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Some Properties of Finite

Summation Formula Coupled

Study of Nonlinear Evolution

Compactness in Bicomplex Space

Discovering Thoughts, Inventing Future

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CEIBS (China Europe International Business School).
Beijing, Shanghai and Shenzhen
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University of Barcelona
BA in Mathematics (Licenciatura)
University of Barcelona

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Davee Department of Neurology and Clinical NeuroscienceNorthwestern University
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Exponential Estimators of Population Mean in Post-Stratified Sampling using Known Value of Some Population Parameters

By Onyeka, A. C., Nlebedim, V. U. & Izunobi, C. H.

Federal University of Technology, Nigeria

Abstract - This paper proposes some exponential estimators of the population mean in post-stratified sampling (PSS) scheme, when using known value of some population parameters. The bias and mean squared error of the proposed estimators are obtained up to first order approximations. Conditions under-which the proposed estimators perform better than other estimators, like the post-stratified sampling mean estimator, the ratio type estimator and the dual to ratio estimator, proposed by Onyeka (2012, 2013), are obtained. The theoretical results are further verified and confirmed using numerical illustrations.

Keywords : *post-stratified sampling, mean squared error, ratio, dual to ratio, exponential, and composite estimators.*

GJSFR-F Classification : *MSC 2010: 97K80, 47N30*

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Exponential Estimators of Population Mean in Post-Stratified Sampling using Known Value of Some Population Parameters

Onyeka, A. C. [α], Nlebedim, V. U. [σ] & Izunobi, C. H. [ρ]

Abstract- This paper proposes some exponential estimators of the population mean in post-stratified sampling (PSS) scheme, when using known value of some population parameters. The bias and mean squared error of the proposed estimators are obtained up to first order approximations. Conditions under-which the proposed estimators perform better than other estimators, like the post-stratified sampling mean estimator, the ratio type estimator and the dual to ratio estimator, proposed by Onyeka (2012, 2013), are obtained. The theoretical results are further verified and confirmed using numerical illustrations.

Keywords: *post-stratified sampling, mean squared error, ratio, dual to ratio, exponential, and composite estimators.*

AMS (2013) Classification: 62D05

I. Introduction

Many authors have contributed to the use of known population parameters of an auxiliary character in constructing estimators of the population parameters of a study variate. Notably is the work carried out by Khoshnevisan et al (2007), who discussed a general family of estimators of $\overline{Y}$ under the SRSWOR scheme using known parameters of the auxiliary variable x, such as standard deviation, correlation coefficient, coefficient of skewness, kurtosis and coefficient of variation. Under the stratified random sampling, Koyuncu and Kadilar (2009) discussed a general family of combined estimators of $\overline{Y}$ that makes use of known population parameters. Recently, Onyeka (2012) developed a general family of estimators of $\overline{Y}$ under the post-stratified sampling scheme, using known values of some population parameters of an auxiliary character. The general family of estimators of the population mean $\overline{Y}$ proposed by Onyeka (2012), under the post-stratified sampling scheme is given by

$$t_1 = \bar{y}_{ps}\left(\frac{a\overline{X}+b}{\alpha\left(a\bar{x}_{ps}+b\right)+(1-\alpha)\left(a\overline{X}+b\right)}\right)^g \qquad (1.1)$$

where,

$y_{hi}(x_{hi})$ is the ith observation for the study (auxiliary) variate in stratum h

Authors α σ ρ : Department of Statistics, Federal University of Technology, PMB 1526, Owerri, Nigeria. E-mail : aloyonyeka@yahoo.com

$\bar{y}_{ps} = \sum_{h=1}^{L} \omega_h \bar{y}_h$ is the post-stratified estimator of $\bar{Y}$

$\bar{x}_{ps} = \sum_{h=1}^{L} \omega_h \bar{x}_h$ is the post-stratified estimator of $\bar{X}$

$\bar{X} = \sum_{h=1}^{L} \omega_h \bar{X}_h$ is the population mean of auxiliary variate, x.

$a(\neq 0)$, b are either constants or functions of known population parameters of the auxiliary variate, such as coefficient of variation (C_x), correlation coefficient (ρ_{yx}), standard deviation (σ_x), skewness $(\beta_1(x))$, and kurtosis $(\beta_2(x))$

ω_h is stratum weight, L is the number of strata in the population, $\bar{X}_h$ is the population mean of the auxiliary variate in stratum h, $\bar{y}_h(\bar{x}_h)$ is the sample mean of the study (auxiliary) variate in stratum h, N_h is the population size of stratum h, and N is the population size.

More recently, Onyeka (2013), still studying estimation of population mean in post-stratified sampling scheme, proposed a class of dual to ratio estimators of the population mean, $\bar{Y}$ in post-stratified sampling, using known population parameters of an auxiliary character x, as:

$$t_2 = \bar{y}_{ps}\left(\frac{\alpha\left(a\bar{x}_{ps}^{*} + b\right) + (1-\alpha)\left(a\bar{X} + b\right)}{a\bar{X} + b}\right)^{g} \tag{1.2}$$

where $\bar{x}_{ps}^{*}$ is a transformed sample mean of the auxiliary variable, x, based on the variable transformation, $\bar{x}_{hi}^{*} = \frac{N\bar{X} - nx_{hi}}{N-n}$ and satisfying the relationship $\bar{X} = f\bar{x}_{ps} + (1-f)\bar{x}_{ps}^{*}$ in line with the transformation, $x_i^{*} = \frac{N\bar{X} - nx_i}{N-n}$, $i = 1, 2, \ldots, N$, used by Srivenkataramana (1980) to obtain a dual to ratio estimate of $\bar{Y}$ under the simple random sampling scheme. Other authors who have used the variable transformation introduced by Srivenkataramana (1980) under the simple random sampling scheme include Singh and Tailor (2005), Tailor and Sharma (2009), and Sharma and Tailor (2010). The present study, however, extends the works carried out by Onyeka (2012, 2013) by introducing the use of exponential estimators in post-stratified sampling scheme when using information on some known population parameters.

II. The Proposed Exponential Estimators

The use of exponential estimators was first discussed by Bahl and Tuteja (1991), who, under the simple random sampling scheme, used an exponential ratio-type estimator of the form

$$t = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{2.1}$$

Notes

Singh and Vishwakama (2007) used some modified ratio-type and product-type exponential estimators of population mean in double sampling scheme, while Singh et al. (2009) proposed a class of ratio-type exponential estimators of the population mean using known values of population parameters of an auxiliary character, under the simple random sampling scheme. The class of estimators proposed by Singh et al. (2009) is of the form:

Notes

$$t^* = \bar{y}\exp\left[\frac{(a\bar{X}+b) - (a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right] = \bar{y}\exp\left[\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right] \quad (2.2)$$

Following Singh et al. (2009), and extending the works carried out by Onyeka (2012, 2013), we propose a class of ratio-type exponential estimators of the population mean in post-stratified sampling scheme, using known values of population parameters of an auxiliary character, as

$$t_3 = \bar{y}_{ps}\exp\left[\frac{(a\bar{X}+b) - (a\bar{x}_{ps}+b)}{(a\bar{X}+b)+(a\bar{x}_{ps}+b)}\right] = \bar{y}_{ps}\exp\left[\frac{a(\bar{X}-\bar{x}_{ps})}{a(\bar{X}+\bar{x}_{ps})+2b}\right] \quad (2.3)$$

To obtain the properties of the proposed exponential estimator, t_3, we expand (2.3) up to first order approximations in expected values to obtain:

$$\left(t_3-\bar{Y}\right) = \bar{Y}\left(e_0 - \tfrac{1}{2}\lambda e_1 - \tfrac{1}{2}\lambda e_0 e_1 + \tfrac{3}{8}\lambda^2 e_1^2\right) \quad (2.4)$$

and

$$\left(t_3-\bar{Y}\right)^2 = \bar{Y}^2\left(e_0^2 + \tfrac{1}{4}\lambda^2 e_1^2 - \lambda e_0 e_1\right) \quad (2.5)$$

where

$$e_0 = \frac{\bar{y}_{ps}-\bar{Y}}{\bar{Y}}, \quad e_1 = \frac{\bar{x}_{ps}-\bar{X}}{\bar{X}} \quad \text{and} \quad \lambda = \frac{a\bar{X}}{a\bar{X}+b} \quad (2.6)$$

Taking the unconditional expectations of (2.4) and (2.5), for repeated samples of fixed size n, gives the approximate bias and mean squared error of the proposed estimator, t_3, respectively as:

$$B(t_3) = E\left(t_3-\bar{Y}\right) = \bar{Y}\left(E(e_0) - \tfrac{1}{2}\lambda E(e_1) - \tfrac{1}{2}\lambda E(e_0 e_1) + \tfrac{3}{8}\lambda^2 E(e_1^2)\right) \quad (2.7)$$

and

$$MSE(t_3) = E\left(t_3-\bar{Y}\right)^2 = \bar{Y}^2\left(E(e_0^2) + \tfrac{1}{4}\lambda^2 E(e_1^2) - \lambda E(e_0 e_1)\right) \quad (2.8)$$

Following Onyeka (2012), we have

$$E\left(e_0\right) = E\left(e_0\right) = 0 \quad (2.9)$$

$$E\left(e_0^2\right) = \frac{V\left(\bar{y}_{ps}\right)}{\bar{Y}^2} = \frac{1}{\bar{Y}^2}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{yh}^2 \quad (2.10)$$

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$$E\left(e_1^2\right)=\frac{V\left(\bar{x}_{ps}\right)}{\bar{X}^2}=\frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{xh}^2 \tag{2.11}$$

and

$$E\left(e_0 e_1\right)=\frac{Cov\left(\bar{y}_{ps},\bar{x}_{ps}\right)}{\bar{Y}\bar{X}}=\frac{1}{\bar{Y}\bar{X}}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{yxh} \tag{2.12}$$

where $f=n/N$ is the population sampling fraction, $S_{yh}^2\left(S_{xh}^2\right)$ is the population variance of y(x) in stratum h, and S_{yxh} is the population covariance of y and x in stratum h. Using (2.9) to (2.12) to make the necessary substitutions in (2.7) and (2.8) gives the bias and mean squared error of the proposed estimator, t_3, respectively as:

$$B(t_3)=\frac{1}{\bar{X}}\left(\frac{1-f}{n}\right)\left[\tfrac{3}{8}\lambda^2 R^2 A_{22}-\tfrac{1}{2}\lambda A_{12}\right] \tag{2.13}$$

and

$$MSE\,(t_3)=\left(\frac{1-f}{n}\right)\left[A_{11}+\tfrac{1}{4}\lambda^2 R^2 A_{22}-\lambda R A_{12}\right] \tag{2.14}$$

where

$$R=\frac{\bar{Y}}{\bar{X}},\ A_{11}=\sum_{h=1}^{L}\omega_h S_{yh}^2,\ A_{22}=\sum_{h=1}^{L}\omega_h S_{xh}^2,\ \text{and}\ A_{12}=\sum_{h=1}^{L}\omega_h S_{yxh} \tag{2.15}$$

III. Modified (Composite) Estimators

A linear function or combination of any two of the estimators, t_k, k = 1, 2, 3 respectively given in (1.1), (1.2) and (2.3) would result in a modified (composite-type) estimator of the population mean in post-stratified sampling, using known values of some population parameters of an auxiliary character. Accordingly, we propose the following modified estimators of $\bar{Y}$, as linear functions of pairs of the three estimators, t_k, k = 1, 2, 3.

$$t_4=\gamma t_1+(1-\gamma)t_2 \tag{3.1}$$

$$t_5=\gamma t_1+(1-\gamma)t_3 \tag{3.2}$$

$$t_6=\gamma t_2+(1-\gamma)t_3 \tag{3.3}$$

where γ is a constant or weighting fraction chosen, in practice, to minimize the mean squared error of the respective estimators in (3.1) to (3.3). Notice that the estimator, t_1 proposed by Onyeka (2012), in line with the estimators proposed by Khoshnevisan et.al (2007) under the simple random sampling scheme, is a ratio-type estimator of $\bar{Y}$ in post-stratified sampling scheme for all positive values of g in (1.1). The estimator, t_2, proposed by Onyeka (2013) is a dual to ratio type estimator, while the estimator, t_3, proposed in

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this study is an exponential type estimator. Consequently, the proposed composite estimator, t_4 is a "ratio cum dual to ratio" type estimator, the proposed estimator, t_5, is a "ratio cum exponential" type estimator, while the proposed composite estimator, t_6, is a "dual to ratio cum exponential" type estimator of the population mean in post-stratified sampling, using known values of some population parameters of an auxiliary character. Following a procedure similar to the procedure for obtaining the bias and mean squared error of the proposed exponential estimator, t_3, as described in Section 2.0, the biases and mean squared errors of the proposed composite estimators, t_k, k = 4, 5, 6, are obtained up to first order approximation as:

$$B(t_4) = \frac{1}{\bar{\bar{X}}}\left(\frac{1-f}{n}\right)\left[\left(\gamma+\pi^2-\gamma\pi^2\right)\tfrac{1}{2}g(g+1)\alpha^2\lambda^2R^2A_{22}-(\gamma+\pi-\gamma\pi)g\alpha\lambda A_{12}\right] \quad (3.4)$$

$$B(t_5) = \frac{1}{\bar{\bar{X}}}\left(\frac{1-f}{n}\right)\left[\left\{\tfrac{1}{2}\alpha^2g(g+1)\gamma+\tfrac{3}{8}-\tfrac{3}{8}\gamma\right\}\lambda^2R^2A_{22}-\left(\alpha g\gamma+\tfrac{1}{2}-\tfrac{1}{2}\gamma\right)\lambda A_{12}\right] \quad (3.5)$$

$$B(t_6) = \frac{1}{\bar{\bar{X}}}\left(\frac{1-f}{n}\right)\left[\left\{\pi^2\alpha^2g(g+1)\gamma+\tfrac{3}{8}-\tfrac{3}{8}\gamma\right\}\lambda^2R^2A_{22}-\left(\pi\alpha g\gamma+\tfrac{1}{2}-\tfrac{1}{2}\gamma\right)\lambda A_{12}\right] \quad (3.6)$$

and

$$MSE(t_4) = \left(\frac{1-f}{n}\right)\left[A_{11}+(\gamma+\pi-\gamma\pi)^2g^2\alpha^2\lambda^2R^2A_{22}-(\gamma+\pi-\gamma\pi)2g\alpha\lambda RA_{12}\right] \quad (3.7)$$

$$MSE(t_5) = \left(\frac{1-f}{n}\right)\left[A_{11}+\left(\alpha g\gamma+\tfrac{1}{2}-\tfrac{1}{2}\gamma\right)^2\lambda^2R^2A_{22}-\left(\alpha g\gamma+\tfrac{1}{2}-\tfrac{1}{2}\gamma\right)2\lambda RA_{12}\right] \quad (3.8)$$

$$MSE(t_6) = \left(\frac{1-f}{n}\right)\left[A_{11}+\left(\pi\alpha g\gamma+\tfrac{1}{2}-\tfrac{1}{2}\gamma\right)^2\lambda^2R^2A_{22}-\left(\pi\alpha g\gamma+\tfrac{1}{2}-\tfrac{1}{2}\gamma\right)\lambda RA_{12}\right] \quad (3.9)$$

where $\pi = f/(1-f) = n/(N-n)$. Notice that Onyeka (2012) obtained the mean squared error of the ratio-type estimator, t_1 as:

$$MSE(t_1) = MSE(\bar{y}_{pss}) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h\left(S_{yh}^2+\alpha^2\lambda^2g^2R^2S_{xh}^2-2\alpha\lambda gRS_{yxh}\right) \quad (3.10)$$

or

$$MSE(t_1) = \left(\frac{1-f}{n}\right)\left[A_{11}+\alpha^2\lambda^2g^2R^2A_{22}-2\alpha\lambda gRA_{12}\right] \quad (3.11)$$

Similarly, Onyeka (2013) obtained the mean squared error of the dual to ratio type estimator, t_2 as:

$$MSE(t_2) = MSE(\bar{y}^*_{pss}) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h\left(S_{yh}^2+\pi^2\alpha^2\lambda^2g^2R^2S_{xh}^2-2\pi\alpha\lambda gRS_{yxh}\right) \quad (3.12)$$

or

$$MSE(t_2) = \left(\frac{1-f}{n}\right)\left[A_{11}+\pi^2\alpha^2\lambda^2g^2R^2A_{22}-2\pi\alpha\lambda gRA_{12}\right] \quad (3.13)$$

It follows, therefore, that the general form of the mean squared errors of the estimators, t_k, $k=1,2,\cdots,6$ could be written as:

$$MSE(t_k) = \left(\frac{1-f}{n}\right)\left[A_{11} + \theta_k^2 R^2 A_{22} - 2\theta_k R A_{12}\right] \tag{3.14}$$

where,

$$\left.\begin{array}{l} \theta_1 = \alpha g\lambda,\, \theta_2 = \pi\alpha g\lambda,\, \theta_3 = \tfrac{1}{2}\lambda,\, \theta_4 = \alpha g\lambda\gamma + \pi\alpha g\lambda - \pi\alpha g\lambda\gamma \\ \theta_5 = \alpha g\lambda\gamma + \tfrac{1}{2}\lambda - \tfrac{1}{2}\lambda\gamma,\, \theta_6 = \pi\alpha g\lambda\gamma + \tfrac{1}{2}\lambda - \tfrac{1}{2}\lambda\gamma \end{array}\right\} \tag{3.15}$$

Notes

IV. Efficiency Comparison

Here, we shall compare the efficiency of the proposed exponential estimator (t_3) with those of the customary post-stratified mean estimator $(\bar{y}_{ps})$, the ratio-type estimator (t_1), the dual to ratio estimator (t_2), the ratio cum dual to ratio estimator (t_4), the ratio cum exponential estimator (t_5), and the dual to ratio cum exponential estimator (t_6).

a) *Efficiency of t_3 over $\bar{y}_{ps}$*

Using (2.10), (3.14) and (3.15), the proposed exponential estimator, (t_3) would perform better than the customary post-stratified mean estimator, $(\bar{y}_{ps})$, in terms of having a smaller mean squared error if:

$$\frac{\beta}{\lambda R} > \frac{1}{4} \tag{4.1}$$

where $\beta = \frac{A_{12}}{A_{22}} = \frac{\sum_{h=1}^{L}\omega_h S_{yxh}}{\sum_{h=1}^{L}\omega_h S_{xh}^2}$ is an expression of the population regression coefficient of the study variate (y) on the auxiliary character (x).

b) *Efficiency of t_3 over the estimators t_k, $k \neq 3$*

Here, we compare the efficiency of the proposed exponential estimator (t_3) with those of the ratio-type estimator (t_1) proposed by Onyeka (2012), the dual to ratio estimator (t_2) proposed by Onyeka (2013), and the ratio cum dual to ratio estimator (t_4), the ratio cum exponential estimator (t_5), and the dual to ratio cum exponential estimator (t_6) proposed in the present study. Using (3.14), the proposed exponential estimator (t_3) would perform better than the other five (5) estimators, t_k, k = 1, 2, 4, 5, 6, in terms of having smaller mean squared error if:

$$\left.\begin{array}{lll} & (1) & \theta_k < \theta_3 < \frac{\beta}{R} \\ \text{or} & & \\ & (2) & \frac{\beta}{R} < \theta_3 < \theta_k \end{array}\right\} \tag{4.2}$$

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where θ_k, k = 1, 2, …, 6 are as given in (3.15).

V. Empirical Illustration

Each of the six estimators, t_k, is a general class of estimators capable of generating an infinite number of combined estimators of $\overline{Y}$ in post-stratified sampling scheme, by making appropriate choices of the values of the constants α, g, a, b and γ in (1.1), (1.2), (2.3), (3.1), (3.2) and (3.3), as the case may be. Authors like Sisodia and Dwivedi (1981), Singh and Kakran (1993), Upadhyaya and Singh (1999), Kadilar and Cingi (2003) and Koyuncu and Kadilar (2009) considered special cases of some of these general classes of estimators using various known population parameters. For illustration and comparison purposes in the present study, we shall consider estimators of the form proposed by Upadhyaya and Singh (1999), by making use of the coefficient of variation (C_x) and coefficient of kurtosis $(\beta_2(x))$, as two known population parameters of the auxiliary variable, x, in estimating the population mean $(\overline{Y})$ in post-stratified random sampling scheme. Consequently, we consider the following special cases of the estimators, t_k, (k = 1, 2, …, 6), of the population mean, $\overline{Y}$, in post-stratified sampling scheme by choosing $a = \beta_2(x)$, $b = C_x$, $\alpha = 1$, $g = 1$ and $0 < \gamma < 1$, following after Upadhyaya and Singh (1999):

Ratio type estimator, $$t_{1US} = \bar{y}_{ps}\left(\frac{\overline{X}\beta_2(x) + C_x}{\bar{x}_{ps}\beta_2(x) + C_x}\right) \tag{5.1}$$

Dual to Ratio type estimator, $$t_{2US} = \bar{y}_{ps}\left(\frac{\bar{x}^*_{ps}\beta_2(x) + C_x}{\overline{X}\beta_2(x) + C_x}\right) \tag{5.2}$$

Exponential type estimator, $$t_{3US} = \bar{y}_{ps}\exp\left(\frac{(\overline{X} - \bar{x}_{ps})\beta_2(x)}{(\overline{X} + \bar{x}_{ps})\beta_2(x) + 2C_x}\right) \tag{5.3}$$

Ratio cum Dual to Ratio type estimator, $$t_{4US} = \gamma t_{1US} + (1-\gamma)t_{2US} \tag{5.4}$$

Ratio cum Exponential type estimator, $$t_{5US} = \gamma t_{1US} + (1-\gamma)t_{3US} \tag{5.5}$$

Dual to Ratio cum Exponential type estimator, $$t_{6US} = \gamma t_{2US} + (1-\gamma)t_{3US} \tag{5.6}$$

where the subscripts (US) indicate that the estimators follow after Upadhyaya and Singh (1999) by utilizing coefficient of variation (C_x) and coefficient of kurtosis $(\beta_2(x))$, as two known population parameters of the auxiliary variable, x. The usual post-stratified mean estimator, is

Usual post-stratified mean estimator, $$\bar{y}_{ps} = \sum_{h=1}^{L}\omega_h\bar{y}_h \tag{5.7}$$

Here, we shall examine the efficiencies of the estimators (5.1) to (5.7) using the data given in Onyeka (2012) on the academic performance of 96 students of Statistics department, Federal University of Technology, Owerri, during the 2008/2009 academic session. From Table 1 of Onyeka (2012), we obtain the following relevant statistics:

Table 1 : Summary Statistics from Onyeka (2012)

$N = 96$	$n = 20$
$\beta_2(x) = 3.83$	$C_x = 0.10$
$\pi = 0.26316$	$\lambda = 0.99962$
$A_{11} = 0.325$	$A_{22} = 49.83$
$A_{12} = 3.26$	$R = 0.03581$

Using the data in Table 1 to make the necessary substitutions in (2.10), (3.14) and (3.15), we obtain the variance of the post-stratified mean estimator, $\bar{y}_{ps}$ and the mean squared errors of the estimators (5.1) to (5.6) as shown in table 2. Using table 2, the percentage relative efficiencies (PRE) of the proposed exponential estimator, t_{3US} over the other estimators are obtained as shown in table 3.

Table 2 : Variance / MSE's of the estimators (5.1) to (5.7)

γ	**Estimators**						
	$\bar{y}_{ps}$	t_{1US}	t_{2US}	t_{3US}	t_{4US}	t_{5US}	t_{6US}
0.10	0.01286	0.00615	0.01061	0.00888	0.01004	0.00855	0.00904
0.20	0.01286	0.00615	0.01061	0.00888	0.00950	0.00823	0.00920
0.30	0.01286	0.00615	0.01061	0.00888	0.00898	0.00793	0.00937
0.32	0.01286	0.00615	0.01061	0.00888	0.00888	0.00787	0.00940
0.40	0.01286	0.00615	0.01061	0.00888	0.00850	0.00764	0.00954
0.50	0.01286	0.00615	0.01061	0.00888	0.00804	0.00736	0.00971
0.60	0.01286	0.00615	0.01061	0.00888	0.00761	0.00709	0.00988
0.70	0.01286	0.00615	0.01061	0.00888	0.00720	0.00684	0.01006
0.80	0.01286	0.00615	0.01061	0.00888	0.00682	0.00660	0.01024
0.90	0.01286	0.00615	0.01061	0.00888	0.00647	0.00637	0.01042

Table 3 : Percentage Relative Efficiencies (PRE) of t_{3US} over others

γ	**Estimators**						
	$\bar{y}_{ps}$	t_{1US}	t_{2US}	t_{3US}	t_{4US}	t_{5US}	t_{6US}
0.10	145	69	119	100	113	96	102
0.20	145	69	119	100	107	93	104
0.30	145	69	119	100	101	89	106
0.32	145	69	119	100	100	89	106
0.40	145	69	119	100	96	86	107
0.50	145	69	119	100	91	83	109
0.60	145	69	119	100	86	80	111
0.70	145	69	119	100	81	77	113
0.80	145	69	119	100	77	74	115
0.90	145	69	119	100	73	72	117

Notes

Notes

Table 3 shows that for the given set of data, the proposed exponential estimator, t_{3US} has relative positive gain in efficiency over (a) the customary post-stratified mean estimator, $\bar{y}_{ps}$, (b) the dual to ratio type estimator t_{2US} proposed by Onyeka (2013), (c) the proposed dual to ratio cum exponential estimator, t_{6US}, for all values of γ, and (d) the proposed ratio cum dual to ratio estimator, t_{4US} for values of γ less than 0.32. On the other hand, the proposed exponential estimator, t_{3US}, is not more efficient than (i) the ratio type estimator, t_{1US}, proposed by Onyeka (2012), (ii) the proposed ratio cum exponential estimator, t_{5US}, for all values of γ, and (iii) the proposed ratio cum dual to ratio estimator, t_{4US}, for values of γ greater than 0.32. For the given set of data, the best of the seven estimators, (5.1) to (5.7), in terms of having the smallest mean squared error, is the ratio type estimator, t_{1US}, proposed by Onyeka (2012), while the least efficient is the customary post-stratified mean estimator, $\bar{y}_{ps}$.

It is worthy of note that the above empirical inferences are consistent with the theoretical results and efficiency conditions obtained in section 4. Using (3.15) and the data in table 1, the values of θ_3 and β/R are obtained as 0.49981 and 1.82686 respectively, indicating that $\theta_3 < \beta/R$. This means that theoretically, condition (1) of (4.2) needs to be satisfied for the proposed exponential estimator, t_{3US}, to perform better than the other estimators, since $\theta_3 < \beta/R$. Obviously, this reduces the efficiency condition (1) of (4.2) to $\theta_k < \theta_3$. From (3.15) and the data in table 1, the values of θ_1 and θ_2, for instance, are obtained as 0.99962 and 0.26306 respectively. Notice that $\theta_1 = 0.99962$ is not less than $\theta_3 = 0.49981$, hence the estimator t_{3US} is not more efficient than the estimator t_{1US}, as indicated by the empirical results. Again, we observe that $\theta_2 = 0.26306$ is less than $\theta_3 = 0.49981$, hence the estimator t_{3US} is more efficient than the estimator t_{2US}, as already indicated and confirmed from the empirical results. Similar examination of other estimators would confirm that all the empirical results obtained in section 5 are consistent with the theoretical results derived in section 4.

