divided between the financial intermediaries (banks, insurance companies and pension funds) and the markets (bond and stock markets). In Nigeria, the banking sector covers a larger percentage of equities listed on the floor of the exchange. Banks are significant to the world economy and make up a good portion of the equity market. The global financial sector has nearly $\$ 6$ trillion in market capitalization, implying that banks account for a significant share in the global economy. In 2008, the financial stock worldwide, specifically equity market capitalization and outstanding bonds and loans, was estimated at US\$ 175 trillion and this increased to US $\$ 212$ trillion by the end of 2010 (Roxburgh et al., 2011). Banks are instrumental in providing capital for infrastructure, innovation, job creation and overall prosperity while playing an integral role in society because they affect the spending of individual consumers and the growth of entire industries (Cogan, 2008). Banks play a delegated role of monitoring investments on behalf of investors (Diamond, 1984) and have the capability of reducing liquidity risk thereby creating investment opportunity (Diamond and Dybvig, 1983).

To facilitate this demand in the financial system, the stock markets have grown considerably in the developed and developing countries over the last two decades.

For some developing economies, the stock exchanges are relatively large. The market capitalization of exchanges in Malaysia and Jordan represents a higher share of GDP than in France or Germany, while India's stock exchange lists more companies than the stock markets of all other countries except the USA. But on the other hand, for many other developing countries, the markets until the mid-1980s generally suffered from the classical defects of bank dominated economies, that is, shortage of equity capital, lack of liquidity, absence of foreign institutional investors, and lack of investor confidence in the stock market (Agarwal 1997).

Chittedi (2009) stated that the reason for studying the behavior of stock prices, is to prevent future capital market crash. The 2008/2009 financial crisis revealed that the power of globalisation can take a problem from one corner of the globe to multiple destinations (Chittedi, 2009). The crisis began in the US subprime mortgage sector, causing house prices to decline, economic activity to reduce and risk aversion to increase. It caused the failure of several large US based financial firms and had ramifications on household consumption (Naudé, 2009). The crisis rapidly spread to other parts in the world thereby causing failure of banks in Europe and decline in various stock indices as well as significant reductions in the market value of equities and commodities (Usman, 2010). The crisis spread further to other developed countries, as well as to emerging markets through a range of financial and real sector channels (Stephanou, 2009). The work of Demirguc-Kunt and Levine (1996), Singh (1997) and Levine and Zervos (1998)
find that stock market development plays an important role in predicting future economic growth in situations where the stock markets are active.

## iII. Methodology

 stock prices of Nigerian breweries from 2008 to 2012 and to check if the series are stationary. The series depend on time and the right assumption is that the series behave the same way with passage of time, the nature or structure of this dependency was measured by using auto-covariance, the auto-correlation and partial autocorrelation. An autocorrelation of +1 represents perfect positive correlation (i.e. an increase in one time series will lead to a proportionate increase in the other time series), while a value of -1 represents perfect negative correlation. The partial autocorrelation was to determine the rate of serial dependence of the stock prices as it moves along with time. An autoregressive model and moving average model were fitted to stationary series to predict or forecast the future stock prices. Alkaike Information Criteria (AIC) was used to determine the order of the fitted autoregressive model. The reasonable fit of the fitted autoregressive model was assessed by carrying out diagnostic checks.
## a) Autocovariance, Autocorrelation and Partial Autocorrelation

The mathematical representation of both the auto-covariance and auto-correlation are given as:

Let $\mathrm{X}_{\mathrm{t}}$ be a stationary time series with mean $\mu$ and variance $\sigma^{2}$, and assume for ease of notation that $t$ takes on integer values $t= \pm 0, \pm 1, \ldots$. The autocovariance function of $\mathrm{X}_{\mathrm{t}}$ at lag k is defined as:

$$
\begin{equation*}
\gamma(k)=E\left(X_{t}-\mu\right)\left(X_{t-k}-\mu\right) \tag{1}
\end{equation*}
$$

The autocorrelation function at lag k is defined as:

$$
\begin{equation*}
\rho(k)=\frac{\gamma(k)}{\gamma(0)}=\frac{\gamma(k)}{\sigma_{x}^{2}} \tag{2}
\end{equation*}
$$

The partial autocorrelation denoted as $\emptyset_{k k}$ was obtained by substituting $\gamma_{k}$ for $\hat{\gamma}_{k}$ by a recursive method given by Durbin (1960) as follows:

$$
\begin{equation*}
\emptyset_{k+1, k+1}=\frac{\widehat{\gamma}_{k+1}-\sum_{j=1}^{k} \widehat{\phi}_{k j} \gamma_{k+1-j}}{1-\sum_{j=1}^{k} \phi_{k j} \widehat{\gamma}_{j k}} \text { and } \widehat{\varnothing}_{k+1, j}=\widehat{\emptyset}_{k j}-\widehat{\emptyset}_{k+1, k+1} \widehat{\phi}_{k, k+1-j} \tag{3}
\end{equation*}
$$

## b) Autoregressive Filter (Ar (P))

The mathematical representation of the autoregressive model of order p is defined below:

$$
\begin{equation*}
X_{t}=\emptyset_{1} X_{t-1}+\emptyset_{2} X_{t-2}+\cdots+\emptyset_{p} X_{t-p}+a_{t} \tag{4}
\end{equation*}
$$

where $a_{t}$ is the white noise, $p$ denote the order of the $A R$ and $\emptyset_{p}$ denote the $A R$ parameter.
c) Moving Average Filter (Ma (Q))

The mathematical representation of the moving average model of order q denoted as MA (q) is given by:

$$
\begin{equation*}
X_{t}=a_{t}-\theta_{1} a_{t-1}-\theta_{2} a_{t-2}-\cdots-\theta_{q} a_{t-q} \tag{5}
\end{equation*}
$$

where $a_{t}$ is the white noise, $q$ denote the order of the $M A$ and $\theta_{q}$ denote the $M A$ parameter.

## d) Autoregressive Moving Average Filter (Arma $(P, Q)$ )

An ARMA ( $\mathrm{p}, \mathrm{q}$ ) where p and q denotes the order of the autoregressive and moving average models respectively of stochastic process $X_{t}$ is given by:

$$
\begin{equation*}
X_{t}=\emptyset_{1} X_{t-1}+\emptyset_{2} X_{t-2}+\cdots+\emptyset_{p} X_{t-p}-a_{t}-\theta_{1} a_{t-1}-\theta_{2} a_{t-2}-\cdots-\theta_{q} a_{t-q} \tag{6}
\end{equation*}
$$

Where $\emptyset_{p}$ and $\theta_{q}$ are the $A R$ and $M A$ parameters and $a_{t}$ is the white noise.

## e) Autoregressive Integrated Moving Average Filter ((Arima ( $P, D, Q$ ))

Box and Jenkins (1976) developed a methodology for fitting ARMA models to different data. These are known as autoregressive integrated moving-average (ARIMA) models. The ARIMA ( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) where p denote the order of the AR, d denote the order of differencing and $q$ denote the order of MA. The mathematical representation of ARIMA ( $\mathrm{p}, \mathrm{d}, \mathrm{q}$ ) model is given by:

$$
\nabla^{d} \emptyset(B) X_{t}=\theta(B) a_{t}
$$

where $\nabla=(1-B), B$ is the backward shift operator given as $B X_{t}=X_{t-1}, \varnothing$ and $\theta$ are the AR and MA parameter respectively.

## f) The Yule-Walker Equations

According to Insightful (July, 2001), let $\gamma(k)$ be the auto-covariance of the AR(p) process $x_{t}$. The autoregressive parameters satisfy the Yule-Walker equations:

$$
\begin{equation*}
\sum_{k=1}^{p} \gamma(k-i) \alpha_{k}=\gamma(i), \quad i=1,2, \ldots, p \tag{7}
\end{equation*}
$$

## g) Burg's Algorithm

As an alternative to using the Yule-Walker equations for fitting autoregressive models is Burg's approach. Burg's approach is based on estimating the kth partial correlation coefficient by minimizing the sum of forward and backward prediction errors. (Insightful: July, 2001):

$$
\begin{equation*}
S S\left(a_{k, k}\right)=\sum_{t=k+1}^{n}\left\{\left[x_{t}-a_{1, k} x_{t-1}-\cdots-a_{k, k} x_{t-k}\right]^{2}+\left[x_{t-k}-a_{1, k} x_{t-k+1}-\ldots-a_{k, k} x_{t}\right]^{2}\right. \tag{8}
\end{equation*}
$$

Given all of the coefficients for the order k-1model, Equation (8) is a function only of $a_{k, k}$. The function essentially measures how well the order k model predicts forwards and backwards. The algorithm is optimal in the sense of maximizing a measure of entropy.

## h) Alkaike Information Criteria (Aic)

A way of selecting the order of the AR process is to find an order that balances the reduction of estimated error variance with the number of parameters being fit. One such measure is Akaike.s Information Criterion (AIC). Simply put, the AIC is a tool for determining the order of the fitted autoregressive model. For an order k model, this criterion can be written as:

$$
\begin{equation*}
\operatorname{AIC}(k)=n \log \left(\hat{\sigma}_{\varepsilon, k}^{2}\right)+2 k \tag{9}
\end{equation*}
$$

If the series is an AR process, then the value of k that minimizes $\operatorname{AIC}(k)$ is an estimate of the order of the autoregression.

## IV. Discussion of Results

The time plot in figure 1 below possesses an irregular upward trend. This means that the series is not stationary. To make it stationary, we need to difference the series at an appropriate time lag k.

TIME PLOT OF THE NIGERIAN BREWERY PLC


Fig. 1: Time Plot for the Stock Price of the Nigerian Brewery Plc
A first difference of the non stationary series in figure 1 yields the figure 2 below. From the fig 2 , the series can be judged to arise from a random process with zero mean and constant variance.

PLOT OF THE DIFFERENCED VALUES


Fig. 2 : Plot of the Differenced Values
The plots of the Autocorrelation and Partial Autocorrelation in figures 3 and 4 respectively also implies that stationarity has been introduced into the original nonstationary series in figure 1 with most of the Plotted points decaying to zero sharply.

## MODEL IDENTIFICATION

Series: nb. diff


$$
\begin{aligned}
& \emptyset_{1}=0.086195609, \emptyset_{2}=-0.050680792, \emptyset_{3}=-0.046775400, \emptyset_{4}=-0.005039509 \\
& \emptyset_{5}=-0.018359770, \emptyset_{6}=-0.063492854, \emptyset_{7}=-0.034724455, \emptyset_{8}=-0.066127158
\end{aligned}
$$

## a) Autoregressive Model Order Identification Process

The identification of the order of the fitted Autoregressive Model was done by plotting its Akaike Information Criterion (AIC) as shown in fig 5 . In detecting the correct
order for the fitted autoregressive model, it is necessary to examine the value at which the plotting its Akaike Information Criterion (AIC) as shown in fig 5 . In detecting the correct
order for the fitted autoregressive model, it is necessary to examine the value at which the AIC gives a minimum value, bearing in mind that the first AIC value is for order zero.
The minimum value at which the plot Akaike Information Criterion gives a minimum is 8 AIC gives a minimum value, bearing in mind that the first AIC value is for order zero.
The minimum value at which the plot Akaike Information Criterion gives a minimum is 8 which make the order of the fitted autoregressive model to be 8 .

Fig. 3 : Plots of the Autocorrelation
Series : nb. diff


Fig. 4 : Partial Autocorrelation
Fitting Autoregressive Model to the stationary series, the following AR parameters were obtained:


Fig. 5 : Plot of the Akaike Information Criterion
The reasonable fit of the fitted autoregressive model was assessed by carrying out diagnostic checks. By observing the plots of Autocorrelation and Partial Autocorrelation of residual, as shown in fig. 6 and fig 7 below, if they all possess the property of stationarity. The plots of the Autocorrelation and Partial Autocorrelation of this residual in figures 6 and fig. 7 respectively also implies that the residual possess stationarity property with most of the plotted points decaying to zero sharply.

PLOT OF THE RESIDUAL


Fig. 6
Series : nb.ar\$resid


Hence we have the Autoregressive model as:

$$
\begin{gathered}
X_{t}=0.086195609 X_{t-1}-0.050680792 X_{t-2}-0.046775400 X_{t-3}-0.005039509 X_{t-4} \\
-0.018359770 X_{t-5}-0.063492854 X_{t-6}-0.034724455 X_{t-7}-0.066127158 X_{t-8}
\end{gathered}
$$

Fitting ARIMA $(8,1,2)$ to the stationary series, we have the following AR and MA parameters respectively as:
AR : $-0.46742,-0.0039,-0.09095,-0.0429,-0.02584,-0.07834,-0.08245,-0.08556$ and MA : 0.39996, 0.54668

The ARIMA model diagnostic check in fig. 9 below contains the plots of resulting residual, the autocorrelation and partial autocorrelation and the p value at each time lag. The plots of acf and pacf of the residual with almost all the plotted points decaying to zero sharply attests to the stationarity condition being met and hence the fitted ARIMA model is a reasonable fit for the original series.

MODEL DIAGNOSIS


Fig. 9

## b) Forecasting

The predicted stock price of the Nigerian Breweries Plc with the corresponding standard error and $95 \%$ Confidence interval are shown in the table below. The predicted price ranges from 138.66 to 141.49

| Time | Forecasted Price | S. E | 95\% C.I |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | Lower | Upper |
| 1 | 138.66 | 6.32 | 126.2728 | 151.0472 |
| 2 | 138.79 | 6.38 | 126.2852 | 151.2948 |
| 3 | 139.41 | 6.38 | 126.9052 | 151.9148 |
| 4 | 140.57 | 6.38 | 128.0652 | 153.0748 |
| 5 | 140.26 | 6.38 | 127.7552 | 152.7648 |
| 6 | 140.39 | 6.38 | 127.8852 | 152.8948 |
| 7 | 140.68 | 6.39 | 128.1556 | 153.2044 |
| 8 | 141.02 | 6.39 | 128.4956 | 153.5444 |
| 9 | 141.49 | 6.39 | 128.9656 | 154.0144 |
| 10 | 141.09 | 6.40 | 128.546 | 153.634 |
| 11 | 141.10 | 6.41 | 128.5364 | 153.6636 |
| 12 | 140.95 | 6.41 | 128.3864 | 153.5136 |
| 13 | 141.02 | 6.41 | 128.4564 | 153.5836 |
| 14 | 140.93 | 6.42 | 128.3468 | 153.5132 |
| 15 | 140.91 | 6.42 | 128.3268 | 153.4932 |
| 16 | 140.88 | 6.43 | 128.2772 | 153.4828 |
| 17 | 140.89 | 6.43 | 128.2872 | 153.4928 |
| 18 | 140.94 | 6.43 | 128.3372 | 153.5428 |
| 19 | 140.93 | 6.43 | 128.3272 | 153.5328 |
| 20 | 140.95 | 6.43 | 128.3472 | 153.5528 |
| 21 | 140.94 | 6.44 | 128.3176 | 153.5624 |
| 22 | 140.95 | 6.44 | 128.3276 | 153.5724 |
| 23 | 140.95 | 6.44 | 128.3276 | 153.5724 |
| 24 | 140.95 | 6.44 | 128.3276 | 153.5724 |
| 25 | 140.94 | 6.45 | 128.298 | 153.582 |
| 26 | 140.94 | 6.45 | 128.298 | 153.582 |
| 27 | 140.94 | 6.45 | 128.298 | 153.582 |
| 28 | 140.94 | 6.45 | 128.298 | 153.582 |
| 29 | 140.94 | 6.46 | 128.298 | 153.582 |
| 30 | 140.94 | 6.46 | 128.298 | 153.582 |

## V. Conclussion

The time plot showed an irregular upward trend. A first difference of the non stationary series made the series stationary. The plots of the Autocorrelation and Partial Autocorrelation showed that stationary has been introduced into the original nonstationary series in with most of the Plotted points decaying to zero sharply. The plot of Akaike Information Criterion showed that the order of the fitted autoregressive model was 8. The ARIMA model diagnostic check showed that the fitted ARIMA model had a reasonable fit for the original series. Predicted stock price ranges from 138.66 to 141.49.

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# Establishment of a Summation Formula in the Light of Contiguous Relation 

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Abstract - The main aim of the present paper is to compute a summation formula in the light of recurrence relation and contiguous relation.

Keywords : gaussian hypergeometric function, contiguous function, recurrence relation, bailey summation theorem and legendre duplication formula.

GJSFR-F Classification : MSC 2010: 33C60, 33C70, 33D15, 33D50, 33D60

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# Establishment of a Summation Formula in the Light of Contiguous Relation 

Abstract - The main aim of the present paper is to compute a summation formula in the light of recurrence relation and contiguous relation.
Keywords : gaussian hypergeometric function, contiguous function, recurrence relation, bailey summation theorem and legendre duplication formula.

## I. Introduction

Generalized Gaussian hypergeometric function of one variable is defined by

$$
{ }_{A} F_{B}\left[\begin{array}{cc}
a_{1}, a_{2}, \cdots, a_{A} & ; \\
b_{1}, b_{2}, \cdots, b_{B} & ;
\end{array}\right]=\sum_{k=0}^{\infty} \frac{\left(a_{1}\right)_{k}\left(a_{2}\right)_{k} \cdots\left(a_{A}\right)_{k} z^{k}}{\left(b_{1}\right)_{k}\left(b_{2}\right)_{k} \cdots\left(b_{B}\right)_{k} k!}
$$

or

$$
{ }_{A} F_{B}\left[\begin{array}{ccc}
\left(a_{A}\right) & ; &  \tag{1}\\
\left(b_{B}\right) & ; & z
\end{array}\right] \equiv{ }_{A} F_{B}\left[\begin{array}{ccc}
\left(a_{j}\right)_{j=1}^{A} & ; & \\
\left(b_{j}\right)_{j=1}^{B} & ; & z
\end{array}\right]=\sum_{k=0}^{\infty} \frac{\left(\left(a_{A}\right)\right)_{k} z^{k}}{\left(\left(b_{B}\right)\right)_{k} k!}
$$

where the parameters $b_{1}, b_{2}, \cdots, b_{B}$ are neither zero nor negative integers and $A, B$ are non-negative integers.