VI. Concluding Remark

The paper first discussed two estimators, the ratio type estimator and the dual to ratio type estimator, earlier proposed by Onyeka (2012, 2013), and then proposed and considered a new exponential type estimator for population mean in post-stratified sampling scheme, using known value of some population parameters of an auxiliary character. The paper further proposed additional three estimators, which are composite or linear functions of the ratio type, the dual to ratio type, and the new exponential type estimators, bringing together, four new estimators of the population mean in post-stratified sampling scheme proposed in the study. Properties of the proposed estimators, including their biases and mean squared errors were obtained up to first order approximations. In particular, theoretical conditions under-which the proposed

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exponential type estimator would perform better than the other estimators, were obtained. The theoretical results were further verified and confirmed using numerical illustrations.

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Notes

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Some Results on Compactness in Bicomplex Space

By Sukhdev Singh & Rajiv K. Srivastava
Lovely Professional University, India

Abstract - In this paper, we have studied the continuity and compactness of the bicomplex space and its subsets. We have studied the compactness of some subsets of the bicomplex space in the idempotent order topology. We have also given a result regarding homeomorphism in the idempotent order topology and the complex order topology on the bicomplex space.

Keywords : Bicomplex net, idempotent order topology, complex order topology, compactness, homeomorphism.

GJSFR-F Classification : FOR Code: 820305, 079999

SOME RESULTS ON COMPACTNESS IN BICOMPLEX SPACE

Strictly as per the compliance and regulations of :

Ref

1. Price, G. B.: An introduction to multicomplex spaces and functions, Marcel Dekker Inc., New York, (1991).

Some Results on Compactness in Bicomplex Space

Sukhdev Singh [α] & Rajiv K. Srivastava [σ]

Abstract - In this paper, we have studied the continuity and compactness of the bicomplex space and its subsets. We have studied the compactness of some subsets of the bicomplex space in the idempotent order topology. We have also given a result regarding homeomorphism in the idempotent order topology and the complex order topology on the bicomplex space.

Keywords : Bicomplex net, idempotent order topology, complex order topology, compactness, homeomorphism.

I. Introduction

Throughout the paper, C_0, C_1 and C_2 denote sets of real numbers, complex numbers and bicomplex numbers, respectively. A bicomplex number is defined as (cf. [1], [3])

$$\xi = x_1 + i_1 x_2 + i_2 x_3 + i_1 i_2 x_4 = z_1 + i_2 z_2, \text{ where } x_p \in C_0, 1 \le p \le 4, \; z_1, z_2 \in C_1, \; i_1^2 = i_2^2 = -1$$

and $i_1 i_2 = i_2 i_1$.

With usual binary compositions, C_2 becomes a commutative algebra with identity. Besides the additive and multiplicative identities 0 and 1, there exist exactly two non-trivial idempotent elements denoted by e_1 and e_2 defined as $e_1 = (1 + i_1 i_2)/2$ and $e_2 = (1 - i_1 i_2)/2$. Note that $e_1 + e_2 = 1$ and $e_1 . e_2 = 0$.

A bicomplex number $\xi = z_1 + i_2 z_2$ can be uniquely expressed as a complex combination of e_1 and e_2 as (cf. [3])

$$\begin{aligned} \xi &= z_1 + i_2 z_2 = (z_1 - i_1 z_2) e_1 + (z_1 + i_1 z_2) e_2 \\ &= {}^1\xi e_1 + {}^2\xi e_2 \end{aligned}$$

where ${}^1\xi = z_1 - i_1 z_2$ and ${}^2\xi = z_1 + i_1 z_2$.

Author α : Department of Mathematics, Lovely Professional University, Phagwara-144411, Punjab, India.
E-mail : ssukhdev1209@yahoo.com
Author σ : Department of Mathematics, Institute of Basic Science, Dr. B. R. Ambedkar University, Agra-282002, India.
E-mail : rajivmathsibs@yahoo.com

The complex coefficients ${}^1\xi$ and ${}^2\xi$ are called the *idempotent components* and the combination ${}^1\xi e_1 + {}^2\xi e_2$ is known as *idempotent representation* of bicomplex number ξ. The auxiliary complex spaces A_1 and A_2 are defined as follows:

$$A_1 = \{z_1 - i_1 z_2 \; ; z_1, z_2 \in C_1\} = \left\{{}^1\xi : \xi \in C_2\right\}$$

and

$$A_2 = \{z_1 + i_1 z_2 \; ; z_1, z_2 \in C_1\} = \left\{{}^2\xi : \xi \in C_2\right\}.$$

The idempotent representation $(z_1 - i_1 z_2)\, e_1 + (z_1 + i_1 z_2) e_2 = {}^1\xi e_1 + {}^2\xi e_2$ associates with each point $\xi = z_1 + i_2 z_2$ in C_2, the points ${}^1\xi = z_1 - i_1 z_2$ and ${}^2\xi = z_1 + i_1 z_2$ in A_1 and A_2, respectively and to each pair of points $(z, w) \in A_1 \times_e A_2$, there corresponds a unique bicomplex point $\xi = z e_1 + w e_2$.

Srivastava [3] initiated the topological study of C_2. He defined three topologies on C_2, viz., norm topology τ_1, complex topology τ_2 and idempotent topology τ_3 and has proved some results on these topological structures.

In the present paper, we shall confine ourselves mainly to C_2 equipped with τ_5 and τ_6. For the sake of ready reference, we give below relevant literature of τ_5 and τ_6 (for details cf. [4]).

Denote by $\prec_C$, the dictionary ordering of the bicomplex numbers expressed in the complex form. The order topology induced by this ordering is called as *Complex Order Topology*. Complex order topology τ_5 is generated by the basis B_5 comprising members of the following families of subsets of C_2:

(i) $K_1 = \{(z_1 + i_2 z_2, w_1 + i_2 w_2)_C : z_1 \prec w_1\}$

(ii) $K_2 = \{(z_1 + i_2 z_2, z_1 + i_2 w_2)_C : z_2 \prec w_2\}$.

Remark 1.1: Note that, since $z_1 \prec w_1$ and $z_2 \prec w_2$ in the dictionary order topology in C_1, K_1 and K_2 can also be described as $K_1 = M_1 \cup M_2$ and $K_2 = M_3 \cup M_4$, where

(i) $M_1 = \left\{(z_1 + i_2 z_2 \,, w_1 + i_2 w_2)_C : \operatorname{Re} z_1 < \operatorname{Re} w_1\right\}$

(ii) $M_2 = \left\{(z_1 + i_2 z_2 \,, w_1 + i_2 w_2)_C : \operatorname{Re} z_1 = \operatorname{Re} w_1, \operatorname{Im} z_1 < \operatorname{Im} w_1\right\}$

(iii) $M_3 = \left\{(z_1 + i_2 z_2 \,, z_1 + i_2 w_2)_C : \operatorname{Re} z_2 < \operatorname{Re} w_2\right\}$

(iv) $M_4 = \left\{(z_1 + i_2 z_2 \,, w_1 + i_2 w_2)_C : \operatorname{Re} z_2 = \operatorname{Re} w_2, \operatorname{Im} z_2 < \operatorname{Im} w_2\right\}$.

Note further that M_1, M_2, M_3 and M_4 are in fact, families of space segments, frame segments, plane segments and line segments, respectively.

Ref

3. Srivastava, Rajiv K.: Certain Topological Aspects of Bicomplex Space, Bull. Pure & Appl. Math., 2 (2008), 222–234.

Similarly, denote by $\prec_{ID}$ the dictionary ordering of the bicomplex numbers expressed in the idempotent from. The order topology induced by this ordering is called as *Idempotent Order Topology*. Hence, idempotent order topology τ_6 is generated by the basis B_6 comprising of members of the following families of subsets of C_2:

Notes

(i) $L_1 = \left\{\left({}^1\xi e_1 + {}^2\xi e_2, {}^1\eta e_1 + {}^2\eta e_2\right)_{ID} : {}^1\xi \prec {}^1\eta\right\}$

(ii) $L_2 = \left\{\left({}^1\xi e_1 + {}^2\xi e_2, {}^1\xi e_1 + {}^2\eta e_2\right)_{ID} : {}^2\xi \prec {}^2\eta\right\}$

the set $(\xi, \eta)_{ID}$ denoting the open interval with respect to the ordering $\prec_{ID}$ and $\prec$ denoting the dictionary order relation in A_1 and A_2.

The set of the type $\{\xi : a < \text{Re } {}^1\xi < b\}$ is called an ID– space segment. A set of the type $\{\xi : \text{Re } {}^1\xi = a\}$ is called an ID – frame and is denoted as $(\text{Re } {}^1\xi = a)$. A set of the type $\{\xi : \text{Re } {}^1\xi = a, b < \text{Im } {}^1\xi < c\}$ is called as an open ID – frame segment. The terms ID – plane, ID –plane segment, ID – line and ID – line segment are define analogously (for details cf. [5]).

Note that L_1 and L_2 can also be described as $L_1 = N_1 \cup N_2$ and $L_2 = N_3 \cup N_4$, where

(i) $N_1 = \left\{\left({}^1\xi e_1 + {}^2\xi e_2, {}^1\eta e_1 + {}^2\eta e_2\right)_{ID} : \text{Re } {}^1\xi < \text{Re } {}^1\eta\right\}$

(ii) $N_2 = \left\{\left({}^1\xi e_1 + {}^2\xi e_2, {}^1\eta e_1 + {}^2\eta e_2\right)_{ID} : \text{Re } {}^1\xi = \text{Re } {}^1\eta, \text{Im } {}^1\xi < \text{Im } {}^1\eta\right\}$

(iii) $N_3 = \left\{\left({}^1\xi e_1 + {}^2\xi e_2, {}^1\xi e_1 + {}^2\eta e_2\right)_{ID} : \text{Re } {}^2\xi < \text{Re } {}^2\eta\right\}$

(iv) $N_4 = \left\{\left({}^1\xi e_1 + {}^2\xi e_2, {}^1\xi e_1 + {}^2\eta e_2\right)_{ID} : \text{Re } {}^2\xi = \text{Re } {}^2\eta, \text{Im } {}^2\xi < \text{Im } {}^2\eta\right\}$.

Note further that N_1, N_2, N_3 and N_4 are families of open ID–space segments, open ID–frame segments, open ID–plane segments and open ID–line segments, respectively.

In other words,

$$B_5 = \bigcup_{p=1}^{2} K_p = \bigcup_{p=1}^{4} M_p$$

and

$$B_6 = \bigcup_{p=1}^{2} L_p = \bigcup_{p=1}^{4} N_p .$$

Theorem 1.1 [7]: Every order topology is Hausdorff.

Remarks 1.2: The auxiliary complex spaces A_1 and A_2 are Hausdorff space with respect to the order topology generated by dictionary order relation $\prec$ on them.

II. Compactness in Bicomplex Space

In this section, we have given some results regarding compactness on some subsets of the bicomplex space.

Lemma 2.1: Suppose that $S = \{ze_1 + we_2 : z \in A_1, w = z^{-1}, 0 \prec z \prec 1\}$.

Any subset $(z_0 e_1 + w_1 e_2 , z_0 e_1 + w_2 e_2)_{ID}$, where $0 \prec z_0 \prec 1$, of C_2 contains at most one point of the set S.

Proof: We have $S = \left\{ze_1 + we_2 : z \in A_1, w = z^{-1}, 0 \prec z \prec 1\right\}$.

Let $z_0 \in C_1$, where $0 \prec z \prec 1$.

As $z_0 \neq 0$.

$\Rightarrow \exists$ an element $w \in C_1$ such that $z_0^{-1} \neq w$.

Therefore, $\exists$ $z_0 e_1 + we_2 \in C_2$ such that

$$z_0 e_1 + we_2 \in (z_0 e_1 + w_1 e_2, z_0 e_1 + w_2 e_2)_{ID},$$

where $w_1, w_2 \in A_2$ and $w_1 \prec w \prec w_2$.

Further, if $(z_0 e_1 + w_1 e_2, ze_1 + w_2 e_2)_{ID}$ is a subset of C_2 such that $w \prec w_1$ where $w = z_0^{-1}$, then

$$(z_0 e_1 + w_1 e_2, ze_1 + w_2 e_2)_{ID} \cap S = \phi.$$

Theorem 2.1: The set $S = \{ze_1 + we_2 : z \in A_1, w = z^{-1}, 0 \prec z \prec 1\}$ is a compact subset of C_2.

Proof: To prove S is a compact, we show that the set S is bounded and closed with respect to the idempotent order topology in C_2. Since, the coefficients of e_1 of the elements of S are bounded by the lower and upper bounds 0 and 1, respectively.

Therefore, the elements of the set S are bounded by the bicomplex numbers $w_1 e_2$ and $e_1 + w_1 e_2$, $w_1 \in A_2$ as lower and upper bounds respectively, i.e., $ze_1 + we_2 \in (w_1 e_2, e_1 + w_1 e_2)_{ID}$, where $0 \prec z \prec 1$.

Hence, the set S is bounded.

To prove that S is closed, we shall show that S^c is open in C_2.

Let $\xi = z_0 e_1 + w_0 e_2 \in S^c$.

Then there are three possibilities:

1) $z_0^{-1} \neq w_0$ and $z_0 \in (0, 1)$

2) $z_0^{-1} = w_0$ and $z_0 \notin (0, 1)$

3) $z_0^{-1} \neq w_0$ and $z_0 \notin (0, 1)$.

Ref

5. Srivastava, Rajiv K. and Sukhdev Singh: on Bicomplex Nets and their Confinements, Amer. J. of Math. and Stat., Vol. 1(1), (2011), 8 – 16.

Case 1): When $z_0^{-1} \neq w_0$ and $z_0 \in (0,1)$.

Since both z_0^{-1} and w_0 are members of the auxiliary complex space A_2 and A_2 is a Hausdorff space under the dictionary order topology.
Therefore, $\exists\ u_1, u_2 \in A_2$ such that

$$z_0^{-1} \in (u_1, u_2).$$

Similarly, $\exists\ v_1, v_2 \in A_2$ such that

$$w_0 \in (v_1, v_2) \text{ and } (u_1, u_2) \cap (v_1, v_2) = \phi.$$

So, we have

$$\xi = z_0 e_1 + w_0 e_2 \in (z_0 e_1 + v_1 e_2, z_0 e_1 + v_2 e_2)_{ID}$$

Now if the interval $(z_0 e_1 + v_1 e_2, z_0 e_1 + v_2 e_2)_{ID}$ has non − empty intersection with the set S then it will contain only one point namely, $z_0 e_1 + z_0^{-1} e_1$.
But

$$z_0 e_1 + z_0^{-1} e_1 \notin (z_0 e_1 + v_1 e_2, z_0 e_1 + v_2 e_2)_{ID}.$$

Hence, $$(z_0 e_1 + v_1 e_2, z_0 e_1 + v\_ e\_)_{ID} \cap S = \phi.$$

$$\Rightarrow \xi = z_0 e_1 + w_0 e_2 \in (z_0 e_1 + v_1 e_2, z_0 e_1 + v_2 e_2)_{ID} \subset S^c$$

Case 2): When $z_0^{-1} = w_0$ and $z_0 \notin (0,1)$.

Two sub cases will arise as follows:

Either $z_0 \preceq 0$ or $z_0 \succeq 1$.

Sub − case 2) (i): If $z_0 \preceq 0$.

If $z_0 = 0$. Then $\xi = z_0 e_1 + w_0 e_2$ does not exist. Therefore we can say that $z_0 \neq 0$.

If $z_0 \prec 0$, then as $z_0 (\neq 0) \in A_1$ and as A_1 is a Hausdorff space under dictionary order topology.
Therefore, there exist $a_1, a_2 \in A_1$ such that

$$0 \in (a_1, a_2)$$

and similarly there exist

$$w_1, w_2 \in A_1 \text{ such that } z_0 \in (w_1, w_2).$$

and $$(a_1, a_2) \cap (w_1, w_2) = \phi.$$

Also, $$(0,1) \cap (w_1, w_2) = \phi.$$

Therefore, $w_1 \prec 0$ as well as $w_2 \prec 0$.

Notes

Hence, for any $b_1, b_2 \in A_2$ we have

$$z_0 e_1 + w_0 e_2 \in (w_1 e_1 + b_1 e_2 ,\ w_1 e_1 + b_2 e_2)_{ID} \subset S^c$$

Sub – case 2(ii): If $z_0 \succeq 1$.

Either $z_0 = 1$ or $z_0 \succ 1$.

If $z_0 = 1$, then $\xi = 1$.

So there exists two bicomplex numbers $\eta = e_1 + \left(\frac{1}{2}\right) e_2$ and $\zeta = e_1 + \left(\frac{3}{2}\right) e_2$

such that $\xi \in (\eta , \zeta)_{ID} \subset S^c$.

If $z_0 \succ 1$, then $z_0^{-1} (= w_0) \prec 1$.

Since $z_0\ (\neq 1) \in A_1$ and as w_0 and 1 are two points of A_2.

Also A_2 is Hausdorff under dictionary order topology.

$\Rightarrow \exists\, u_1, u_2 \in A_2$ such that $w_0 \in (u_1 , u_2)$

and similarly, $\exists\, v_1 , v_2 \in A_2$ such that

$$1 \in (v_1 , v_2)$$

and

$$(u_1 , u_2) \cap (v_1 , v_2) = \phi .$$

Therefore,

$$z_0 e_1 + w_0 e_2 \in (z_0 e_1 + u_1 e_2 , z_0 e_1 + u_2 e_2)_{ID}$$

and

$$(z_0 e_1 + u_1 e_2 ,\ z_0 e_1 + u_2 e_2)_{ID} \cap S \neq \phi$$

$\Rightarrow (z_0 e_1 + u_1 e_2 ,\ z_0 e_1 + u_2 e_2)_{ID} \subset S^c$.

Case 3): When $z_0^{-1} \neq w_0$ and $z_0 \notin (0, 1)$.

As $z_0^{-1} \neq w_0$ and $z_0^{-1} , w_0 \in A_2$ and A_2 is Hausdorff space in the dictionary order topology.

Therefore, there exist $c_1 , c_2 \in A_2$ such that

$$z_0^{-1} \in (c_1 , c_2)$$

Similarly there exist $d_1 , d_2 \in A_2$ such that

$$w_0 \in (d_1 , d_2) \text{ and } (c_1 , c_2) \cap (d_1 , d_2) = \phi .$$

Therefore we have obtained an interval

$$(z_0 e_1 + d_1 e_2 ,\ z_0 e_1 + d_2 e_2)_{ID}$$

Notes

such that

$$z_0 e_1 + w_0 e_2 \in (z_0 e_1 + d_1 e_2 ,\ z_0 e_1 + d_2 e_2)_{ID}$$

and $$(z_0 e_1 + d_1 e_2 , z_0 e_1 + d_2 e_2)_{ID} \subset S^c .$$

We conclude that set S is closed as well as bounded.
Hence, S is compact subset of C_2.

N

Theorem 2.2: The set
$S = \{(z , \sin z^{-1}) : 0 \prec z \prec 1\}$ is a compact subset of C_2.

Proof : W have $S = \{(z , \sin z^{-1}) : 0 \prec z \prec 1\}$

$\Rightarrow S = \{(z , \sin w) : 0 \prec z \prec 1 ,\ w = z^{-1}\}$

$S = \{z e_1 + (\sin w) e_2 : 0 \prec z \prec 1 ,\ w = z^{-1}\}$.

$\Rightarrow \xi = z_0 e_1 + w_0 e_2 \in (z_0 e_1 + v_1 e_2 ,\ z_0 e_1 + v_2 e_2)_{ID} \subset S^c$

To prove S is a compact subset of C_2, we show that S is closed and bounded.

Since, $0 \prec z \prec 1$,

$0 \prec z$ as well as $z \prec 1$.

Therefore, z is bounded with lower and upper bounds 0 and 1, respectively.

Hence, S is bounded subset of C_2 under the idempotent order relation with lower and upper bounds $w_1 e_2$ and $e_1 + w_2 e_2$, respectively.

Now to show S is closed in C_2.

Let $\xi = u e_1 + v e_2 \in S^c$.

Then there are three possibilities as follows:

(i) $u \notin (0, 1)$ and $v = \sin u^{-1}$

(ii) $u \in (0, 1)$ and $v \neq \sin u^{-1}$

(iii) $u \notin (0, 1)$ and $v \neq \sin u^{-1}$.

Case (i): If $u \notin (0, 1)$ and $v = \sin u^{-1}$

Now as $u \notin (0, 1)$.

Therefore, either $u \preceq 0$ or $1 \preceq u$.

If u = 0, then $\sin u^{-1}$ is not defined. So that, $u \neq 0$.

Now suppose that $u < 0$.

Since u and 0 are two distinct points of A_1 and A_1 is Hausdorff space with respect to the dictionary order topology.

Therefore, there exists $u_1, u_2 \in A_1$ such that

$$u \in (u_1, u_2).$$

Similarly there exists $z_1, z_2 \in A_1$ such that

$0 \in (z_1, z_2)$ and also $(u_1, u_2) \cap (z_1, z_2) = \phi$.

So that

$$ue_1 + ve_2 \in (u_1 e_1 + v_1 e_2, u_2 e_1 + v_2 e_2)_{ID}$$

and
$$(u_1 e_1 + v_1 e_2, u_2 e_1 + v_2 e_2)_{ID} \cap S = \phi.$$

$$ue_1 + ve_2 \in (u_1 e_1 + v_1 e_2, u_2 e_1 + v_2 e_2)_{ID} \subset S^c$$

Therefore, S^c is an open set. Hence S is a closed set.

Similarly we can prove that S is closed if $1 \preceq u$.

Case b): If $u \in (0 + i_1 0, 1 + i_1 0)$ and $v \neq \sin u^{-1}$.

Then v and $\sin u^{-1}$ are two distinct points of A_2 and A_2 is Hausdorff space with respect to the dictionary order topology.

Therefore there exists $v_1, v_2 \in A_2$ such that

$$v \in (v_1, v_2).$$

Similarly, there exists $w_1, w_2 \in A_2$ such that $\sin u^{-1} \in (w_1, w_2)$

and

$$(v_1, v_2) \cap (w_1, w_2) = \phi.$$

Therefore,
$$ue_1 + ve_2 \in (ue_1 + v_1 e_2, ue_1 + v_2 e_2)_{ID}$$

and
$$(ue_1 + v_1 e_2, ue_1 + v_2 e_2)_{ID} \cap S = \phi$$

$\Rightarrow ue_1 + ve_2$

$\in (ue_1 + v_1 e_2, ue_1 + v_2 e_2)_{ID} \subset S^c$

So that S^c is an open set.

Hence, S is a closed set.

Case c): If $u \notin (0 + i_1 0, 1 + i_1 0)$ and $v \neq \sin u^{-1}$.

By the similar procedure as in case (a), we have S is a closed subset of C_2.

Hence we conclude that S is a closed and bounded subset of C_2.

Therefore, S is a compact subset of C_2.

Notes

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Something New in Math: Meaningful Mathematics Courses for Liberal Arts Undergraduates

By Gary Stogsdill

Prescott College, United States

Abstract - For over 100 years a vital if little known movement has been underway to allow liberal arts undergraduates to meet their math requirement with more meaningful and relevant options than the traditional skills courses in algebra, geometry, trigonometry, and calculus. Often referred to as *liberal arts mathematics*, and with a subset called *humanistic mathematics*, such courses may explore mathematics as a realm of ideas that are essential to understanding the world we live in and what it means to be human. Although resistance to this movement has been vigorous and tenacious, it is now widely recognized that liberal arts undergraduates deserve access to such courses in order to meet their math requirement. The author describes a century-long argument in favor of meaningful mathematics courses for liberal arts undergraduates, traces the evolution of liberal arts math courses, justifies such courses in a discussion of what mathematics really is, and presents his own innovative pedagogy with a humanistic math course, *Mathematical Explorations*, which provides liberal arts undergraduates with the opportunity to alleviate math anxiety, improve reasoning ability, engage in experiential learning, and explore math-related ideas that are meaningful, relevant, useful, and inspiring.

Keywords : humanistic mathematics, liberal arts mathematics, Mathematical Explorations, liberal arts undergraduates, innovative pedagogy, prescott college.

GJSFR-F Classification : MSC 2010: 00A05, 00A66

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I. Introduction

Those of us with sufficiently gray hair will remember the failed experiment in the 1960s called *new math*. What many of us may not know is that a vital and now successful movement has been underway for over 100 years to provide liberal arts undergraduates with more meaningful and relevant options than the traditional skills courses in algebra, geometry, trigonometry, and calculus. I will refer to these "new" courses as *liberal arts mathematics* and in some cases as *humanistic mathematics*. The *Encyclopedia of Mathematics Education* offers this definition of liberal arts mathematics: "College courses in mathematics appreciation for students in non-science fields" (Bumcrot, 2001, as cited in Sporn, 2010, p. 1). By *liberal arts undergraduates* I mean students who are working toward a Bachelor of Arts degree in a field other than science, technology, engineering, or mathematics.

a) *A Century-Long Argument for Liberal Arts Mathematics*

Awareness of the need for liberal arts mathematics began in the early 1900s soon after skills courses like *College Algebra* were becoming standardized in the 1890s as a requirement for liberal arts undergraduates (George, 2007). By 1911 the International

Author : Prescott College, United State. E-mail : gary.stogsdill@prescott.edu

Commission on the Teaching of Mathematics was able to reflect that "the average [college] student who does not find mathematics [skills] an attractive subject obtains little advantage of any sort in the required study of it" (as cited in George, p. 36).

Criticism of traditional skills-based college math courses would grow over the next several decades. As early as 1923 critics could confidently assert that "most teachers who have given serious thought to the needs of [the liberal arts] student, are…convinced that the traditional courses in trigonometry, college algebra, and analytic geometry can no longer be justified as best meeting their needs" (Young, 1923, as cited in George, 2007, p. 24).

Traditional educators responded to this criticism the way traditional educators often do…by doing nothing. In 1937 a piece in the American Mathematical Monthly captured the growing frustration of progressive educators: "It is deplorable…to contemplate the prevalence even today of traditional, hidebound 'trigonometry' and 'college algebra'…with excessive emphasis on mechanical manipulative processes and petty details, and almost no indication whatever of their possible relation to vital problems [in other human endeavors]" (Schaaf, 1937, as cited in George, 2007, pp. 37-8). The same author would deliver the following timeless lament: "[Given the nature of traditional skills courses,] are students to be blamed if they voice distaste, disgust, or abhorrence for mathematics?" (Schaaf, as cited in George, p. 39).

Michael George (2007) authored a doctoral dissertation on the history of liberal arts mathematics and summarized the first half of the twentieth century as follows:

The traditional [undergraduate math skills] curriculum was being increasingly attacked as unappealing to and overly difficult for many students. The drill-based classroom experience was repetitive, unimaginative, and stifling to students' creative sensibilities. More importantly, the traditional [skills] curriculum was only *one* aspect of the larger body of mathematics in the world. Ironically, it was in some ways the least accessible form of mathematics…. (p. 51).

Although liberal arts math courses were being offered by then, the second half of the twentieth century would continue to generate strong criticism against traditional math courses. Morris Kline, long time professor of mathematics at New York University, did not mince words in his 1954 response to the still prevalent practice of requiring math skills courses for liberal arts undergraduates:

These subjects comprise nothing but a series of dry, boring, unmotivated, disconnected, and, to the student, unimportant techniques. The subjects are taught as techniques and the students are expected to master and reproduce them in parrot-like fashion….By persisting in the teaching of college algebra and trigonometry to the liberal arts student, college teachers of mathematics have been…teaching brick laying instead of architecture, and color mixing instead of painting. Such teaching has discouraged latent interest in mathematics and has embittered many young people towards mathematics, and in many cases, toward all learning. (as cited in Sporn, 2010, p. 141).

And, yes, the same criticism continues into the twenty-first century: "The mathematics courses commonly used by students to satisfy a [liberal arts] education requirement—college algebra or calculus—have almost nothing to do with anything that will affect their future quality of life or their ability to contribute to…society" (Steen, 2004, as cited in Sporn, 2010, p. 169).

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Recall that this century-long criticism began very soon after skills courses like *College Algebra* were becoming standardized in the 1890s as a requirement for liberal arts undergraduates. The chief rationale for this requirement was, and continues to be, *mental discipline* (George, 2007). The argument of mental discipline goes like this: "Learning 'hard' subjects like mathematics [skills] and Latin develops one's raw mental 'powers'" (George, p. 30). In other words, we need to set the bar high and require liberal arts undergraduates to take skills courses such as *College Algebra* because the mental discipline inherent in these courses benefits students in other ways. Like a bitter medicine, *College Algebra* or another skills course needs to be required of students because it is good for them.

This same argument had been used to justify the requirement of Greek and Latin for American college students for some two-and-a-half centuries (Winterer, 2002). Finally in the 1880s colleges began to drop the Greek and Latin requirement (Winterer), and this is precisely the time period when math skills courses like *College Algebra* began to be required. The mental discipline advocates, having just lost the battle for Greek and Latin, were determined to promote a requirement of challenging math skills.

However, the argument of mental discipline was dealt a serious blow as early as 1923 with the research of the prominent psychologist Edward Thorndike, who found in repeated experimental studies that "improvement in any particular mental function rarely improved other functions" (George, 2010, p. 31). Specific to mathematics, Thorndike (1923) concluded, "The old notion that Latin or mathematics made the mind more effective [in other human endeavors] was largely superstition….Mathematics improves mathematical reasoning but not the power to reason in general" (as cited in Sporn, 2010, p. 64). Thorndike's studies inspired an abundance of follow-up research, with an occasional result seeming to contradict certain aspects of Thorndike's conclusion—for example, findings "suggesting that sense-discrimination could be transferred from one sense to another" (George, 2007, p. 31)—but with most of the subsequent research continuing to corroborate Thorndike (Sporn).

In spite of the experimental evidence against it, the mental discipline argument in favor of setting the bar high by requiring *College Algebra* for liberal arts undergraduates has a Hydra-like existence and continues to raise new heads into the twenty-first century, which prompted David Lemire, editor of the *Journal of College Reading and Learning*, to review all of the accumulated experimental evidence regarding mental discipline and to conclude in 2002 that "there is little or no relationship between [traditional] math problem solving and real life problem solving….[therefore,] math skills (algebra, geometry, trigonometry, and calculus)…should never be required within the normal…college curriculum" (as cited in Sporn, p. 172).

Even if a grain of truth existed in the mental discipline argument, which seems unlikely, a more obvious and compelling truth is that "a student's interest in a subject [is] much more instrumental in that student's achieving valuable learning than the subject's perceived inherent value [as mental discipline]" (George, 2007, p. 31). Clearly all of the above signals an end to the era of requiring liberal arts undergraduates to suffer through skills courses like *College Algebra*.

b) *What is Mathematics Really?*

At this point it may be helpful to explore what mathematics really is because some of us will be thinking, "Wait, if liberal arts undergraduates are not working math skills, then how can they be 'doing math'?" Recall the previous definition of liberal arts mathematics: "College courses in mathematics appreciation for students in non-science fields" (Bumcrot, 2001, as cited in Sporn, 2010, p. 1). A pertinent analogy would be to consider undergraduate courses in literature, music appreciation, and art appreciation. Imagine enrolling in a music appreciation course and being told that you will have to master piano technique in order to receive credit. That would be absurd. Skill at the piano is neither a prerequisite nor a corequisite for understanding and appreciating great music. The same is true with literature and art. Technical proficiency in skills is a separate issue from understanding and appreciating a subject from a liberal arts perspective.

While we can easily see this distinction with music, art, and literature, it may not be as obvious with math because often all we know of math is the skills part: doing operations with numbers and solving equations. Yet this is merely the grammar of mathematics. The traditional skills approach to teaching math to liberal arts undergraduates is like trying to teach Shakespeare by requiring students to diagram selected sentences of that magnificent prose into their grammatical components. By doing this students would miss the essential experience of Shakespeare, and we can certainly understand and appreciate Shakespeare even if we cannot distinguish an adjective from an adverb. It is the same with mathematics. Focusing on technical math skills tends to obscure what mathematics really is, and we can understand and appreciate mathematics from a liberal arts perspective whether or not we can factor a quadratic equation.

Mathematics is not numbers, algorithms, and equations any more than literature is a set of grammar rules. Yes, of course skills are one component of mathematics just as grammar is one component of writing, but would it not seem odd if we continued into college to teach writing as merely a set of grammar rules and taught nothing of the actual purposes for writing and nothing of the masterpieces of literature that comprise a significant portion of the intellectual heritage of our culture?

What, then, is mathematics really? Mathematics is a powerful symbolic language that has the ability to discover and convey important ideas about the patterns and relationships that surround us, patterns and relationships that not only serve the mundane necessities of daily commerce but also the loftier aims of natural science, social science, technology, human development, philosophy, esthetics, the arts, and virtually every other realm of human endeavor. Therefore, liberal arts undergraduates are actually "doing math" more faithfully and more rigorously when they work with ideas than when they practice skills. As Harald Ness, professor of mathematics at the University of Wisconsin observed:

What is not [commonly] realized is that mathematics deals primarily with ideas and that those ideas are some of the most profound and important in our culture….We need to communicate to our students and the public in general that mathematics is *not* a set of skills (bag of tricks, if you will) to solve mundane problems, but a vast body of knowledge that is an integral part of our culture. (1990, as cited in Sporn, 2010, p. 146).

c) *Evolution of Liberal Arts Math Courses*

In order to have widespread liberal arts math courses there had to first be liberal arts math textbooks. According to George (2007), the prototype for such a text was

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Introduction to Mathematics, published in 1911 by Alfred North Whitehead, one of the great mathematicians of his time and a professor at both Cambridge and Harvard. Whitehead belonged to an age when a breadth of liberal learning was the norm, and he contributed seminal ideas to not only mathematics but also the fields of philosophy, science, and education. In mathematics Whitehead co-authored with Bertrand Russell the monumental work *Principia Mathematica*, a comprehensive attempt to systematize the logical basis for all of mathematics. This was hardly a math teacher who could be accused of wanting to "set the bar low." Rather, this was a scholar who possessed sufficient breadth of learning to fully understand that the essence of mathematics is ideas and who wanted to convey this essence to students. Being about ideas, Whitehead's text did not contain skills problems at the end of each chapter (George), which is the hallmark of every traditional math textbook. Perhaps this reveals the reason Whitehead's text failed to catch on in the classroom: because math teachers, then as now, tend not to know how to teach mathematics without basing it on skills problems.

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The next liberal arts math textbooks would not appear until the 1930s when it was well established that "clearly students who did not have majors in mathematics, science, and technology would be better off terminating their mathematical education with some kind of [liberal arts] mathematics course, designed specifically for them" (George, 2007, p. 42). Even with several textbooks on the market, the inertia of tradition caused colleges to drag their feet in creating the new liberal arts math courses. In many cases colleges chose to abolish altogether any math requirement for liberal arts undergraduates rather than offer them meaningful courses (George). This trend would grow over the next few decades, with a survey of all American colleges in 1957 finding that only half of the colleges offering liberal arts degrees required any mathematics at all of their liberal arts students (Sporn, 2010).

The liberal arts math courses that *were* created in the 1930s and 1940s were slow to catch on, with an unsuitable text being a frequently cited criticism (George, 2007). Although well intentioned, math educators seemed unable to create truly new textbooks that did not mirror the traditional pattern of requiring skills problems at the end of each chapter. Further, as may have been the downfall of Whitehead's 1911 text, math teachers tend not to know how to teach mathematics without basing it on problem sets. The result was that the early liberal arts math courses were little improvement over traditional math skills courses.

This focused widespread attention in the 1940s and 1950s on how to create a successful and meaningful liberal arts math course, with a number of interesting outcomes. One outcome was the recommendation that all undergraduates, not just liberal arts students, would benefit from being required to take such a course because "the student completing [a liberal arts math course] will actually have a type of mathematical knowledge and an understanding of mathematical concepts that the typical major in the field does not possess" (Newsom, 1949, as cited in George, 2007, p. 81). Other outcomes included the general consensus that "there is not a single course that meets the needs of all students and all instructors" and that "the choice of topics is not as important for the success of the course as are the attitudes and interests of the teachers, and the attitudes and emotions of the students" (Special Committee on College Mathematics for Non-Science Students, 1956, as cited in Sporn, 2010, p. 13).

As a result several new liberal arts math courses were created in the late 1950s and 1960s that did meet with success, and available data from community colleges actually show a greater enrollment in liberal arts math courses than in *College Algebra* during the late 1960s and early 1970s (Sporn, 2010). However, liberal arts math courses began a sharp decline in the mid 1970s that would continue through the early 1990s (Sporn). Perhaps liberal arts math courses, like so many other innovative trends that thrived in the 1960s and early 1970s, were a casualty of the societal and educational pendulum that swings toward tradition and conservatism after a prolonged period of innovation and liberalism.

However, a sharp increase in offerings and enrollments in liberal arts math began in the 1990s and continues to grow into the twenty-first century (George, 2007; Sporn, 2010). Currently a majority of American colleges and universities offer a liberal arts math course as meeting the math requirement for liberal arts undergraduates, with many institutions offering several such options (Sporn, 2010).

There is no consensus for what the ideal liberal arts math course should look like (George, 2007; Sporn, 2010), and even a quick survey of American institutions reveals a wide spectrum of liberal arts math courses, with some being traditional skills courses masquerading as liberal arts math, some being remarkably innovative idea-based courses, and everything in between. However, one promising approach to liberal arts math is called humanistic mathematics. Humanistic math courses focus on understanding the great math-related ideas that have helped to define the world we live in and that "inspire the imagination and sense of wonder in students" (Sporn, 2010, p. 185). In addition, according to the *Journal of Humanistic Mathematics*, humanistic math courses explore "the human face of mathematics" through "the esthetic, cultural, historical, literary, pedagogical, philosophical, psychological, and sociological aspects [of mathematics] as a human endeavor" (http://ccms.claremont.edu/publications/jhm)

The Mathematical Association of America (MAA) has begun emphasizing the importance of humanistic mathematics not only for liberal arts students but for all undergraduates. In 2004 the MAA's Committee on the Undergraduate Program in Mathematics published the following recommendation:

Students should encounter some meaningful ideas of mathematics. College students study the best paintings, the most glorious music, the most influential philosophy, and the greatest literature of all time. Mathematics departments can compete on that elevated playing field by offering and making accessible to all students intriguing and powerful mathematical ideas….[T]he mathematical component of every student's education [should] contain some of the most profound and useful ideas that the student learns in college. (as cited in Sporn, 2010, p. 140)

d) An Example of a Humanistic Math Course: Mathematical Explorations

My own contribution to humanistic mathematics is the course *Mathematical Explorations* at Prescott College. Seeds for this course were sown as long ago as 1985 when I first began teaching math at the local community college. There I taught a sequence of three remedial skills courses whose purpose was to prepare liberal arts students to pass the required *College Algebra.* Like a factory worker my job was to crank out skills instruction to terrified students whose fervent hope was that they could retain these memorized skills long enough to pass chapter tests and then the final exam. If they could repeat this stressful process through the capstone course, *College Algebra,* then they would be done forever with the torture. I made a point of asking all of my students about the quality of their learning, and without exception they told me that they found no use in the algebra skills and quickly began to forget them as soon as each course ended, if not

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sooner—a fact that I can corroborate since I often had the same students in all three courses.

I vowed that if I ever had the privilege of teaching in a private liberal arts college I would do all I could to create meaningful learning experiences for math students. My good fortune was to be given exactly that opportunity, starting in 1990, to teach mathematics at Prescott College, a progressive student-centered institution. However, on the very first day of my first course, two of the nine enrolled students came separately to the classroom door to tell me they were sorry but they just could not enter a math classroom because of being so traumatized by math. That is when I knew my work was cut out for me. I quickly learned that students who are terrified of math or have a history of being unsuccessful in math or are just not interested in math need innovative pedagogy in order to become engaged and successful. Unfortunately, in my experience a majority of liberal arts undergraduates fall into one or more of those three categories of math avoidance.

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For the 13 years that I taught in the resident program of [the author's institution] I was required to teach traditional math skills, but I discovered that my students would become much more engaged if I added humanistic math components to the class. For the first half of every class period I began to provide students with idea-based discussions and projects. As an example of our discussions, we would put a human and cultural face on mathematics by exploring the intriguing math-related ideas and creations of ancient and indigenous cultures. As an example of our projects, I would introduce the golden ratio and its relationship to art by bringing prints of Impressionist paintings and having students pair up and search for golden ratios within their paintings. I found that students who had previously hated math responded well to these discussions and projects, and in many cases actually became interested in math.

Then in 2004, I was asked by the limited-residency program of Prescott College to create a new kind of online math course for liberal arts undergraduates that would be the antithesis of traditional *College Algebra* and that would engage our students, many of whom are long past traditional college age, with meaningful and relevant learning. I eagerly adapted my previous experiments with humanistic mathematics to create *Mathematical Explorations*, a course that does not rely on a textbook. The three major components of this course are reasoning exercises that enable students to develop better quantitative thinking abilities, a self-chosen experiential project, and self-chosen research into an interdisciplinary math-related topic of vital importance in the human quest to understand the world we live in and what it means to be human.

However, the first thing we do is a math therapy exercise where students journal extensively and then discuss their past experiences with math, many of which tend to be decidedly negative. On the surface this may look like an innocuous activity, but I have been doing it for 23 years—that is how I got those two traumatized students to finally enter my classroom in 1990—and it always enables the anxious students to enter *Mathematical Explorations* with an improved mindset. Other students find benefit in the exercise, too. Please see Stogsdill (2013) for a detailed description of the math therapy exercise.

The quantitative reasoning exercises are a weekly series of brainteasers that help students improve their general reasoning ability. These brainteasers are very different from traditional math skills problems and word problems, which we have learned do little or nothing to improve reasoning in general. Unlike traditional math problems where everything hinges on knowing the right equation and the right skills to produce the one right answer, these reasoning exercises cannot be solved with an equation and instead emphasize the *process* of learning how to reason more creatively and effectively in situations involving quantitative concepts. Students post to an online forum their thoughts about that week's reasoning exercises—just answers are not allowed, students must write about their process—and then they interact with each other's thought processes while I provide clues as needed toward the effective way, or ways, to approach the reasoning. Then students continue their thought processes based on my clues and feedback from other students until at the end of the week I provide not just the answers but also a final explanation about the thought process for each brainteaser. Students are graded minimally on the right answer and primarily on their willingness to engage in the process of creative problem solving.

The experiential project provides students with an opportunity to do something hands-on that interests them and that involves math skills and/or math ideas. Students choose from a list of potential projects and at the end of the course share their finished product with the rest of the class through an online forum. Following are examples of experiential projects that have been completed recently. One student designed and constructed an elaborate tool shed, documenting the extensive use of mathematics at every stage of this process. An adventure education student documented a wide variety of mathematical patterns in nature through a 23-page original photo essay. An expressive arts student who was exploring chaos theory and fractals for her research topic purchased a computer fractal program and created several beautiful pieces of original fractal art. A creative writing student wrote and illustrated an original story for children on his research topic of cosmology. A sustainability student designed a passive solar home, complete with calculations pertaining to solar gain. A human development student wrote a substantial business/financial plan for her future work as an aromatherapist. A teacher preparation student who worked as an aide in a classroom created and taught a series of lesson plans on his research topic of indigenous mathematics.

The math-related research topic is the heart of *Mathematical Explorations.* Students choose from a list of potential topics that include the history of great mathematical ideas, cosmology, quantum theory, chaos theory and complexity, fractals, sacred geometry, the golden ratio, infinity, space and time, and ancient and indigenous mathematics. They investigate several resources, including at least two books, in order to synthesize and reflect on their learning in a 7-10 page paper. As with the experiential project, at the end of the course students share this paper with the rest of the class through an online forum so that each student benefits from the research of every other student. This research is often where students become fascinated with mathematics and inspired to pursue further learning because they realize that math is about ideas that directly relate to their lives in meaningful ways.

Students respond to *Mathematical Explorations* exceptionally well. As with other courses at Prescott College, at the end of Mathematical Explorations all students complete a narrative self-evaluation. Following are excerpts from three self-evaluations in recent offerings of *Mathematical Explorations* that capture the most prevalent feedback I receive about the course: that it changes their relationship with math in a positive way.

"This course completely changed my perspective about math. I went from disliking math intensely and feeling it had no relevance to my life, to discovering it had everything to do with my life!"

"This course has truly ignited an interest in mathematics in me that I thought had been crushed in high school."

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"The most compelling result of this course was a stunning realization of how interconnected mathematics is to every element of our existence: a paradigm shift as vivid as if I had been living in the black-and-white world of Kansas and was transported through the whirlwind of this semester's explorations to the fantastical and extravagantly Technicolor realm of Oz….I will never again look at numbers without seeing their infinite capacity as an exquisite language of discovery, invention, and imagination."

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Another common outcome of *Mathematical Explorations* is that students are inspired to continue learning about math-related topics, as illustrated below in excerpts from three recent self-evaluations:

"I became utterly engrossed while learning about [my research topic] and am inspired and passionate about continuing my studies."

"I…discovered immensely practical applications for math in my life and work that I will continue to explore with great interest."

"My chosen research topic of chaos theory and fractals provided me with not only some fascinating insights into the world of math and science, but also some new perspectives on psychology, art, and philosophy, which contributed in many unexpected ways to the evolution of my understanding of human development. I expect that my new views on the balance between order and chaos, and the all-encompassing interconnectedness of earthly and cosmic things, will have a lasting influence on the way I perceive natural cycles, systems theory, human behavior, thought process, and social interaction."

Another common outcome, one that may be even more surprising, is that students talk about enjoying math and having fun with it, as illustrated below in three more excerpts from recent self-evaluations:

"The course was very exciting, thought provoking, and led to great discussions with my classmates."

"*Math Explorations* showed how enjoyable math can be if approached in the right way. I felt no hesitation in doing my best for the work for this course because it was fun."

"I always knew that math was interesting and I even suspected that it could be fun. I just always assumed that the fun, interesting parts were for other people, not for me. Taking this math class has changed everything for me."

For those who find it unsavory to combine the words *fun* and *rigor* in the same sentence, students also complete a course questionnaire consisting of a five-point Likert-type scale where one equals *not at all* and five equals *very much so*. One of the questions asks about rigor: "Please rate the level of difficulty or challenge you faced from readings, assignments, and discussions." The average response of all students who have taken *Mathematical Explorations* is 4.3, or somewhere between *challenging* and *very challenging*.

And for those who still doubt that it is really possible to provide meaningful math courses for liberal arts students, the average of all responses to the question "How relevant and useful were the required readings, dialogues, and assignments?" is 4.8, which is getting pretty close to *very relevant and very useful*.

Mathematical Explorations is but one example of the many innovative possibilities in meaningful math courses for liberal arts undergraduates. These possibilities are only limited by the imagination and breadth of learning of individual mathematics instructors.

It will truly be something new in math when all liberal arts undergraduates have the option of meeting their math requirement with a course that alleviates math anxiety, improves reasoning ability, provides experiential learning opportunities, and allows the exploration of math-related ideas that are meaningful, relevant, useful, and inspiring.

II. Acknowledgements

I am indebted to Michael George and Howard B. Sporn for their outstanding dissertations on liberal arts mathematics. I am grateful to Merrill Glustrom, Bill Walton, and Steve Walters for believing in my potential as a math educator.

Notes

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Radiating Heat Transfer on Unsteady Free Convective Flow through Rotating Porous Medium with Fluctuating Heat Flux

By Pawan Kumar Sharma & Sushil Kumar Saini

Amity School of Engineering and Tchnology, India

Abstract - This communication investigates the effect of radiating heat transfer on unsteady free convection flow past a vertical surface in a rotating porous medium. It is assumed that surface is rotating with angular velocity Ω. The variable heat flux is assumed on the vertical surface varies with time; the governing equations are solved by adopting complex variable notations. The analytical expressions for velocity and temperature fields are obtained. The effects of various parameters on mean velocity, mean temperature, transient velocity and transient temperature have been discussed and shown graphically.

Keywords : *porous medium, incompressible fluid, heat flux, radiating heat transfer.*

GJSFR-F Classification : *MSC 2010: 00A69*

RADIATING HEAT TRANSFER ON UNSTEADY FREE CONVECTIVE FLOW THROUGH ROTATING POROUS MEDIUM WITH FLUCTUATING HEAT FLUX

Strictly as per the compliance and regulations of :

Radiating Heat Transfer on Unsteady Free Convective Flow through Rotating Porous Medium with Fluctuating Heat Flux

Pawan Kumar Sharma [α] & Sushil Kumar Saini [σ]

Abstract - This communication investigates the effect of radiating heat transfer on unsteady free convection flow past a vertical surface in a rotating porous medium. It is assumed that surface is rotating with angular velocity Ω. The variable heat flux is assumed on the vertical surface varies with time; the governing equations are solved by adopting complex variable notations. The analytical expressions for velocity and temperature fields are obtained. The effects of various parameters on mean velocity, mean temperature, transient velocity and transient temperature have been discussed and shown graphically.

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I. Introduction

The buoyancy-induced flows in fluid-saturated porous media have been a prime topic of many studied during the past several years. This is now considered to be an important field in the general areas of fluid mechanics and heat transfer through radiation. In view of the importance in various engineering and technological applications such as in the field of agriculture engineering to study underground water resources, seepage of water under a dam, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purification processes, in underground coal gasification, heat recovery from geothermal systems, etc.

In view of geophysical applications of the flow through porous medium, a series of investigations has been made by Raptis et.al [1-3], where the porous medium is either bounded by horizontal, vertical surfaces or parallel porous plates. Singh et.al [4] and Lai and Kulacki [5] heve been studied the free convective flow past vertical wall. Nield [6] studied convection flow through porous medium with inclined temperature gradient. The oscillatory Couette flows in a rotating system have been studied by

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(1) Raptis,A., Perdikis,G. and Tzivanidis,G.: Free convection flow through a porous medium bounded by a vertical surface. J. Phys. D. Appl. Phys. 14 (1981), 99-102.

Author α : *Department of Applied Mathematics, Amity School of Engineering and Technology, New-Delhi 110061 India.*
E-mail : drpawanksharma@yahoo.com
Author σ : *Department of Mathematics, Dronacharya Government College, Gurgaon, Haryana 122001 India.*
E-mail: sksainihes29@rediffmail.com

Muzumder [7] and Ganapathy [8]. Singh et sl. [9] also studied periodic solution on oscillatory flow through channel in rotating porous medium. Further due to increasing scientific and technical applications on the effect of radiation on flow characteristic has more importance in many engineering processes occurs at very high temperature and acknowledge radiative heat transfer such as nuclear power plant, gas turbine and various propulsion devices for aircraft, missile and space vehicles. The effect of radiation on flow past different geometry a series of investigation have been made by Hassan [10], Seddeek [11] and Sharma et al [12].

Therefore the object of the present paper is to investigate the effect of radiation on unsteady free convection flow past a porous vertical surface in a rotating porous medium. Assuming periodic thermal diffusion at the plate, the analytical solution is obtained using regular perturbation technique and discussed graphically.

a) Mathematical Formulation of the Problem

We consider the unsteady viscous incompressible fluid through a porous medium, occupying a semi-infinite region of the space bounded by a vertical infinite porous surface in a rotating system, when the temperature of the surface, varies with time. We assume the effect of radiation on vertical surface which is subjected to uniform constant suction velocity in the direction perpendicular to surface. We consider the vertical infinite porous surface rotating with constant angular velocity Ω about an axis which is perpendicular to the vertical plane confined with a viscous fluid occupying the porous region. Vertical porous plane is taken to be $z^* = 0$ plane with z^* axis normal to it. X^* axis is selected vertically upwards and y^* axis in the perpendicular direction in $z^* = 0$ plane. The flow is assumed to be along the plane $z^*=0$. With the above frame of reference and assumptions, with physical variables, except the pressure p, are the function of z^* and t^* only. The flow in porous medium involves small velocities permitting the neglect of heat due to viscous dissipation in governing equation. The equation expressing the conservation of mass and energy transfer in rotating frame of reference are given by

$$\frac{\partial w^*}{\partial z^*} = 0, \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} - 2\Omega v^* = g\beta(T^* - T^*) + \nu \frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\nu u^*}{k^*}, \tag{2}$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2\Omega u^* = \nu \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\nu v^*}{k^*}, \tag{3}$$

Ref
(7) Muzumder,B.S.: An exact solution of oscillatory Couette flow in a rotating system. J. Appl. Mech. 58 (1991), 1104-1107.

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} - \frac{\nu w^*}{k^*}, \tag{4}$$

$$\frac{\partial T^*}{\partial t^*} + w^*\frac{\partial T^*}{\partial z^*} = \alpha\frac{\partial^2 T^8}{\partial z^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial z^*}, \tag{5}$$

Notes

where u*, v* w^* are components of velocity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, T* is the temperature, T^*_∞ is the temperature in free stream, ν is the kinematic viscosity, Ω is the angular velocity, K^* is the permeability, C_p is the specific heat at constant pressure, q_r^* is radiative heat flux, p* is the pressure, ρ is the density, t* is the time and α is the thermal diffusivity.

The boundary conditions of the problem are

$$\left.\begin{array}{l} z = 0:\ u^* = 0,\quad v^* = 0,\quad \dfrac{\partial T^*}{\partial z^*} = -\dfrac{q_w^*}{\kappa}(1+\varepsilon\, e^{i\omega^* t^*}) \\ z\to\infty:\ u^*\to 0,\ v^*\to 0,\quad T^*\to T^*_\infty\ . \end{array}\right\} \tag{6}$$

where q_w^* is the heat flux at wall ω^* is the frequency of fluctuation and κ is the thermal conductivity of the plate, For constant suction, we have from equation (1) ,

$$w = -w_0 \tag{7}$$

Considering u + iv = U and taking into account equation (7), the equations (2) and (3) can be written as

$$\frac{\partial U^*}{\partial t^*} - w_0\frac{\partial U^*}{\partial z^*} + 2\iota\Omega U^* = g\beta(T^* - T^*_\infty) + \nu\frac{\partial^2 U^*}{\partial z^{*2}} - \frac{\nu U^*}{k^*}, \tag{8}$$

We introduce the following non-dimensional quantities as:

$$z = \frac{w_0 z^*}{\nu},\quad U = \frac{U^*}{w_0},\quad t = \frac{t^* w_0^2}{\nu},\quad \omega = \frac{\nu\omega^*}{w_0^2},\quad \theta = \frac{\kappa(T^* - T^*_\infty)w_0}{q_w^*\nu}, \tag{9}$$

$$k = \frac{w_0^2 k^*}{\nu^2},\ \text{R (rotation parameter)} = \frac{\Omega\nu}{w_0^2},\ \alpha\,(\text{thermal diffusivity}) = \frac{\kappa}{\rho C_p},$$

$$\text{Gr (Grashof number)}\quad \frac{g\beta q_w^*\nu^2}{w_0^4\kappa},\ \text{Pr (Prandtal number)} = \frac{\nu}{\alpha}.$$

$$F\,(\text{ radiation parameter }) = \frac{4\,\nu\, I}{\rho\, C_p\, w_0^2}\,,$$

The Radiative heat flux Cogley [13] $\frac{\partial q_r^*}{\partial z^*} = 4\,(T^* - T_\infty^*)\, I^*$,

$I^* = \int_0^\infty \kappa_{\lambda\omega} \frac{\partial e_{b\lambda}}{\partial T^*}\, d\lambda$, $\kappa_{\omega\lambda}$ is the absorption coeffieient at the wall and $e_{b\lambda}$ is Planck's function.