Contiguous Relation[E. D. p.51(10), Andrews p.363(9.16)] is defined as follows

$$
(a-b){ }_{2} F_{1}\left[\begin{array}{cc}
a, b ; & z  \tag{2}\\
c ; & ;
\end{array}\right]=a_{2} F_{1}\left[\begin{array}{ccc}
a+1, & b ; & z \\
c & ; &
\end{array}\right]-b_{2} F_{1}\left[\begin{array}{cc}
a, b+1 ; & z \\
c & ;
\end{array}\right]
$$

Recurrence relation of gamma function is defined as follows

$$
\begin{equation*}
\Gamma(z+1)=z \Gamma(z) \tag{3}
\end{equation*}
$$

[^3]Legendre duplication formula[Bells \& Wong p.26(2.3.1)] is defined as follows

$$
\begin{align*}
\sqrt{\pi} \Gamma(2 z) & =2^{(2 z-1)} \Gamma(z) \Gamma\left(z+\frac{1}{2}\right)  \tag{4}\\
\Gamma\left(\frac{1}{2}\right) & =\sqrt{\pi}=\frac{2^{(b-1)} \Gamma\left(\frac{b}{2}\right) \Gamma\left(\frac{b+1}{2}\right)}{\Gamma(b)}  \tag{5}\\
= & \frac{2^{(a-1)} \Gamma\left(\frac{a}{2}\right) \Gamma\left(\frac{a+1}{2}\right)}{\Gamma(a)} \tag{6}
\end{align*}
$$

Bailey summation theorem [Prud, p.491(7.3.7.8)]is defined as follows


$$
=\frac{2^{(a-1)} \Gamma\left(\frac{a}{2}\right) \Gamma\left(\frac{a+1}{2}\right)}{\Gamma(a)}
$$

$$
{ }_{2} F_{1}\left[\begin{array}{cc}
a, 1-a & ; \frac{1}{2}  \tag{7}\\
c & ; 2
\end{array}\right]=\frac{\Gamma\left(\frac{c}{2}\right) \Gamma\left(\frac{c+1}{2}\right)}{\Gamma\left(\frac{c+a}{2}\right) \Gamma\left(\frac{c+1-a}{2}\right)}=\frac{\sqrt{\pi} \Gamma(c)}{2^{c-1} \Gamma\left(\frac{c+a}{2}\right) \Gamma\left(\frac{c+1-a}{2}\right)}
$$

iI. Main Result of Summation Formula

$$
{ }_{2} F_{1}\left[\begin{array}{ccc}
a, & -a-53 & ; \\
c & & \frac{1}{2}
\end{array}\right]
$$

$$
=\frac{\sqrt{\pi} \Gamma(c)}{2^{c+53}}\left[\frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a+53}{2}\right)}\{399999419504807160803447104415465472000000\right.
$$

-763033261517433920897597569086329978880000a $+487163496069626070324753370776182685696000 a^{2}$
$-143832143876600302074189687617077741977600 a^{3}$
$+20987561774364491960260633222019397550080 a^{4}$
$-1271374592866991393388819201546481815552 a^{5}$ $-18789752346646864497619955170616736768 a^{6}$ $+5153012575994221993585253534049912960 a^{7}-50647459165569255057532106709855360 a^{8}$ $-10064089426419076584549603581511520 a^{9}+76715995873543269330396087956320 a^{10}$ $+12692345509362566082812462131160 a^{11}+45127363184557440811632211240 a^{12}$ $-8818789087985317941134937250 a^{13}-138408104538705232557946850 a^{14}$ $+2024132107455001498032260 a^{15}+73008911009011774625140 a^{16}$
$+421352406408459129890 a^{17}-8606639734270768190 a^{18}-151442351224861840 a^{19}$ $-644877760772960 a^{20}+4589390615810 a^{21}+60465999490 a^{22}+220163060 a^{23}+41860 a^{24}$
$-1378 a^{25}-2 a^{26}+1044332121459330560843393654665909370880000 c$ -1510531506038982108827199299081478340608000ac

$$
\begin{aligned}
& +773509337340182957477212135487050796236800 a^{2} c \\
& -184939284969800446520380971948981157232640 a^{3} c \\
& +21169783097800753645457139898394498727936 a^{4} c \\
& -813984784077918996962679465051059625984 a^{5} c \\
& -43730083119624850497605526147963377664 a^{6} c \\
& +3713685601685463874080829801657890816 a^{7} c \\
& +46116706523248276200672333384069632 a^{8} c-6601955941720744588391412660125184 a^{9} c \\
& -77754065179325079683018172333184 a^{10} c+6216316854597276984170004533376 a^{11} c \\
& +122722747931747414737050763232 a^{12} c-2364622480019908086179545344 a^{13} c \\
& -86493693622931781133065664 a^{14} c-250322249822469715972224 a^{15} c \\
& +19940855680183352094752 a^{16} c+281037850910223154176 a^{17} c+152908004151009536 a^{18} c \\
& -27025608641775744 a^{19} c-236084454013408 a^{20} c-427452689664 a^{21} c+4158330176 a^{22} c \\
& +24076416 a^{23} c+37856 a^{24} c+1118472038587258227545710874039345479680000 c^{2} \\
& -1296588237185272406228909112633567215616000 a c^{2} \\
& +544168995816011123337092396202666047569920 a^{2} c^{2} \\
& -105959102971397653597364173753914148356096 a^{3} c^{2} \\
& +9351697806317337152737653621133977526272 a^{4} c^{2} \\
& -166685478605933101957562251082304817152 a^{5} c^{2} \\
& -25468751864498946047784212962435886592 a^{6} c^{2} \\
& +998331657471978340027029233431492096 a^{7} c^{2} \\
& +39241159767069161448921317767290496 a^{8} c^{2}-1510131341825478978100703337704832 a^{9} c^{2} \\
& -50280397853116662182839952385952 a^{10} c^{2}+893012465473570435007591621280 a^{11} c^{2} \\
& +41973105675398989440474765272 a^{12} c^{2}+32450125871385998774235328 a^{13} c^{2} \\
& -15445763913552254279562032 a^{14} c^{2}-190727900887510386999456 a^{15} c^{2} \\
& +1073652386244648160808 a^{16} c^{2}+40011925428345032448 a^{17} c^{2}+262080379253895488 a^{18} c^{2} \\
& -761944668316832 a^{19} c^{2}-17770368191576 a^{20} c^{2}-77078793792 a^{21} c^{2}-38142832 a^{22} c^{2} \\
& +463008 a^{23} c^{2}+728 a^{24} c^{2}+687491523990620632276591559393635467264000 c^{3} \\
& -656585803081935271390527662141701525340160 a c^{3} \\
& +228404831956792673010895209847112092090368 a^{2} c^{3} \\
& -36192934306554045978185830581695959007232 a^{3} c^{3} \\
& +2383211453979296760825044701672492105728 a^{4} c^{3} \\
& +7405094859574742990001466511237480448 a^{5} c^{3}
\end{aligned}
$$

$-7305018539998303150895812386151374848 a^{6} c^{3}$ $+106635767396254423194007901969854464 a^{7} c^{3}$
$+11257440640192196065645211014569984 a^{8} c^{3}-119517295902722793899604676386816 a^{9} c^{3}$ $-11181568050528212256752674902528 a^{10} c^{3}-10894704455756680578794541312 a^{11} c^{3}$ $+5888062077890877413477847808 a^{12} c^{3}+69318194648180696615324928 a^{13} c^{3}$
$-106273003226322915840 a^{16} c^{3}+1968283947482012160 a^{17} c^{3}+24123132054461440 a^{18} c^{3}$ $+67070875872000 a^{19} c^{3}-364730446080 a^{20} c^{3}-2648405760 a^{21} c^{3}-4542720 a^{22} c^{3}$

$$
\begin{aligned}
& +279570397854965736127589440792924520448000 c^{4} \\
& -223326287538418582067568918685436463808512 a c^{4} \\
& +64729887412573742648795733935169546485760 a^{2} c^{4}
\end{aligned}
$$

$$
-8295593761202708425053011797263030091776 a^{3} c^{4}
$$

$$
+383488372052315931163471867173472026624 a^{4} c^{4}
$$

$$
+10624320316121034711079896219836092416 a^{5} c^{4}
$$

$$
-1244485250661318204919565786060923904 a^{6} c^{4}
$$

$$
-4625973359831572256925964685992448 a^{7} c^{4}
$$

$$
+1722331662035355724645251373048832 a^{8} c^{4}+9776545699971389593318869814272 a^{9} c^{4}
$$

$$
-1268115756709425207724714478400 a^{10} c^{4}-16969271858274845050494143136 a^{11} c^{4}
$$ $+380424665047891848806347744 a^{12} c^{4}+9474057481040657036275744 a^{13} c^{4}$ $+8513965071540770482784 a^{14} c^{4}-1526425859773962438720 a^{15} c^{4}$

$-14776617316398400320 a^{16} c^{4}+6152270772648000 a^{17} c^{4}+821534820346240 a^{18} c^{4}$

$$
+4317368255200 a^{19} c^{4}+3422138720 a^{20} c^{4}-25465440 a^{21} c^{4}-43680 a^{22} c^{4}
$$ $+81363502230003144803467941815434421993472 c^{5}$ $-54826170667472787699011447620471414063104 a c^{5}$ $+13255776543566365717340142897365375778816 a^{2} c^{5}$ $-1357710619635928283425181631167588204544 a^{3} c^{5}$ $+38807646930001802359869127436284723200 a^{4} c^{5}$ $+2557974357363323848650212859635957760 a^{5} c^{5}$ $+2557974357363323848650212859635957760 a^{5} c^{5}$ $-134093843700679130709689468637347840 a^{6} c^{5}$ $-2902376818146657564248450626682880 a^{7} c^{5}$ $+155464629071164619281383620542464 a^{8} c^{5}+3157683186011838247045682749440 a^{9} c^{5}$

$$
\begin{aligned}
& -75708856990708722777861795840 a^{10} c^{5}-2171738207283631890537553920 a^{11} c^{5} \\
& +5127833402372048512528384 a^{12} c^{5}+600811767333472209715200 a^{13} c^{5} \\
& +4774529179177044418560 a^{14} c^{5}-34391981767441858560 a^{15} c^{5}-680152042060947456 a^{16} c^{5} \\
& -2632218703994880 a^{17} c^{5}+8705426329600 a^{18} c^{5}+84748984320 a^{19} c^{5} \\
& +159903744 a^{20} c^{5}+17806447135039486775404419433232829775872 c^{6} \\
& -10165996100355694083684155039836790587392 a c^{6} \\
& +2046194505136873631499525988075614240768 a^{2} c^{6} \\
& -164331105684939481546784144226870886400 a^{3} c^{6} \\
& +1990615964669433906975826311141785600 a^{4} c^{6} \\
& +356678676215071985433937655662673920 a^{5} c^{6} \\
& -8863201024276738854806897171251200 a^{6} c^{6} \\
& -423202849337835803893891732750336 a^{7} c^{6} \\
& +7806429046107883873009395712000 a^{8} c^{6}+339652929834603484315247216640 a^{9} c^{6} \\
& -1407990157615911504706606080 a^{10} c^{6}-140778608668536044943120384 a^{11} c^{6} \\
& -920822379908476250196992 a^{12} c^{6}+19046678037474221854720 a^{13} c^{6} \\
& +287286972442231224320 a^{14} c^{6}+453621694218620928 a^{15} c^{6}-14020331989407744 a^{16} c^{6} \\
& -91791298959360 a^{17} c^{6}-101256995840 a^{18} c^{6}+543262720 a^{19} c^{6}+1025024 a^{20} c^{6} \\
& +3030664692950145710229442392094880563200 c^{7} \\
& -1468267437390458223999118111195728445440 a c^{7} \\
& +244843777107785893401520229032369586176 a^{2} c^{7} \\
& -14966313582657878425013530198661922816 a^{3} c^{7} \\
& -62835263742845886731037182841061376 a^{4} c^{7} \\
& +33679158694519332609069451056513024 a^{5} c^{7} \\
& -255441313211072316791942670450688 a^{6} c^{7}-35995269296382044465943436197888 a^{7} c^{7} \\
& +88617999474498865643861901312 a^{8} c^{7}+21089858727463774831003779072 a^{9} c^{7} \\
& +129154285658568506654343168 a^{10} c^{7}-5155546888600334430437376 a^{11} c^{7} \\
& -71727717006594465529856 a^{12} c^{7}+161825725058728034304 a^{13} c^{7} \\
& +8239537258675666944 a^{14} c^{7}+43167645797253120 a^{15} c^{7}-83632399319040 a^{16} c^{7} \\
& -1234913771520 a^{17} c^{7}-2588917760 a^{18} c^{7}+410876303062523347534291888774525747200 c^{8} \\
& -168819864219779026938445868318014308352 a c^{8} \\
& +23145840262428122894173413157825937408 a^{2} c^{8} \\
& -1026231310050230148311146544556146688 a^{3} c^{8}
\end{aligned}
$$

$-22691948965122338384170275518087168 a^{4} c^{8}$
$+2263977681014601327046533492916224 a^{5} c^{8}+13235099282792920190860904480768 a^{6} c^{8}$ $-2018657016368668808539350687744 a^{7} c^{8}-17091512223538729278717702144 a^{8} c^{8}$ $+810055893983986278362612736 a^{9} c^{8}+11861048880200065643615232 a^{10} c^{8}$ $-89063981330582659534848 a^{11} c^{8}-2570898847669165764608 a^{12} c^{8}$
$-9270846648058963968 a^{13} c^{8}+114629968150444032 a^{14} c^{8}+978122952069120 a^{15} c^{8}$ $+1415341424640 a^{16} c^{8}-5937085440 a^{17} c^{8}-12446720 a^{18} c^{8}$ $+45155672153311413752907757374396170240 c^{9}$ $-15698093486831634464277134090535174144 a c^{9}$ $+1750482025210672478319692413189226496 a^{2} c^{9}$ $-51649471465987798684963456054984704 a^{3} c^{9}$
$-2369993210358181085163864782536704 a^{4} c^{9}+109012062924046946295615722618880 a^{5} c^{9}$ $+2045308405607025296831272058880 a^{6} c^{9}-75372909102362994534649036800 a^{7} c^{9}$ $-1431445924167232892214067200 a^{8} c^{9}+17428396504567221237841920 a^{9} c^{9}$ $+489501076293217811496960 a^{10} c^{9}+676712533724347760640 a^{11} c^{9}$
$-51019249412221009920 a^{12} c^{9}-365692545332674560 a^{13} c^{9}+323314405539840 a^{14} c^{9}$ $+9879310172160 a^{15} c^{9}+23300259840 a^{16} c^{9}+4075564271410352135422620389857034240 c^{10}$ $-1194044418190373393703397371645263872 a c^{10}$ $+106702555728454458642255440052748288 a^{2} c^{10}$ $-1732113329700293971646800215932928 a^{3} c^{10}$
$-158866194296066946708417963687936 a^{4} c^{10}+3598684727455839294668923207680 a^{5} c^{10}$ $+129191692350168053994386718720 a^{6} c^{10}-1691280135733809338273267712 a^{7} c^{10}$ $-61614761685361660501659648 a^{8} c^{10}+55250537574570676715520 a^{9} c^{10}$
$+11978292264714652631040 a^{10} c^{10}+73692947175279624192 a^{11} c^{10}$ $-495741973411356672 a^{12} c^{10}-5961530399784960 a^{13} c^{10}-11034166640640 a^{14} c^{10}$ $+37997346816 a^{15} c^{10}+89616384 a^{16} c^{10}+305005891400829760278355120436019200 c^{11}$ $-74890887532628040341097459241451520 a c^{11}$ $+5256480440302154985751985881350144 a^{2} c^{11}$
$-21603263664240325434452320714752 a^{3} c^{11}-7706637734457726594130705907712 a^{4} c^{11}$ $+66147310464952547319179378688 a^{5} c^{11}+5256837649286902835380224000 a^{6} c^{11}$ $-9142511603742486691577856 a^{7} c^{11}-1690925332263568927358976 a^{8} c^{11}$ $-8996831263425077968896 a^{9} c^{11}+172843003943708786688 a^{10} c^{11}$
$-8996831263425077968896 a^{9} c^{11}+172843003943708786688 a^{10} c^{11}$ $+1779277054168793088 a^{11} c^{11}+108481490976768 a^{12} c^{11}-47779936468992 a^{13} c^{11}$ $-128786890752 a^{14} c^{11}+19057992829842637736683968344883200 c^{12}$ $-3893655110495585354702463352963072 a c^{12}$
$+208739533566121377212362576101376 a^{2} c^{12}+1538194659121745502941420191744 a^{3} c^{12}$ $-280597390713121922098126913536 a^{4} c^{12}-404757859250133287500120064 a^{5} c^{12}$ $+149165603489920243877740544 a^{6} c^{12}+830324106833601694892032 a^{7} c^{12}$ $-30133026908678867550208 a^{8} c^{12}-305400285369739542528 a^{9} c^{12}$ $+1119785780923170816 a^{10} c^{12}+22266522380304384 a^{11} c^{12}+52434839568384 a^{12} c^{12}$ $-153140822016 a^{13} c^{12}-412778496 a^{14} c^{12}+998899412450453819652330299064320 c^{13}$ $-168281696954328847326650296172544 a c^{13}+6616883610341473269935553642496 a^{2} c^{13}$ $+125612602112053274188537921536 a^{3} c^{13}-7710644284135944377612632064 a^{4} c^{13}$
$-74019299747342409573335040 a^{5} c^{13}+2959955941401219872849920 a^{6} c^{13}$ $+34098636370174720081920 a^{7} c^{13}-315721702452174520320 a^{8} c^{13}$ $-5237416112947200000 a^{9} c^{13}-5478858510827520 a^{10} c^{13}+147015189135360 a^{11} c^{13}$ $+462311915520 a^{12} c^{13}+44035623813709020644612841144320 c^{14}$ $-6049266193348017741042320146432 a c^{14}+163721970706465902026270179328 a^{2} c^{14}$ $+5287737971080255697886117888 a^{3} c^{14}-156916893550025647689039872 a^{4} c^{14}$ $-2744491382099947502960640 a^{5} c^{14}+38666834713059135324160 a^{6} c^{14}$ $+717130569599169331200 a^{7} c^{14}-965355249547345920 a^{8} c^{14}-52612454979993600 a^{9} c^{14}$ $-159293126737920 a^{10} c^{14}+403887882240 a^{11} c^{14}+1270087680 a^{12} c^{14}$ $+1634071061774805071752803123200 c^{15}-180541499307501456448919961600 a c^{15}$
$+3002414913590002686269325312 a^{2} c^{15}+154932346935146715115683840 a^{3} c^{15}$ $-2224386491239426299002880 a^{4} c^{15}-62071375036348744335360 a^{5} c^{15}$ $+255849679669696659456 a^{6} c^{15}+9460133390482145280 a^{7} c^{15}+21247855637299200 a^{8} c^{15}$ $-294030378270720 a^{9} c^{15}-1109548597248 a^{10} c^{15}+50985684411786655115195187200 c^{16}$ $-4454184348290973381167153152 a c^{16}+34894874466770437625872384 a^{2} c^{16}$
$+3373390240474944118456320 a^{3} c^{16}-17678012543716208148480 a^{4} c^{16}$ $-959168384030242111488 a^{5} c^{16}-1012566911574736896 a^{6} c^{16}+78802497389199360 a^{7} c^{16}$ $+315518991728640 a^{8} c^{16}-706803793920 a^{9} c^{16}-2667184128 a^{10} c^{16}$ $+1333460180866043535561850880 c^{17}-90170359708404698855768064 a c^{17}$ $+46914003514968565088256 a^{2} c^{17}+55391522588023949623296 a^{3} c^{17}$