Substituting these non-dimensional quantities in equations (8) and (5), we get

$$\frac{\partial U}{\partial t} - \frac{\partial U}{\partial z} - 2iRU = Gr\,\theta + \frac{\partial^2 U}{\partial z^2} - \frac{U}{k}\,, \tag{10}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial z} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2} - F\,\theta\,, \tag{11}$$

and the boundary conditions (6) become

$$\left.\begin{aligned} z = 0:\ U = 0, \quad & \frac{\partial \theta}{\partial z} = -(1+\varepsilon\, e^{i\omega t}) \\ z \to \infty:\ U \to 0, \quad & \theta \to 0\,. \end{aligned}\right\} \tag{12}$$

II. Solution of the Problem

In order to solve the problem, we assume the solutions of the following form because amplitude ε (< < 1) of the variation of temperature is very small

$$\left.\begin{aligned} U(z,t) &= U_0(z) + \varepsilon\, U_1(z)\, e^{\iota\omega t} + \ldots\ldots\ldots\ldots \\ \theta(z,t) &= \theta_0(z) + \varepsilon\,\theta_1(z)\, e^{\iota\omega t} + \ldots\ldots\ldots\ldots \end{aligned}\right\} \tag{13}$$

Substituting (13) in equations (10) and (11), and equating the coefficient of identical powers of ε and neglecting those of ε^2, ε^3 etc., we get

$$U_0'' + U_0' - 2\,\iota\, R\, U_0 - \frac{U_0}{k} = -\,Gr\,\theta_0\,, \tag{14}$$

$$U_1'' + U_1' - 2\,\iota\, R\, U_1 - \iota\,\omega\, U_1 - \frac{U_1}{k} = -\,Gr\,\theta_1\,, \tag{15}$$

$$\theta_0'' + Pr\,\theta_0' - F\,Pr\,\theta_0 = 0\,, \tag{16}$$

$$\theta_1'' + Pr\,\theta_1' - (F + \iota\,\omega)\,\theta_1\, Pr = 0\,. \tag{17}$$

The corresponding boundary conditions (12) reduce to

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$$\left.\begin{array}{l} z = 0:\ U_0 = 0,\ U_1 = 0,\ \dfrac{\partial \theta_0}{\partial z} = -1,\ \dfrac{\partial \theta_1}{\partial z} = -1 \\ z \to \infty:\ U_0 \to 0,\ U_1 \to 0,\ \theta_0 \to 0,\ \theta_1 \to 0. \end{array}\right\} \tag{18}$$

Solving equations (14) to (17) under corresponding boundary conditions (18), we get

$$U_0(z) = c_4\,(e^{-c_1 z} - e^{-c_3 z}) \tag{19}$$

$$U_1(z) = c_6\,(e^{-c_2 z} - e^{-c_5 z}) \tag{20}$$

$$\theta_0(z) = \frac{1}{c_1} e^{-c_1 z}, \tag{21}$$

$$\theta_1(z) = \frac{1}{c_2} e^{-c_2 z}. \tag{22}$$

where

$$c_1 = \frac{1}{2}\left[\Pr + \sqrt{\Pr^2 + 4F\Pr}\right]$$

$$c_2 = \frac{1}{2}\left[\Pr + \sqrt{\Pr^2 + 4\Pr(F + \iota\omega)}\right]$$

$$c_3 = \frac{1}{2}\left[1 + \sqrt{1 + 8\iota R + \frac{4}{k}}\right]$$

$$c_4 = -\frac{Gr}{c_1\,(c_1^2 - c_1 - 2\iota R - \frac{1}{k})}$$

$$c_5 = \frac{1}{2}\left[1 + \sqrt{1 + 4(2\iota R + \iota\omega) + \frac{4}{k}}\right]$$

$$c_6 = -\frac{Gr}{c_2\,(c_2^2 - c_2 - 2\iota R - \iota\omega - \frac{1}{k})}$$

III. RESULTS

a) Steady Flow

We take $U_0 = u_0 + \iota v_0$ in equation (19) and subsequent comparison of the real and imaginary parts gives the mean primary $\frac{u_0}{w_0}$ and mean secondary $\frac{v_0}{w_0}$ velocity fields as

$$\left.\begin{aligned}\frac{u}{w_0} &= a_5\, e^{-c_1 z} - a_5\, e^{-a_3 z} \cos a_4 z - a_6\, e^{-a_3 z} \sin a_4 z \\ \frac{v}{w_0} &= a_6\, e^{-c_1 z} + a_5\, e^{-a_3 z} \sin a_4 z - a_6\, e^{-a_3 z} \cos a_4 z\end{aligned}\right\}, \tag{23}$$

b) *Unsteady Flow*

Replacing the unsteady parts

$U_1(z,t) = M_r + \iota M_i$, and $\theta_1(z,t) = T_r + \iota T_i$ respectively in equation (20), we get

$$[U(z,t), \theta(z,t)] = [U_0(z), \theta_0(z)] + \varepsilon\, e^{\iota\omega t}\, [(M_r + \iota M_i), (T_r + \iota T_i)] \tag{24}$$

The primary, secondary velocity fields in terms of the fluctuating components are

$$\frac{u}{w_0}(z,t) = u_0 + \varepsilon\,(M_r \cos\omega t - M_i \sin\omega t) \tag{25}$$

$$\frac{v}{w_0}(z,t) = v_0 + \varepsilon\,(M_r \sin\omega t + M_i \cos\omega t) \tag{26}$$

Taking $\omega t = \frac{\pi}{2}$ in equations (24) , (25) and (26), we get the expression for transient primary velocity, transient secondary velocity and transient temperature as

$$\frac{u}{w_0}\left(z, \frac{\pi}{2\omega}\right) = u_0(z) - \varepsilon\, M_1(z), \tag{27}$$

$$\frac{v}{w_0}\left(z, \frac{\pi}{2\omega}\right) = v_0(z) + \varepsilon\, M_r(z), \tag{28}$$

$$\theta\left(z, \frac{\pi}{2\omega}\right) = \theta_0(z) - \varepsilon\, T_i(z). \tag{29}$$

where

$$M_r = a_{11}\,[e^{-a_1 z} \cos a_2 z - e^{-a_7 z} \cos a_8 z] - a_{12}\,[e^{-a_7 z} \sin a_8 z - e^{-a_1 z} \sin a_2 z]$$

$$M_i = a_{12}\,[e^{-a_1 z} \cos a_2 z - e^{-a_7 z} \cos a_8 z] + a_{11}\,[e^{-a_7 z} \sin a_8 z - e^{-a_1 z} \sin a_2 z]$$

$$T_i = -\frac{e^{-a_1 z}}{a_1^2 + a_2^2}\,[a_1 \sin a_2 z + a_2 \cos a_2 z]$$

Notes

$$c_2 = a_1 + \imath a_2 \ , \ c_3 = a_3 + \imath a_4 \ , \ c_4 = a_5 + \imath a_6 \ , \ c_5 = a_7 + \imath a_8 \ , \ c_6 = a_{11} + \imath a_{12}$$

$$a_1 = \frac{1}{2}\left[Pr + \sqrt{\sqrt{(Pr^2 + 4 F Pr)^2 + 16 \omega^2 Pr^2}} \ . \ \frac{(Pr^2 + 4 F Pr)^2 - 16 \omega^2 Pr^2}{(Pr^2 + 4 F Pr)^2 + 16 \omega^2 Pr^2} \right]$$

$$a_2 = \frac{1}{2}\left[\sqrt{\sqrt{(Pr^2 + 4 F Pr)^2 + 16 \omega^2 Pr^2}} \ . \ \frac{8 \omega Pr (Pr^2 + 4 F Pr)}{(Pr^2 + 4 F Pr)^2 + 16 \omega^2 Pr^2} \right]$$

$$a_3 = \frac{1}{2}\left[1 + \sqrt{\sqrt{(1+\frac{4}{k})^2 + 64 R^2}} \ . \ \frac{(1+\frac{4}{k})^2 - 64 R^2}{(1+\frac{4}{k})^2 + 64 R^2} \right]$$

$$a_4 = \frac{1}{2}\left[\sqrt{\sqrt{(1+\frac{4}{k})^2 + 64 R^2}} \ . \ \frac{16 R (1+\frac{4}{k})}{(1+\frac{4}{k})^2 + 64 R^2} \right]$$

$$a_5 = -\frac{Gr (c_1^3 - c_1^2 - \frac{c_1}{k})}{(c_1^3 - c_1^2 - \frac{c_1}{k})^2 + 4 R^2 c_1^2} ,$$

$$a_6 = -\frac{2 Gr R c_1}{(c_1^3 - c_1^2 - \frac{c_1}{k})^2 + 4 R^2 c_1^2} ,$$

$$a_7 = \frac{1}{2}\left[1 + \sqrt{\sqrt{(1+\frac{4}{k})^2 + 16 (2R + \omega)^2}} \ . \ \frac{(1+\frac{4}{k})^2 - 64 (2R + \omega)^2}{(1+\frac{4}{k})^2 + 64 (2R + \omega)^2} \right]$$

$$a_8 = \frac{1}{2}\left[\sqrt{\sqrt{(1+\frac{4}{k})^2 + 16 (2R + \omega)^2}} \ . \ \frac{8 (2R + \omega)(1+\frac{4}{k})}{(1+\frac{4}{k})^2 + 64 (2R + \omega)^2} \right]$$

$$a_9 = a_1^2 - a_2^2 - a_1 - \frac{1}{k} \ ,$$

$$a_{10} = 2 a_1 a_2 - a_2 - 2R - \omega \ ,$$

$$a_{11} = -\frac{Gr (a_1 a_9 - a_2 a_{10})}{(a_1 a_9 - a_2 a_{10})^2 + (a_2 a_9 + a_1 a_{10})^2} ,$$

$$a_{12} = \frac{Gr (a_2 a_9 + a_1 a_{10})}{(a_1 a_9 - a_2 a_{10})^2 + (a_2 a_9 + a_1 a_{10})^2} .$$

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IV. Discussion and Conclusions

From equation (23), it has been found that steady part of the mean primary velocity field has a two layer character. These layers may be identified as suction layer and thermal layer. The suction layer is due to rotation and porosity of the medium. The thermal layer is arising due to interaction of the thermal field due to radiation and the velocity field and is dependent on Prandtl Number and Radiation Parameter.

For physical interpretation of the problem, the numerical values of the mean primary and mean secondary velocity profiles have been computed for fixed values of physical parameter, for Grashof number Gr = 2, Prandtl number Pr= 0.71 (air), frequency of fluctuation ω = 0.5 and for different values of Rotation parameter R, Radiation parameter F and permeability of porous medium k. From fig.1 we observe that the mean primary velocity increases with increase in either rotation parameter R or permeability k. This shows that the viscosity and rotation of porous medium exert retarding influence on the primary flow. It has also been observed that it decreases with increasing radiation parameter F. the mean primary velocity increases in the vicinity of the surface and than decreases with perpendicular distance from the surface.

The mean secondary velocity is given in fig.2 for fixed values of Gr = 2, Pr = 0.71 (air) and ω =0.5. It is observed that it decreases sufficiently higher with increasing rotation parameter F. It is interesting to note that mean secondary velocity increases with rotation parameter and permeability for z<0.5, while reverse phenomena is observed for z>0.5, showing that the effect of flow parameter is more significant for relatively small values of z. The transient primary velocity profiles are presented in fig.3 for fixed values of Gr = 2, Pr = 0.71 (air) and ω =0.5. It is observed that the

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transient primary velocity increases with increasing either permeability or rotation parameter. It is interesting to note that it increases with increasing radiation parameter for $z<0.9$ than it decreases for $z \geq 0.9$.

The transient secondary velocity is given in fig.4. It is observed that transient velocity increases in the vicinity of vertical surface while, reverse effect is observed with increasing radiation parameter. It is also observed that it increases with increasing rotation parameter and permeability for small value of z, while it decreases with higher value of z.

Fig.5 shows the variation of mean and transient temperature profiles for different values of ω and F. The mean temperature and transient temperature both decreases with increasing radiation parameter. Also the transient temperature increases with increasing ω. It is interesting to note that both are decreasing exponentially with distance far away from the vertical surface.

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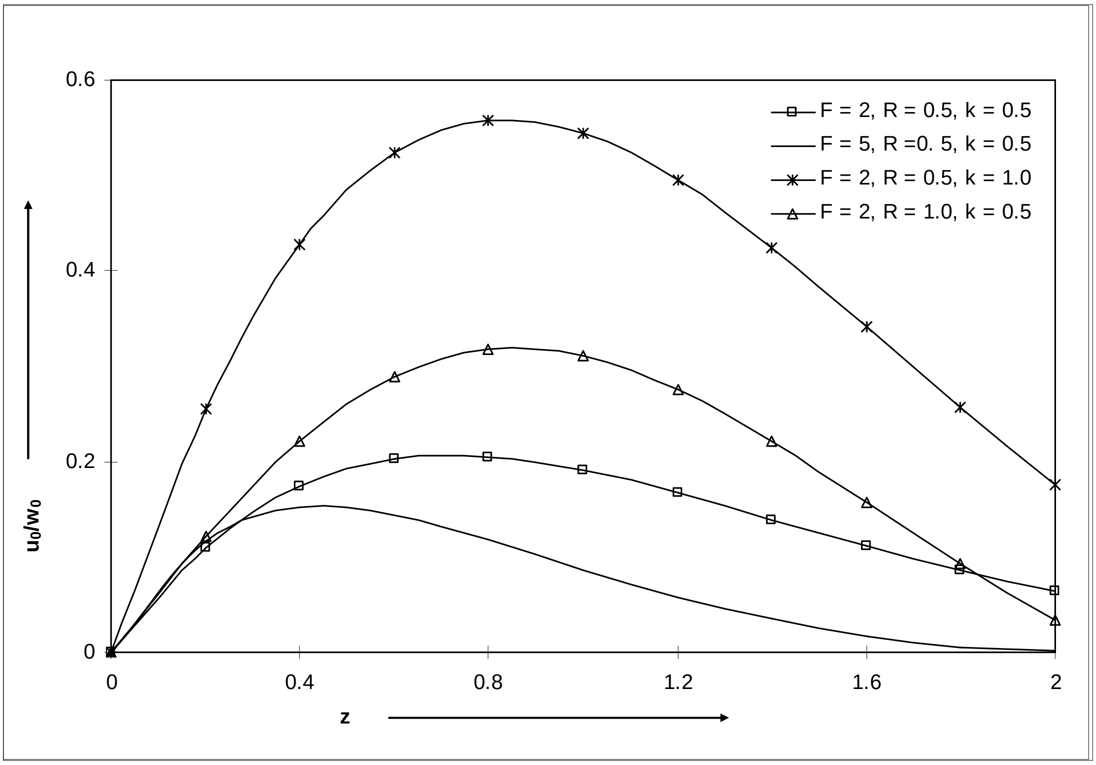

Figure 1 : Mean primary velocity for Pr=0.71, Gr=2 and ω=0.5

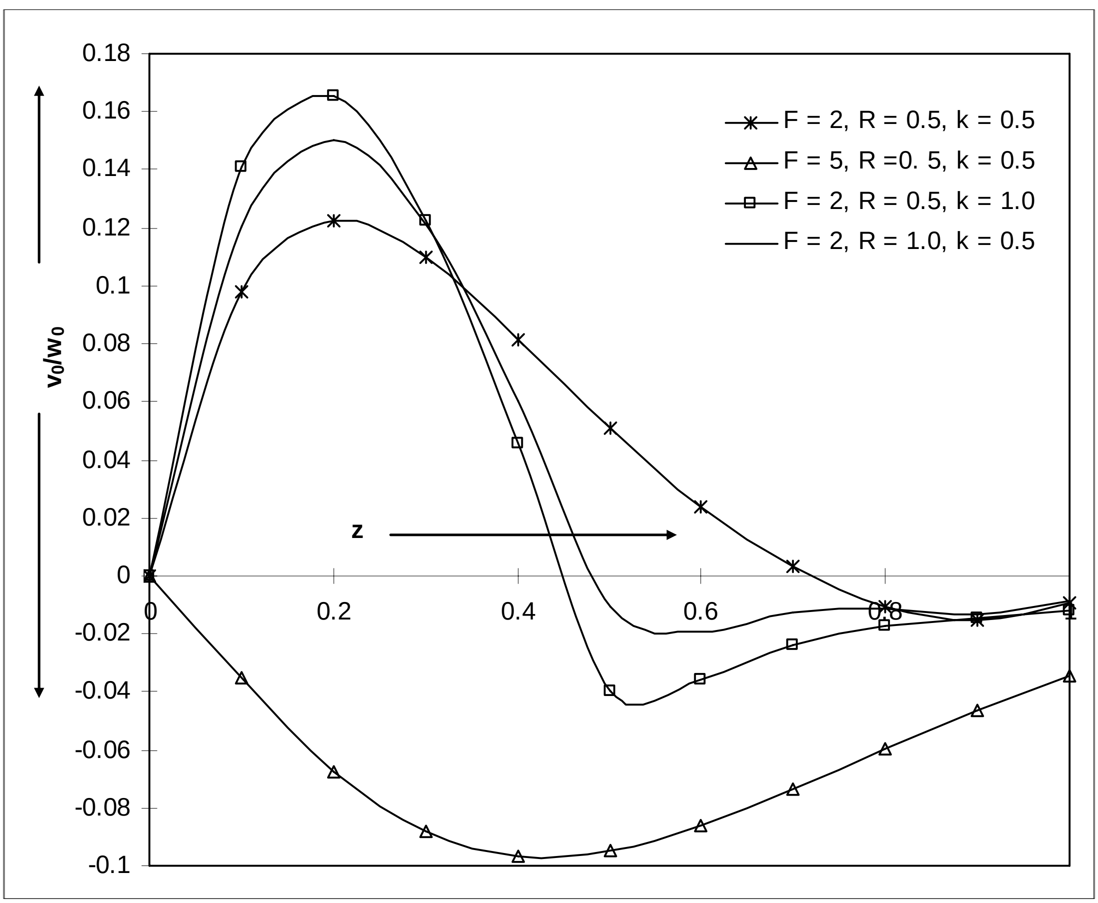

Figure 2 : Mean secondary velocity for Gr=2, Pr=0.71 and ω=0.5

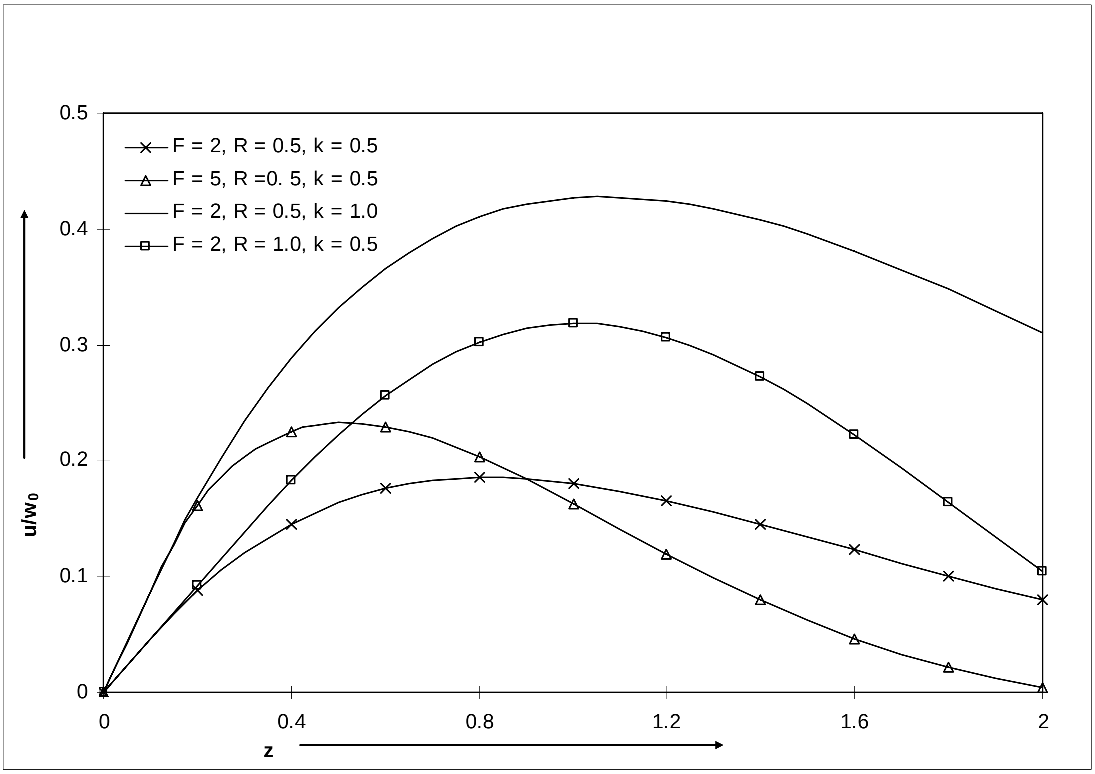

Figure 3 : Transient primary velocity for Pr=0.71, Gr = 2 and ω = 0.5

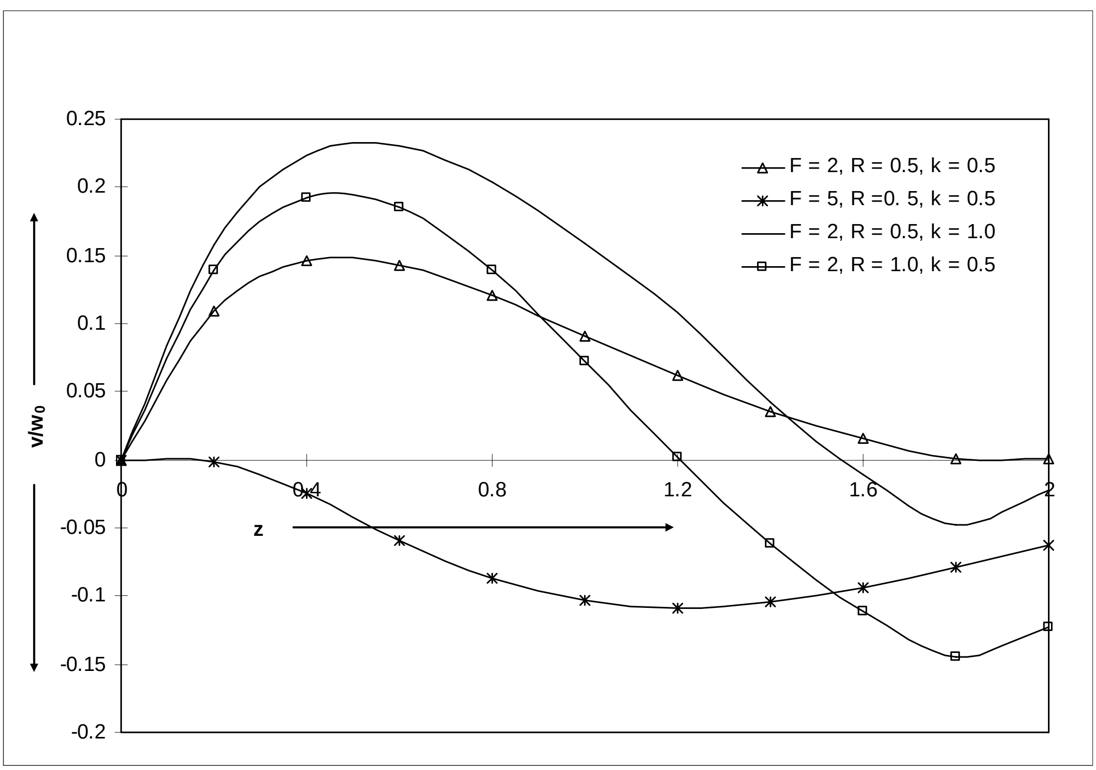

Figure 4 : Transient secondary velocity for Pr=0.71, Gr=2 and ω = 0.5

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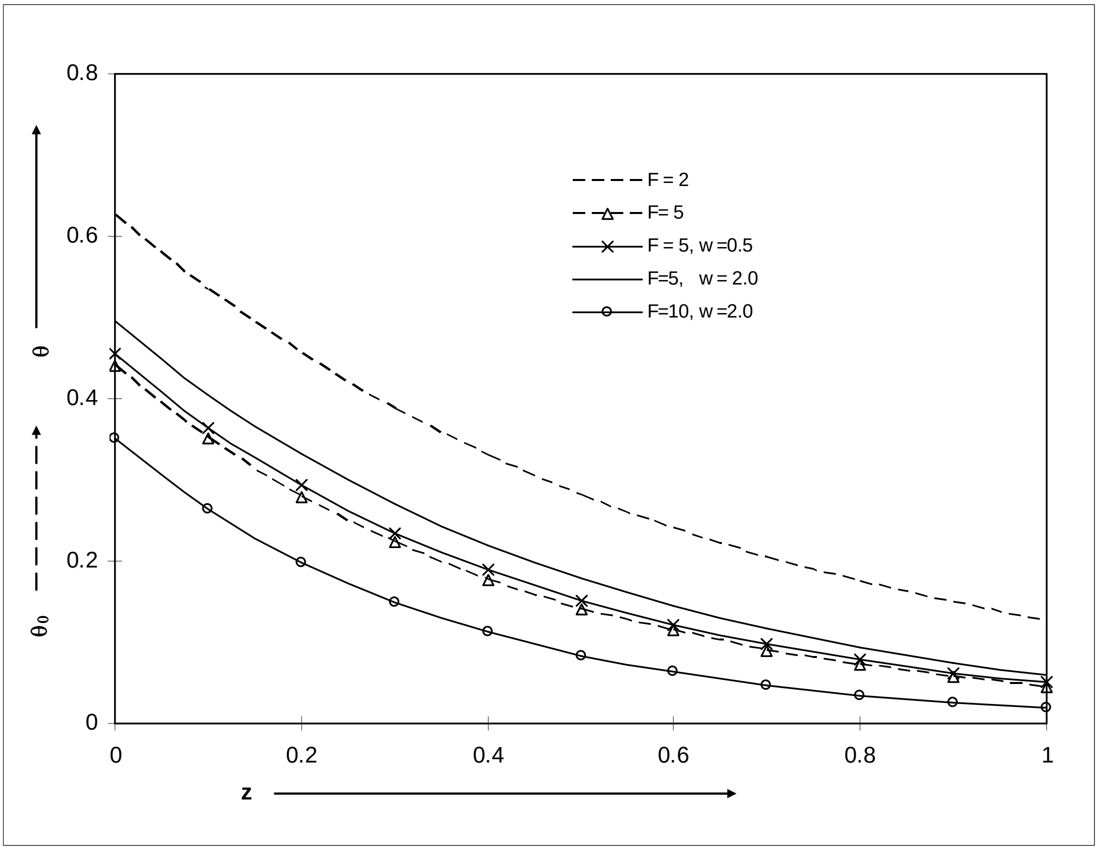

Figure 5 : Mean temperature θ_0 and Transient temperature (ε= 0.2 and $\omega t=\pi/2$) for air Pr = 0.71

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Study of Nonlinear Evolution Equations in Mathematical Physics

By Sadia Marzan, Fatima Farhana, Md. Tanjir Ahmed, Kamruzzaman Khan & M. Ali Akbar

Pabna University of Science and Technology, Bangladesh

Abstract - In the present paper, we construct the traveling wave solutions involving parameters for some nonlinear evolution equations in the mathematical physics via the Konopelchenko-Dubrovsky Coupled System equation and the (1+1)-dimensional nonlinear Ostrovsky equation by using the Bernoulli Sub-ODE method. By using this method exact solutions involving parameters have been obtained. When the parameters are taken as special values, solitary wave solutions have been originated from the hyperbolic function solutions. It has been shown that the method is effective and can be used for many other NLEEs in mathematical physics.