$$
\begin{aligned}
& +37673993243644133376 a^{4} c^{17}-10230389206123806720 a^{5} c^{17}-40813874915573760 a^{6} c^{17} \\
& +380509901291520 a^{7} c^{17}+1794858024960 a^{8} c^{17}+29073465895558637819002880 c^{18} \\
& -1481035666753646548221952 a c^{18}-8068995167853417070592 a^{2} c^{18} \\
& +680050063391208767488 a^{3} c^{18}+3146170885897453568 a^{4} c^{18}-72425343989841920 a^{5} c^{18} \\
& -405231385968640 a^{6} c^{18}+813055344640 a^{7} c^{18}+3835166720 a^{8} c^{18} \\
& +524066711681035875123200 c^{19}-19411371410490843463680 a c^{19} \\
& -197826263019998412800 a^{2} c^{19}+6068171637428060160 a^{3} c^{19}+43684283243560960 a^{4} c^{19} \\
& -307078165954560 a^{5} c^{19}-1931309219840 a^{6} c^{19}+7716371772743037747200 c^{20} \\
& -198091776945408704512 a c^{20}-2736638549586608128 a^{2} c^{20}+37218464248627200 a^{3} c^{20} \\
& +325035627642880 a^{4} c^{20}-590534934528 a^{5} c^{20}-3714056192 a^{6} c^{20} \\
& +91216012561570856960 c^{21}-1515948395036934144 a c^{21}-24882767267364864 a^{2} c^{21} \\
& +140378590150656 a^{3} c^{21}+1324326322176 a^{4} c^{21}+844195024559144960 c^{22} \\
& -8180325104484352 a c^{22}-147842203123712 a^{2} c^{22}+245417115648 a^{3} c^{22}+2315255808 a^{4} c^{22} \\
& +5888453849907200 c^{23}-27742804377600 a c^{23}-523449139200 a^{2} c^{23}+29095047987200 c^{24} \\
& \left.-44459622400 a c^{24}-838860800 a^{2} c^{24}+90731184128 c^{25}+134217728 c^{26}\right\} \\
& +\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+54}{2}\right)}\{-1816263274051483610082302082123694080000000 a \\
& +2188444307722497708867373846544910581760000 a^{2} \\
& -972649591306496379223112097769247244288000 a^{3} \\
& +203014872950617792589030945654112536371200 a^{4} \\
& -19334396725906001884339179306749629962240 a^{5} \\
& +354939277311645059750305917863704425984 a^{6} \\
& +66231858907546829601146760822431513472 a^{7} \\
& -2831466643791990973002848162852379648 a^{8} \\
& -129489777954333998108105338847554848 a^{9}+5583491046025870674622092671192832 a^{10} \\
& +215321829142006455650308069861032 a^{11}-4459820731034537665320289653888 a^{12} \\
& -244230034535300889153801194238 a^{13}-161516730081872262368219598 a^{14} \\
& +129376412646637834962638172 a^{15}+1917641404501908471635652 a^{16} \\
& -14259946067841588097698 a^{17}-649159782175846879938 a^{18}-5474868380045249328 a^{19} \\
& +22975501539656952 a^{20}+733680618372702 a^{21}+4827916801422 a^{22}+3982604652 a^{23} \\
& -100550268 a^{24}-483678 a^{25}-702 a^{26}+1816263274051484013373763208729329664000000 c
\end{aligned}
$$

-5688764794393161212695198424835285319680000 ac $+4497504192339789956233037053703087554560000 a^{2} c$ $+4497504192339789956233037053703087554560000 a^{2} c$ $-1514020803092432433815354203917672382464000 a^{3} c$ $+243623318913501013444770627033397864550400 a^{4} c$ $-16353392222216778426951922368654485806080 a^{5} c$ $-146714519620043524162069120818258686208 a^{6} c$ $+65052238560049255412741795855023598784 a^{7} c$ $-849632688774728588933726724168109056 a^{8} c$
$-129014700250825653360443585620882128 a^{9} c+1253968007810571901961380803098880 a^{10} c$ $+167223997128887354767090762706724 a^{11} c+422755160181760693313408964384 a^{12} c$ $-119140372318604979918225783483 a^{13} c-1798955547786802804452208515 a^{14} c$ $+28174433593447143774511974 a^{15} c+980696049761332089338634 a^{16} c$ $+5508244409983362257067 a^{17} c-117363820433445104805 a^{18} c-2041015801934651256 a^{19} c$ $-8625861574526196 a^{20} c+62303467453227 a^{21} c+816462360675 a^{22} c+2970117774 a^{23} c$ $+561834 a^{24} c-18603 a^{25} c-27 a^{26} c+3500320486670665058282383725852654305280000 c^{2}$ $-6532892771979693939419880260348400893952000 a c^{2}$ $+3872026776553966044635216868757782547660800 a^{2} c^{2}$
$-1024404339452508135686525036498285273579520 a^{3} c^{2}$
$+127423786395286739038967185304395883962368 a^{4} c^{2}$
$-5506387024539392525279259403425491245056 a^{5} c^{2}$ $-247962324866381944918572176330320452096 a^{6} c^{2}$ $+24304345904184318787447470766440605184 a^{7} c^{2}$ $+237741704940782140165252370329282944 a^{8} c^{2}$
$-16419648243837705724927794816 a^{13} c^{2}-577021526191118051208197136 a^{14} c^{2}$
$-1513602039311816934576096 a^{15} c^{2}+135307412063697187550808 a^{16} c^{2}$ $+1883678205689150352768 a^{17} c^{2}+869435177693404608 a^{18} c^{2}-182874251947619232 a^{19} c^{2}$ $-1591105615595496 a^{20} c^{2}-2867428916352 a^{21} c^{2}+28099392048 a^{22} c^{2}+162515808 a^{23} c^{2}$ $+255528 a^{24} c^{2}+3008038170946403034084544373245964451840000 c^{3}$
$-4115499037925746947081435748595542523904000 a c^{3}$


$$
\begin{gathered}
+1006673687713276347325324146864712261632 a^{4} c^{5} \\
+24067169446277828202018859641128712192 a^{5} c^{5} \\
-3239396080830659830877666613136608768 a^{6} c^{5}
\end{gathered}
$$

$-6953510306817729044870715183787776 a^{7} c^{5}+4543452454777736949661448542662144 a^{8} c^{5}$ $+21882756077492827212293566726656 a^{9} c^{5}-3403868544602545992750544899168 a^{10} c^{5}$
$-43924439672590216187262355632 a^{11} c^{5}+1039351360008810888581422608 a^{12} c^{5}$
$+25322012344697562222608880 a^{13} c^{5}+19119451159307428527312 a^{14} c^{5}$
$-4127387475621056755680 a^{15} c^{5}-39707467514313936480 a^{16} c^{5}+17849789971165152 a^{17} c^{5}$ $+2219509886577984 a^{18} c^{5}+11649560242320 a^{19} c^{5}+9225936720 a^{20} c^{5}-68756688 a^{21} c^{5}$ $-117936 a^{22} c^{5}+140864629492372916818919828999084936527872 c^{6}$
$-102199642229036592757571065030519944118272 a c^{6}$
$+26192891094830979382009368992500414414848 a^{2} c^{6}$
$-2825926530687292011889061175765420736512 a^{3} c^{6}$
$+88004948849941316204013917080752488448 a^{4} c^{6}$ $+5239314565073002738287856752606609408 a^{5} c^{6}$
$-297425347522921001692733340139241472 a^{6} c^{6}-5973379325037395427107731326124032 a^{7} c^{6}$ $+348212888595802960260667020877824 a^{8} c^{6}+6774941347810463584124978448384 a^{9} c^{6}$
$-172314612597159685752747088896 a^{10} c^{6}-4805124300520721529886550016 a^{11} c^{6}$
$+12660586696363550283795456 a^{12} c^{6}+1349229413525202504142848 a^{13} c^{6}$ $+10612960913443544967168 a^{14} c^{6}-78060542365315141632 a^{15} c^{6}-1529021801723480064 a^{16} c^{6}$ $-5903042192087040 a^{17} c^{6}+19627984696320 a^{18} c^{6}+190685214720 a^{19} c^{6}+359783424 a^{20} c^{6}$ $+27667479345381849862403977071788436750336 c^{7}$ $-16782815275880282060459567164233753821184 a c^{7}$ $+3549232799999849329752105953152305463296 a^{2} c^{7}$ $-298783272144733274132436585409184268288 a^{3} c^{7}$ $+4218204903730573982424238705373872128 a^{4} c^{7}$ $+647809788597672077368823137076822016 a^{5} c^{7}$ $-17307521311903518017838191672991744 a^{6} c^{7}-780162898530773385326540098658304 a^{7} c^{7}$ $+15351337266925540708080875188224 a^{8} c^{7}+640475173167756453442675289088 a^{9} c^{7}$ $-2926647525777582282688276992 a^{10} c^{7}-269884581355083108519705600 a^{11} c^{7}$ $-1729689386526874992840192 a^{12} c^{7}+36898627985539086514176 a^{13} c^{7}$ $+551988023054514748416 a^{14} c^{7}+857242807211501568 a^{15} c^{7}-27064670902004736 a^{16} c^{7}$

$$
\begin{gathered}
-176919209026560 a^{17} c^{7}-195057308160 a^{18} c^{7}+1047720960 a^{19} c^{7}+1976832 a^{20} c^{7} \\
+4274984634541941913218477566771960217600 c^{8} \\
-2178574198304936421174811869818891796480 a c^{8} \\
+379064631568388508401540674276420485120 a^{2} c^{8} \\
-24215889370915350351834696612362059776 a^{3} c^{8} \\
\\
-60012151613764697274784836802707456 a^{4} c^{8} \\
+54643586782932729131758702159970304 a^{5} c^{8}
\end{gathered}
$$

$$
-471121219806982161740973881966592 a^{6} c^{8}-59218236330436678999629233332224 a^{7} c^{8}
$$

$$
+178091576414645849397118550016 a^{8} c^{8}+35233895883090307520125172736 a^{9} c^{8}
$$$+208080401147893670363016192 a^{10} c^{8}-8713227042139208949854208 a^{11} c^{8}$

            \(-120007356432638056132608 a^{12} c^{8}+280473768268600338432 a^{13} c^{8}\)
            \(-120007356432638056132608 a^{12} c^{8}+280473768268600338432 a^{13} c^{8}\)
    \(+13897638538809772032 a^{1} 4 c^{8}+72645346251878400 a^{15} c^{8}-141601504112640 a^{16} c^{8}\)
    $-2083916989440 a^{17} c^{8}-4368798720 a^{18} c^{8}+530933225436046364909516153483480268800 c^{9}$
$-227673444364841185253178298563116924928 a c^{9}$
$+32389198212892040381449124988677259264 a^{2} c^{9}$
$-1499876043592737947469008719497265152 a^{3} c^{9}$
$-1499876043592737947469008719497265152 a^{3} c^{9}$
$-30559620558398797416617023538921472 a^{4} c^{9}$
$+3314873959644189871872476980862976 a^{5} c^{9}+16997264449357998946247253712896 a^{6} c^{9}$
$-2989648769879639675510468333568 a^{7} c^{9}-24261370820529808154842718208 a^{8} c^{9}$
$+1213735232412663978721347072 a^{9} c^{9}+17522450808900806325173760 a^{10} c^{9}$
$-135246350642965478332416 a^{11} c^{9}-3845175916875530704896 a^{12} c^{9}$
$-13772188601388112896 a^{13} c^{9}+172193077947331584 a^{14} c^{9}+1466329487800320 a^{15} c^{9}$
$+2120995768320 a^{16} c^{9}-8905628160 a^{17} c^{9}-18670080 a^{18} c^{9}$
$+53845282698700673493044561724875735040 c^{10}$
$-19412903333361790917782480854274015232 a c^{10}$
$+2236003486457846678390689107733905408 a^{2} c^{10}$
$-69137053507996484366675565561249792 a^{3} c^{10}$
$-3012202613139577875629503281561600 a^{4} c^{10}$
$+145496413295612386860791202840576 a^{5} c^{10}+2632390364672740020006571769856 a^{6} c^{10}$
$-101511884647250204034500296704 a^{7} c^{10}-1890744023925809572034211840 a^{8} c^{10}$
$+23748650593579942751305728 a^{9} c^{10}+656546131763336146894848 a^{10} c^{10}$
$+869591036429137870848 a^{11} c^{10}-68878463374997692416 a^{12} c^{10}$
$-492502382771503104 a^{13} c^{10}+439661923024896 a^{14} c^{10}+13337068732416 a^{15} c^{10}$
$+31455350784 a^{16} c^{10}+4511782079473825145251907958755819520 c^{11}$
$-1363693554550409481001988443261108224 a c^{11}$
$+125410521099580042584605710738784256 a^{2} c^{11}$
$-2164323878294156987991592570454016 a^{3} c^{11}$ $-187485720553571360376769177485312 a^{4} c^{11}+4425936548044371058926593114112 a^{5} c^{11}$ $+154548151519816195593133473792 a^{6} c^{11}-2097670747590124321406238720 a^{7} c^{11}$ $-74805082901288920733829120 a^{8} c^{11}+76023133171280028205056 a^{9} c^{11}$ $+14670305915647187460096 a^{10} c^{11}+89840379705225117696 a^{11} c^{11}$ $-609825616259198976 a^{12} c^{11}-7312288870268928 a^{13} c^{11}-13530786766848 a^{14} c^{11}$ $+46633107456 a^{1} 5 c^{1} 1+109983744 a^{16} c^{11}+315057804071553077594228732303769600 c^{12}$ $-79457550046907145913828284636856320 a c^{12}$ $+5722147149446209424646833845567488 a^{2} c^{12}$
$-27867945938612712473653709635584 a^{3} c^{12}-8446025899828394222297549635584 a^{4} c^{12}$ $+76536516935664043287120838656 a^{5} c^{12}+5827547829895659111529119744 a^{6} c^{12}$
$-11275939120566955067670528 a^{7} c^{12}-1893040949144959846023168 a^{8} c^{12}$ $-9968451459569335369728 a^{9} c^{12}+194607840917850587136 a^{10} c^{12}$ $+1997386491657682944 a^{11} c^{12}+108519053819904 a^{12} c^{12}-53752428527616 a^{13} c^{12}$ $-144885252096 a^{1} 4 c^{1} 2+18448020907486504811430073388236800 c^{13}$ $-3856745853450338227021757376626688 a c^{13}$
$+211641738970982432884384258326528 a^{2} c^{13}+1433748264612439560591551496192 a^{3} c^{13}$ $-286462397082332455760368631808 a^{4} c^{13}-334717916773476330204364800 a^{5} c^{13}$ $+153683855236170251244306432 a^{6} c^{13}+839739517126519279140864 a^{7} c^{13}$ $-31261302613119321686016 a^{8} c^{13}-315489781260729040896 a^{9} c^{13}$ $+1167869845840674816 a^{10} c^{13}+23110204618948608 a^{11} c^{13}+54411556405248 a^{12} c^{13}$ $-159030853632 a^{13} c^{13}-428654592 a^{14} c^{13}+909482581712513207385940159365120 c^{14}$ $-156274186020921147699410472271872 a c^{14}+6279292679174525654448344137728 a^{2} c^{14}$ $+115956561233432791587999449088 a^{3} c^{14}-7360152423160490040436457472 a^{4} c^{14}$ $-69291789307316134281216000 a^{5} c^{14}+2845564665449629306060800 a^{6} c^{14}$ $+32561955952558276608000 a^{7} c^{14}-305161120199410974720 a^{8} c^{14}$
$-5040442012218163200 a^{9} c^{14}-5245737727426560 a^{10} c^{14}+141764646666240 a^{11} c^{14}$
$+445800775680 a^{12} c^{14}+37833546707041895399564047810560 c^{15}$
$-5285882183619194304423027277824 a c^{15}+146083328446676129035956781056 a^{2} c^{15}$ $+4638394209673830244650319872 a^{3} c^{15}-140584769016834228427948032 a^{4} c^{15}$ $-2435605232707110241566720 a^{5} c^{15}+34836083038449580769280 a^{6} c^{15}$ $+642581667671970152448 a^{7} c^{15}-880173479338573824 a^{8} c^{15}-47325764545413120 a^{9} c^{15}$
$-143267795435520 a^{10} c^{15}+363499094016 a^{11} c^{15}+1143078912 a^{12} c^{15}$ $+1328536279489011201768790425600 c^{16}-148912396569253327076390338560 a c^{16}$ $+2531192885146930883095166976 a^{2} c^{16}+128614693591513820120481792 a^{3} c^{16}$ $-1878200551872092467888128 a^{4} c^{16}-51981878954804943126528 a^{5} c^{16}$ $+217431350420978663424 a^{6} c^{16}+7967433044494909440 a^{7} c^{16}+17859224874516480 a^{8} c^{16}$ $-248088131665920 a^{9} c^{16}-936181628928 a^{10} c^{16}+39324937420565516206787788800 c^{17}$ $-3477519408228495235951362048 a c^{17}+28032529202838965591212032 a^{2} c^{17}$ $+2651393497096599815061504 a^{3} c^{17}-14165258247809019150336 a^{4} c^{17}$ $-758760374133196259328 a^{5} c^{17}-787692050765512704 a^{6} c^{17}+62545525067612160 a^{7} c^{17}$ $+250403875061760 a^{8} c^{17}-561285365760 a^{9} c^{17}-2118057984 a^{10} c^{17}$ $+977881700980828090851655680 c^{18}-66801106085793434248937472 a c^{18}$ $+43590510705713325539328 a^{2} c^{18}+41284521661569630732288 a^{3} c^{18}$ $+26390765956634247168 a^{4} c^{18}-7659664312998297600 a^{5} c^{18}-30527842717532160 a^{6} c^{18}$ $+285382425968640 a^{7} c^{18}+1346143518720 a^{8} c^{18}+20312269369057937813667840 c^{19}$
$-1043420520990729981394944 a c^{19}-5610443146844898852864 a^{2} c^{19}$ $+481519230461779378176 a^{3} c^{19}+2220816220809854976 a^{4} c^{19}-51433538762833920 a^{5} c^{19}$ $-287760431185920 a^{6} c^{19}+577697218560 a^{7} c^{19}+2724986880 a^{8} c^{19}$ $+349451482591594650009600 c^{20}-13031124524113644748800 a c^{20}$
$-132358476854886137856 a^{2} c^{20}+4089382966879322112 a^{3} c^{20}+29424316770680832 a^{4} c^{20}$ $-207277762019328 a^{5} c^{20}-1303633723392 a^{6} c^{20}+4918829777203023052800 c^{21}$ $-126939929386332192768 a c^{21}-1751952124085796864 a^{2} c^{21}+23914007441178624 a^{3} c^{21}$ $+208836869750784 a^{4} c^{21}-379629600768 a^{5} c^{21}-2387607552 a^{6} c^{21}$
$+55669063440673013760 c^{22}-928805370464305152 a c^{22}-15241882330202112 a^{2} c^{22}$ $+86141407592448 a^{3} c^{22}+812654788608 a^{4} c^{22}+493935192315002880 c^{23}$ $-4799094350413824 a c^{23}-86731646828544 a^{2} c^{23}+144049176576 a^{3} c^{23}+1358954496 a^{4} c^{23}$ $+3307151661465600 c^{24}-15605327462400 a c^{24}-294440140800 a^{2} c^{24}+15703474176000 c^{25}$

$$
\begin{equation*}
\left.\left.-24008196096 a c^{25}-452984832 a^{2} c^{25}+47110422528 c^{26}+67108864 c^{27}\right\}\right] \tag{8}
\end{equation*}
$$

Derivation of main result (8):
Substituting $b=-a-53, z=\frac{1}{2}$ in given result (2), we get

$$
\begin{gathered}
(2 a+53)_{2} F_{1}\left[\begin{array}{ccc}
a & ,-a-53 & ; \frac{1}{2} \\
c & ; & \\
=a_{2} F_{1}\left[\begin{array}{ccc}
a+1 \\
c
\end{array},\right. & -a-53 & ; \frac{1}{2}
\end{array}\right]+(a+53)_{2} F_{1}\left[\begin{array}{ccc}
a, & -a-52 & ; \frac{1}{2} \\
c & & ;
\end{array}\right]
\end{gathered}
$$

Now using same parallel method which is used in Ref[6], we can prove the main result.

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# Modelling Optimum Response in a Longitudinal Survey 

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#### Abstract

Non-response rates in surveys have been recognized as important indicators of data quality since they introduce bias in the estimates which increases the mean square error. In order to reduce this error, previous studies have examined the effects of response predictors on response rates. There is dearth of information about models which focus on the interaction effects of response predictors on response rates. The study was therefore designed to develop and validate a model which would reduce non-response and achieve optimum response by the introduction of interaction effects of the response predictors that have been broken down into levels.

A two-stage stratified random sampling scheme was used in selecting 750 households in Oyo town. Households were interviewed in five waves. An interviewer-administered questionnaire was used to collect data on demographic characteristics and response predictors including age, gender, educational qualification, religion, employment status, family size, and duration of interview. Demographic characteristics were analyzed using summary statistics. Incidence Rate Ratio was used to examine the response rate at various levels of response predictors. Odd ratio was used to examine the relationship between response rate and each of the response predictors.


Keywords : Iongitudinal survey, response predictors, non-response rate. model validation.
GJSFR-F Classification : MSC 2010: 97K80

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# Modelling Optimum Response in a Longitudinal Survey 

Olayiwola O. M. ${ }^{\alpha}$, Apantaku F. S. ${ }^{\circ}$, Bisira H. O. ${ }^{\rho}$ \& Adewara A. A. ${ }^{\omega}$

Abstract - Non-response rates in surveys have been recognized as important indicators of data quality since they introduce bias in the estimates which increases the mean square error. In order to reduce this error, previous studies have examined the effects of response predictors on response rates. There is dearth of information about models which focus on the interaction effects of response predictors on response rates. The study was therefore designed to develop and validate a model which would reduce non-response and achieve optimum response by the introduction of interaction effects of the response predictors that have been broken down into levels.

A two-stage stratified random sampling scheme was used in selecting 750 households in Oyo town. Households were interviewed in five waves. An interviewer-administered questionnaire was used to collect data on demographic characteristics and response predictors including age, gender, educational qualification, religion, employment status, family size, and duration of interview. Demographic characteristics were analyzed using summary statistics. Incidence Rate Ratio was used to examine the response rate at various levels of response predictors. Odd ratio was used to examine the relationship between response rate and each of the response predictors. A model was developed by breaking the predictors of response into levels and their interaction effects were introduced into Denise and Lan model.

The respondents' mean age and modal family size were $51.8 \pm 6.9$ and 3 respectively, $64.8 \%$ were females, $52.8 \%$ were muslims and majority ( $88.9 \%$ ) were employed. The family size, duration of interview, education, number of visit, Language of interview, familiarity, gender, house ownership, Nationality and duration of residence in a community are positively related to the response rate. Age is negatively related to the response rate and there is no association between employment status and response rate. The developed model showed that family size ( $\mathrm{x}_{1}$ ), duration of interview $\left(x_{2}\right)$, and their interaction $\left(x_{1} x_{2}\right)$ significantly ( $p<0.05$ ) determined the response rate.

The developed model established that both main and interaction effects of response predictors play key roles in improving response rate in a longitudinal survey.
Keywords : Iongitudinal survey, response predictors, non-response rate. model validation.

## I. Introduction

Gaining valid answers to sensitive questions, questions pertaining to private and socially frowned upon or illegal behavior is difficult. People typically underreport sensitive behavior while over-reporting socially desirable behaviors (Warner 1965). Various techniques have been developed to guarantee anonymity and minimize the respondent's feelings of jeopardy, so that more honest answers can be expected. Two such techniques are: The randomized response technique RRT Warner (1965), Fox and Tracy (1986) and

[^4]the unmatched count technique UCT; also called item count technique, unmatched block design, or block total response Dalton et al. (1994), Raghavarao and Federer (1979).

Brunner and Carroll (1969), in their study titled the effect of prior notification on the refusal rate in surveys'. They studied the effects of survey sponsor on the response rate and they found that an advance letter printed on university stationary increased response by $30 \%$ over samples who received no advance letter, while an advance letter on stationary from a relatively unknown business decreased response rate by $6 \%$.

Heberlein and Baumgartner (1978) worked on factors affecting response rates to mail questionnaires. They compared response rates for 98 published mail questionnaire surveys and found higher response rates for government-sponsored surveys.

Kalton et al (1978) studied the effects of general and specific questions on response rate. Respondents were asked about driving standards generally and about driving standards among younger drivers. When the general question was asked first, 34 per cent of the respondents said that general driving standards were lower than they used to be. When that question followed the more specific question about younger drivers, the corresponding percentage fell by 7 per cent.

Schuman et al (1981). studied the context effects on survey response to questions with two opinion questions on abortion and they found that the distributions of answers to the more specific questions were the same whether the specific question was asked before or after the general question, but the distributions of answers to the general questions differed according to the question position.

Groves, et al. (1992) examined the effect of interviewer interaction with the respondents on the response rate and they found that tailoring the interaction was important. However, they also found that mentioning the survey sponsor was rated as a highly efficient means of securing cooperation. Successful interviewers also felt that the agency should pay more attention to public relations and thus, "the image of the agency is seen as a tool to work with and attain a better response rate."

Harris-Kojetin and Tucker (1999) in their study titled' exploring relation of economic and political conditions with refusal rates to government survey', found that in times of more positive public opinion regarding the government and government leaders, cooperation rates were higher. Survey respondents representing establishments (such as schools, hospitals, factories, farms or other businesses) may be somewhat different from respondents representing themselves or their households. These differences may make attitudes toward the survey sponsor more important than in general household surveys.

Snijkers et al (1999) studied the tactics that high performing survey interviewers can use to gain cooperation. Similar to Groves, et al. (1992), they found that tailoring the interaction was important. However, they also found that mentioning the survey sponsor was rated as a highly efficient means of securing cooperation. Successful interviewers also felt that the agency should pay more attention to public relations and thus, "the image of the agency is seen as a tool to work with and attain a better response rate."

Mac Elroy (2000) worked on 'variable influencing drop-out rate in web based surveys. The reviewed 19 studies done by Modalis Research Technology (USA) involving business-to-business technology related decisions. He found that drop-out rates decrease with incentives and increase with questionnaire length.

Sheehan (2001) in his study 'E-mail survey response rates' he studied the influence of five factors (the year the study was undertaken, the number of questions in the survey,
the number of pre-notification contacts, the number of follow-up contacts and survey topic salience) on response rates in 31 email surveys undertaken since 1986. She showed that the year the survey was undertaken and the number of follow-up contacts had the most influence on response rates to the survey questions: response rates decrease with time and increase with the number of follow-ups.

Knapp and Heidingsfelder (2001) conducted research on 'Drop out analysis' he reviewed nine unrestricted self-selected surveys done by Internet Rogator (Germany) in order to identify factors influencing drop-out rates. They found that longer surveys, sensitive topics and lack of incentives lead to higher drop-out rates.

Ariel Rubinstein (2004) carried out research on response time to survey questions. Lecture audiences and students were asked to respond to virtual decision and game situation at gametheory.tau.ac.il. Several thousand observations were collected and the response time for each answer was recorded. He showed that emotional response, require less response time than choices that require the use of cognitive reasoning.

Cheti and Franco (2005) worked on survey response and survey characteristics using probit model and they found that the number of children in the household, home ownership and the length of residence at the current address were positively influenced contact of the respondents in the survey. Women, people with college education respond more to the survey questions.

Sigrid Haunberger (2006) carried out research on the effects of interviewer and respondents characteristics on response behaviour in panel surveys. The logistic regression analysis provides results that several respondents' characteristics as well as interviewer characteristics has an impact on the refusal rate. For older interviewers, female interviewer, interviewers with high experience and interviewers with higher education lower refusal rates has been found. Older respondents agreed more than the younger ones to cooperate.

Fitzgerald et al. (1998), Campanelli et al. (1997) and Lepkowski and Couper (2002), offers suggestions about which variables are likely to help to predict contact and cooperation. These variables include both survey features and household and personal characteristics.

We may distinguish between two sets of variables that explain the probability of contact: household-specific variables that are linked to the probability of contacting a household, and person-specific variables that are linked to the probability of contacting a person.

The probability of contacting a household is inversely related to its degree of geographical mobility and to the probability of finding someone at home. Moreover, because people may pretend to be absent when an interviewer knocks at the door, the contact probability may also be related to a household's willingness to cooperate.

Predictors of contact include the number of adults, number of children in a household, home ownership, year of residence, high number of visit, duration of household interview in the last wave.

Once a household has been successfully contacted, a lack of cooperation is mainly the result of a personal decision that reflects personal characteristics. The personal characteristics that we consider include age, gender, education, employment status, couple living relationship.

To capture a person's past experience with the survey, we include features of the personal interview process in the current wave, namely duration of personal interview, mode of interview, language of interview, interviewer familiarity (a person who is
contacted by the same interviewer as in previous waves is likely to be more willing to cooperate again Groves and Couper, (1998); Laurie et al., (1999); Hox and de Leeuw (2002).

Denise and Lan (2006) modelled probability of a contact and the probability of response conditional on a contact was modeled using

$$
\begin{equation*}
\log \left(\sum_{m>c} \pi_{m i 2}^{(s)} / \pi_{c i 2}^{(s)}\right)=\beta_{0}+\sum_{p} \beta_{c p} x_{p i}+\sum_{q} \delta_{c q} x_{q i 1}^{*(s-1)}+\sum_{r} \delta_{c r} z_{r i 2}^{(s)} \tag{1}
\end{equation*}
$$

where
$x_{p i}$ are fixed characteristics of subject i measured at sweep $0, \mathrm{p}=0, \ldots \ldots \ldots . . \mathrm{P}\left(x_{0 i}=1\right.$ for all i)
$x_{q i, t-k}^{*}=$ time varying characteristics of subject I , measured at time $\mathrm{t}-1 \quad(\mathrm{q}=1 \ldots . . . \mathrm{Q}$ and $\mathrm{k}=1$....., often k will be 1)
$z_{\text {ri,t-k. }}=$ time-varying characteristics of the data collection process, measured for subject i at times $\mathrm{t}-\mathrm{k}(\mathrm{r}=1, \ldots . ., \mathrm{R}$ and $\mathrm{k}=0 \ldots$;often k will be 1 but can be 0 for variables such as the number of contacts before a response is obtained)

## II. Justification for the Research

Previous studies have examined the effects of response predictors on response rates. It was observed that authors examined the main effect of both extraneous response predictors and inherent response predictors on response rate. However, the existing models did not consider the interaction effect of response predictors. Hence, there is a need for a model that will capture both main and interactions effects of response predictors on response rate. This research work critically examined and provided solution to the above 'gaps' by the introduction of interaction effects of the response predictors into Denise and Lan model.

## iil. Choice of Response Predictors

Fitzgerald et al. (1998), Campanelli et al. (1997), and Lepkowski and Couper (2000), offered suggestions about which variables are likely to help in predicting contact and cooperation. These variables include both survey features and household and personal characteristics.

We may distinguish between two sets of variables that explained the probability of contact: household-specific variables that are linked to the probability of contacting a household, and person-specific variables that are linked to the probability of contacting a person.

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## IV. Methodology

A two-stage stratified random sampling scheme was used in selecting 750 households in Oyo town. Households were interviewed in five waves. An intervieweradministered questionnaire was used to collect data on demographic characteristics and response predictors including age, gender, educational qualification, religion, employment status, family size, and duration of interview. Demographic characteristics were analyzed using summary statistics. Incidence Rate Ratio was used to examine the response rate at various levels of response predictors. Odd ratio was used to examine the relationship between response rate and each of the response predictors. A model was developed by breaking the predictors of response into levels and their interaction effects were introduced into Denise and Lan model.

## a) Model Development

The model was developed by introducing interaction of response predictors into Denise and Lan (2006) model, model (1).
The developed model is given as:
where,

$$
\begin{gather*}
\pi_{i j}^{(s)}=\log \left(\sum_{m>c} \pi_{m i 2}^{(s)} / \pi_{c i 2}^{(s)}\right)  \tag{3}\\
\beta_{1}^{(s)^{f c}}=\sum_{p} \beta_{c p} x_{p i}  \tag{4}\\
\beta_{2}^{(s)^{t v s}}=\sum_{q} \gamma_{c q} x_{q i 1}^{*(s-1)}  \tag{5}\\
\beta_{3}^{(s)^{t v d}}=\sum_{r} \delta_{c r} z_{r i 2}^{(s)}  \tag{6}\\
\beta_{12}^{(s)^{f c t v s}}=\sum_{p} \sum_{q}(\beta \gamma)_{c p q} x_{p i} x_{i 1}^{*(s-1)}  \tag{7}\\
\beta_{13}^{(s)^{f c t v d}}=\sum_{p} \sum_{r}(\beta \delta)_{c p r} x_{p i} z_{r i 2}^{s}  \tag{8}\\
\beta_{23}^{(s)^{t v s t v v i}}=\sum_{q} \sum_{r}(\gamma \delta)_{c q r} x_{q i 1}^{*(s-1)} z_{r i 2}^{(s)}  \tag{9}\\
\beta_{123}^{(s)^{f c t v s}+t v d}=\sum_{p} \sum_{q} \sum_{r}(\beta \gamma \delta)_{c p q r} x_{p i} x_{q i 1}^{*(s-1)} z_{r i 2}^{(s)} \tag{10}
\end{gather*}
$$

$\beta_{c p^{\prime}}, \gamma_{c q}, \quad \delta_{c r r^{\prime}}(\beta \gamma)_{c p q^{\prime}},(\beta \delta)_{c p r^{\prime}}(\gamma \delta)_{c q r},(\beta \gamma \delta)_{c p q r}$ are the parameters of the model to be estimated.

## b) Validation of the Developed Model

The developed model [model (2)] was validated and compared with the existing model [model (1)] by using a set of data collected from 750 house heads in Oyo town.