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STUDY OF NONLINEAR EVOLUTION EQUATIONS INMATHEMATICAL PHYSICS

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Study of Nonlinear Evolution Equations in Mathematical Physics

Sadia Marzan [α], Fatima Farhana [σ], Md. Tanjir Ahmed [ρ], Kamruzzaman Khan [ω] & M. Ali Akbar [¥]

Abstract - In the present paper, we construct the traveling wave solutions involving parameters for some nonlinear evolution equations in the mathematical physics via the Konopelchenko-Dubrovsky Coupled System equation and the (1+1)-dimensional nonlinear Ostrovsky equation by using the Bernoulli Sub-ODE method. By using this method exact solutions involving parameters have been obtained. When the parameters are taken as special values, solitary wave solutions have been originated from the hyperbolic function solutions. It has been shown that the method is effective and can be used for many other NLEEs in mathematical physics.

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I. Introduction

NLEEs are encountered in various fields of mathematics, physics, chemistry, biology, engineering and numerous applications. Exact solutions of NLEEs play an important role in the proper understanding of qualitative features of many phenomena and processes in various areas of natural science. Exact solutions of nonlinear equations graphically demonstrate and allow unscrambling the mechanisms of many complex nonlinear phenomena such as spatial localization of transfer processes, multiplicity or absence steady states under various conditions, existence of peaking regimes and many others. Even those special exact solutions that do not have a clear physical meaning can be used as test problems to verify the consistency and estimate errors of various numerical, asymptotic, and approximate analytical methods. Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving NLEEs. It is significant that many equations of physics, chemistry, and biology contain empirical parameters or empirical functions. Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore, investigation exact traveling wave solutions are becoming successively attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the (G'/G)-expansion method [1-7], the Hirota's bilinear transformation method [8,9], the modified simple equation method [10-13], the tanh-function method [14], the first integral method[15], the Exp-function method [16-18], the Jacobi elliptic function method [19], the

Authors α σ ρ ω : Department of Mathematics, Pabna University of Science and Technology, Pabna-6600, Bangladesh.
E-mail : k.khanru@gmail.com
Author ¥ : Department of Applied Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh.

homotopy perturbation method [20-22], the Bernoulli Sub-ODE method [23-24], the enhanced (G'/G)-expansion Method [25-27], the $\exp(-\Phi(\xi))$-expansion method [28] and so on.

The objective of this paper is to find the exact solutions then the solitary wave solutions for the Konopelchenko-Dubrovsky Coupled System equation and the (1+1)-dimensional nonlinear Ostrovsky equation through Bernoulli Sub-ODE method.

The article is arranged as follows: In section II, the Bernoulli Sub-ODE method is discussed. In section III, we apply this method to the nonlinear evolution equations pointed out above; in section IV, graphical representation and in section V, conclusions are given.

II. Methodology

In this section, we describe the Bernoulli Sub-ODE method for finding traveling wave solutions of NLEEs. Suppose that a nonlinear partial differential equation, say in two independent variables x and t is given by

$$\Re(u,u_t,u_x,u_{tt},u_{xx},u_{xt},\cdots\cdots\cdots\cdots)=0\,, \tag{1}$$

where $u(\xi)=u(x,t)$ is an unknown function, $\Re$ is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [23, 24]:

Step 1. Combining the independent variables x and t into one variable ξ, we suppose that

$$u(\xi)=u(x,t)\,, \qquad \xi = x\pm\omega t\,. \tag{2}$$

The traveling wave transformation Eq. (2) permits us to reduce Eq. (1) to the following ODE:

$$\Re(u,u',u'',\cdots\cdots\cdots)=0\,, \tag{3}$$

where $\Re$ is a polynomial in $u(\xi)$ and its derivatives, while $u'(\xi)=\dfrac{du}{d\xi}$, $u''(\xi)=\dfrac{d^2u}{d\xi^2}$ and so on.

Step 2. We suppose that Eq.(3) has the formal solution

$$u(\xi)=\sum_{i=0}^{n} a_i G^i\,, \tag{4}$$

where $G=G(\xi)$ satisfy the equation $G'+\lambda G=\mu G^2$, (5)

in which $a_i\,(-n\le i\le n; n\in \mathrm{N})$ are constants to be determined later, and $\mu\neq 0\,,\lambda\neq 0$.

then the Eq. (5) is the type of Bernoulli equation, and we can obtain the solution as

$$G=-\frac{\lambda}{2\mu}\left(\tanh\left(\frac{\lambda}{2}\xi\right)-1\right). \tag{6}$$

Ref

20. Mohiud-Din S.T., Homotopy perturbation method for solving fourth-order boundary value problems, *Math. Prob. Engr.* Vol. 2007, 1-15, Article ID 98602, doi:10.1155/2007/98602.

$$G=-\frac{\lambda}{2\mu}\left(\coth\left(\frac{\lambda}{2}\xi\right)-1\right). \tag{7}$$

Step 3. The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq.(1) or Eq.(3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi))=n$ which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right)=n+q,\ D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right)=np+s(n+q). \tag{8}$$

Therefore we can find the value of n in Eq.(4), using Eq.(1).

Step 4. We substitute Eq. (4) into Eq.(3) using Eq. (5) and then collect all terms of same powers of $G(\xi)$ together, then set each coefficient of them to zero to yield a system of algebraic equations, solving this system we obtain the values of a_i and ω.

Finally, substituting a_i, ω and Eq. (6), Eq. (7) into Eq. (4) we obtain exact traveling wave solutions of Eq. (1).

III. Applications

a) *The Konopelchenko-Dubrovsky Coupled System equation*

In this section, we will consider the following the Konopelchenko-Dubrovsky Coupled System equation:

$$u_t-u_{xxx}-6buu_x+\frac{3}{2}a^2u^2u_x-3v_y+3au_xv=0 \tag{9}$$

$$u_y=v_x \tag{10}$$

This system was studied by Taghizadeh N. and Mirzazadeh M. by the first integral method [15].

Suppose that

$$u(x,t)=u(\xi),v(x,t)=v(\xi)\ \xi=kx+\alpha y+\omega t, \tag{11}$$

where k,α,ω are constants that to be determined later.

By Eq. (9), Eq. (10) and Eq. (11) are converted into the following ODEs,

$$\omega u'-k^3u'''-6bkuu'+\frac{3}{2}a^2ku^2u'-3\alpha v'+3aku'v=0, \tag{12}$$

$$\alpha u'=kv'. \tag{13}$$

Integrating Eq.(13) once with zero constant, Eq. (13) reduces to

$$v=\frac{\alpha}{k}u, \tag{14}$$

Ref

15. Taghizadeh and Mirzazade M., (2011), Exact Travelling Wave solutions for Konopelchenko-Dubrovsky equation by the First Integral Method, Applications and Applied Mathematics Vol. 6, Issue 11 pp. 1893-1901.

Substituting Eq. (14) in Eq. (12), then

$$\omega u' - k^3 u''' + \frac{3}{2} a^2 k u^2 u' - 3\frac{\alpha^2}{k} u' + 3[a\alpha - 2bk] u u' = 0. \tag{15}$$

Integrating Eq. (15) once, Eq. (15) reduces to

$$\omega u - k^3 u'' + \frac{1}{2} a^2 k u^3 - 3\frac{\alpha^2}{k} u + \frac{3}{2}[a\alpha - 2bk] u^2 + R = 0, \tag{16}$$

where R is the integration constant.
Suppose that the solution of Eq. (16) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i, \tag{17}$$

where a_i are constants, and $G = G(\xi)$ satisfies the following Bernoulli equation:

$$G' + \lambda G = \mu G^2 \tag{18}$$

Balancing the order of u'' and u^3 in Eq. (16), we have $3m = m + 2, m = 1$. So Eq. (17) can be rewritten as

$$u(\xi) = a_1 G + a_0, a_1 \neq 0, \tag{19}$$

where a_1, a_0 are constants to be determined later.

Substituting Eq. (19) into Eq. (16) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0 : 2\omega a_0 k - 6bk^2 a_0{}^2 + 3a\alpha a_0^2 k + 2Rk + a^2 k^2 a_0^3 - 6\alpha^2 a_0$$

$$G^1 : -12bk^2 a_0 a_1 + 3a^2 k^2 a_0^2 a_1 + 6a\alpha a_0 a_1 k - 6\alpha^2 a_1 - 2k^4 a_1 \lambda^2 + 2\omega a_1 k$$

$$G^2 : 6k^4 a_1 \mu\lambda + 3a^2 k^2 a_0 a_1^2 - 6bk^2 a_1^2 + 3a\alpha a_1^2 k$$

$$G^3 : -4k^4 a_1 \mu^2 + a^2 k^2 a_1^3$$

Solving the above system of algebraic equations, we get the following two sets of solutions:

Set-1: $R = -\frac{1}{2}\frac{1}{k^2 a^4}\left((k^2\lambda a - 2bk + a\alpha)(-4b^2k^2 + 4bka\alpha - \alpha^2 a^2 - 2bk^3\lambda a + k^2\lambda a^2 \alpha)\right)$,

$$\omega = -\frac{1}{2}\left(\frac{-12b^2k^2 + 12bka\alpha + k^4\lambda^2 a^2 - 9a^2\alpha^2}{a^2 k}\right), \quad a_0 = -\frac{k^2\lambda a - 2bk + a\alpha}{ka^2}, \quad a_1 = \frac{2k\mu}{a}.$$

Set-2: $R = -\frac{1}{2}\frac{1}{k^2 a^4}\left((k^2\lambda a + 2bk - a\alpha)(4b^2k^2 - 4bka\alpha + \alpha^2 a^2 - 2bk^3\lambda a + k^2\lambda a^2 \alpha)\right)$,

$$\omega = -\frac{1}{2}\left(\frac{-12b^2k^2 + 12bka\alpha + k^4\lambda^2 a^2 - 9a^2\alpha^2}{a^2 k}\right), \quad a_0 = \frac{k^2\lambda a + 2b \ \ -a\alpha}{ka^2}, \quad a_1 = -\frac{2k\mu}{a},$$

Notes

Substituting Set-1 and Set-2 into Eq. (19) along with Eq. (6) and Eq. (7), we get the following exact traveling wave solutions:

Case 1: When $R=-\frac{1}{2}\frac{1}{k^2a^4}\left(\left(k^2\lambda a-2bk+a\alpha\right)\left(-4b^2k^2+4bka\alpha-\alpha^2a^2-2bk^3\lambda a+k^2\lambda a^2\alpha\right)\right)$,

$\omega=-\frac{1}{2}\left(\frac{-12b^2k^2+12bka\alpha+k^4\lambda^2a^2-9a^2\alpha^2}{a^2k}\right)$, $a_0=-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}$, $a_1=\frac{2k\mu}{a}$

$$u_1(x,t)=-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}-\frac{1}{a}\left(k\lambda\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)\right) \tag{20}$$

$$u_2(x,t)=-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}-\frac{1}{a}\left(k\lambda\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)\right), \tag{21}$$

where $\xi=kx+\alpha y+\left(-\frac{1}{2}\left(\frac{-12b^2k^2+12bka\alpha+k^4\lambda^2a^2-9a^2\alpha^2}{a^2k}\right)\right)t$,

Substituting Eq. (20) and Eq. (21) into Eq. (14), yields

$$v_1(x,t)=\frac{\alpha}{k}\left(-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}-\frac{1}{a}\left(k\lambda\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)\right)\right). \tag{22}$$

$$v_2(x,t)=\frac{\alpha}{k}\left(-\frac{k^2\lambda a-2bk+a\alpha}{ka^2}-\frac{1}{a}\left(k\lambda\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)\right)\right). \tag{23}$$

Case 2: When $R=-\frac{1}{2}\frac{1}{k^2a^4}\left(\left(k^2\lambda a+2bk-a\alpha\right)\left(4b^2k^2-4bka\alpha+\alpha^2a^2-2bk^3\lambda a+k^2\lambda a^2\alpha\right)\right)$,

$\omega=-\frac{1}{2}\left(\frac{-12b^2k^2+12bka\alpha+k^4\lambda^2a^2-9a^2\alpha^2}{a^2k}\right)$, $a_0=\frac{k^2\lambda a+2bk-a\alpha}{ka^2}$, $a_1=-\frac{2k\mu}{a}$,

$$u_3(x,t)=\frac{k^2\lambda a+2bk-a\alpha}{ka^2}+\frac{1}{a}\left(k\lambda\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)\right). \tag{24}$$

$$u_4(x,t)=\frac{k^2\lambda a+2bk-a\alpha}{ka^2}+\frac{1}{a}\left(k\lambda\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)\right). \tag{25}$$

Substituting Eq. (24) and Eq. (25) into Eq. (14), yields

$$v_3(x,t)=\frac{\alpha}{k}\left(\frac{k^2\lambda a+2bk-a\alpha}{ka^2}+\frac{1}{a}\left(k\lambda\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)\right)\right). \tag{26}$$

Notes

$$v_4(x,t)=\frac{\alpha}{k}\left(\frac{k^2\lambda a+2bk-a\alpha}{ka^2}+\frac{1}{a}\left(k\lambda\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)\right)\right). \tag{27}$$

b) The (1+1)-dimensional nonlinear Ostrovsky equation

Consider the (1+1)-dimensional nonlinear Ostrovsky equation

$$uu_{xxt}-u_x u_{xt}+u^2u_t=0, \tag{28}$$

This equation is a model for weakly nonlinear surface and internal waves in a rotation ocean. Following the above procedure we transform Eq. (28) into ODE:

$$-(uu'')'+2u'u''-u^2u''=0, \tag{29}$$

obtained upon using $\xi=x-ct$. Integrating Eq.(29) with respect to ξ one has

$$3uu''-3(u')^2+u^3+R=0, \tag{30}$$

where R is the integration constant.

Balancing the nonlinear term u^3 with the highest order derivative uu'' that gives

$3m=m+m+2$,

so that $m=2$.

So Eq. (4) can be rewritten as

$$u(\xi)=a_2G^2+a_1G+a_0,\quad a_1,a_2\neq 0, \tag{31}$$

where a_0,a_1,a_2 are constants to be determined later.

Substituting Eq. (31) into Eq. (30) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$G^0: a_0^3$

$G^1: 3a_0a_1\lambda^2+3a_0^2a_1$

$G^2: 3a_0^2a_2+12a_0a_2\lambda^2+3a_0a_1^2-9a_0a_1\mu\lambda$

$G^3: 6a_0a_1\mu^2-3a_1^2\mu\lambda+a_1^3-30a_0a_2\mu\lambda+6a_0a_1a_2+3a_1a_2\lambda^2$

$G^4 8:1\ a_0a_2\mu^2 + 53a_0a_2^2-1\ a_1a_2\mu\lambda+3a_1^2\mu^2+3a_1^2a_2$

$G^5: 3a_1a_2^2-6a_2^2\mu\lambda+12a_1a_2\mu^2$

$G^6: 6a_2^2\mu^2+a_2^3$

Solving the above system of algebraic equations, we get the following solution:

$$R=R, c=c, a_0=0, a_1=6\mu\lambda, a_2=-6\mu^2$$

Substituting these values into Eq. (31) along with Eq. (6) and Eq. (7), we get the following exact traveling wave solutions:

Notes

$$u_1(x,t) = -3\lambda^2\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)-\frac{3}{2}\lambda^2\left(\tanh\left(\frac{1}{2}\lambda\xi\right)-1\right)^2, \tag{32}$$

$$u_2(x,t) = -3\lambda^2\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)-\frac{3}{2}\lambda^2\left(\coth\left(\frac{1}{2}\lambda\xi\right)-1\right)^2, \tag{33}$$

Notes

where $\xi = x - ct$.

IV. Graphical Illustration of Some Obtained Solutions

We make graphs of obtained solutions, so that they can represent the importance of each obtained solution and physically interpret the importance of parameters. Some of our obtained traveling wave solutions are represented in Figure 1-Figure 4 with the aid of Maple:

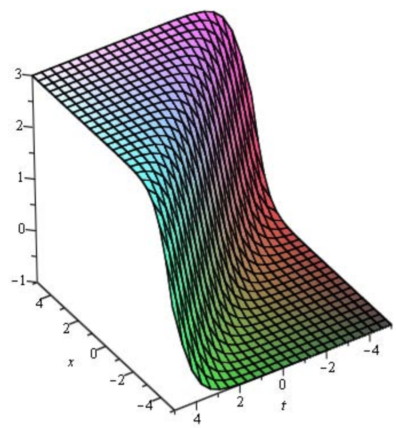

Figure (1) : Profile of Eq. (24) with $k=2, \lambda=1, a=1, b=1, \alpha=2, \omega=2$ and $y=0$.

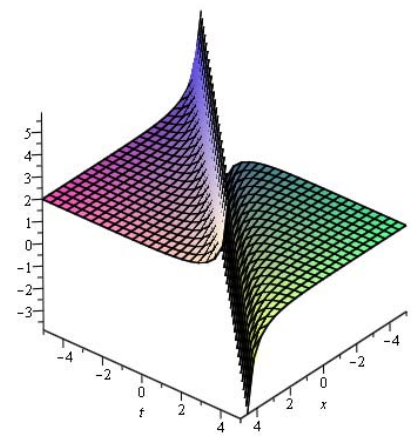

Figure (2) : Profile of Eq. (27) with $k=1, \lambda=-1, a=1, b=1, \alpha=1, \omega=-1$ and $y=0$.

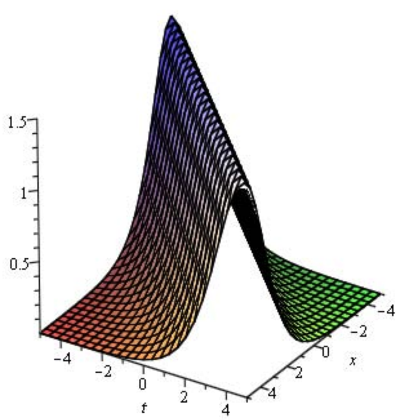

Figure (3) : Profile of Eq. (32) with $c=1, \lambda=1$.

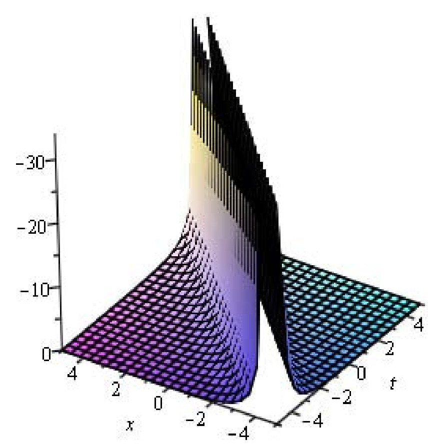

Figure (4) : Profile of Eq. (33) with $c=1, \lambda=1$.

V. Conclusion

The Bernoulli Sub-ODE method presented in this article has been applied to the NLEEs through the Konopelchenko-Dubrovsky Coupled System equation and the (1+1)-dimensional nonlinear Ostrovsky equation for finding the exact solutions and the solitary wave solutions of these equations which appeal the attention of many Mathematicians. This simple and powerful method can be more successfully applied to study nonlinear partial differential equations, which frequently arise in engineering sciences, mathematical physics and other scientific real-time application fields.

Notes

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Statistical Analysis for the Presence of pH Content of Ground Water at Different Locations of Industrial area at Ghazipur in India

By Farid Ansari & Salahuddin
P.D.M College of Engineering, India

Abstract - In this paper, we discuss about the recently collected sample of ground water at different locations at Ghazipur and its experimental analysis in laboratory for the presence of pH content. Also, we represents the data graphically and interpreted the data using the method called analysis of variance. Further, we analyze our findings with the established results. Lastly we concluded that the samples does not depend on locations but it depends on months.

Keywords : *ground water; analysis of variance; graphical representation.*

GJSFR-F Classification : *MSC 2010: 62J10, 62H86, 62P12*

STATISTICAL ANALYSIS FOR THE PRESENCE OF PH CONTENT OF GROUND WATER AT DIFFERENT LOCATIONS OF INDUSTRIAL AREA AT GHAZIPUR IN INDIA

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Statistical Analysis for the Presence of pH Content of Ground Water at Different Locations of Industrial area at Ghazipur in India

Farid Ansari [α] & Salahuddin [σ]

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I. Introduction

a) Quality of Ground Water

The chemical and biological character of ground water is acceptable for most uses. The quality of ground water in some parts of the country, particularly shallow ground water, is changing as a result of human activities. Ground water is less susceptible to bacterial pollution than surface water because the soil and rocks through which ground water flows screen out most of the bacteria. Bacteria, however, occasionally find their way into ground water, sometimes in dangerously high concentrations. But freedom from bacterial pollution alone does not mean that the water is fit to drink. Many unseen dissolved mineral and organic constituents are present in ground water in various concentrations. Most are harmless or even beneficial; though occurring infrequently, others are harmful, and a few may be highly toxic. Water is a solvent and dissolves minerals from the rocks with which it comes in contact.

Ground water may contain dissolved minerals and gases that give it the tangy taste enjoyed by many people. Without these minerals and gases, the water would taste flat. The most common dissolved mineral substances are sodium, calcium, magnesium, potassium, chloride, bicarbonate, and sulfate. In water chemistry, these substances are called common constituents.

Water typically is not considered desirable for drinking if the quantity of dissolved minerals exceeds 1,000 mg/L (milligrams per liter). Water with a few thousand mg/L of dissolved minerals is classed as slightly saline, but it is sometimes used in areas where less-mineralized water is not available. Water from some wells and springs contains very large concentrations of dissolved minerals and cannot be tolerated by humans and other animals or plants.

Authors α σ : P.D.M College of Engineering, Bahadurgarh, Haryana, India. E-mails : checkfarid@gmail.com, vsludn@gmail.com

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Dissolved mineral constituents can be hazardous to animals or plants in large concentrations; for example, too much sodium in the water may be harmful to people who have heart trouble. Boron is a mineral that is good for plants in small amounts, but is toxic to some plants in only slightly larger concentrations. Water that contains a lot of calcium and magnesium is said to be hard. The hardness of water is expressed in terms of the amount of calcium carbonate-the principal constituent of limestone-or equivalent minerals that would be formed if the water were evaporated. Water is considered soft if it contains 0 to 60 mg/L of hardness, moderately hard from 61 to 120 mg/L, hard between 121 and 180 mg/L, and very hard if more than 180 mg/L. Very hard water is not desirable for many domestic uses; it will leave a scaly deposit on the inside of pipes, boilers, and tanks. Hard water can be softened at a fairly reasonable cost, but it is not always desirable to remove all the minerals that make water hard. Extremely soft water is likely to corrode metals, although it is preferred for laundering, dishwashing, and bathing.

Ground water, especially if the water is acidic, in many places contains excessive amounts of iron. Iron causes reddish stains on plumbing fixtures and clothing. Like hardness, excessive iron content can be reduced by treatment. A test of the acidity of water is pH, which is a measure of the hydrogen-ion concentration. The pH scale ranges from 0 to 14. A pH of 7 indicates neutral water; greater than 7, the water is basic; less than 7, it is acidic. A one unit change in pH represents a 10-fold difference in hydrogen-ion concentration. For example, water with a pH of 6 has 10 times more hydrogen-ions than water with a pH of 7. Water that is basic can form scale; acidic water can corrode. According to U.S. Environmental Protection Agency criteria, water for domestic use should have a pH between 5.5 and 9.

b) ANOVA is Defines as

ANOVA is a statistical tool used in several ways to develop and confirm an explanation for the observed data. It is an extension of the t-test, which is used in determining the nonsignificance of difference of three or more group of values. In practice , there are several types of ANOVA depending on the number of treatments and the way they are applied to the subject in the experiment.

(i) One way ANOVA
(ii) Two way ANOVA
(iii) Factorial ANOVA
(iv) Mixed design ANOVA
(v) Multivariate analysis of variance(MANOVA)

The calculations of ANOVA can be characterized as computing a number of means and variances, dividing two variances and comparing the ratio to a handbook value to determine statistical significance.

The F-test is used for comparisons of the components of the total deviation. For example, in one-way or single factor ANOVA, statistical significance is tested for by comparing the F test statistic
F=Variance between samples/ Variance within samples.

The textbook method of concluding the hypothesis test is to compare the observed value of F with the critical value of F determined from tables. The critical value of F is a function of the numerator degrees of freedom, the denominator degrees of freedom and the significance level (α). If $F \geq F_{Critical}$ (Numerator DF, Denominator DF, α) then reject the null hypothesis.

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II. Main Data of the Samples (After Laboratory Analysis)

Presence of pH content in ground water at different locations of Ghazipur

Months / Area Codes												
CODE	**JAN**	**FEB**	**MAR**	**APR**	**MAY**	**JUN**	**JUL**	**AUG**	**SEP**	**OCT**	**NOV**	**DEC**
GWS1	7.9	7.74	7.4	7.64	7.1	7.25	7.6	7.8	7.81	7.84	7.83	7.8
GWS2	7.9	7.64	7.5	7.4	7.45	7.8	7.71	7.8	7.85	7.9	8	7.9
GWS3	7.34	7.2	6.6	6.64	6.78	7	7.3	7.4	7.32	7.1	7.12	7
GWS4	7.2	7.2	7.6	7.42	7.38	7.9	7.84	7.45	7.4	7.4	7.2	7.8
GWS5	7.84	7.54	7.87	7.1	7.64	7.34	7.92	7.94	8.2	8.24	8.2	7.75
GWS6	7.65	7.34	7.5	7.8	7.22	7.83	7.64	7.56	7.64	7.54	7.32	7.4
GWS7	7.82	7.6	7.75	7.8	7.9	7.8	7.85	7.7	7.65	7.65	7.44	7.64
GWS8	7.2	7.16	7.6	7.6	7.85	7.45	7.42	7.8	7.45	7.6	7.2	7.45
GWS9	7.6	7.5	7.82	7.8	7.2	7.82	7.62	7.62	7.6	7.45	7.55	7.64
GWS10	7.2	7.65	7.6	7.27	7.6	7.25	7.51	7.65	7.2	7.25	7.4	7.4

GWS1= Distillery campus, GWS2 = Bio-composting Yard, GWS3= Rampur Bantara village ,
GWS4 = Nandganj Railway Station, GWS5 = Attarsua Village
GWS6 = Reonsa village , GWS7 = Dhamupur, GWS8 = Saheri village
GWS9 = Kusmhi Kala village, GWS10 = Husainpur village

III. Graphical Representation of the Data

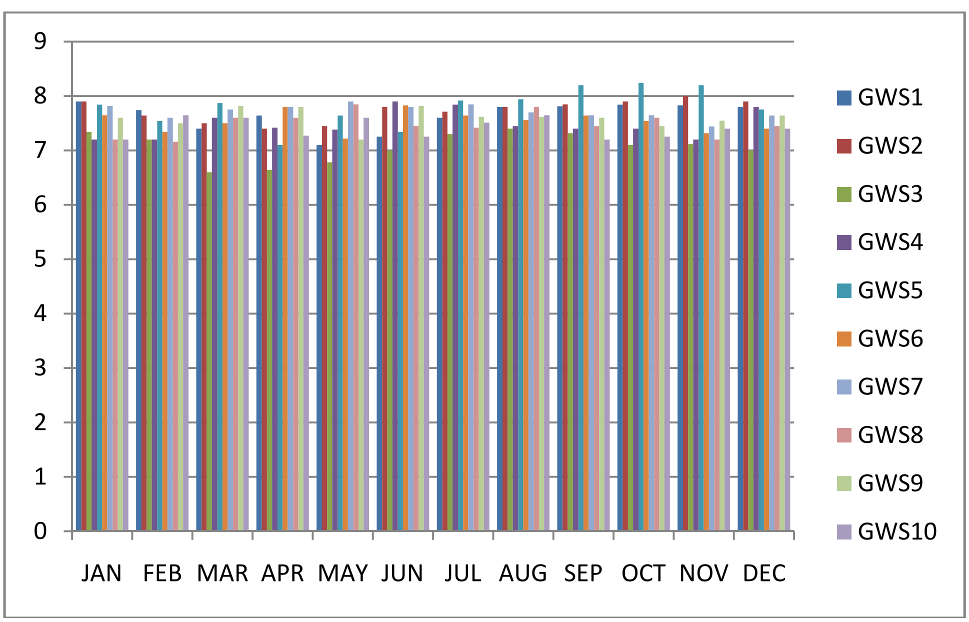

Fig : Clustered Bar representation of pH content

Now we have analysis the data by using statistical tool anova that is analysis of variance. We have analysis two way anova to conclude that if there is any significant difference between the samples or not.