## V. Discussion of Results

## Table 1 : Numbers of Non-Missing Observations, Means and Standard Deviations of the Response Predictors

| Response <br> predictors | Number of non-missing <br> observations | Mean | Standard <br> deviation | Mode |
| :---: | :---: | :---: | :---: | :---: |
| Family size | 750 | 3 | 2.0515 | 3 |
| Duration of <br> interview | 750 | 7.2067 | 4.7008 | 11 |
| Education | 750 | 2.4200 | 1.1068 | Secondary |
| Number of visit | 750 |  | 3 |  |
| Language of <br> interview | 750 |  |  | English <br> language |
| Familiarity with <br> interviewer | 750 |  | Not <br> familiar |  |
| House ownership | 750 | 10.2667 | 9.2357 | Tenants |
| Nationality | 750 |  |  | 7 |
| Duration of <br> residence | 750 | 51.8 | 6.9 | Yoruba |
| Tribe | 750 |  |  | Females |
| Religion | 750 |  |  |  |
| Age | 750 | 750 |  |  |
| Gender |  |  |  |  |

## a) Response Rate at Each Level of the Response Predictor

Family size was classified into six categories. Family without children, family with one child, family with two children, family with three children, family with four children and more than four children. The first category was taken as reference level and its incidence rate ratio (IRR) is 1, which was compared with other levels. Any IRR value greater than 1 means higher response rate compared with reference level and value lower than 1 implies low response rate. Response from the family with one child was $27 \%$ higher compared with the family without child, response from the family with two, three, four, and more than four were higher in the following percentage respectively, $56 \%, 66 \% 75 \%$ and $84 \%$.

Duration of interview was classified into four levels. Respondents that were interviewed for not more than five minutes, 5-10 minutes, 11-15 minutes and more than fifteen minutes. Less than 5 minutes was taken as reference level and this was compared with other levels. The response from those that were interviewed for $5-10$ minutes was $23 \%$ higher compared with those that were interviewed for less than 5 minutes. Response from 11-15 minutes was $58 \%$ higher and response from those that were interviewed for more than 16 minutes was $59 \%$ higher compared with those that were interview for less than 5 minutes.

Among the respondents, some were living with their spouse and some were not. Non living with the spouse was taken as reference level. From the analysis, the result showed that the response rate from those that were living with their spouse is $14 \%$ higher compared with those that were not living with their spouse as at the time of interview.

Both English and Yoruba language were used during the survey. Yoruba language was taken as reference level. The response from those that were interviewed with English language is $53 \%$ higher compared with those that were interviewed with Yoruba language.

The respondents' ages were grouped into three age categories: 30-50, 51-70 and 71 and above years. $30-50$ years is the reference age group and the result shows that the response from the respondents between ages 51-70 years is $94 \%$ higher compare with the response from respondents between ages 30-50 and response from the respondents between ages 71 years and above is $12 \%$ higher compare with the response from respondents between ages 30-50 (table 2). This implies that respondents at the middle age respond better to survey questions compared with youth and old age respondents.

Some respondents were familiar with the interviewer and some were not. The response rate from those that are familiar with the Interviewer is $21 \%$ higher compare with those that are not familiar with the interviewer (table 2) the more the familiarity, the higher the response rate. Levels of education were categorised into three; (primary, secondary and tertiary). Primary was used as reference level and the result showed that the response from the respondents with secondary educatiobn was $27 \%$ higher compared with those with primaty education and the response from the respondents with Tertiary educatiobn was $54 \%$ higher compared with those with primaty education. The higher the educational level, the higher the response rate.

During the survey, before the interviewer succeeded in getting response from the respondents, some respondents were visited one time, two times, three, four and five times. The response rate increased till fourth visits and at fifth visit, it declined (table 2). The response rate increased from the first to fourth visit, but at the fifth visit, the response obtained was $98 \%$ lower compared with the first visit.

Both male and female participated in the survey. Females were taken as reference level. The result of the analysis portrayed that response from the female was $15 \%$ higher than the response rate from the males.

Majority of the respondents are tenants while minorities are owner occupiers. Being a tenant was taken as reference level. The result from the analysis showed that the response rate from tenants was $7 \%$ higher than the owner occupiers.

There was no significant difference in the response rate from unemployed respondents and employed respondents. The number of years in which the respondents have been living in their communities varies. This was classified into four categories; 1-5 years, 6-10 years, 11-15 years, 16 years and above. 1-5 years was used as reference year. The response from those that have been living in their community within 6-10 years is $33 \%$ higher compared with the response from those that have been living in their community within 1-5 years. 11-15 years is $58 \%$ higher, for more than 15years is $81 \%$ higher compared with the response from those that have been living in their community within 1-5 years. The more the number of years a respondent has spent in his/her community, the more they response to survey questions.

The response from Nigerians was $52 \%$ higher compared with response from non Nigerians.

Table 2 : Incidence Rate Ratios for Various Levels of Response Predictors

|  | Incidence Rate Ratio (IRR) |
| :---: | :---: |
| Level of family size | 1.27134 |
| One | 1.564931 |
| Two | 1.664464 |
| Three | 1.75134 |
| Four | 1.844516 |
| More than four | Incidence Rate Ratio (IRR) |
| Duration of interview | 1.0000 |
| $1-5$ minutes | 1.234665 |
| 6-10 minutes | 1.58666465 |
| $11-15$ minutes | Incidence Rate Ratio (IRR) |
| More than 15 minutes | 1.0000 |
| Language of interview | 1.5326667 |
| Yoruba language | Incidence Rate Ratio (IRR) |
| English language | 1.0000 |
| Age categories | 1.9417225 |
| $30-50$ years | 1.1235789 |
| $51-70$ years | Incidence Rate Ratio (IRR) |
| $71-90$ years | 1.0000 |
| Familiarity | 1.209975 |
| Unfamiliar Respondents | Incidence Rate Ratio (IRR) |
| Familiar Respondents | 1.0000 |
| Level of Education | 1.2712579 |
| Primary | 1.5419527 |
| Secondary | Tertiary |

$$
I R R=\frac{\text { response from target population }}{\text { response from reference level }}
$$

## b) Individual Effects of Response Predictors on Response Rate

The effect of the predictors of response is explained with respect to their odd ratios. Odd ratio greater than 1 means positive association, less than 1 means negative association and 1 means no association between the variables.

The family size, duration of interview, education, number of visit, Language of interview, familiarity, gender, house ownership, Nationality and duration of residence in a community are positively related to the response rate. Age is negatively related to the response rate and there is no association between employment status and response rate. See table 3.

Table 3 : Odd Ratios For Predictors Of Response

| Predictors of Response and Response Rate | Odd ratios |
| :---: | :---: |
| Response rate * tribe | 1.266 |
| Response rate * age | 0.7596 |
| Response rate * language of interview | 1.1411 |
| Response rate * familiarity with interviewer $^{\text {Response rate }}$ * education | 1.4064 |
| Response rate * number of visit | 2.7511 |
| Response rate * gender | 1.7899 |
| Response rate * house ownership | 1.1853 |
| Response rate * family size | 1.7402 |
| Response rate * duration of interview | 1.1185 |
| Response rate * spouse kind of settlement | 1.3298 |
| Response rate * employment status | 1.007 |
| Response rate * year of reciding | 1.137 |

c) Results of existing and developed Models

Table 4 compares the estimates and p-values that were obtained for two alternative models. model 1 (existing model), and model 2 (developed model) are for the probability of conditional cooperation given contact with main effects of response predictors and conditional cooperation given contact with both main and interaction effects of response predictors respectively.

The intercept $\alpha_{2,}$ and $\alpha_{3}$ are directly interpretable as the inverse transforms of the probabilities response given contact (main effect) and response given contact (main and interaction effect) for the reference case respectively.

From model 2 (developed model), both family size and duration of interview contributed significantly ( $\mathrm{p}<0.05$ ). By eliminating other response predictors from model 2, we have a parsimonious model i.e a model with least variables, but with a reasonable fit (adjusted $R^{2}=0.811$ and lack of fit insignificant at $5 \%$ ), which contained family size and duration of interview with their interaction as shown in table 4.

Table 4: Parameter Estimate for Model 1 and Model 2

| Preditor of Response | Model 1 |  | Model 2a |  | Parsimonious Model |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | estimate | p-value | Estimate | p-value | estimate | p-value |
| Intercept | 0.0291 |  | 0.0310 |  | 0.0110 |  |
| Tribe | 0.2359+ | 0.0673+ | 0.1951\# | 0.5720 |  |  |
| language of interview | 0.1320 | 0.6390 | 0.1053 | 0.0701 |  |  |
| familiarity with interviewer | 0.3410+ | 0.0852+ | 0.2546\# | 0.0811 |  |  |
| Education | 1.0120+ | 0.06260+ | 0.9022\# | 0.823 |  |  |
| number of visit | 1.0260+ | 0.0924+ | 0.6366\# | 0.0627 |  |  |
| Gender | $0.1700^{+}$ | 0.0799+ | 0.0040 | 0.0661 |  |  |
| house ownership | 0.1150 | 0.0873 | 0.0213 | 0.0793 |  |  |
| family size | 0.5540+ | 0.005+ | 1.0647\# | 0.026** | 0.043 | 0.025** |
| duration of interview | 0.1120+ | 0.0103+ | 1.0438\# | 0.016* | 0.032 | 0.002* |
| spouse kind of settlement | 0.2850+ | 0.0614+ | 0.2151\# | 0.7310 |  |  |
| year of residing | $0.1284+$ | 0.0.691+ | 0.1133 | 0.9210 |  |  |
| family size* duration of interview |  |  | 0.9594\# | 0.032* | 0.021 | 0.037* |
| p-value | 0.042 |  | 0.036 |  | 0.024 |  |
| adjusted $R^{2}$ | 0.684 |  | 0.691 |  | 0.811 |  |

-     + means positive association with response given contact (main effects)
- \# means positive association with response given contact (main and interaction effects)
-     * means significant at $5 \%$ level of significance


## VI. Conclusions

Consideration of interaction effects of response predictors is a useful technique to improve response rate in a longitudinal survey. Two major effective response predictors were identified- namely; duration of interview and family size. The developed model established that both main and interaction effects of response predictors play key roles in improving response rate.

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# A Summation Formula in the Shade of Contiguous Relation 

By Salahuddin, M. P. Chaudhary \& Vinesh Kumar

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Abstract - The main aim of the present paper is to compute a summation formula in the shade of recurrence relation and contiguous relation.

Keywords : contiguous function, recurrence relation, bailey summation theorem and legendre duplication formula.

GJSFR-F Classification : MSC 2010: 40A25, 2010: 33C60, 33C70

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# A Summation Formula in the Shade of Contiguous Relation 

Salahuddin ${ }^{\text {a }}$, M. P. Chaudhary ${ }^{\circ}$ \& Vinesh Kumar ${ }^{\text { }}$

Abstract - The main aim of the present paper is to compute a summation formula in the shade of recurrence relation and contiguous relation.
Keywords : contiguous function, recurrence relation, bailey summation theorem and legendre duplication formula.

## I. Introduction

Generalized Gaussian hypergeometric function of one variable is defined as follows

$$
{ }_{A} F_{B}\left[\begin{array}{cc}
a_{1}, a_{2}, \cdots, a_{A} & ; \\
\\
b_{1}, b_{2}, \cdots, b_{B} & ;
\end{array}\right]=\sum_{k=0}^{\infty} \frac{\left(a_{1}\right)_{k}\left(a_{2}\right)_{k} \cdots\left(a_{A}\right)_{k} z^{k}}{\left(b_{1}\right)_{k}\left(b_{2}\right)_{k} \cdots\left(b_{B}\right)_{k} k!}
$$

or

$$
{ }_{A} F_{B}\left[\begin{array}{ccc}
\left(a_{A}\right) & ; &  \tag{1}\\
\left(b_{B}\right) & ; & z
\end{array}\right] \equiv{ }_{A} F_{B}\left[\begin{array}{ccc}
\left(a_{j}\right)_{j=1}^{A} & ; & \\
\left(b_{j}\right)_{j=1}^{B} & ; & z
\end{array}\right]=\sum_{k=0}^{\infty} \frac{\left(\left(a_{A}\right)\right)_{k} z^{k}}{\left(\left(b_{B}\right)\right)_{k} k!}
$$

where the parameters $b_{1}, b_{2}, \cdots, b_{B}$ are neither zero nor negative integers and $A, B$ are non-negative integers.

Contiguous Relation[E. D. p.51(10), Andrews p.363(9.16)] is defined as follows

$$
(a-b){ }_{2} F_{1}\left[\begin{array}{ccc}
a, b ; & z  \tag{2}\\
c & ; & z
\end{array}\right]=a_{2} F_{1}\left[\begin{array}{ccc}
a+1, & b ; & z \\
c & ; & z-b{ }_{2} F_{1}\left[\begin{array}{cc}
a, b+1 ; & z \\
c & ;
\end{array}\right] .\left[\begin{array}{cc} 
\\
c
\end{array}\right]
\end{array}\right.
$$

Recurrence relation of gamma function is defined by

$$
\begin{equation*}
\Gamma(\eta+1)=\eta \Gamma(\eta) \tag{3}
\end{equation*}
$$

Legendre duplication formula[Bells \& Wong p.26(2.3.1)] is defined by

$$
\begin{equation*}
\sqrt{\pi} \Gamma(2 z)=2^{(2 z-1)} \Gamma(z) \Gamma\left(z+\frac{1}{2}\right) \tag{4}
\end{equation*}
$$

[^5]\[

$$
\begin{align*}
\Gamma\left(\frac{1}{2}\right) & =\sqrt{\pi}=\frac{2^{(b-1)} \Gamma\left(\frac{b}{2}\right) \Gamma\left(\frac{b+1}{2}\right)}{\Gamma(b)}  \tag{5}\\
& =\frac{2^{(a-1)} \Gamma\left(\frac{a}{2}\right) \Gamma\left(\frac{a+1}{2}\right)}{\Gamma(a)} \tag{6}
\end{align*}
$$
\]

Bailey summation theorem [Prudnikov, p.491(7.3.7.8)]is defined as follows

$$
\begin{align*}
& { }_{2} F_{1}\left[\begin{array}{lll}
a, 1-a & ; & \frac{1}{2} \\
c & ; 2
\end{array}\right]=\frac{\Gamma\left(\frac{c}{2}\right) \Gamma\left(\frac{c+1}{2}\right)}{\Gamma\left(\frac{c+a}{2}\right) \Gamma\left(\frac{c+1-a}{2}\right)}=\frac{\sqrt{\pi} \Gamma(c)}{2^{c-1} \Gamma\left(\frac{c+a}{2}\right) \Gamma\left(\frac{c+1-a}{2}\right)}  \tag{7}\\
& \text { iI. Main Summation Formula } \\
& { }_{2} F_{1}\left[\begin{array}{ccc}
a & , & -a-54
\end{array} \quad ; \frac{1}{2}\right] \\
& =\frac{\sqrt{\pi} \Gamma(c)}{2^{c+54}}\left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+54}{2}\right)}\{-3632526548102967623456065290853023744000000 a\right. \\
& +4446799967986720070907003058844676587520000 a^{2} \\
& -2029423777265418667267401072793003032576000 a^{3} \\
& +443393446729261561819047053333687728742400 a^{4} \\
& -46492623077576826452271912851638915860480 a^{5} \\
& +1470471643076035263532029552407442835968 a^{6} \\
& +114572503003776862831268755822471937664 a^{7} \\
& -7813304623052902580133509116890000960 a^{8} \\
& -153416538212733096592297056887202080 a^{9}+14724690149357492747415014222077040 a^{10} \\
& +247244392290154740728936296785320 a^{11}-14345567277256921190022015881260 a^{12} \\
& -352277952633068085930977771230 a^{13}+5374257859606163400930658835 a^{14} \\
& +250042385428957947954454805 a^{15}+1139963388129223673994290 a^{16} \\
& -61210783668006255693280 a^{17}-1021346729033060962675 a^{18}-1391519338833706225 a^{19} \\
& +112554717038947040 a^{20}+1183227483550970 a^{21}+2786824170845 a^{22}-28308195565 a^{23} \\
& -222901510 a^{24}-523900 a^{25}-13 a^{26}+a^{27}+3632526548102968026747526417458659328000000 c \\
& \text {-11447440941328048633017211362987705630720000ac } \\
& +9211118326702870702420837367532908052480000 a^{2} c \\
& -3197626066461919106455613933982917492736000 a^{3} c \\
& +544034877784552685444825886335003267481600 a^{4} c
\end{align*}
$$

$-41849976638850767561406485048985094164480 a^{5} c$
$+338674010210218811784293916756559709184 a^{6} c$
$+131177023711791865111053764775734350080 a^{7} c$
$-3822922134862208518793922528110579200 a^{8} c$

$$
\begin{aligned}
- & 221198552564911668866275091063488448 a^{9} c+6183887309666073264552858303047296 a^{10} c \\
& +285768752072250779724688685696560 a^{11} c-3414217135501837933664628119840 a^{12} c
\end{aligned}
$$

$$
-237613364550194685192697112228 a^{13} c-894245510448316634134671164 a^{14} c
$$

$$
+89111818360260555305584840 a^{15} c+1385921634828462900823720 a^{16} c
$$

$$
-4972943401975613435228 a^{17} c-304935890554278164324 a^{18} c-2506978617169563680 a^{19} c
$$

$$
+5089885996932880 a^{20} c+200697003839332 a^{21} c+1143581839036 a^{22} c+1058348200 a^{23} c
$$

$$
-11899160 a^{24} c-38948 a^{25} c-28 a^{26} c+7000640973341330116564767451705308610560000 c^{2}
$$

$$
-13197770891330255431647935972400430055424000 a c^{2}
$$

$$
+7986872888777489932100412993498267687321600 a^{2} c^{2}
$$

$$
-2191093384173710840496892578048324355031040 a^{3} c^{2}
$$

$$
+292154091798287466806755221034252982747136 a^{4} c^{2}
$$

$$
-15649191339713282193316734250744178614272 a^{5} c^{2}
$$

$$
-284595469676096712179994160702559262720 a^{6} c^{2}
$$

$$
+55466321801378218467154018711649242880 a^{7} c^{2}
$$

$$
-306363689623909032803221252328698880 a^{8} c^{2}
$$

$$
-91180302417289209324391443624747584 a^{9} c^{2}+327218529522563287674096864919680 a^{10} c^{2}
$$

$$
+92049324521481933164800386706640 a^{11} c^{2}+534370623670037201750663618304 a^{12} c^{2}
$$

$$
-48422887139692156312515351692 a^{13} c^{2}-788954231874934003008584480 a^{14} c^{2}
$$

$$
+7296299288795158190543800 a^{15} c^{2}+285520511362272310663968 a^{16} c^{2}
$$

$$
+1737586861039344518060 a^{17} c^{2}-21717164092308484800 a^{18} c^{2}-366771778900223840 a^{19} c^{2}
$$

$$
-1460471963158208 a^{20} c^{2}+6258948547852 a^{21} c^{2}+72721914720 a^{22} c^{2}+201033560 a^{23} c^{2}
$$

$$
+43680 a^{24} c^{2}-364 a^{25} c^{2}+6016076341892806068169088746491928903680000 c^{3}
$$