IV. Analysis the Data using Two Way Anova

Sum of squares between areas= 4.764304
Sum of squares between months = 0.697649
Total sum ofsquares =10.60318

Sources of value	Sum of squares	Degrees of freedom	Mean square(variance)
Between Area	4.764304	11	0.063423
Between month	0.697649	9	0.529367
Residual	5.141226	99	0.051932
Total	10.60318	119	

Let us take the Hypothesis that there is no signaficance difference of pH content between the areas and months.
First we compare the variance of areas with the variance of residual.
F_1= =1.22173.
The table value of F for υ_1=11 and υ_2=99 at 5% level of significance is 1.886684.
.The calculated value is less than table value and we conclude that the pH content of different areas are same.
Now , let us compare the variance according to months with the variance of residuals.
F_2= 10.19355.
The table value of F for υ_3=9 and υ_2=99 at 5% level of significance is 1.975806.
The calculated value is greater than table value and we conclude that pH content of different areas changes according to months. That is pH content depends on month.That is pH content in different month is different.

V. Conclusion

It is concluded that the pH content of ground water does not depend on locations but it depends on months. That is only at different months pH content changes.

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Some Properties of Finite Boolean Algebra

By Md. Ekramul Islam, Md. Rezwan Ahamed Fahim, Arjuman Ara & Md. Hannan Miah

Pabna University of Science and Technology, Bangladesh

Abstract - In this paper we examine the relationship between the Ideal and Boolean Algebra of Lattice. Here the main result is that principal ideal (atom), principal dual ideal (filter) and also their product are Boolean algebra.

Keywords : *principal ideal, principal dual ideal, boolean algebra.*

GJSFR-F Classification : *MSC 2010: 03G05, 94C10*

SOME PROPERTIES OF FINITE BOOLEAN ALGEBRA

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Some Properties of Finite Boolean Algebra

Md. Ekramul Islam [α], Md. Rezwan Ahamed Fahim [σ], Arjuman Ara [ρ] & Md. Hannan Miah [ω]

Abstract - In this paper we examine the relationship between the Ideal and Boolean Algebra of Lattice. Here the main result is that principal ideal (atom), principal dual ideal (filter) and also their product are Boolean algebra.

Keywords : *principal ideal, principal dual ideal, boolean algebra.*

I. Introduction

Boolean algebra, as developed in 1854 by George Boole in his book An Investigation of the Laws of Thought, is a variant of ordinary elementary algebra differing in its values, operations, and laws. Instead of the usual algebra of numbers, Boolean algebra is the algebra of truth values 0 and 1, or equivalently of subsets of a given set. The operations are usually taken to be conjunction $\wedge$, disjunction $\vee$, and negation $\neg$, with constants 0 and 1. And the laws are definable as those equations that hold for all values of their variables, for example $x \vee (y \wedge x) = x$. Applications include mathematical logic, digital logic, computer programming, set theory, and statistics.

Boole's algebra predated the modern developments in abstract algebra and mathematical logic; it is however seen as connected to the origins of both fields. In an abstract setting, Boolean algebra was perfected in the late 19th century by Jevons, Schröder, Huntington, and others until it reached the modern conception of an (abstract) mathematical structure. For example, the empirical observation that one can manipulate expressions in the algebra of sets by translating them into expressions in Boole's algebra is explained in modern terms by saying that the algebra of sets is *a* Boolean algebra (note the indefinite article). In fact, M. H. Stone proved in 1936 that every Boolean algebra is isomorphic to a field of sets.

In the 1930s, while studying switching circuits, Claude Shannon observed that one could also apply the rules of Boole's algebra in this setting, and he introduced **switching algebra** as a way to analyze and design circuits by algebraic means in terms of logic gates. Shannon already had at his disposal the abstract mathematical apparatus, thus he cast his switching algebra as the two-element Boolean algebra. In circuit engineering settings today, there is little need to consider other Boolean algebras, thus "switching algebra" and "Boolean algebra" are often used interchangeably. Efficient implementation of Boolean functions is a fundamental problem in the design of combinatorial logic circuits.

Authors α σ : Department of Mathematics, Pabna University of Science and Technology, Bangladesh.
Author ρ : Department of Textile Engineering, City University, Bonani, Dhaka-1213, Bangladesh.
Author ω : Department of Mathematics, Begum Rokeya University, Rangpur, Bangladesh.

II. Material and Method

2.1 Definition: A sub-lattice H of a lattice L is called a **convex sub-lattice** if for any $a, b \in H$, $a < c < b$ implies $c \in H$.

Example: in the fig below (fig-1) $\{0, a, c\}, \{0, b, c\}$ are convex lattices.

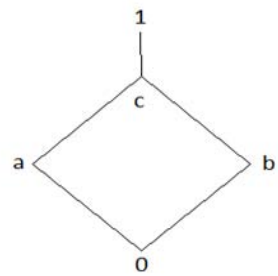

Fig-1 : Convex Lattice

2.2 Definition: Let L be a lattice and I a non-empty subset of L. then I is called an **ideal** of L if

i. I is a sub-lattice of L ii. for any $a \in I, x \in L, x \wedge a \in I$.

Example: in fig below (fig-2) $\{0, a\}, \{0, b\}, \{0, a, b, c\}$ are ideals:

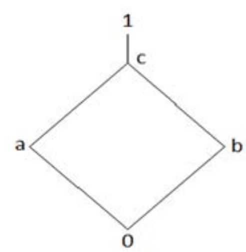

Fig-2 : Ideal

2.3 Definition: An ideal of a lattice L is generated by a single element is called a **principal ideal**. If $a \in L$ then the ideal generated by a principal ideal denoted by $(a]$. Infact $(a] = \{x \in L : x \leq a)$. The set of all ideals of a lattice L is denoted by $I(L)$.

2.4 Definition: A non-empty subset D of a lattice L is called a **dual ideal** or **filters** if

i. For all $a, b \in D, a \wedge b \in D$ ii. For $a \in D, a \leq t, t \in L$ implies $t \in D$.

Example: In the lattice below (fig-3) $\{1\}, \{1, c\}, \{1, c, a\}, \{1, c, b\}, \{1, c, a, b, 0\}$ are dual lattice:

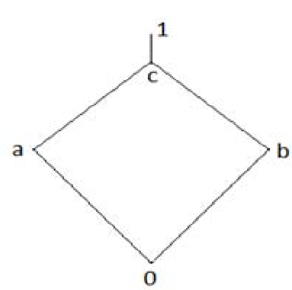

Fig-3 : Dual Lattice

2.5 Definition: Let L be a lattice and $a \in L$. Then $[a) = \{x \in L : a \leq x\}$ form a dual ideal. This ideal is called a **principal dual ideal**.

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2.6 Definition: Let L_1 and L_2 be two lattices then the mapping $f: L_1 \to L_2$ is called a **meet homomorphism** if for all $a, b \in L$, $f(a \wedge b) = f(a) \wedge f(b)$ and is called **join homomorphism** if for all $a, b \in L$, $f(a \vee b) = f(a) \vee f(b)$ and f called a **homomorphism** if f is both meet and join homomorphism.

2.7 Definition: A mapping $f: L_1 \to L_2$ is called an **isomorphism** if it is homomorphism, one-one and onto. It is denoted by $L_1 \cong L_2$.

2.8 Definition: Let L be a bounded lattice that is a lattice L with 0 and 1. for an element $a \in L$, an element $b \in L$ is called a **compliment** of a if $a \wedge b = 0$ and $a \vee b = 1$ and is denoted by $b = \neg a$. Obviously $\neg 0 = 1$ and $\neg 1 = 0$.

2.9 Definition: In a lattice if every element has a complimented, then it is called a **complimented lattice**.

2.10 Definition: A complimented distributive lattice is called a **Boolean lattice**.

2.11 Algebraic definition of Boolean algebra: An algebra $B = < L, \wedge, \vee, \neg, 0, 1 >$, where L is a non-empty set together with two binary operations $\wedge$ and $\vee$ and a unary operation $\neg$ and null operations 0 and 1, is called a **Boolean algebra** if it satisfies the following conditions:

a) $\wedge, \vee$ are independent

b) $\wedge, \vee$ are associative

c) $\wedge, \vee$ are commutative

d) $\wedge, \vee$ are satisfy absorption law

e) For all $a, b, c \in L$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$.

f) for all $a \in L$, there exists $\neg a \in L$ such that $a \wedge \neg a = 0$ and $a \vee \neg a = 1$.

Example : Let $B = \{0, a, b, 1\}$. Define $\wedge, \vee, \neg$ by following:

$\vee$	0	a	b	1		$\wedge$	0	a	b		$\neg$	
0	0	a	b	1		0	0	0	0		0	1
a	a	a	1	1		a	0	a	0		a	b
b	b	1	b	1		b	0	0	b		b	a
1	1	1	1	1		1	0	a	b		1	0

Then B form a Boolean algebra under those operations.

III. Results and Discussion

3.1 Theorem: A principal ideal of a Boolean algebra is again a Boolean algebra.

Proof: Let $B = < B, \leq, \neg, \wedge, \vee, 0, 1 >$ be a Boolean algebra and $a \in B$.

A principal ideal of B generated by a, $(a]$, is: $(a] = \{x \in B : x \leq a\}$

Now let an algebra $((a] =< (a], \leq_{(a]}, \neg_{(a]}, \wedge_{(a]}, \vee_{(a]}, 0_{(a]}, 1_{(a]} >)$ with the relation of B as

1. $\leq_{(a]} = \leq \partial(a]$
2. $\neg_{(a]} = \{< x, \neg x \wedge a >: x \in (a]\}$
3. $\wedge_{(a]} = \wedge\partial(a]$
4. $\vee_{(a]} = \vee\partial(a]$
5. $0_{(a]} = 0$
6. $1_{(a]} = a$

Now if $x, y \in (a]$, then $x, y \leq a$ hence $(x \wedge y) \leq a$ and $(x \vee y) \leq a$, hence $(x \wedge y) \in (a]$ and $(x \vee y) \in (a]$. Hence $< (a], \leq_{(a]}, \wedge_{(a]}, \vee_{(a]}, 0_{(a]} >$ is a substructure of $< B, \leq, \wedge, \vee, 0 >$ and $< (a], \leq_{(a]}, \wedge_{(a]}, \vee_{(a]} >$ is a sub-lattice of $< B, \leq, \wedge, \vee >$.

Again $0_{(a]}$ is the minimum of (a]. $1_{(a]} = a$, which is, obviously, the maximum of (a].

Hence $< (a], \leq_{(a]}, \wedge_{(a]}, \vee_{(a]}, 0_{(a]}, 1_{(a]} >$ is bounded by $0_{(a]}$ and $1_{(a]}$.

Now as $< B, \leq, \wedge, \vee >$ is distributive, it follows that $< (a], \leq_{(a]}, \wedge_{(a]}, \vee_{(a]} >$ is distributive. Hence (a] is a bounded distributive lattice. So we only need to prove that (a] is complemented, i.e. that $\neg_{(a]}$ is complementation on (a].

First: (a] is closed under $\neg_{(a]}$. Because $\neg x \wedge a \leq a$, for any $x \in B$, hence also for any $x \leq a$.

Secondly: $\neg_{(a]}$ holds the laws of $0_{(a]}$ and $1_{(a]}$: Let $x \in (a]$ then

$$x \wedge_{(a]} \neg_{(a]}(x) = x \wedge (\neg x \wedge a) \quad = (x \wedge \neg x) \wedge (x \wedge a) = 0 \wedge (x \wedge a) = 0 = 0_{(a]}$$

And $$x \vee_{(a]} \neg_{(a]}(x) = x \vee (\neg x \wedge a) = (x \vee \neg x) \wedge (x \vee a) = 1 \wedge (x \vee a) = x \vee a = a = 1_{(a]}$$

Thus, indeed $\neg_{(a]}$ is complementation on (a]. (Proved)

3.2 Theorem: Principal dual ideal of a Boolean algebra is again a Boolean algebra.

Proof: Let $B =< B, \leq, \neg, \wedge, \vee, 0, 1 >$ be a Boolean algebra and $a \in B$. A principal dual ideal of B generated by a, $[a)$, is: $[a) = \{x \in B : a \leq x\}$. Now an algebra

$[a) =< [a), \leq_{[a)}, \neg_{[a)}, \wedge_{[a)}, \vee_{[a)}, 0_{[a)}, 1_{[a)} >)$ with the relation of B as

1. $\leq_{[a)} = \leq \partial[a)$
2. $\neg_{[a)} = \{< x, \neg x \vee a >: x \in [a)\}$
3. $\wedge_{[a)} = \wedge\partial[a)$

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4. $\vee_{[a)} = \vee\partial[a)$

5. $0_{[a)} = a$

6. $1_{[a)} = 1$

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Let $x, y \in [a)$, then $a \leq x, y$, hence $a \leq (x \wedge y)$ and $a \leq (x \vee y)$, hence $(x \wedge y) \in [a)$ and $(x \vee y) \in [c)$. Hence $< [a), \leq_{[a)}, \wedge_{[a)}, \vee_{[a)}, 1_{[a)} >$ is a substructure of $< B, \leq, \wedge, \vee, 1 >$ and also $< [a), \leq_{[a)}, \wedge_{[a)}, \vee_{[a)} >$ is a sub-lattice of $< B, \leq, \wedge, \vee >$. Now $1_{[a)}$ is the maximum of [a) and $0_{[a)} = a$, which is, obviously, the minimum of [a). Hence $< [a), \leq_{[a)}, \wedge_{[a)}, \vee_{[a)}, 0_{[a)}, 1_{[a)} >$ is bounded by $0_{[a)}$ and $1_{[a)}$.

As $< B, \leq, \wedge, \vee >$ is distributive, it follows that $< [a), \leq_{[a)}, \wedge_{[a)}, \vee_{[a)} >$ is distributive.

Hence [a) is a bounded distributive lattice. So we only need to prove that [a) is complemented, i.e. that $\neg_{[a)}$ is complementation on [a).

First: [a) is closed under $\neg_{[a)}$. This is obvious, since obviously $a \leq \neg x \vee a$ for any $x \in B$, hence also for any x such that $a \leq x$.

Secondly, $\neg_{[a)}$ respects the laws of $0_{[a)}$ and $1_{[a)}$:

Let $x \in [a)$ then,

$x \wedge_{[a)} \neg_{[a)}(x) = x \wedge (\neg x \vee a) = (x \wedge \neg x) \vee (x \wedge a) = 0 \vee (x \wedge a) = x \wedge a = a = 0_{[a)}$

$x \vee_{[a)} \neg_{[a)}(x) = x \vee (\neg x \vee a) \;= (x \vee x\neg) \vee (x \vee a) = 1 \vee (x \vee a) = 1 = 1_{[a)}$

Thus, indeed $\neg_{[a)}$ is complementation on [a). (proved).

3.3 Lemma: If $a \neq 1$ **then** $(a] \cap [\neg a) = \phi$

Proof: Let $x \in (a] \cap [\neg a)$. Then

$x \leq a$ and $\neg a \leq x$. Then $x \leq a$ and $\neg x \leq a$.

Then $x \vee \neg x \leq a \Rightarrow 1 \leq a \Rightarrow a = 1$

Hence if $a \neq 1$ then $(a] \cap [\neg a) = \phi$.

3.4 Theorem: (a] **and** $[\neg a)$ **are isomorphic**.

Proof: If $a = 1$, then $(a] = [\neg a) = B$. So clearly they are isomorphic. So let $a \neq 1$. We define: $f : (a] \to [\neg a)$ by: for every $x \in (a] : f(x) = x \vee \neg a$.

1. f is a function:

Since for every $x \in B$, $\neg a \leq x \vee a$, also for every $x \in (a] : \neg a \leq x \vee a$. Hence for every $x \in (a] : f(x) \in [\neg a)$, hence f is indeed a function from (a] into $[\neg a)$.

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2. f is onto:

Let $y \in [\neg a)$.

Then $\neg a \leq y \Rightarrow \neg y \leq a$ .... (1). Hence $\neg y \in (a]$.

Now $\neg_{[a)} y = \neg(\neg y) \wedge a = y \wedge a$. Hence $(y \wedge a) \in (a]$.

Again $f(y \wedge a) = (y \wedge a) \vee \neg a = (y \vee \neg_{[a)} a) \wedge (a \vee \neg a) = (y \vee \neg a) \wedge 1 = y \vee \neg a = y$ [from (1)]

Hence f is onto.

3. f is one-one:

Let $f(x_1) = f(x_2)$.

Then $x_1 \vee \neg a = x_2 \vee \neg a \Rightarrow \neg(x_1 \vee \neg a) = \neg(x_2 \vee \neg a)$

$\Rightarrow \neg x_1 \wedge a = \neg x_2 \wedge a$ [by De-Morgan's law]

$\Rightarrow \neg_{(a]} x_1 = \neg_{(a]} x_2$

Hence these are the relative complements of x_1 and x_2 in Boolean algebra $(a]$.

Hence $x_1 = x_2$.

Hence h is one-one.

4. f is homomorphism:

for $0_{(a]} = 0$, $f(0) = 0 \vee \neg a = \neg a = 0_{(\neg a]}$

for $1_{(a]} = a$, $f(a) = a \vee \neg a = 1 = 1_{[\neg a)}$

$f(x \wedge y) = (x \wedge y) \vee \neg a = (x \vee \neg a) \wedge (y \vee \neg a) = f(x) \wedge f(y)$

$f(x \vee y) = (x \vee y) \vee \neg a = (x \vee \neg a) \vee (y \vee \neg a) = f(x) \vee f(y)$

And $f(\neg_{(a]} x) = f(\neg x \wedge a) = (\neg x \wedge a) \vee \neg a = \neg(x \vee \neg a) \vee \neg a = \neg_{(\neg a]}(x \vee \neg a) = \neg_{(\neg a]}(f(x))$

Hence f is homomorphism.

Thus, indeed, h is an isomorphism. (Proved).

3.5 Lemma: If a is an atom in B, then for every $x \in B - \{0\} : a \leq x$ or $a \leq \neg x$

Proof: Let a be an atom in B. Suppose that $\neg(a \leq x)$. Then $(a \wedge x) \neq a$. But $a \wedge x \leq a$. Since a is an atom, that means that $a \wedge x = 0$. But that means that $a \leq \neg x$.

3.6 Corollary: If a is an atom in B, then $(\neg a] \cup [a) = B$.

Proof: This follows from lemma 5: Let $x \in B$ and $x \notin [a)$. Then $\neg(a) \leq x$. Hence by lemma 5 $a \leq \neg x$, and that means that $x \leq \neg a$, hence $x \in (\neg a]$.

3.7 Theorem: Let A and B be two Boolean algebra, then the product of A and B, $A \times B$, is a Boolean algebra.

Notes

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Proof: Since **A** and **B** are Boolean algebras. The **product** of **A** and **B**, $A \times B$, is given by:

$A \times B =< B_\times, \leq_\times, \neg_\times, \wedge_\times, \vee_\times, 0_\times, 1_\times >$ where

1. $B_\times = A \times B$

2. $\leq_\times = \{<< a_1, b_1 >, < a_2, b_2 >>: a_1 \leq_A a_2 \text{ and } b_1 \leq_B b_2\}$

3. For every

$< a, b > \in A \times B : \neg_\times(< a, b >) =< \neg_A a, \neg_B b >$

4. For every

$<< a_1, b_1 >, < a_2, b_2 >> \in A \times B :<< a_1, b_1 > \wedge_\times < a_2, b_2 >>=< a_1 \wedge_A a_2, b_1 \wedge_B b_2 >$

5. For every $<< a_1, b_1 >, < a_2, b_2 >> \in A \times B :<< a_1, b_1 > \vee_\times < a_2, b_2 >>=< a_1 \vee_A a_2, b_1 \vee_B b_2 >$

6. $0_\times =< 0_A, 0_B >$

7. $1_\times =< 1_A, 1_B >$

(a) $\leq_\times$ is a partial order.

Reflexive: Since for every $a \in A : a \leq_A a$ and for every $b \in B : b \leq_B b$, for every

$< a, b > \in A \times B :< a, b > \leq_\times < a, b >$

Anti-symmetric: Let $< a_1, b_1 > \leq_\times < a_2, b_2 >$ and . $< a_2, b_2 > \leq_\times < a_1, b_1 >$

Then $a_1 \leq_A a_2$ and $b_1 \leq_B b_2$ and $a_2 \leq_A a_1$ and $b_2 \leq_B b_1$, Hence $a_1 = a_2$ and $b_1 = b_2$, hence $< a_1, b_1 > = < a_2, b_2 >$

Transitive: Let $< a_1, b_1 > \leq_\times < a_2, b_2 >$ and. $< a_2, b_2 > \leq_\times < a_3, b_3 >$. Then $a_1 \leq_A a_2$ and $b_1 \leq_B b_2$ and $a_2 \leq_A a_3$ and $b_2 \leq_B b_3$, hence $a_1 \leq_A a_3$ and $b_1 \leq_B b_3$, hence $< a_1, b_1 > \leq_\times < a_3, b_3 >$

(b) $< a_1, b_1 > \wedge_\times < a_2, b_2 > = < a_1 \wedge_A a_2, b_1 \wedge_B b_2 >$:

$a_1 \wedge_A a_2 \leq_A a_1$, $a_1 \wedge_A a_2 \leq_A a_2$, $b_1 \wedge_B b_2 \leq_B b_1$, $b_1 \wedge_B b_2 \leq_B b_2$

Hence

$< a_1, b_1 > \wedge_\times < a_2, b_2 > \leq_\times < a_1, b_1 >$

$< a_1, b_1 > \wedge_\times < a_2, b_2 > \leq_\times < a_2, b_2 >$

Let $< a, b > \leq_\times < a_1, b_1 >$ and, $< a, b > \leq_\times < a_2, b_2 >$

Then $a \leq_A a_1$ and $b \leq_B b_1$ and $a \leq_A a_2$ and $b \leq_B b_2$,

hence $a \leq_A a_1 \wedge_A a_2$ and $b \leq_B b_1 \wedge_B b_2$, hence $< a, b > \leq_\times < a_1, b_1 > \wedge_\times < a_2, b_2 >$

Hence $\wedge_\times$ is meet in $\leq_\times$.

Similarly $\vee_\times$ is join in $\leq_\times$.

(c). $0_\times = <0_A, 0_B>$:

Since for every $a \in A$, $0_A \leq_A a$ and for every $b \in B$, $0_B \leq_B b$, for every $<a,b> \in A \times B$, $<0_A, 0_B> \leq_\times <a,b>$. Hence $0_\times$ is the minimum under $\leq_\times$.

Similarly $1_\times$ is the maximum under $\leq_A$.

So **A** $\times$ **B** is a bounded lattice.

(d). $<a_1, b_1> \wedge_\times (<a_2, b_2> \vee_\times <a_3, b_3>) = <a_1 \wedge_A (a_2 \vee_A a_3), b_1 \wedge_B (b_2 \vee_B b_3) = <(a_1 \wedge_A a_2) \vee_A (a_1 \wedge_A a_3), (b_1 \wedge_B b_2) \vee_B (b_1 \wedge_B b_3)> = <a_1 \wedge_A a_2, b_1 \wedge_B b_2> \vee_\times <a_1 \wedge_A a_3, b_1 \wedge_B b_3> = (<a_1, b_1> \wedge_\times <a_2, b_2>) \vee_\times (<a_1, b_1> \wedge_\times <a_3, b_3>)$

So **A** $\times$ **B** is distributive.

(e). $\neg_\times$ satisfies the laws of $0_\times$ and $1_\times$:

$<a,b> \wedge_\times \neg_\times(<a,b>) = <a,b> \wedge_\times <\neg_A a, \neg_B b> = <a \wedge_A \neg_A a, b \wedge_B \neg_B b> = <0_A, 0_B> = 0_\times$.

$<a,b> \vee_\times \neg_\times(<a,b>) = <a,b> \vee_\times <\neg_A a, \neg_B b> = <a \vee_A \neg_A a, b \vee_B \neg_B b> = <1_A, 1_B> = 1_\times$.

So **A** $\times$ **B** is a Boolean algebra.

3.8 Theorem: Let B_1 and B_2 be isomorphic Boolean algebras such that $B_1 \cap B_2 = \phi$ and let h be an isomorphism between B_1 and B_2. The product of B_1 and B_2 under h, $B^h{}_{1+2}$, is a Boolean algebra.

Proof: Let us definite the **product of B_1 and B_2 under h**, $B^h{}_{1+2}$ as,

$B^h{}_{1+2} = < B^h{}_{1+2}, \leq_{1+2}, \neg_{1+2}, \wedge_{1+2}, \vee_{1+2}, 0_{1+2}, 1_{1+2} >$ where:

1. $B^h{}_{1+2} = B_1 \cup B_2$

2. $\leq_{1+2} = \leq_1 \cup \leq_2 \cup \{< b_1, b_2 >: h(b_1) \leq_2 b_2\}$

3. $\neg_{1+2}$ is defined by:

$$b \in B_{1+2} : \neg_{1+2}(b) = \begin{cases} \neg_1(h(b)), \text{if } b \in B_1 \\ \\ \neg_1(h^{-1}(b)), \text{if } b \in B_2 \end{cases}$$

4. $\wedge_{1+2}$ is defined by:

$$a, b \in B_{1+2} : a \wedge_{1+2} b = \begin{cases} a \wedge_1 b, & \text{if } a, b \in B_1 \\ a \wedge_2 b, & \text{if } a, b \in B_2 \\ a \wedge_1 h^{-1}(b), & \text{if } a \in B_1, b \in B_2 \end{cases}$$

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5. $\vee_{1+2}$ is defined by:

$$a,b \in B_{1+2} : a \vee_{1+2} b = \begin{cases} a \vee_1 b, & \text{if } a,b \in B_1 \\ a \vee_2 b, & \text{if } a,b \in B_2 \\ h(a) \vee_1 b, & \text{if } a \in B_1, b \in B_2 \end{cases}$$

Notes

6. $0_{1+2} = 0_1$.

7. $1_{1+2} = 1_1$.

1. $\leq_{1+2}$ is a partial order:

$\leq_{1+2}$ is reflexive:

If $a \in B_1 : a \leq_1 a$, hence, $a \leq_{1+2} a$

If $a \in B_2 : a \leq_2 a$, hence, $a \leq_{1+2} a$

$\leq_{1+2}$ is anti-symmetric.

Let $a \leq_{1+2} b$ and $b \leq_{1+2} a$. This is only possible if $a,b \in B_1$ or $a,b \in B_2$.

In the first case $a \leq_1 b$ and $b \leq_1 a$, hence a=b. In the second case $a \leq_2 b$ and $b \leq_2 a$, hence a=b.

$\leq_{1+2}$ is transitive.

Let $a \leq_{1+2} b$ and $b \leq_{1+2} c$

If $a,b,c \in B_1$, then $a \leq_1 b$ and $b \leq_1 c$, hence $a \leq_1 c$, and $a \leq_{1+2} c$.

If $a,b,c \in B_2$, then $a \leq_2 b$ and $b \leq_2 c$, hence $a \leq_2 c$, and $a \leq_{1+2} c$.

If $a \in B_1$ and $b,c \in B_2$, then $h(a) \leq_2 b$ and $b \leq_2 c$. Then $h(a) \leq_2 c$ and $a \leq_{1+2} c$.

If $a,b \in B_1$ and $c \in B_2$, then $a \leq_1 b$ and $h(b) \leq_2 c$.

Since h is an isomorphism, this means that $h(a) \leq_2 h(b)$, and hence $h(a) \leq_2 c$. Hence $a \leq_{1+2} c$.

2. $\wedge_{1+2}$ is meet under $\leq_{1+2}$.

If $a,b \in B_1 : a \wedge_{1+2} b = a \wedge_1 b$, which is meet under $\leq_1$, and $\leq_1 = \leq_{1+2} \partial B_1$. If $a,b \in B_2 : a \wedge_{1+2} b = a \wedge_2 b$, which is meet under $\leq_2$, and $\leq_2 = \leq_{1+2} \partial B_2$.

If $a \in B_1, b \in B_2$, then $a \wedge_{1+2} b = a \wedge_1 h^{-1}(b)$, $a \wedge_1 h^{-1}(b) \leq_1 a$ and $a \wedge_1 h^{-1}(b) \leq_1 h^{-1}(b)$.

By definition of $\leq_{1+2}$, $h^{-1}(b) \leq_{1+2} h(h^{-1}(b))$. So $h^{-1}(b) \leq_{1+2} b$. Then $a \wedge_1 h^{-1}(b) \leq_{1+2} b$. This means that $a \wedge_{1+2} b \leq_{1+2} a$ and $a \wedge_{1+2} b \leq_{1+2} b$.

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If $x \le_{1+2} a$ and $x \le_{1+2} b$, then $x \le_1 a$ and $h(x) \le_2 b$. Since h is an isomorphism, then $h^{-1}(h(x)) \le_1 h^{-1}(b)$ i.e. $x \le_1 h^{-1}(b)$, then $x \le_1 a \wedge_1 h^{-1}(b)$. Hence $x \le_{1+2} a \wedge_{1+2} b$. So indeed $\wedge_{1+2}$ is meet under $\le_{1+2}$.

3. $\vee_{1+2}$ is join under $\le_{1+2}$.

If $a,b \in B_1 : a \vee_{1+2} b = a \vee_1 b$, which is join under $\le_1$, and $\le_1 = \le_{1+2} \partial B_1$. If $a,b \in B_2 : a \vee_{1+2} b = a \vee_2 b$, which is join under $\le_2$, and $\le_2 = \le_{1+2} \partial B_2$. If $a \in B_1, b \in B_2$, then $a \vee_{1+2} b = h(a) \vee_2 b$.

$b \le_2 h(a) \vee_2 b$ and $h(a) \le_2 h(a) \vee_2 b$.

As we have seen $a \le_{1+2} h(a)$, hence $a \le_{1+2} h(a) \vee_2 b$. So $a \le_{1+2} a \vee_{1+2} b$ and $b \le_{1+2} a \vee_{1+2} b$. If $a \le_{1+2} x$ and $b \le_{1+2} x$, then $h(a) \le_{1+2} x$, hence $h(a) \vee_2 b \le_2 x$.

Hence $a \vee_{1+2} b \le_{1+2} x$. So indeed $\vee_{1+2}$ is join under $\le_{1+2}$.

4. $0_{1+2} = 0_1$

If $a \in B, 0_1 \le_1 a$. Hence $0_{1+2} \le_{1+2} a$. h is an isomorphism, so $h(0_1) = 0_2$. If $a \in B_2$, then $h(0_1) \le_2 a$, hence $0_{1+2} \le_{1+2} a$. So indeed 0_{1+2} is the minimum under $\le_{1+2}$.

Similarly, 1_{1+2} is the maximum under $\le_{1+2}$. We have proved so far that $B^h{}_{1+2}$ is a bounded lattice.

5. Distributivity: $a \wedge_{1+2} (b \vee_{1+2} c) = (a \wedge_{1+2} b) \vee_{1+2} (a \wedge_{1+2} c)$

a. Let $a,b,c \in B_1$ or $a,b,c \in B_2$, then distributivity follows from distributivity of $\wedge_1$ and $\vee_1$ and of $\wedge_2$ and $\vee_2$.

b. Let $a \in B_1$ and $b,c \in B_2$

$$a \wedge_{1+2} (b \vee_{1+2} c) = a \wedge_1 h^{-1}(b \vee_2 c) = a \wedge_1 (h^{-1}(b) \vee_1 h^{-1}(c)) = (a \wedge_1 (h^{-1}(b)) \vee_1 (a \wedge_1 h^{-1}(c))$$

$$= (a \wedge_{1+2} b) \vee_{1+2} (a \wedge_{1+2} c)$$

c. Let $b \in B_1$ and $a,c \in B_2$

$$a \wedge_{1+2} (b \vee_{1+2} c) = a \wedge_2 (h(b) \vee_2 c) = a \wedge_2 h(b)) \vee_2 (a \wedge_2 c)$$

If $a \in B_2$ and $b \in B_1$, then $a \wedge_{1+2} b = h^{-1}(a) \wedge_1 b$.

Then $h(a \wedge_{1+2} b) = h(h^{-1}(a) \wedge_1 b) = h(h^{-1}(a)) \wedge_2 h(b)) = a \wedge_2 h(b))$

Hence $(a \wedge_2 h(b)) \vee_2 (a \wedge_2 c) = h(a \wedge_{1+2} b) \vee_2 (a \wedge_2 c) = (a \wedge_{1+2} b) \vee_{1+2} (a \wedge_{1+2} c)$

d. Let $b,c \in B_1$ and $a \in B_2$

$$a \wedge_{1+2} (b \vee_{1+2} c) = a \wedge_{1+2} (b \vee_1 c) = h^{-1}(a) \wedge_1 (b \vee_1 c) = (b \wedge_1 h^{-1}(a)) \vee_1 (c \wedge_1 h^{-1}(a))$$

$$= (a \wedge_{1+2} b) \vee_{1+2} (a \wedge_{1+2} c)$$

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e. Let $a,b\in B_1, c\in B_2$

$a\wedge_{1+2}(b\vee_{1+2}c)=a\wedge_1(h^{-1}(b\vee_{1+2}c))$

If $b\in B_1, c\in B_2$, then $b\vee_{1+2}c=h(b)\vee_2 c$

Hence $h^{-1}(b\vee_{1+2}c)=h^{-1}(h(b)\vee_2 c)=h^{-1}(h(b))\vee_1 h^{-1}(c)=b\vee_1 h^{-1}(c)$

So $a\wedge_1(h^{-1}(b\vee_{1+2}c))=a\wedge_1(b\vee_1 h^{-1}(c))=(a\wedge_1 b)\vee_1(a\wedge_1 h^{-1}(c))$

$=(a\wedge_{1+2}b)\vee_{1+2}(a\wedge_{1+2}c)$

These are all the relevant cases, so $B^h{}_{1+2}$ is a distributive bounded lattice.

6. $\neg_{1+2}$ satisfies the laws of 0_{1+2} and 1_{1+2}.

If $a\in B_1$, $a\wedge_{1+2}\neg_{1+2}(a)=a\wedge_1 h^{-1}(\neg_{1+2}(a))=a\wedge_1 h^{-1}(\neg_2(h(a)))=a\wedge_1 h^{-1}(h(\neg_1 a))$

$=a\wedge_1\neg_1 a=0_1=0_{1+2}$

$a\vee_{1+2}\neg_{1+2}(a)=h(a)\vee_2\neg_{1+2}(a)=h(a)\vee_2\neg_2(h(a))=1_2=1_{1+2}$

If $a\in B_2$,

$a\wedge_{1+2}\neg_{1+2}(a)=h^{-1}(a)\wedge_1\neg_{1+2}(a)=h^{-1}(a)\wedge_1\neg_1(h^{-1}(a))=0_1=0_{1+2}$

$a\vee_{1+2}\neg_{1+2}(a)=a\vee_2 h(\neg_{1+2}(a))=a\vee_2 h(\neg_1(h^{-1}(a))=a\vee_2\neg_2(h(h^{-1}(a))=a\vee_2\neg_2 a$

$=1_2=1_{1+2}$

Thus $B^h{}_{1+2}$ is a Boolean algebra.

3.9 Theorem: Let B_1 and B_2 be isomorphic Boolean algebras such that $B_1\cap B_2=\phi$, and let h be an isomorphism between B_1 and B_2. Let {0,1} be a Boolean algebra of cardinality 2. Prove that $B^h{}_{1+2}$ is isomorphic to $B_1\times\{0,1\}$.

Proof: We define function k from $B_1\cup B_2$ into $B_1\times\{0,1\}$:

For all $x\in B_1: k(x)=<x,0>$

For all $x\in B_2: k(x)=<h^{-1}(x),1>$

1. Since h is an isomorphism between B_1 and B_2, and $B_1\cap B_2=\phi$, k is obviously a bijection between $B_1\cup B_2$ and $B_1\times\{0,1\}$.

2. If $x\in B_1$,

$k(\neg_{1+2}(x))=<h^{-1}(\neg_{1+2}(x)),1> \;=<h^{-1}(\neg_2(h(x)),\neg_{\{0,1\}}0>=h^{-1}(h(\neg_1(x))),\neg_{\{0,1\}}0>$

$=<\neg_1(x),\neg_{\{0,1\}}0$

$=\neg_\times<x,0>$

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3. $k(0_{1+2}) = k(0_1) = < 0_1, 0 > = 0_\times$

$k(1_{1+2}) = k(1_2) = < h^{-1}(1_2), 1) > \ = 1_\times$

4. k preserves meet:

If $a, b \in B_1$ then

$k(a \wedge_{1+2} b) = k(a \wedge_1 b) . = < a \wedge_1 b, 0 > = < a, 0 > \wedge_\times < b, 0 > = k(a) \wedge_\times k(b)$

If $a, b \in B_2$ then

$k(a \wedge_{1+2} b) = k(a \wedge_2 b) = < h^{-1}(a \wedge_2 b), 1 > = < h^{-1}(a) \wedge_1 h^{-1}(b), 1 >$

$= < h^{-1}(a), 1 > \wedge_\times h^{-1}(b), 1 > = k(a) \wedge_\times k(b)$

If $a \in B_1$ and $b \in B_2$ then

$k(a \wedge_{1+2} b) = k(a \wedge_1 h^{-1}(b)) = < a \wedge_1 h^{-1}(b), 0 > = < a \wedge_1 h^{-1}(b), 0 \wedge_{\{0,1\}} 1 >$

$= < a, 0 > \wedge_\times < h_{-1}(b), 1 >$

$= k(a) \wedge_\times k(b)$

5. k preserves join:

If $a, b \in B_1$ then If $a, b \in B_1$ then

$k(a \vee_{1+2} b) = k(a \vee_1 b) = < a \vee_1 b, 0 > = < a, 0 > \vee_\times < b, 0 > = k(a) \vee_\times k(b)$

If $a, b \in B_2$ then

$k(a \vee_{1+2} b) = k(a \vee_2 b) = < h^{-1}(a \vee_2 b), 1 > = < h^{-1}(a) \vee_1 h^{-1}(b), 1 >$

$= < h^{-1}(a), 1 > \vee_\times h^{-1}(b), 1 > = k(a) \vee_\times k(b)$

If $a \in B_1$ and $b \in B_2$ then

$k(a \vee_{1+2} b) = k(h(a) \vee_2 b) = < h^{-1}(h(a) \vee_2 b), 1 > = < a \vee_1 h^{-1}(b), 1 >$

$= < a \vee_1 h^{-1}(b), 0 \vee_{\{0,1\}} 1 > = < a, 0 > \vee_\times < h_{-1}(b), 1 > = k(a) \wedge_\times k(b)$

Thus indeed k is an isomorphism.

3.10 Theorem: Let B_1 and B_2 be Boolean algebras, $\neg a_1$ an atom in B_1 and $\neg a_2$ an atom in B_2, and let $(a_1]$ be isomorphic to $[a_2)$. Then B_1 and B_2 are isomorphic.

Proof: Let h_1 be the isomorphism between $(a_1]$ and $[\neg a_1)$ defined by:

for all $x \in (a_1] : h_1(x) = x \vee_{B_1} \neg_{B_1}(a_1)$

Let h_2 be the isomorphism between $(a_2]$ and $[\neg a2)$ defined by:

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for all $x \in (a_2] : h_2(x) = x \vee_{B_2} \neg_{B_2}(a_2)$

Let k the isomorphism between $(a_1]$ and $(a_2]$. Say –0,1" be a two element Boolean algebra. Then $B_1 = B_{(a_1]+[\neg a_1)}^{h_1}$ and $B_2 = B_{(a_2]+[\neg a_2)}^{h_2}$, by Theorem11.

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B_1 is isomorphic to $(a_1] \times \{0,1\}$ and B_2 is isomorphic to $(a_2] \times \{0,1\}$, by Theorem 10.

Define $g : (a_1] \times \{0,1\} \to (a_2] \times \{0,1\}$ by:

for every $< a, b > \in (a_1] \times \{0,1\} : g(< a, b >) = < k(a), b >$

It is straightforward to prove that g is an isomorphism between B_1 and B_2.

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A Summation Formula Coupled with Contiguous Relation and Recurrence Relation

By Salahuddin, M. P. Chaudhary & Upendra Kumar Pandit

P.D.M College of Engineering, India

Abstract - The main aim of the present paper is to develop a summation formula coupled with recurrence relation and contiguous relation.

Keywords : *gaussian hypergeometric function, contiguous function, recurrence relation, bailey summation theorem and legendre duplication formula.*

GJSFR-F Classification : *MSC 2010 : 33C60 , 33C70 , 33D15 , 33D50 , 33D60*

A SUMMATION FORMULA COUPLED WITH CONTIGUOUS RELATION AND RECURRENCE RELATION

Notes

A Summation Formula Coupled with Contiguous Relation and Recurrence Relation

Salahuddin [α], M. P. Chaudhary [σ] & Upendra Kumar Pandit [ρ]

Abstract - The main aim of the present paper is to develop a summation formula coupled with recurrence relation and contiguous relation.

Keywords : *gaussian hypergeometric function, contiguous function, recurrence relation, bailey summation theorem and legendre duplication formula.*

I. Introduction

Generalized Gaussian hypergeometric function of one variable is defined by

$$
{}_AF_B\left[\begin{matrix} a_1, a_2, \cdots, a_A \ ; \\ \\ b_1, b_2, \cdots, b_B \ ; \end{matrix} \quad z\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1}
$$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are positive integers.

Contiguous Relation[Andrews p.363(9.16)] is defined as follows

$$
(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; \\ c\ ; \end{matrix} \quad z\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b\ ; \\ c\ ; \end{matrix} \quad z\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1\ ; \\ c\ ; \end{matrix} \quad z\right] \tag{2}
$$

Recurrence relation of gamma function is defined as follows

$$
\Gamma(z+1) = z\ \Gamma(z) \tag{3}
$$

Legendre duplication formula[Bells & Wong p.26(2.3.1)] is defined as follows

$$
\sqrt{\pi}\ \Gamma(2z) = 2^{(2z-1)}\ \Gamma(z)\ \Gamma\left(z+\frac{1}{2}\right) \tag{4}
$$

Author α : P.D.M College of Engineering, Bahadurgarh, Haryana, India. E-mails : sludn@yahoo.com, vsludn@gmail.com
Author σ : International Scientific Research and Welfare Organization, New Delhi, India. E-mail : mpchaudhary_2000@yahoo.com
Author ρ : Former Research Scholar, Department of Mathematics, University of Delhi, New Delhi, India.

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{5}$$

$$= \frac{2^{(a-1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{6}$$

Notes

Bailey summation theorem [Prudnikov, p.491(7.3.7.8)]is defined as follows

$${}_2F_1\left[\begin{matrix} a, 1-a & ; & \frac{1}{2} \\ c & ; & 2 \end{matrix}\right] = \frac{\Gamma(\frac{c}{2})\ \Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-1}\ \Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} \tag{7}$$

II. Main Result of Summation Formula

$${}_2F_1\left[\begin{matrix} a \quad , & -a-55 & ; & \frac{1}{2} \\ c & & ; & 2 \end{matrix}\right]$$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+55}}\Big[\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+55}{2})}\Big\{42399938467509559045165393068039340032000000$$
$$-816981582916094980266931943229097771008000000a$$
$$+531794777481477000506923383221433070387200000a^2$$
$$-162082018398215746618714832524625839472640000a^3$$
$$+24988202318687598683166889415013525098496000a^4$$
$$-1724580102239552952733281118846613546803200a^5$$
$$-5972403528891101949572462638155844608a^6$$
$$+602428259830734229227406154866707724800a^7$$
$$-13700388831005615109859897326289532160a^8$$
$$-1086825613829916422011720567308912000a^9$$
$$+2405920545751233600038387495689632 0a^{10} + 1442211065939340613049446496383200a^{11}$$
$$-124157805250123397064698550309 60a^{12} - 1232305555998371359599497403000a^{13}$$
$$-71427331222549738317995778 00a^{14} + 485326427545976056159719600a^{15}$$
$$+8818391979507020394687840a^{16} - 231507529126158663378 00a^{17}$$
$$-212933521907486023644 0a^{18} - 20301435778111488000a^{19} + 35915175974517840a^{20}$$
$$+1981897331614200a^{21} + 13573636844040a^{22} + 16140524400a^{23} - 223292160a^{24}$$
$$-1081080a^{25} - 1512a^{26} + 111499203713698653771006621603417324257280000c$$
$$-163742052796897112820242552072096619429888000ac$$
$$+859440686639359141176706557714229034876928 00a^2c$$

Notes

$$-21370396371652979215143578801590652211363840a^3c$$
$$+2623550525769583347331377063702319807021056a^4c$$
$$-124497318095462860387905918553038985236480a^5c$$
$$-3612097702465706840440965975322699226112a^6c$$
$$+492218605823122896902587070587500049920a^7c$$
$$-275805202029035686979482288671812864a^8c$$
$$-859547133037731586445840086808643200a^9c$$
$$+53849592348935331827315036697152a^{10}c+907578552960077055195680523419680a^{11}c$$
$$+7651803335938703791473697804496a^{12}c-500906035361948659636478818280a^{13}c$$
$$-9640252794712151486284261528a^{14}c+74956460074277553035389840a^{15}c$$
$$+3727566043077344399337536a^{16}c+26910119291435727816040a^{17}c$$
$$-318378226807212008008a^{18}c-6425222117022051200a^{19}c-30386831619814384a^{20}c$$
$$+140938447609960a^{21}c+2091326625752a^{22}c+7603195600a^{23}c+2344160a^{24}c$$
$$-40040a^{25}c-56a^{26}c+120646700333168033241532139957502439587840000c^2$$
$$-142659480406997066194151084608764679028736000ac^2$$
$$+61711333840473436052348085502526307395174400a^2c^2$$
$$-12603390708055126796601073388588938679746560a^3c^2$$
$$+1217475722312962109665401816041455663202304a^4c^2$$
$$-33851050611751939558979043571994270208000a^5c^2$$
$$-2674577215164854362954317293171437747200a^6c^2$$
$$+156561528713541741738032523108156748800a^7c^2$$
$$+3244821684646720859831682549389687040a^8c^2$$
$$-245702704791967536868878693236832000a^9c^2$$
$$-4318390995349206755511571721014080a^{10}c^2+191052409939284375354316492255680a^{11}c^2$$
$$+4694456764257273131605381750896a^{12}c^2-51975812984276457325998748800a^{13}c^2$$
$$-2502538219153419403669670880a^{14}c^2-12633112393114472300733120a^{15}c^2$$
$$+443114118765699647275152a^{16}c^2+6911664342731463744000a^{17}c^2$$
$$+11923507217435299200a^{18}c^2-514563941781489600a^{19}c^2-4687868618505072a^{20}c^2$$
$$-9563925571200a^{21}c^2+66245830560a^{22}c^2+389188800a^{23}c^2+589680a^{24}c^2$$
$$+75111045620180303476410127043804050489344000c^3$$
$$-73507508824541531903012227142592915843317760ac^3$$
$$+26513867296308549533659020799938205718151168a^2c^3$$

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$$-4450858307928320966637967059709533429104640a^3c^3$$
$$+3305285038964107806981740434367116327157 76a^4c^3$$
$$-28905706913735072174696339880330209894 40a^5c^3$$
$$-879055550623931467328348348096780532736a^6c^3$$
$$+23987662118376858165743025588752184320a^7c^3$$
$$+1256494416477929651719763990568806656a^8c^3$$
$$+1256494416477929651719763990568806656a^8c^3$$
$$-31785915326939604073792309364524800a^9c^3$$
$$-1346820308106917157002616520689472a^{10}c^3+1392611216392405505267312989 5360a^{11}c^3$$
$$+89936751524883885517170334 8592a^{12}c^3+3278792338038966450112611200a^{13}c^3$$
$$-262042670867732864765158880a^{14}c^3-3635009226005737914459840a^{15}c^3$$
$$+1061218968502319027 0864a^{16}c^3+586314123608120793600a^{17}c^3$$
$$+4074779563518801280a^{18}c^3-70181603315824 00a^{19}c^3-217313554425968a^{20}c^3$$
$$-940039900800a^{21}c^3-540772960a^{22}c^3+4804800a^{23}c^3+7280a^{24}c^3$$
$$+3100944522060760929407766384283727010 2016000c^4$$
$$-255043067184998321565642088722965428646707 20ac^4$$
$$+77153658640320476654131443191961125608488 96a^2c^4$$
$$-10602391901822120370759380831847438876672 00a^3c^4$$
$$+58084988558317385445910172270528900628480a^4c^4$$
$$+679527066067646201260702440036684595200a^5c^4$$
$$-16891639841885131093534255431925235712 0a^6c^4$$
$$+1254038894284672678898791754193715200a^7c^4$$
$$+232415090630368193886748227788636160a^8c^4$$
$$-11043561480184674534849934869504 00a^9c^4$$
$$-1950237322507000741054497547714 56a^{10}c^4-8428104129063405660330352896 00a^{11}c^4$$
$$+837814503765656875196625868 80a^{12}c^4+1144468524142304687226700800a^{13}c^4$$
$$-104641743377402297302579 20a^{14}c^4-3273690576878959311360 00a^{15}c^4$$
$$-15780949882657749504 00a^{16}c^4+19248546775736678400a^{17}c^4+251291800496056320a^{18}c^4$$
$$+736078831488000a^{19}c^4-3056913699840a^{20}c^4-22832409600a^{21}c^4-37739520a^{22}c^4$$
$$+918367203209026482142278071402189777220 4032c^5$$
$$-640377953384091073459530643925355359895 5520ac^5$$
$$+162784932489901864293620546672936189794 7136a^2c^5$$