$-8341596652072026236116897582764406603776000 a c^{3}$ $+4001850939872167416046145885931356848128000 a^{2} c^{3}$

$$
-882909674743446524173608599777725893672960 a^{3} c^{3}
$$

$$
+91719417630240307850190627433238113886208 a^{4} c^{3}
$$

$$
-3030075016607756348233826719977981757440 a^{5} c^{3}
$$

$$
-173820633392837756153941434187607498240 a^{6} c^{3}
$$

$$
\begin{gathered}
+12052902463886094478736396397386594816 a^{7} c^{3} \\
+179211164796489899280806791716326528 a^{8} c^{3}
\end{gathered}
$$

$-17689253888817289402312817855802240 a^{9} c^{3}-240371634387184730061267844952480 a^{10} c^{3}$ $+13051174294265468921628655481568 a^{11} c^{3}+268149543807457100423126853176 a^{12} c^{3}$ $-3506848186957057037827076800 a^{13} c^{3}-135546005721612414552672880 a^{14} c^{3}$ $-528820386212205619824864 a^{15} c^{3}+21595857596853711168712 a^{16} c^{3}$ $+289190509268314368000 a^{17} c^{3}+359351206947761600 a^{18} c^{3}-17481680880183520 a^{19} c^{3}$ $-132017981959864 a^{20} c^{3}-214502608320 a^{21} c^{3}+1177867600 a^{22} c^{3}+4673760 a^{23} c^{3}$ $+3640 a^{24} c^{3}+3108956383027196609961018663628097716224000 c^{4}$ -3403275361361867397249725598578987897978880 ac $^{4}$ $+1327648455477171001265285669064535605510144 a^{2} c^{4}$
$-237160981070123058238520435247431296155648 a^{3} c^{4}$ $-237160981070123058238520435247431296155648 a^{3} c^{4}$ $+18850751279207715678790005363904287621120 a^{4} c^{4}$ $-261640087246922633182405157556223119360 a^{5} c^{4}$ $-45123447992756330854753251540074511360 a^{6} c^{4}$ $+1443041941697296626639826391982532608 a^{7} c^{4}$ $+59628588305938998879313815892337920 a^{8} c^{4}$
$-1767598985950292309360185568642560 a^{9} c^{4}-61124582565088698074200484687424 a^{10} c^{4}$
$+766815382229693289212279824512 a^{11} c^{4}+38593680320702363002767107760 a^{12} c^{4}$ $+89225877125513221657352880 a^{13} c^{4}-10260855114125709525489200 a^{14} c^{4}$
$-122718103062957204794992 a^{15} c^{4}+407744008975486840800 a^{16} c^{4}$ $+16850102246081676000 a^{17} c^{4}+99279261403402080 a^{18} c^{4}-163715674842720 a^{19} c^{4}$ $-3782106408080 a^{20} c^{4}-12432019600 a^{21} c^{4}-4957680 a^{22} c^{4}+21840 a^{23} c^{4}$ $+1094975550956436907775227234736756332953600 c^{5}$
$-978242626520982044335005379387204753686528 a c^{5}$ $+314382894870084342272975280469724425617408 a^{2} c^{5}$ $+314382894870084342272975280469724425617408 a^{2} c^{5}$ $-45454946163560463741237267355567427420160 a^{3} c^{5}$ $+2669511295085729778934788292515291463680 a^{4} c^{5}$ $+15215933139642940814219084054305505280 a^{5} c^{5}$ $-7040593004867965687307122744729190400 a^{6} c^{5}$
$+75399710027748838930155070184529920 a^{7} c^{5}+9012916787329830998102911056609280 a^{8} c^{5}$ $-63894020453791883956249988308992 a^{9} c^{5}-7078079241920872802082102033408 a^{10} c^{5}$
$-18629767924026911261867712000 a^{11} c^{5}+2798908250775976586690526720 a^{12} c^{5}$ $+32733357651798646617302528 a^{13} c^{5}-321346071229017041732096 a^{14} c^{5}$ $-8311257138557809413120 a^{15} c^{5}-33548870426968765440 a^{16} c^{5}+396609173489937408 a^{17} c^{5}$ $+4255851775279104 a^{18} c^{5}+10002781268480 a^{19} c^{5}-32659827200 a^{20} c^{5}-164516352 a^{21} c^{5}$ $-139776 a^{22} c^{5}+281729258984745833637839657998169873055744 c^{6}$
$-209080345294465611912222942147234697838592 a c^{6}$
$+55692691665926505552344748667950606581760 a^{2} c^{6}$
$+55692691665926505552344748667950606581760 a^{2} c^{6}$
$-6477265249649587988555005706548805632000 a^{3} c^{6}$
$+263184454198672052320334817852793815040 a^{4} c^{6}$
$+7673136695076054465444274968035983360 a^{5} c^{6}$
$-726110490382890780475421756065955840 a^{6} c^{6}$
$-4094012565858695299205893049016320 a^{7} c^{6}+818909079613223534437199798853632 a^{8} c^{6}$ $+5710868286108996384692584837120 a^{9} c^{6}-468389717000632029209582976000 a^{10} c^{6}$
$-6305860736193246128193646080 a^{11} c^{6}+101277666501689938085489664 a^{12} c^{6}$ $+2457836711874854440306176 a^{13} c^{6}+4007889101414888335360 a^{14} c^{6}$
$-267513470116291210240 a^{15} c^{6}-2304006099624413184 a^{16} c^{6}-225581682213888 a^{17} c^{6}$ $+73950361605120 a^{18} c^{6}+294553459200 a^{19} c^{6}+171179008 a^{20} c^{6}-512512 a^{21} c^{6}$
$+55334958690763699724807954143576873500672 c^{7}$
$-34447159283462405716080438791869908910080 a c^{7}$
$+7617041445747475208379546665120495042560 a^{2} c^{7}$
$-703925501781924796792921184356876681216 a^{3} c^{7}$
$+17182302149333813134188359503972401152 a^{4} c^{7}$
$+1156460144680239181549889424647290880 a^{5} c^{7}$
$-50431434993551669525735959029268480 a^{6} c^{7}-1120069756485964371181742622064640 a^{7} c^{7}$ $+46790039614326326448784373620736 a^{8} c^{7}+950003665497301164987217643520 a^{9} c^{7}$ $-17091259823830483812712688640 a^{10} c^{7}-481108627237604745654392832 a^{11} c^{7}$ $+437789460524645894286336 a^{12} c^{7}+93938074872709567385600 a^{13} c^{7}$ $+689205725948291368960 a^{14} c^{7}-3334171641254817792 a^{15} c^{7}-60023169624557568 a^{16} c^{7}$ $-191490123601920 a^{17} c^{7}+386283955200 a^{18} c^{7}+2663598080 a^{19} c^{7}+2489344 a^{20} c^{7}$

$$
\begin{array}{r}
+8549969269083883826436955133543920435200 c^{8} \\
-4487183204544222436034062184445158359040 a c^{8} \\
+822114172191171112034830120588798853120 a^{2} c^{8} \\
-59181159832105079310133153622735454208 a^{3} c^{8} \\
+550044067632828817223912346687897600 a^{4} c^{8} \\
+109023784117871692197935887207874560 a^{5} c^{8}
\end{array}
$$

$$
+1511366511083621520561289605120 a^{8} c^{8}+65855223709980656410872990720 a^{9} c^{8}
$$

$$
-154217179022484930147214848 a^{10} c^{8}-20047675270005757092026880 a^{11} c^{8}
$$

$$
-127426565851832978565120 a^{12} c^{8}+1843523156073250263040 a^{13} c^{8}
$$

$$
+24900217195887221760 a^{14} c^{8}+39810865313117184 a^{15} c^{8}-711704868526080 a^{16} c^{8}
$$

$$
-3539660467200 a^{17} c^{8}-2744501760 a^{18} c^{8}+6223360 a^{19} c^{8}
$$

$$
+1061866450872092729819032306966960537600 c^{9}
$$

$$
-470696240064046834277696783394214510592 a c^{9}
$$

$$
+71103696052438540093900938903524737024 a^{2} c^{9}
$$

$$
-3864504631206492457221863820699893760 a^{3} c^{9}
$$

$$
-21995920522328475231687734988636160 a^{4} c^{9}
$$

$$
+7248737630521058763769325842563072 a^{5} c^{9}-38046526497982897282868169113600 a^{6} c^{9}
$$

$$
-62074163138858550811355559557120 a^{7} c^{9}+5507876868597106789294080000 a^{8} c^{9}
$$

$$
+2771338457332736937353410560 a^{9} c^{9}+17232256326831465899206656 a^{10}
$$

$$
-484438884641380103086080 a^{11} c^{9}-6129597332128243916800 a^{12} c^{9}
$$

$$
+7788074391423406080 a^{13} c^{9}+444688699425484800 a^{14} c^{9}+1897803475476480 a^{15} c^{9}
$$

$$
-2194107801600 a^{16} c^{9}-23972382720 a^{17} c^{9}-24893440 a^{18} c^{9}
$$

$$
+107690565397401346986089123449751470080 c^{10}
$$

$$
-40298450154600440670724427588101472256 a c^{10}
$$

$$
+4979603450912020846561803936569753600 a^{2} c^{10}
$$

$$
-194179767204215386689686960160112640 a^{3} c^{10}
$$

$$
-4323989352193297132571413343502336 a^{4} c^{10}
$$

$$
+352004913867257185711954839502848 a^{5} c^{10}+2325331762312550390227270041600 a^{6} c^{10}
$$ $-246675815364740511758225817600 a^{7} c^{10}-2101196022185623846193897472 a^{8} c^{10}$ $+73603627098759322888587264 a^{9} c^{10}+998773609274911009013760 a^{10} c^{10}$

$-5348026267333513420800 a^{11} c^{10}-144246398268992667648 a^{12} c^{10}$

$$
\begin{gathered}
-461009858663337984 a^{13} c^{10}+3791169745059840 a^{14} c^{10}+24705743462400 a^{15} c^{10} \\
+24554889216 a^{16} c^{10}-44808192 a^{17} c^{10}+9023564158947650290503815917511639040 c^{11} \\
-2843545835937856596678364782091304960 a c^{11} \\
+284198508171590530008832493071892480 a^{2} c^{11} \\
-7248241056630633388858805809840128 a^{3} c^{11} \\
-324151866977318962673592068800512 a^{4} c^{11}
\end{gathered}
$$

$$
+12431410271322423440262424166400 a^{5} c^{11}+226207740881347167069894574080 a^{6} c^{11}
$$

$$
-6607004619365771071955632128 a^{7} c^{11}-117830225573088588420157440 a^{8} c^{11}
$$

$$
+1082046359950354800967680 a^{9} c^{11}+28191030879356703621120 a^{10} c^{11}
$$

$$
+44332110912917667840 a^{11} c^{11}-1884347620113014784 a^{12} c^{11}
$$

$$
-10948008471429120 a^{13} c^{11}+5038993489920 a^{14} c^{11}+132501897216 a^{15} c^{11}
$$

$$
+154791936 a^{16} c^{11}+630115608143106155188457464607539200 c^{12}
$$

$$
-166510427814889927691865087069388800 a c^{12}
$$

$$
+13254125875165976604292282062995456 a^{2} c^{12}
$$

$$
-179009416137093751386906480869376 a^{3} c^{12}
$$

$-16178425644598072775547752284160 a^{4} c^{12}+301830441774802757314128117760 a^{5} c^{12}$ $+10297013217762243808331431936 a^{6} c^{12}-105037265307476334520303616 a^{7} c^{12}$ $-3603433122060384811827200 a^{8} c^{12}+644195867700000112640 a^{9} c^{12}$

$$
+476709376875333009408 a^{10} c^{12}+2509653629563551744 a^{11} c^{12}
$$

$$
-11611606660792320 a^{12} c^{12}-107841477918720 a^{13} c^{12}-135184957440 a^{14} c^{12}
$$

$$
+206389248 a^{15} c^{12}+36896041814973009622860146776473600 c^{13}
$$

$$
-8127548367078871350477937109368832 a c^{13}
$$

$$
+504510698528027682635538468175872 a^{2} c^{13}
$$

$-1323497087192695421657310822400 a^{3} c^{13}-593509864332248030778527580160 a^{4} c^{13}$ $+3917288549075718277986516992 a^{5} c^{13}+309083758364887855724232704 a^{6} c^{13}$ $-252346848943346139136000 a^{7} c^{13}-71007214363386055884800 a^{8} c^{13}$ $-341282202560481853440 a^{9} c^{13}+4687277210526941184 a^{10} c^{13}$ $+39130920692613120 a^{11} c^{13}+7356982886400 a^{12} c^{13}-475647836160 a^{13} c^{13}$ $-635043840 a^{14} c^{13}+1818965163425026414771880318730240 c^{14}$ $-331432963543570716694361319407616 a c^{14}$
$+15580190190278883223001177784320 a^{2} c^{14}+118449248389281046014155816960 a^{3} c^{14}$ $-16463738644881958686063329280 a^{4} c^{14}-32868538758870739034046464 a^{5} c^{14}$
$+6508230784490380925337600 a^{6} c^{14}+33973338199764975616000 a^{7} c^{14}$ $-902948500535452631040 a^{8} c^{14}-7739300305016586240 a^{9} c^{14}+19394646571745280 a^{1} 0 c^{14}$ $+306227665305600 a^{11} c^{14}+483903406080 a^{12} c^{14}-635043840 a^{13} c^{14}$ $+75667093414083790799128095621120 c^{15}-11293404558545472311847330775040 a c^{15}$ $+385237786782572649258114088960 a^{2} c^{15}+6886709650612546798746599424 a^{3} c^{15}$ $-345278343889697170514771968 a^{4} c^{15}-3084842618713655468359680 a^{5} c^{15}$ $+95447961247798364733440 a^{6} c^{15}+960076865110754721792 a^{7} c^{15}$ $-6570638002018320384 a^{8} c^{15}-89151322946273280 a^{9} c^{15}-78726384844800 a^{10} c^{15}$ $+1141554806784 a^{11} c^{15}+1778122752 a^{12} c^{15}+2657072558978022403537580851200 c^{16}$ $-320916602122447722645301493760 a c^{16}+7427791929876820972380618752 a^{2} c^{16}$ $+219997932356643978991894528 a^{3} c^{16}-5340463733044059954216960 a^{4} c^{16}$ $-83160599985153711144960 a^{5} c^{16}+906544454419543425024 a^{6} c^{16}$ $+14239272428932497408 a^{7} c^{16}-11566861868728320 a^{8} c^{16}-574066930483200 a^{9} c^{16}$ $-1158891503616 a^{10} c^{16}+1333592064 a^{11} c^{16}+78649874841131032413575577600 c^{17}$ $-7572140988774201526449078272 a c^{17}+105350453272102346707435520 a^{2} c^{17}$ $+4931686436606781711974400 a^{3} c^{17}-56778065888046124892160 a^{4} c^{17}$ $-1382070427809348845568 a^{5} c^{17}+4120776126092279808 a^{6} c^{17}$ $+129464570538885120 a^{7} c^{17}+226140605644800 a^{8} c^{17}-1846632775680 a^{9} c^{17}$ $-3451650048 a^{10} c^{17}+1955763401961656181703311360 c^{18}$
$-147305106472175452601450496 a c^{18}+920659167165155083550720 a^{2} c^{18}$ $+81832124421204329103360 a^{3} c^{18}-323835095122234572800 a^{4} c^{18}$ $-15473132463511568384 a^{5} c^{18}-15563091209093120 a^{6} c^{18}+704270840627200 a^{7} c^{18}$ $+1871561359360 a^{8} c^{18}-1917583360 a^{9} c^{18}+40624538738115875627335680 c^{19}$ $-2337655079141249877278720 a c^{19}+50833154644377600000 a^{2} c^{19}$ $+1011830277655997972480 a^{3} c^{19}+819470699905679360 a^{4} c^{19}-115852889062113280 a^{5} c^{19}$ $-349631964774400 a^{6} c^{19}+1987020062720 a^{7} c^{19}+4642570240 a^{8} c^{19}$ $+698902965183189300019200 c^{20}-29802154983947784683520 a c^{20}$ $-145433881681353244672 a^{2} c^{20}+9157714227866107904 a^{3} c^{20}$ $+34546098681937920 a^{4} c^{20}-543412846592000 a^{5} c^{20}-2011161427968 a^{6} c^{20}$ $+1857028096 a^{7} c^{20}+9837659554406046105600 c^{21}-298541969464415485952 a c^{21}$ $-2547571969167982592 a^{2} c^{21}+58160273550213120 a^{3} c^{21}+312638638653440 a^{4} c^{21}$ $-1362528043008 a^{5} c^{21}-4244635648 a^{6} c^{21}+111338126881346027520 c^{22}$

$$
\begin{aligned}
& -2274252794338738176 a c^{22}-24961255211008000 a^{2} c^{22}+239086820392960 a^{3} c^{22} \\
& +1377577205760 a^{4} c^{22}-1157627904 a^{5} c^{22}+987870384630005760 c^{23} \\
& -12521586242355200 a c^{23}-152383258624000 a^{2} c^{23}+538548633600 a^{3} c^{23} \\
& +2516582400 a^{4} c^{23}+6614303322931200 c^{24}-45713719296000 a c^{24}-544001228800 a^{2} c^{24} \\
& +419430400 a^{3} c^{24}+31406948352000 c^{25}-93348429824 a c^{25}-872415232 a^{2} c^{25} \\
& +94220845056 c^{26}-67108864 a c^{26}+134217728 c^{27} \\
& +\frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a+55}{2}\right)}\{21199969233754779522582696534019670016000000 \\
& \text {-40448776569893792647187579604874997268480000a } \\
& +25826596045904710375921437584297480257536000 a^{2} \\
& -7617288571919285564041299415651496708505600 a^{3} \\
& +1105717245212559253666448811710381636321280 a^{4} \\
& -65198666231202923543092772909741710344192 a^{5} \\
& -1303759126114916549487350691909861771264 a^{6} \\
& +287646393691962023619209274652050389376 a^{7} \\
& -2006518267127267611741367694369814080 a^{8} \\
& -601516203312960815787418024113714400 a^{9}+3132469382094702103722831125866160 a^{10} \\
& +803356044653660754693952888636120 a^{11}+4499989549943790094483885629620 a^{12} \\
& +803356044653660754693952888636120 a^{11}+4499989549943790094483885629620 a^{12} \\
& -590609016348862598837540541530 a^{13}-10687667755074448785878046175 a^{14} \\
& +139536045020284365648854635 a^{15}+5985018757194802795579070 a^{16} \\
& +40988950496274255765160 a^{17}-781798396049215232545 a^{18} \\
& -16018530946657517135 a^{19}-80133122523677080 a^{20}+591940995332830 a^{21} \\
& +9569336149295 a^{22}+43184642605 a^{23}+11461190 a^{24}-517868 a^{25} \\
& -1471 a^{26}-a^{27}+55749601856849326885503310801708662128640000 c \\
& \text {-80833783247988401574359777417003470946304000ac } \\
& +41476398068994392438730217052111173445222400 a^{2} c \\
& -9921634533992418677488328705681181381918720 a^{3} c \\
& +1129595111683638115022135175310497910038528 a^{4} c \\
& -41343328174634653153662633890779951933440 a^{5} c \\
& -2639694442821787683316818271580389664256 a^{6}{ }^{6} \\
& +207834375376397381922779060804046362880 a^{7} c \\
& +3333378777625425671664232987653922688 a^{8} c
\end{aligned}
$$

$$
-395571815692872397925104084755335104 a^{9} c
$$

$-5780540101266474467498356406851424 a^{10} c+395624883991889288907869023160240 a^{11} c$ $+9023851675658406720440350190728 a^{12} c-156462341668600986057545383084 a^{13} c$ $-6718839727553073164364445964 a^{14} c-26627438922785568610966600 a^{15} c$ $+1704366830961932425345408 a^{16} c+27470936711130374480876 a^{17} c$ $+28495095622372575196 a^{18} c-3131651836634189440 a^{19} c-32144429064282392 a^{20} c$ $-71582391764884 a^{21} c+798787884076 a^{22} c+6128671640 a^{23} c+14145040 a^{24} c$ $-364 a^{25} c-28 a^{26} c+60323350166584016620766069978751219793920000 c^{2}$ $-70230677927029756689822554021247997968384000 a c^{2}$ $+29588020646630203497526259209931170487992320 a^{2} c^{2}$
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$+90730899846950167637976 a^{16} c^{2}+4201446753669130768660 a^{17} c^{2}$ $+31923266313082372800 a^{18} c^{2}-94469328416045440 a^{19} c^{2}-2808326092708136 a^{20} c^{2}$
$-15103121325292 a^{21} c^{2}-9978543120 a^{22} c^{2}+174002920 a^{23} c^{2}+535080 a^{24} c^{2}$
$+364 a^{25} c^{2}+37555522810090151738205063521902025244672000 c^{3}$
$-36086962155562346088593064788079569094574080 a^{3}$
$+12624484686579638081195696884493025164918784 a^{2} c^{3}$
$-2006909249127744247092247682853222496174080 a^{3} c^{3}$ $+130817519827749334524808566239762816114688 a^{4} c^{3}$
$+833207918706630930417419847854688860160 a^{5} c^{3}$
$-437921291451502172458374504921889730048 a^{6} c^{3}$
$+5536162854522065088533808278401735168 a^{7} c^{3}$
$+730539070818246811076412510944409728 a^{8} c^{3}$
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$+15504722610303804647038831921418635051008000 c^{4}$
$-12485062293329100668610095394935688734441472 a c^{4}$
$-12485062293329100668610095394935688734441472 a c^{4}$
$+3646162610973168607846804453818678300377088 a^{2} c^{4}$
$-469334239117153564603665910909716414922752 a^{3} c^{4}$
$+21302825274247445266194114141898496655360 a^{4} c^{4}$
$+700891150729743525544010289689624481792 a^{5} c^{4}$
$-75774356577447521278620767470287406080 a^{6} c^{4}$
$-457136252619319463221888858765682688 a^{7} c^{4}$
$+113694623129340461500350786789825280 a^{8} c^{4}$
$+858824919407441295916797097346560 a^{9} c^{4}-90876308973769058608989296814528 a^{10} c^{4}$
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$$
\begin{gathered}
+24807108755263883129546912497144745164800 c^{8} \\
-10373447682342453865843499426738499223552 a c^{8} \\
+1446381707533354580030354276139102371840 a^{2} c^{8} \\
-64299990599507355351434732642978496512 a^{3} c^{8} \\
-1651975424235901649273697522561515520 a^{4} c^{8} \\
+157281314309815720715753632977600512 a^{5} c^{8} \\
+1183901686020266626775687943315456 a^{6} c^{8}
\end{gathered}
$$

$$
-155918120504861856723705862897664 a^{7} c^{8}-1556609873489413309212692766720 a^{8} c^{8}
$$

$$
+70699044648010635087888983040 a^{9} c^{8}+1181768337222779098240740864 a^{10} c^{8}
$$

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$$
+8706381066240 a^{16} c^{9}-224040960 a^{17} c^{9}-24893440 a^{18} c^{9}
$$

$+261160578538060076930306638036818984960 c^{10}$ $-78387768847952887293758504765912776704 a c^{10}$ $+7166292074617072033249448450006712320 a^{2} c^{10}$ $-112617224725178712600722303123193856 a^{3} c^{10}$ $-11982723916445526351990498616737792 a^{4} c^{10}$
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$+1301466755257956911022080 a^{2} c^{17}+6659689450897346097315840 a^{3} c^{17}$ $+6579466103696940072960 a^{4} c^{17}-1637520544983052713984 a^{5} c^{17}$ $-8090946429801725952 a^{6} c^{17}+96859018580459520 a^{7} c^{17}+670678615326720 a^{8} c^{17}$ $-17258250240 a^{9} c^{17}-3451650048 a^{10} c^{17}+2874353873330651339969003520 c^{18}$ $-159073689633643520719847424 a c^{18}-1021489772229901334609920 a^{2} c^{18}$ $+92925400795406857666560 a^{3} c^{18}+516696194466166865920 a^{4} c^{18}$ $-14138170007488823296 a^{5} c^{18}-103604398016430080 a^{6} c^{18}+305543897415680 a^{7} c^{18}$ $+2803506872320 a^{8} c^{18}+1917583360 a^{9} c^{18}+56849001614653539200532480 c^{19}$ $-2325059740350123259985920 a c^{19}-27632354991831263477760 a^{2} c^{19}$ $+970381213262613053440 a^{3} c^{19}+8619226588404776960 a^{4} c^{19}-78166917990318080 a^{5} c^{19}$ $-721668973527040 a^{6} c^{19}+18570280960 a^{7} c^{19}+4642570240 a^{8} c^{19}$ $+933034415636416875724800 c^{20}-27017015283590765740032 a c^{20}$ $-443307042360266850304 a^{2} c^{20}+7263786745898991616 a^{3} c^{20}+83064838878658560 a^{4} c^{20}$ $-221920428556288 a^{5} c^{20}-2713118048256 a^{6} c^{20}-1857028096 a^{7} c^{20}$ $+12550820438506293166080 c^{21}-242230221954063794176 a c^{21}-4895505123997908992 a^{2} c^{21}$ $+35733443979509760 a^{3} c^{21}+494860847022080 a^{4} c^{21}-12733906944 a^{5} c^{21}-4244635648 a^{6} c^{21}$ $+135958348863205539840 c^{22}-1596202130262196224 a c^{22}-37768382119936000 a^{2} c^{22}$ $+92226285731840 a^{3} c^{22}+1690136739840 a^{4} c^{22}+1157627904 a^{5} c^{22}$ $+1156283078604226560 c^{23}-7061944611635200 a c^{23}-195598011596800 a^{2} c^{23}$

$$
\begin{gather*}
+5033164800 a^{3} c^{23}+2516582400 a^{4} c^{23}+7430491393228800 c^{24}-16707590553600 a c^{24} \\
-611948953600 a^{2} c^{24}-419430400 a^{3} c^{24}+33903800745984 c^{25}-872415232 a c^{25} \\
\left.\left.-872415232 a^{2} c^{25}+97844723712 c^{26}+67108864 a c^{26}+134217728 c^{27}\right\}\right] \tag{8}
\end{gather*}
$$

Derivation of main formula:
Putting $b=-a-54, z=\frac{1}{2}$ in given result (2), we get

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# Enhanced $\left(G^{\prime} / G\right)$-Expansion Method to Find the Exact Complexiton Soliton Solutions of (3+1)-Dimensional ZakhrovKuznetsov Equation 

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Abstract - In this article, an enhanced $\left(G^{\prime} / G\right)$-expansion method has been applied to find the traveling wave solutions of the ( $3+1$ )-dimensional Zakhrov-Kuznetsov (ZK) equation. The efficiency of this method for finding these exact solutions has been demonstrated. As a result, a set of complexiton soliton solutions are derived, which are expressed by the combinations of rational, hyperbolic and trigonometric functions involving several parameters. It is shown that the method is effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics.

Keywords : enhanced ( $G^{\prime} / G$ )-expansion method; zk equation; complexiton soliton solutions; traveling wave solutions.

GJSFR-F Classification : MSC 2010: 13D02

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# Enhanced $\left(G^{\prime} / G\right)$-Expansion Method to Find the Exact Complexiton Soliton Solutions of (3+1)-Dimensional Zakhrov-Kuznetsov 

 EquationRafiqul Islam ${ }^{\alpha}$, Kamruzzaman Khan ${ }^{\circ}$, M. Ali Akbar ${ }^{\text { }}$ \& Ekramul Islam ${ }^{\omega}$


#### Abstract

In this article, an enhanced ( $\left.\boldsymbol{G}^{\prime} / \boldsymbol{G}\right)$-expansion method has been applied to find the traveling wave solutions of the (3+1)-dimensional Zakhrov-Kuznetsov (ZK) equation. The efficiency of this method for finding these exact solutions has been demonstrated. As a result, a set of complexiton soliton solutions are derived, which are expressed by the combinations of rational, hyperbolic and trigonometric functions involving several parameters. It is shown that the method is effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics. keywords : enhanced $\left(G^{\prime} / G\right)$-expansion method; zk equation; complexiton soliton solutions; traveling wave solutions.


## I. Introduction

Nowadays NLEEs have been the subject of all-embracing studies in various branches of nonlinear sciences. A special class of analytical solutions named traveling wave solutions for NLEEs have a lot of importance, because most of the phenomena that arise in mathematical physics and engineering fields can be described by NLEEs. NLEEs are frequently used to describe many problems of protein chemistry, chemically reactive materials, in ecology most population models, in physics the heat flow and the wave propagation phenomena, quantum mechanics, fluid mechanics, plasma physics, propagation of shallow water waves, optical fibers, biology, solid state physics, chemical kinematics, geochemistry, meteorology, electricity etc. Therefore investigation traveling wave solutions is becoming more and more attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the Hirota's bilinear transformation method [1, 2], the tanh-function method [3, 4], the extended tanh-method [5, 6], the Exp-function method [7-14], the Adomian decomposition method [15], the F-expansion method [16], the auxiliary equation method [17], the Jacobi elliptic function method [18], Modified Exp-function method [19], the $\left(G^{\prime} / G\right)$-expansion method [20-29], Weierstrass elliptic function method [30], the

[^6]homotopy perturbation method [31-35], the homogeneous balance method [36, 37], the Modified simple equation method [38-42], He's polynomial [43], asymptotic methods and nanomechanics [44], the variational iteration method [45, 46], the casoration formulation [47], the frobenius integrable decomposition [48], the extended multiple Riccati equations expansion method [49, 50], the enhanced $\left(G^{\prime} / G\right)$-expansion method [51] and so on.

The objective of this article is to apply the enhanced $\left(G^{\prime} / G\right)$-expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the ZK equation. The ZK equation is completely integrable and has N -soliton solutions.

The article is prepared as follows: In section II, an enhanced $G^{\prime} G$-expansion method is discussed. In section III, we apply this method to the nonlinear evolution equations pointed out above; in section IV, physical explanations and in section V conclusions are given.

## II. Enhanced $G^{\prime} G$-Expansion Method

In this section we describe enhanced $\left(G^{\prime} / G\right)$-expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear evolution equation, say in two independent variables $x$ and $t$, is given by

$$
\begin{equation*}
\mathcal{R}\left(u, u_{t}, u_{x}, u_{t t}, u_{x x}, u_{x t}, \ldots \ldots \ldots \ldots\right)=0 \tag{2.1}
\end{equation*}
$$

where $u(\xi)=u(x, t)$ is an unknown function, $\mathcal{R}$ is a polynomial of $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method:

Step 1. Combining the independent variables $x$ and $t$ into one variable $\xi=x \pm$ $\omega t$, we suppose that

$$
\begin{equation*}
u(\xi)=u(x, t), \quad \xi=x \pm \omega t \tag{2.2}
\end{equation*}
$$

The traveling wave transformation Eq. (2.2) permits us to reduce Eq. (2.1) to the following ODE:

$$
\begin{equation*}
F\left(u^{\prime} u^{\prime \prime} \ldots \ldots \ldots \ldots\right) \tag{2.3}
\end{equation*}
$$

where $F$ is a polynomial in $u(\xi)$ and its derivatives, while $u^{\prime}(\xi)=\frac{d u}{d \xi}, u^{\prime \prime}(\xi)=\frac{d^{2} u}{d \xi^{2}}$, and so on.
Step 2. We suppose that Eq. (2.3) has the formal solution

$$
\begin{equation*}
u(\xi)=\sum_{i=-n}^{n}\left(\frac{a_{i}\left(G^{\prime} / G\right)^{i}}{\left(1+\lambda\left(G^{\prime} / G\right)\right)^{i}}+b_{i}\left(G^{\prime} / G\right)^{i-1} \sqrt{\sigma\left(1+\frac{\left(G^{\prime} / G\right)^{2}}{\mu}\right)}\right) \tag{2.4}
\end{equation*}
$$

where $G=G(\xi)$ satisfy the equation $G^{\prime \prime}+\mu G=0$,
in which $a_{i}, b_{i}(-n \leq i \leq n ; n \in \mathbb{N})$ and $\lambda$ are constants to be determined later, and $\sigma= \pm 1, \mu \neq 0$.

Step 3. We determine the positive integer $n$ in Eq. (2.4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (2.3).

Step 4. We substitute Eq. (2.4) into Eq.(2.3) using Eq. (2.5) and then collect all terms of same powers of $\left(G^{\prime} / G\right)^{j}$ and $\left(G^{\prime} / G\right)^{j} \sqrt{\sigma\left(1+\frac{1}{\mu}\left(G^{\prime} / G\right)^{2}\right)}$ together, then set each coefficient of them to zero to yield a over-determined system of algebraic equations, solve this system for $a_{i}, b_{i}(-n \leq i \leq n ; n \in \mathbb{N})$ and $\lambda, \omega$.

Step 5. The solution of Eq. (2.5) can be written as follows:
When $\mu<0$, we get
and

$$
\begin{align*}
\frac{G^{\prime}}{G} & =\sqrt{-\mu} \tanh \left(\xi_{0}+\sqrt{-\mu} \xi\right)  \tag{2.6}\\
\frac{G^{\prime}}{G} & =\sqrt{-\mu} \operatorname{coth}\left(\xi_{0}+\sqrt{-\mu} \xi\right) \tag{2.7}
\end{align*}
$$

Again, when $\mu>0$, the solutions are
and

$$
\begin{align*}
& \frac{G^{\prime}}{G}=\sqrt{\mu} \tan \left(\xi_{0}-\sqrt{\mu} \xi\right) .  \tag{2.8}\\
& \frac{G^{\prime}}{G}=\sqrt{\mu} \cot \left(\xi_{0}+\sqrt{\mu} \xi\right) . \tag{2.9}
\end{align*}
$$

where $\xi_{0}$ is an arbitrary constant. Finally, substituting $a_{i}, b_{i}(-n \leq i \leq n ; n \in \mathbb{N}), \lambda, \omega$ and Eqs. (2.6)-(2.9) into Eq. (2.4) we obtain traveling wave solutions of Eq. (2.1).

## III. Application

In this section, we will exert enhanced $\left(G^{\prime} / G\right)$-expansion method to solve the ZK equation in the form,

$$
\begin{equation*}
u_{t}+a u u_{x}+u_{x x}+u_{y y}+u_{z z}=0 \tag{3.1}
\end{equation*}
$$

where $a$ is a positive constant.
The traveling wave transformation equation $u(\xi)=u(x, y, z, t), \xi=x+y+z-\omega t$ transform Eq. (3.1) to the following ordinary differential equation:

$$
\begin{equation*}
-\omega u^{\prime}+a u u^{\prime}+3 u^{\prime \prime}=0 . \tag{3.2}
\end{equation*}
$$

Now integrating Eq. (3.2) with respect to $\xi$ once, we have

$$
\begin{equation*}
C-\omega u+\frac{1}{2} a u^{2}+3 u^{\prime}=0 \tag{3.3}
\end{equation*}
$$

where $C$ is a constant of integration. Balancing the highest-order derivative $u^{\prime}$ and the nonlinear term $u^{2}$, from Eq. (3.3), yields $2 n=n+1$ which gives $n=1$.
Hence for $n=1$ Eq. (2.4) reduces to

$$
\begin{align*}
u(\xi)=a_{0} & +\frac{a_{1}\left(G^{\prime} / G\right)}{1+\lambda\left(G^{\prime} / G\right)}+\frac{a_{-1}\left[1+\lambda\left(G^{\prime} / G\right)\right]}{\left(G^{\prime} / G\right)}+b_{0}\left(G^{\prime} / G\right)^{-1} \sqrt{\sigma\left[1+\frac{\left(G^{\prime} / G\right)^{2}}{\mu}\right]} \\
& +b_{1} \sqrt{\sigma\left[1+\frac{\left(G^{\prime} / G\right)^{2}}{\mu}\right]}+b_{-1}\left(G^{\prime} / G\right)^{-2} \sqrt{\sigma\left[1+\frac{\left(G^{\prime} / G\right)^{2}}{\mu}\right]} . \tag{3.4}
\end{align*}
$$

where $G=G(\xi)$ satisfies Eq. (2.5).
Substitute Eq. (3.4) along with Eq. (2.5) into Eq. (3.3). As a result of this substitution, we get a polynomial of $\left(G^{\prime} / G\right)^{j}$ and $\left(G^{\prime} / G\right)^{j} \sqrt{\sigma\left[1+\frac{\left(G^{\prime} / G\right)^{2}}{\mu}\right]}$. From this polynomial, we equate the coefficients of $\left(G^{\prime} / G\right)^{j}$ and $\left(G^{\prime} / G\right)^{j} \sqrt{\sigma\left[1+\frac{\left(G^{\prime} / G\right)^{2}}{\mu}\right]}$ and setting
them to zero, we get a over-determined system that consists of twenty-five algebraic equations. Solving these over determined system of equations, we obtain the following valid sets.

Set $-1: C=\frac{9 \mu+a_{0}^{2} a^{2}}{2 a}, \omega=a a_{0}, \lambda=0, a_{-1}=0, a_{0}=a_{0}, a_{1}=\frac{3}{a}, b_{-1}=0, b_{0}=0, b_{1}= \pm \frac{3 \sqrt{\mu}}{a \sqrt{\sigma}}$.
Set -2: $\quad C=\frac{\left(a_{0}^{2} a^{2}+12 a a_{0} \lambda \mu+36 \mu+36 \mu^{2} \lambda^{2}\right)}{2 a}, \omega=6 \mu \lambda+a a_{0}, \lambda=\lambda, a_{-1}=0, a_{0}=a_{0}$,

$$
a_{1}=\frac{6\left(1+\mu \lambda^{2}\right)}{a}, b_{1}=0, b_{0}=0, b_{-1}=0
$$

Set-3: $\quad C=\frac{\left(a_{0}^{2} a^{2}-12 a a_{0} \lambda \mu+36 \mu+36 \mu^{2} \lambda^{2}\right)}{2 a}, \omega=-6 \mu \lambda+a a_{0}, \lambda=\lambda, a_{-1}=-\frac{6 \mu}{a}, a_{0}=a_{0}$,

$$
a_{1}=0, b_{1}=0, b_{0}=0, b_{-1}=0
$$

Set-4:

$$
\begin{gathered}
C=\frac{\left(a_{0}^{2} a^{2}+144 \mu\right)}{2 a}, \omega=a a_{0}, \lambda=0, a_{-1}=-\frac{6 \mu}{a}, a_{0}=a_{0}, a_{1}=\frac{6}{b}, \\
b_{1}=0, b_{0}=0, b_{-1}=0
\end{gathered}
$$

Set-5:

$$
\begin{gathered}
C=\frac{\left(a_{0}^{2} a^{2}-6 a a_{0} \lambda \mu+9 \mu+9 \mu^{2} \lambda^{2}\right)}{2 a}, \omega=-3 \mu \lambda+a a_{0}, \lambda=\lambda, a_{-1}=-\frac{3 \mu}{a}, a_{0}=a_{0} \\
a_{1}=0, b_{1}=0, b_{0}= \pm \frac{3 \mu}{a \sqrt{\sigma}}, b_{-1}=0
\end{gathered}
$$

Now for $\mu<0$, substituting the values of $C, \omega, a_{-1}, a_{0}, a_{1}, b_{1}, b_{0}, b_{-1}$ into Eq. (3.4) from the above Set-1 to Set-5, we get the following hyperbolic function solutions of ZK equation.

Family -1:

$$
\begin{aligned}
& u_{1}(\xi)=a_{0}+\frac{3}{a} \sqrt{-\mu} \tanh \left(\xi_{0}+\sqrt{-\mu} \xi\right) \pm \frac{3 \sqrt{\mu}}{a} \operatorname{sech}\left(\xi_{0}+\sqrt{-\mu} \xi\right) \\
& u_{2}(\xi)=a_{0}+\frac{3}{a} \sqrt{-\mu} \operatorname{coth}\left(\xi_{0}+\sqrt{-\mu} \xi\right) \pm \frac{3 i \sqrt{\mu}}{a} \operatorname{cosech}\left(\xi_{0}+\sqrt{-\mu} \xi\right)
\end{aligned}
$$

where

$$
\xi=x+y+z-a a_{0} t
$$

Family-2:

$$
\begin{aligned}
& u_{3}(\xi)=a_{0}+\frac{6\left(1+\mu \lambda^{2}\right) \sqrt{-\mu} \tanh \left(\xi_{0}+\sqrt{-\mu} \xi\right)}{a\left(1+\lambda \sqrt{-\mu} \tanh \left(\xi_{0}+\sqrt{-\mu} \xi\right)\right)} \\
& u_{4}(\xi)=a_{0}+\frac{6\left(1+\mu \lambda^{2}\right) \sqrt{-\mu} \operatorname{coth}\left(\xi_{0}+\sqrt{-\mu} \xi\right)}{a\left(1+\lambda \sqrt{-\mu} \operatorname{coth}\left(\xi_{0}+\sqrt{-\mu} \xi\right)\right)}
\end{aligned}
$$

where

$$
\xi=x+y+z-\left(6 \mu \lambda+a a_{0}\right) t
$$

Family-3:

$$
\begin{gathered}
u_{5}(\xi)=a_{0}-\frac{6 \mu \lambda}{a}+\frac{6 \sqrt{-\mu} \operatorname{coth}\left(\xi_{0}+\sqrt{-\mu} \xi\right)}{a} . \\
u_{6}(\xi)=a_{0}-\frac{6 \mu \lambda}{a}+\frac{6 \sqrt{-\mu} \tanh \left(\xi_{0}+\sqrt{-\mu} \xi\right)}{a} .
\end{gathered}
$$

where

$$
\xi=x+y+z-\left(a a_{0}-6 \mu \lambda\right) t
$$

Family-4:

$$
\begin{aligned}
& u_{7}(\xi)=a_{0}+\frac{6}{a} \sqrt{-\mu}\left(\tanh \left(\xi_{0}+\sqrt{-\mu} \xi\right)+\operatorname{coth}\left(\xi_{0}+\sqrt{-\mu} \xi\right)\right) \\
& u_{8}(\xi)=a_{0}+\frac{6}{a} \sqrt{-\mu}\left(\operatorname{coth}\left(\xi_{0}+\sqrt{-\mu} \xi\right)+\tanh \left(\xi_{0}+\sqrt{-\mu} \xi\right)\right)
\end{aligned}
$$

where

$$
\xi=x+y+z-a a_{0} t
$$

Family-5: $\quad u_{9}(\xi)=a_{0}-\frac{3 \mu \lambda}{a}+\frac{3 \sqrt{-\mu}}{a}\left(\operatorname{coth}\left(\xi_{0}+\sqrt{-\mu} \xi\right) \mp \operatorname{cosech}\left(\xi_{0}+\sqrt{-\mu} \xi\right)\right)$,

$$
u_{10}(\xi)=a_{0}-\frac{3 \mu \lambda}{a}+\frac{3 \sqrt{-\mu}}{a}\left(\tanh \left(\xi_{0}+\sqrt{-\mu} \xi\right) \mp i \operatorname{sech}\left(\xi_{0}+\sqrt{-\mu} \xi\right)\right)
$$

where

$$
\xi=x+y+z-\left(a a_{0}-3 \mu \lambda\right) t
$$

Similarly for $\mu>0$, we get the following periodic solutions of ZK equation.
Family -6:

$$
\begin{gathered}
u_{11}(\xi)=a_{0}+\frac{3}{a} \sqrt{\mu}\left(\tan \left(\xi_{0}-\sqrt{\mu} \xi\right) \pm \sec \left(\xi_{0}-\sqrt{\mu} \xi\right)\right) \\
u_{12}(\xi)=a_{0}+\frac{3}{a} \sqrt{\mu}\left(\cot \left(\xi_{0}+\sqrt{\mu} \xi\right) \pm \operatorname{cosec}\left(\xi_{0}+\sqrt{\mu} \xi\right)\right)
\end{gathered}
$$

where

$$
\xi=x-a a_{0} t
$$

Family-7:

$$
\begin{aligned}
& u_{13}(\xi)=a_{0}+\frac{6\left(1+\mu \lambda^{2}\right) \sqrt{\mu} \tan \left(\xi_{0}-\sqrt{\mu} \xi\right)}{a\left(1+\lambda \sqrt{\mu} \tan \left(\xi_{0}-\sqrt{\mu} \xi\right)\right)} \\
& u_{14}(\xi)=a_{0}+\frac{6\left(1+\mu \lambda^{2}\right) \sqrt{\mu} \cot \left(\xi_{0}+\sqrt{\mu} \xi\right)}{a\left(1+\lambda \sqrt{\mu} \cot \left(\xi_{0}+\sqrt{\mu} \xi\right)\right)}
\end{aligned}
$$

where

$$
\xi=x+y+z-\left(6 \mu \lambda+a a_{0}\right) t
$$

Family-8:

$$
\begin{aligned}
& u_{15}(\xi)=a_{0}-\frac{6 \mu \lambda}{a}+\frac{6 \sqrt{\mu} \cot \left(\xi_{0}-\sqrt{\mu} \xi\right)}{a} \\
& u_{16}(\xi)=a_{0}-\frac{6 \mu \lambda}{a}+\frac{6 \sqrt{\mu} \tan \left(\xi_{0}+\sqrt{\mu} \xi\right)}{a}
\end{aligned}
$$

where

$$
\xi=x+y+z-\left(a a_{0}-6 \mu \lambda\right) t
$$

Family-9:

$$
u_{17}(\xi)=a_{0}+\frac{6}{a} \sqrt{\mu}\left(\tan \left(\xi_{0}-\sqrt{\mu} \xi\right)-\cot \left(\xi_{0}-\sqrt{\mu} \xi\right)\right)
$$

$$
u_{18}(\xi)=a_{0}+\frac{6}{a} \sqrt{\mu}\left(\cot \left(\xi_{0}+\sqrt{\mu} \xi\right)-\tan \left(\xi_{0}+\sqrt{\mu} \xi\right)\right)
$$

where

$$
\xi=x+y+z-a a_{0} t .
$$

Family-10:
where

$$
\begin{gathered}
u_{19}(\xi)=a_{0}-\frac{3 \mu \lambda}{a}-\frac{3 \sqrt{\mu}}{a}\left(\cot \left(\xi_{0}-\sqrt{\mu} \xi\right) \mp \operatorname{cosec}\left(\xi_{0}-\sqrt{\mu} \xi\right)\right) \\
u_{20}(\xi)=a_{0}-\frac{3 \mu \lambda}{a}-\frac{3 \sqrt{\mu}}{a}\left(\tan \left(\xi_{0}+\sqrt{\mu} \xi\right) \mp \sec \left(\xi_{0}+\sqrt{\mu} \xi\right)\right)
\end{gathered}
$$

$$
\xi=x+y+z-\left(a a_{0}-3 \mu \lambda\right) t
$$

## IV. Physical Explanation

## a) Explanation

In this section we will discuss the physical explanations of obtained solutions of ZK equation. It is interesting to point out that the delicate balance between the nonlinearity effect of $u u_{x}$ and the dissipative effect of $u_{x x}, u_{y y}$ and $u_{z z}$ gives rise to solitons, that after a fully interaction with others, the solitons come back retaining their identities with the same speed and shape. The ZK equation has solitary wave solutions that have exponentially decaying wings. If two solitons of the ZK equation collide, the solitons just pass through each other and emerge unchanged.

The determined solutions from Family-1 to Family-10 are complexiton solution. That is the combinations of rational functions, hyperbolic functions and trigonometric functions.

For $\mu<0$, Family- $1\left(u_{1}(\xi)\right)$ and Family- $3\left(u_{6}(\xi)\right)$ are kink solutions represented in Fig. 1 and Fig. 3 for $\mu=-1, \xi_{0}=1, a_{0}=2, a=3, y=0, z=0$ and $\mu=-1, \xi_{0}=1, a=$ $3, a_{0}=2, \lambda=2, y=0, z=0$ within the interval $-3 \leq x, t \leq 3$.

Fig. 2 and Fig. 5 correspond to Family-2 $\left(u_{3}(\xi)\right)$ and Family-5 $\left(u_{9}(\xi)\right)$ for $\mu=$ $-1, \xi_{0}=1, a=3, a_{0}=2, \lambda=2, y=0, z=0 \quad$ and $\quad \mu=-3, \xi_{0}=1, a_{0}=2, a=1, \lambda=-3$, $y=0, z=0$ within the interval $-3 \leq x, t \leq 3$ are complexiton soliton solutions.

Family- $4\left(u_{7}(\xi)\right)$ provides singular kink solution for $\mu=-3, \xi_{0}=1, a_{0}=1, a=$ $2, y=z=0$ within the interval $-3 \leq x, t \leq 3$, represented in Fig. 4.

Consequently, for $\mu>0$, Family-6-Family-10 are combinations of rational and trigonometric functions solutions, also said to be traveling wave solutions that are periodic.

The wave speed $\omega$ plays an important role in the physical structure of the solutions obtained above. For the positive values of wave speed $\omega$ the disturbance represented by $u(\xi)=x+y+z-\omega t$ moves in the positive direction. Consequently, the negative values of wave speed $\omega$ the disturbance represented by $u(\xi)=x+y+z-\omega t$ moves in the negative direction.

Furthermore, the graphical demonstrations of some obtained solutions are shown in Figure-1 to Figure-10 in the following subsection.

## b) Graphical representation

Some of our obtained traveling wave solutions are represented in the following figures with the aid of commercial software Maple:


Fig. 1 : Shape of $u_{1}(\xi)$ for $\mu=-1, \xi_{0}=1$, $a_{0}=2, a=3, y=0, z=0,-3 \leq x, t \leq 3$.


Fig. 3 : Shape of $u_{6}(\xi)$ for $\mu=-1, \xi_{0}=1$,

$$
\begin{aligned}
a_{0}=2, a & =3, \lambda=2, y=0, z=0 \\
& -3 \leq x, t \leq 3
\end{aligned}
$$



Fig. 5 : Shape of $u_{9}(\xi)$ for $\mu=-3, \xi_{0}=1$,

$$
\begin{aligned}
a_{0}=2, a & =1, \lambda=-3, y=0, z=0 \\
& -3 \leq x, t \leq 3
\end{aligned}
$$



Fig. 2 : Profile of $u_{3}(\xi)$ for $\mu=-1, \xi_{0}=1$,

$$
\begin{aligned}
a=3, a_{0} & =2, \lambda=2, y=0, z=0, \\
& -3 \leq x, t \leq 3 .
\end{aligned}
$$



Fig. 4: Shape of $u_{7}(\xi)$ for $\mu=-3, \xi_{0}=1$, $a_{0}=1, a=2, y=0, z=0,-3 \leq x, t \leq 3$.


Fig. 6 : Shape of $u_{11}(\xi)$ for $\mu=1, \xi_{0}=1$,

$$
\begin{gathered}
a_{0}=2, a=1, y=0, z=0,-3 \leq \\
x, t \leq 3 .
\end{gathered}
$$

Fig. 7 : Shape of $u_{13}(\xi)$ for $\mu=0.5, \xi_{0}=2$,

$$
\begin{gathered}
a_{0}=2, a=1, \lambda=0, y=0, z=0,-3 \leq \\
x, t \leq 3 .
\end{gathered}
$$



Fig. 9: Shape of $u_{17}(\xi)$ for $\mu=3, \xi_{0}=1$,

$$
\begin{gathered}
a_{0}=1, a=2, y=0, z=0,-3 \leq \\
x, t \leq 3 .
\end{gathered}
$$



Fig. 8 : Shape of $u_{15}(\xi)$ for $\mu=3, \xi_{0}=0$,

$$
\begin{gathered}
a_{0}=2, a=3, \lambda=0, y=0, z= \\
0,-3 \leq x, t \leq 3 .
\end{gathered}
$$



Fig. 1o: Shape of $u_{19}(\xi)$ for $\mu=3, \xi_{0}=1$, $a_{0}=2, a=1, \lambda=0, y=0, z=0,-3 \leq$ $x, t \leq 3$.

## V. Conclusion

In short, we have illustrated the Enhanced $\left(G^{\prime} / G\right)$-expansion method and utilized it to find the exact solutions of nonlinear equations with the help of commercial software Maple. We have successfully obtained some Complexiton soliton solutions of the ZK equation. When the parameters are taken as special values, the solitary wave solutions and the periodic wave solutions are obtained. Taken as a whole, it is worthwhile to mention that this method is effective for solving other nonlinear evolution equations in mathematical physics.

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