Notes

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$$-1815159162257947595274557383772899482009600a^3c^5$$

$$+6792737576962099744152294871100484747264a^4c^5$$

$$+257779633154015279523202642490892288000a^5c^5$$

$$-2089177739450846255263134400165478400 0a^6c^5$$

$$-188636076024651959589928141704478720a^7c^5$$

$$+25485007317188026296684505764429824a^8c^5$$

$$+2387910103821953543846399671296 00a^9c^5$$

$$-15800217898757585985744897478656a^{10}c^5-2505708559113873649172065228 80a^{11}c^5$$

$$+37198220556924972815422064 64a^{12}c^5+106985735764193136469498880a^{13}c^5$$

$$+2450029072474773663610 88a^{14}c^5-1384470895340556401664 0a^{15}c^5$$

$$-14073945878904022 4256a^{16}c^5-3712213132852224 0a^{17}c^5+6365795521333248a^{18}c^5$$

$$+33399305139200a^{19}c^5+28168684544a^{20}c^5-169128960a^{21}c^5-279552a^{22}c^5$$

$$+20502104007741918877998043435535488002293 76c^6$$

$$-1217835348394760572528362576953365600665600ac^6$$

$$+2598683049192144001959297094629956832460 80a^2c^6$$

$$-2317824624844099891846334951275718246400 0a^3c^6$$

$$+498487460366720876820279906956024217600a^4c^6$$

$$+428512694287759741841484345283067904 00a^5c^6$$

$$-16968759920092278761892620250105446 40a^6c^6$$

$$-459412146795476791443047567361638 40a^7c^6$$

$$+171990983610209839007962585320652 8a^8c^6+412882310245173920205559464960 00a^9c^6$$

$$-6813737687118893367035520921 60a^{10}c^6-22746667844576820510733086720a^{11}c^6$$

$$+48558231894677047098716 16a^{12}c^6+51264151311694387562496 00a^{13}c^6$$

$$+4397195695662529284096 0a^{14}c^6-221004611051349565440a^{15}c^6$$

$$-4951333592042213376a^{16}c^6-19626465303244800a^{17}c^6+50487854376960a^{18}c^6$$

$$+517534617600a^{19}c^6+940972032a^{20}c^6+35686335172279441883512973242852299925 0944c^7$$

$$-18094500441626389526906609470896039002112 0ac^7$$

$$+3229615299191913491642119743903997991321 6a^2c^7$$

$$-2254220785881957994666547888263093288960a^3c^7$$

$$+142054985922883580971031641612696944 64a^4c^7$$

$$+459888683967687270587315077814976512 0a^5c^7$$

$$-83566533000820774996279478533455872a^6c^7$$

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$$-489062081952935490047454640357376 0a^{7}c^{7}+626887373082392906028029622353 92a^{8}c^{7}$$

$$+332772704884970078962988451840 0a^{9}c^{7}-485898487300020876582494822 4a^{10}c^{7}$$

$$-114631270699002425824794624 0a^{11}c^{7}-859583933797760222630092 8a^{12}c^{7}$$

$$+1254405359301639514112 00a^{13}c^{7}+2017193098838664581120a^{14}c^{7}$$

$$+40389761507041689 60a^{15}c^{7}-815527166141276 16a^{16}c^{7}-535024001740800a^{17}c^{7}$$

$$-603939747840a^{18}c^{7}+2738278400a^{19}c^{7}+4978688a^{20}c^{7}$$

$$+496142175105277662590938249942894903296 00c^{8}$$

$$-214734674710162144930073387580732211200 00ac^{8}$$

$$+318666351542636086754435313131873894400 0a^{2}c^{8}$$

$$-168285651438753038920586402328005836800a^{3}c^{8}$$

$$-155678312069289731794530774999564288 0a^{4}c^{8}$$

$$+349589468880467806151666468395745280a^{5}c^{8}$$

$$-1220261782508141587497818317651968a^{6}c^{8}-329978303761249672303380634828800a^{7}c^{8}$$

$$-111546118466324524454330327040a^{8}c^{8}+1649868711673409220952138752 00a^{9}c^{8}$$

$$+1232479859042085975913033728a^{10}c^{8}-3352259261939197596180480 0a^{11}c^{8}$$

$$-495739319236863235522560a^{12}c^{8}+5739240913252374528 00a^{13}c^{8}$$

$$+4706067746778697728 0a^{14}c^{8}+249294059425382400a^{15}c^{8}-366648408023040a^{16}c^{8}$$

$$-5988614860800a^{17}c^{8}-12098211840a^{18}c^{8}$$

$$+560825385437605655287680605923504553984 0c^{9}$$

$$-206845970795097476506817608754602704896 0ac^{9}$$

$$+253022475529584422947369665576386428928a^{2}c^{9}$$

$$-956672863750053570716710425361121280 0a^{3}c^{9}$$

$$-260129829340107041750280546971156480a^{4}c^{9}$$

$$+193871593545608335567753489858560 00a^{5}c^{9}$$

$$+168691460736650122383844386406400a^{6}c^{9}-151667830611521560810709114880 00a^{7}c^{9}$$

$$-154718166160869330135982694400a^{8}c^{9}+517090057529522249064314880 0a^{9}c^{9}$$

$$+81433682692985274259525632a^{10}c^{9}-437901715324452619468800a^{11}c^{9}$$

$$-1454478929871996993536 0a^{12}c^{9}-563954591856015360 00a^{13}c^{9}$$

$$+541697212588646400a^{14}c^{9}+4666979314483200a^{15}c^{9}+6766335713280a^{16}c^{9}$$

$$-24644505600a^{17}c^{9}-49786880a^{18}c^{9}+522321157076120153860613276073637969920c^{10}$$

$$-163647792587518877982477527908614144000ac^{10}$$

$$+163079556932872410340580738915106816 00a^{2}c^{10}$$

Notes

$$-3985796753667242043684268057716326 40a^3c^{10}$$

$$-2112746467712363628546196680435302 4a^4c^{10}$$

$$+779247090966224629410095392358400a^5c^{10}+1659794937961840026100355137536 0a^6c^{10}$$

$$-46928574198862674559894388736 0a^7c^{10}-969833037121569759470623948 8a^8c^{10}$$

$$+886455521220857807831040 00a^9c^{10}+276258998568815820619776 0a^{10}c^{10}$$

$$+55935462457598666342 40a^{11}c^{10}-2431016792315245854 72a^{12}c^{10}$$

$$-175392067876945920 0a^{13}c^{10}+1007119677358080a^{14}c^{10}+40456420392960a^{15}c^{10}$$

$$+91946409984a^{16}c^{10}+40481753031308658860350883545932103680c^{11}$$

$$-10725110026159612123685588670957486080ac^{11}$$

$$+857114048355464113931507191716511744a^2c^{11}$$

$$-105941205169879570413812082435686 40a^3c^{11}$$

$$-11795969892408368928734174533386 24a^4c^{11}$$

$$+213307787400803051834967510220 80a^5c^{11}+85090976647286454200934118195 2a^6c^{11}$$

$$-8487609109832727347945472000a^7c^{11}-347207988376452059934105600a^8c^{11}$$

$$-6614495300674649456640 0a^9c^{11}+57449752944624938287104a^{10}c^{11}$$

$$+363695726767329116160a^{11}c^{11}-19824076419390996 48a^{12}c^{11}$$

$$-2444185308364800 0a^{13}c^{11}-447555084288 00a^{14}c^{11}+136216903680a^{15}c^{11}$$

$$+309583872a^{16}c^{11}+2630159022764979120645210885429657600c^{12}$$

$$-5857146512186439527160397473565900 80ac^{12}$$

$$+3674919737522309182155043660785254 4a^2c^{12}$$

$$-4385519086424528956105854812160 0a^3c^{12}-48986767464583866687368000962560a^4c^{12}$$

$$+294921215175544926013552066560a^5c^{12}+2943200841098824154366764646 4a^6c^{12}$$

$$-98364207033837633208320 00a^7c^{12}-81681042513431511760896 00a^8c^{12}$$

$$-46746892913580815155200a^9c^{12}+712012507861762179072a^{10}c^{12}$$

$$+7397223537947443200a^{11}c^{12}+1962860815319040a^{12}c^{12}-171633298636800a^{13}c^{12}$$

$$-445800775680a^{14}c^{12}+143999323379433380356514948390584320c^{13}$$

$$-2675119427768663828815922402951168 0ac^{13}$$

$$+1279452820163332319562034707431424a^2c^{13}$$

$$+122227087337033294663895744512 00a^3c^{13}-15505605891112457980085062860 80a^4c^{13}$$

$$-4527511264702931816918548480a^5c^{13}+724049505600340612520869888a^6c^{13}$$

$$+4498821108356642747187200a^7c^{13}-12634352158823136231424 0a^8c^{13}$$

$$-1299837145463016652800a^9c^{13}+3824730977850359808a^{10}c^{13}$$

Notes

$$
\begin{gathered}
+8096152959713280 0a^{11}c^{13}+187067404124160a^{12}c^{13}-488983756800a^{13}c^{13}\\
-1270087680a^{14}c^{13}+66655749491540638276336217594265 60c^{14}\\
-102341448119293183959759134392320 0ac^{14}+357403938126320322493617379737 60a^{2}c^{14}\\
+739459344349328301708174950400a^{3}c^{14}-373969420582031835640469913 60a^{4}c^{14}\\
-39090978739438208955187200 0a^{5}c^{14}+1257745452842326453125120 0a^{6}c^{14}\\
+148760297535389486284800a^{7}c^{14}-1143660207840268124160a^{8}c^{14}\\
-19422816227426304000a^{9}c^{14}-2270312464121856 0a^{10}c^{14}+475292211609600a^{11}c^{14}\\
+1440279429120a^{12}c^{14}+261282780175547378895022813347840c^{15}\\
-327797552814837943594054477414 40ac^{15}+780719920307802965212651323392a^{2}c^{15}\\
+26491449680928736548948541440a^{3}c^{15}-671178458823345516258000896a^{4}c^{15}\\
-121227473764621652695449 60a^{5}c^{15}+1437585704600559560949 76a^{6}c^{15}\\
+2722080379543344906240a^{7}c^{15}-247221217520045260 8a^{8}c^{15}-173434714796851200a^{9}c^{15}\\
-512337621024768a^{10}c^{15}+1173561016320a^{11}c^{15}+3556245504a^{12}c^{15}\\
+86726246711989955857162960896 00c^{16}-876737959547991925898304552960ac^{16}\\
+12566031457838852964643504128a^{2}c^{16}+68218321616804874304880640 0a^{3}c^{16}\\
-83609084848146907240857 60a^{4}c^{16}-2405343496287229024665 60a^{5}c^{16}\\
+794460117519589441536a^{6}c^{16}+31936837677141196800a^{7}c^{16}+73279496981053440a^{8}c^{16}\\
-871369054617600a^{9}c^{16}-3168614744064a^{10}c^{16}+24331814799537392499994656768 0c^{17}\\
-19483075333612069236978483200ac^{17}+12305076276087295181848576 0a^{2}c^{17}\\
+1324817133864431045836800 0a^{3}c^{17}-5584621178095620587520 0a^{4}c^{17}\\
-33185458770256566681 60a^{5}c^{17}-4312411139621781504a^{6}c^{17}+23934236169338880 0a^{7}c^{17}\\
+931301204951040a^{8}c^{17}-1898407526400a^{9}c^{17}-6903300096a^{10}c^{17}\\
+57487077466613026799380070 40c^{18}-356870776571747724636979200ac^{18}\\
-131073276130850966077440a^{2}c^{18}+19580138645735929282560 0a^{3}c^{18}\\
+215420772438858792960a^{4}c^{18}-319175764343783424 00a^{5}c^{18}\\
-126198262592962560a^{6}c^{18}+1047920954572800a^{7}c^{18}+4763277066240a^{8}c^{18}\\
+1136980032293070784010649 60c^{19}-532506692029938988482560 0ac^{19}\\
-3106500254785226145792 0a^{2}c^{19}+2177952413300503347200a^{3}c^{19}\\
+10197417129300459520a^{4}c^{19}-205175977620275200a^{5}c^{19}-1112415540346880a^{6}c^{19}\\
+2042730905600a^{7}c^{19}+9285140480a^{8}c^{19}+18660688312728337514496 00c^{20}\\
-6363657048005252481024 0ac^{20}-6482816830073921863 68a^{2}c^{20}
\end{gathered}
$$

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$$+1769905400032788480 0a^{3}c^{20}+124499026174279680a^{4}c^{20}-794213776097280a^{5}c^{20}$$

$$-4813416824832a^{6}c^{20}+25101640877012586332160c^{21}-5940674240531608371 20ac^{21}$$

$$-803188976677971558 4a^{2}c^{21}+992907293097984 00a^{3}c^{21}+838442879549440a^{4}c^{21}$$

$$-1400729763840a^{5}c^{21}-8489271296a^{6}c^{21}+2719166977264110796 80c^{22}$$

$$-417128409936691200 0ac^{22}-66386603173478400a^{2}c^{22}+343815487488000a^{3}c^{22}$$

$$+3125595340800a^{4}c^{22}+231256615720845312 0c^{23}-20708839129088000ac^{23}$$

$$-361299024281600a^{2}c^{23}+553648128000a^{3}c^{23}+5033164800a^{4}c^{23}+14860982786457600c^{24}$$

$$-64776830976000ac^{24}-1177760563200a^{2}c^{24}+67807601491968c^{25}-95965675520ac^{25}$$

$$-1744830464a^{2}c^{25}+195689447424c^{26}+268435456c^{27}\Big\}+$$

$$+\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+56}{2})}\Big\{-1961564335975602625554969761244154429440000 00a$$

$$+24019854151585796545125529684426430251008 0000a^{2}$$

$$-10960189236768388520857180440439412844134 4000a^{3}$$

$$+2389529185977790341949085127968287327580 1600a^{4}$$

$$-248146647230615507164969315073869697436672 0a^{5}$$

$$+72783255762659198551645140635473544812032a^{6}$$

$$+68193847280627872946525677717526636332 80a^{7}$$

$$-431637831606287563218538442674021152576a^{8}$$

$$-104161295649582565332690342641330280 00a^{9}$$

$$+86726462535416079762683142970418008 0a^{10}+17540616105165807466720125817999400a^{11}$$

$$-889238066833797297385859418147820a^{12}-252318388445946504708056612081 50a^{13}$$

$$+335988871954753140866090646935a^{14}+1880288152651518635962725635 0a^{15}$$

$$+109450371377684173117028915a^{16}-4965714405817745751521350a^{17}$$

$$-94625294414346874055605a^{18}-189257120780300823250a^{19}+1195960403822477793 5a^{20}$$

$$+145801888274792750a^{21}+414572115739925a^{22}-4549117122550a^{23}$$

$$-44916108055a^{24}-138668530a^{25}-3367a^{26}+770a^{27}+a^{28}$$

$$+196156433597560273444366426542767603712000000c$$

$$-621865680624392717398935707543022442905600000ac$$

$$+501923481897635701910088815990891080581120000a^{2}c$$

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$$+29670954062579487684192070256600222598758400a^{4}c$$

$$-2252845991953731291870738134610483906723840a^{5}c$$

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$$+113290661129521201883570066535441399767704a^{6}c$$
$$+77124790712465204804849297640604983552 00a^{7}c$$
$$-205465047201313839101708021868058275840a^{8}c$$
$$-1407699492851347292315682161344460 1600a^{9}c$$
$$+3557214374964525289644504679589621 76a^{10}c$$
$$+192943367670380521035972961519920 00a^{11}c$$
$$-19518117895935744629818305467520 0a^{12}c-1700235267167776729793328863784 0a^{13}c$$
$$-8838902113780983779802977438 4a^{14}c+68513902959024033342007176 00a^{15}c$$
$$+12137941388893619734750224 0a^{16}c-3562388123000016943432 80a^{17}c$$
$$-2986469537248253230574 4a^{18}c-28181400414593025600 0a^{19}c$$
$$+52466919258001248 0a^{20}c+2779104512352168 0a^{21}c+18972152204241 6a^{22}c$$
$$+224151127200a^{23}c-3128842080a^{24}c-15135120a^{25}c-21168a^{26}c$$
$$+381667139108534794321244968809545324298240000c^{2}$$
$$-72419035296550460080954410734275623754137 6000ac^{2}$$
$$+44039349826441482730002452321592030399037 4400a^{2}c^{2}$$
$$-12124350974187353397216636466024930033729 5360a^{3}c^{2}$$
$$+16160942803379150883877403242122078054580 224a^{4}c^{2}$$
$$-84735927042749870108719391584442993590272 0a^{5}c^{2}$$
$$-19466397805942879517229810425632123527168a^{6}c^{2}$$
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$$-103144571640071760650719240193078297 60a^{8}c^{2}$$
$$-5790666217965358497006091605721091200a^{9}c^{2}$$
$$+95470136386755058740047528377944 32a^{10}c^{2}$$
$$+62515571091416151669914475056180 80a^{11}c^{2}+47328421857762953710451680975776a^{12}c^{2}$$
$$-35275554636835337024542925567 60a^{13}c^{2}-6564361477385204670017492557 6a^{14}c^{2}$$
$$+5499138651916905314063544 80a^{15}c^{2}+26016384131839847061981392a^{16}c^{2}$$
$$+184266068533572296591320a^{17}c^{2}-2257067026276515116664a^{18}c^{2}$$
$$-4492688237291814560 0a^{19}c^{2}-211186166729329792a^{20}c^{2}+993146650135640a^{21}c^{2}$$
$$+14643067224968a^{22}c^{2}+53188735600a^{23}c^{2}+16358160a^{24}c^{2}-280280a^{25}c^{2}$$
$$-392a^{26}c^{2}+331868763435552857797695559762269469409280000c^{3}$$
$$-46362211349022764238063456537891210421862 4000ac^{3}$$
$$+22387863729325241220625205127740923474083 8400a^{2}c^{3}$$

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$$
\begin{gathered}
-49645660451710213411526895091940600726814720a^{3}c^{3}\\
+5155974607959493321901288693858659515138048a^{4}c^{3}\\
-162980067628706585806272136075258763059200a^{5}c^{3}\\
-10959974378167974636989566019938161530880a^{6}c^{3}\\
+717478173028347744465591515639413032960a^{7}c^{3}\\
+13029481870434045005384162448872851968a^{8}c^{3}\\
-1134988214850457919749278477323712000a^{9}c^{3}\\
-18354760857175694977904334865073280a^{10}c^{3}\\
+898707561658975041369752220036480a^{11}c^{3}+21125758510235690722494303778656a^{12}c^{3}\\
-253068151075625143350258566400a^{13}c^{3}-11579418720650296117702376640a^{14}c^{3}\\
-55906347460864133169256320a^{15}c^{3}+2082433101452290294579872a^{16}c^{3}\\
+32074140520110648960000a^{17}c^{3}+53299295146921824000a^{18}c^{3}\\
-2408151557943331200a^{19}c^{3}-21848158286738784a^{20}c^{3}-44418550176000a^{21}c^{3}\\
+309499444800a^{22}c^{3}+1816214400a^{23}c^{3}+2751840a^{24}c^{3}\\
+173899721025361423006064096582409205579776000c^{4}\\
-192062251276326945198326045095569208136171520ac^{4}\\
+75547504720379442438309847871903389560078336a^{2}c^{4}\\
-13585085420594054247705399858485557176729600a^{3}c^{4}\\
+1078520487348185762924670746372533758320640a^{4}c^{4}\\
-12869832841551429897256272899481149798400a^{5}c^{4}\\
-2837803351832646833163751356888925237760a^{6}c^{4}\\
+85884531683922334835856504984562603520a^{7}c^{4}\\
+4104054538253380583659522054118662016a^{8}c^{4}\\
-113848608096266985812769801977782400a^{9}c^{4}\\
-4529848036472267973875525186192736a^{10}c^{4}+51599380170979438887502260318240a^{11}c^{4}\\
+3103903286636783784094844440328a^{12}c^{4}+10234801639350213310500315200a^{13}c^{4}\\
-919892634398125873650190480a^{14}c^{4}-12560781816563392467255200a^{15}c^{4}\\
+38779959218077538380920a^{16}c^{4}+2052501949317538118400a^{17}c^{4}\\
+14187426354285504320a^{18}c^{4}-24953063182247200a^{19}c^{4}-760925790528904a^{20}c^{4}\\
-3288166481600a^{21}c^{4}-1889443920a^{22}c^{4}+16816800a^{23}c^{4}+25480a^{24}c^{4}\\
+62237636134674789629823289339412939695718400c^{5}\\
-56191079871989990772100935814466466333327360ac^{5}
\end{gathered}
$$

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$$+182426098894474184008518162433863366192660 48a^2c^5$$

$$-265922151698629227557051388437859579985920 0a^3c^5$$

$$+155534521615927662589208498640168844001280a^4c^5$$

$$+13340609543376062814697863758721515520 00a^5c^5$$

$$-449335329328840808047035972340585267200a^6c^5$$

$$+41247110864758040919759495652614144 00a^7c^5$$

$$+626681635856224082418957175542251520a^8c^5$$

$$-36552191383399535308388247109632 00a^9c^5-5364168762834266226028648798740 48a^{10}c^5$$

$$-20444867746149416582183947776 00a^{11}c^5+23447494694164398346669759488 0a^{12}c^5$$

$$+31327960710747009357470822 40a^{13}c^5-29910991939177606651570176a^{14}c^5$$

$$-9135165299909271727104 00a^{15}c^5-4349380912772783247360a^{16}c^5$$

$$+541701537276688588 80a^{17}c^5+702909104844570624a^{18}c^5+2053775243520000a^{19}c^5$$

$$-8572531968000a^{20}c^5-63930746880a^{21}c^5-105670656a^{22}c^5$$

$$+163083555361327119242185687666379294779637 76c^6$$

$$-122541052855123476834973063526660533596979 20ac^6$$

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$$-5660028797310332284885152307 20a^{11}c^6+88179911201367052579831388 16a^{12}c^6$$

$$+24757682278862730886008320 0a^{13}c^6+5388003550092894213683 20a^{14}c^6$$

$$-32369650742279036426240a^{15}c^6-327059114954454378496a^{16}c^6$$

$$-7788296229762048 0a^{17}c^6+14863380401565696a^{18}c^6+77886986777600a^{19}c^6$$

$$+65645612032a^{20}c^6-394634240a^{21}c^6-652288a^{22}c^6$$

$$+326981702828598561877746918175132104209203 2c^7$$

$$-206536074301200339378751788034828617646080 0ac^7$$

$$+463350133563898009968853435444850054922240a^2c^7$$

$$-432931255524222738470026053153916767436 80a^3c^7$$

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$$+1022025633856068398478198448636676800512a^4c^7$$
$$+79328777075504375144832989760916684800a^5c^7$$
$$-3367800217935218934915705799524679680a^6c^7$$
$$-858147800022081980651916474649804 80a^7c^7$$
$$+34400232180044857926971642785628 16a^8c^7+79514208187732328003456246784000a^9c^7$$
$$-138521430400936706322960144384 0a^{10}c^7-448665044473176441961597747 20a^{11}c^7$$
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$$+870656465454531736780 80a^{14}c^7-446654781753575915520a^{15}c^7$$
$$-98958050589716398 08a^{16}c^7-3914114312908 8000a^{17}c^7+101201542041600a^{18}c^7$$
$$+1035069235200a^{19}c^7+1881944064a^{20}c^7$$
$$+5170332992212934263524035313549485770 01472c^8$$
$$-275991007508535914378181157194935618764800ac^8$$
$$+51430190282300069814658339707260229386240a^2c^8$$
$$-3746891881530224698281343564418518876160a^3c^8$$
$$+300112573535465120238651837626649149 44a^4c^8$$
$$+7646689191196072390966423555337338880a^5c^8$$
$$-1495578292383412244419657962185441 28a^6c^8$$
$$-8242391084914308786312287348940800a^7c^8+1128613590685938903726260635033 60a^8c^8$$
$$+5714921342987833399345341619200a^9c^8-101978651597476724634921477 12a^{10}c^8$$
$$-1996753644002932387326673920a^{11}c^8-147378133539779184805002 24a^{12}c^8$$
$$+2206986576089434979840 00a^{13}c^8+3518700225524078566400a^{14}c^8$$
$$+697023840858369 0240a^{15}c^8-142859263691848704a^{16}c^8-935774468428800a^{17}c^8$$
$$-1055849034240a^{18}c^8+4791987200a^{19}c^8+8712704a^{20}c^8$$
$$+6589075761617689123666469970975978946 5600c^9$$
$$-2978827942859454564394635736348473950 2080ac^9$$
$$+458947405324331703001803493755463296 6144a^2c^9$$
$$-252498230766276619469325838331727052800a^3c^9$$
$$-19460345721359971257983300174925004 80a^4c^9$$
$$+525732038405361521167066812039168000a^5c^9$$
$$-22749811748523068016859061649408 00a^6c^9-502354137946146197697338056704000a^7c^9$$
$$+49851017742381405230525644800a^8c^9+254561183892775408429683302400a^9c^9$$
$$+18529864646902932247612047 36a^{10}c^9-5227411647330675003801600 0a^{11}c^9$$

$$
\begin{gathered}
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+731818386330071040000a^{14}c^{9} + 38684677596364800 0a^{15}c^{9} - 572487384268800a^{16}c^{9} \\
-9315623116800a^{17}c^{9} - 18819440640a^{18}c^{9} \\
+6877156982331765467067844973253539921920c^{10} \\
-26327286398864451455530768901747612057 60ac^{10} \\
+3328218404863498582793678992142164295 68a^{2}c^{10} \\
-1311606492086195445511491222452568064 0a^{3}c^{10} \\
-33462283063666724000899043320686182 4a^{4}c^{10} \\
+265925425967521026590130328269619 20a^{5}c^{10} \\
+2146647671286505220963494653460 48a^{6}c^{10} - 21013574189838515145468136488960a^{7}c^{10} \\
-207752754353935015586289696768a^{8}c^{10} + 724035312201911090859878400 0a^{9}c^{10} \\
+1125344040680584858342809 60a^{10}c^{1}0 - 62235945075099327307776 0a^{11}c^{1}0 \\
-20311735873429425897472a^{12}c^{10} - 78326530384640778240a^{13}c^{10} \\
+759524229040152576a^{14}c^{10} + 6530274806415360a^{15}c^{10} + 9464924012544a^{16}c^{10} \\
-34502307840a^{17}c^{10} - 69701632a^{18}c^{10} + 594963029980574462673295182995379978240c^{11} \\
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+19747662192046084362032151156668497920a^{2}c^{11} \\
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-2553997884579975604965404857152307 2a^{4}c^{11} \\
+983282425826229844894352277504000a^{5}c^{11} + 20308032314909648742195354009600a^{6}c^{11} \\
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+6912219176240177479680a^{11}c^{11} - 3094562410124575703 04a^{12}c^{11} \\
-2227456949708390400a^{13}c^{11} + 1294271101992960a^{14}c^{11} + 51489989591040a^{15}c^{11} \\
+117022703616a^{16}c^{11} + 43049806998675382670680519006318755840c^{12} \\
-1172512498233940178277848073201778688 0ac^{12} \\
+961733880221956544247541510081347584a^{2}c^{12} \\
-127111559573303735979754822225100 80a^{3}c^{12} \\
-13296604157975106084021450947297 28a^{4}c^{12} \\
+250143043957859098919520737689 60a^{5}c^{12} + 970872153544862037549175373824a^{6}c^{12} \\
-10036657008894153897643704320a^{7}c^{12} - 40124542339306130594554265 6a^{8}c^{12} \\
-3786956886462604574720 0a^{9}c^{12} + 669083143239640811601 92a^{10}c^{12}
\end{gathered}
$$

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$$+421856720315532902400a^{11}c^{12}-2318480650637598720a^{12}c^{12}-28500662756966400a^{13}c^{12}$$
$$-52176233840640a^{14}c^{12}+158919720960a^{15}c^{12}+361181184a^{16}c^{12}$$
$$+2622501866151648674822905390537113600c^{13}$$
$$-5981295485544737380099730212202186240ac^{13}$$
$$+3842003603787463439855796830286643 2a^{2}c^{13}$$
$$-707661269317285670619382284288 00a^{3}c^{13}-515410005648693456397974424780 80a^{4}c^{13}$$
$$+330158114505788084643912744960a^{5}c^{13}+31285875928414161162992615424a^{6}c^{13}$$
$$-1542339086800180838400000 0a^{7}c^{13}-875899351045607738572800 0a^{8}c^{13}$$
$$-49709126101886396006400a^{9}c^{13}+767521437952865992704a^{10}c^{13}$$
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$$-480093143040a^{14}c^{13}+135120160639924436020541683987906560c^{14}$$
$$-25625329787265844219567098319339520ac^{14}$$
$$+12522655217425432693634724381327 36a^{2}c^{14}$$
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$$-41068926895192089782124544 00a^{5}c^{14}+719013210522774123396464640a^{6}c^{14}$$
$$+440381404284022156820480 0a^{7}c^{14}-12625596016878282735616 0a^{8}c^{14}$$
$$-1293769038943086182400a^{9}c^{14}+384265512732092006 4a^{10}c^{14}+80920454961561600a^{11}c^{14}$$
$$+186942935531520a^{12}c^{14}-488983756800a^{13}c^{14}-1270087680a^{14}c^{14}$$
$$+5904988207785551117924797482270720c^{15}-9229275481632755861155875913728 00ac^{15}$$
$$+32888428808620478837542449315840a^{2}c^{15}+664953901102407023723670405120a^{3}c^{15}$$
$$-34591391532099664608554385408a^{4}c^{15}-3559116831813558210134016 00a^{5}c^{15}$$
$$+117091297754792258804121 60a^{6}c^{15}+137652032428798503813120a^{7}c^{15}$$
$$-10701456133758690263 04a^{8}c^{15}-18095430700892160000a^{9}c^{15}-21071288048025600a^{10}c^{15}$$
$$+443606064168960a^{11}c^{15}+1344260800512a^{12}c^{15}+219149011598897000590157461585920c^{16}$$
$$-27917843636235046982222883061760ac^{16}+678200311751466484764894035968a^{2}c^{16}$$
$$+226579477622888528998694912 00a^{3}c^{16}-585049837700042874668974080a^{4}c^{16}$$
$$-104760929351236066285977 60a^{5}c^{16}+125958048102699766317056a^{6}c^{16}$$
$$+2372329651450126663680a^{7}c^{16}-22001125721716162 56a^{8}c^{16}-15168007194869760 0a^{9}c^{16}$$
$$-448021587492864a^{10}c^{16}+1026865889280a^{11}c^{16}+3111714816a^{12}c^{16}$$
$$+69041658003990981538706620416 00c^{17}-707149036748253787339515494400ac^{17}$$
$$+10352800487418100623259729920a^{2}c^{17}+553607208947840288764723200a^{3}c^{17}$$

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$$
\begin{aligned}
&-689427018508392713945088 0a^{4}c^{17}-196748052074755475374080a^{5}c^{17}\\
&+6595526376982031892 48a^{6}c^{17}+2625691246578892 8000a^{7}c^{17}+601477634064384 00a^{8}c^{17}\\
&-717598044979200a^{9}c^{17}-2609447436288a^{10}c^{17}+1842610985470604662255543910 40c^{18}\\
&-149187131834067396468722892 80ac^{18}+970659578479893705434071 04a^{2}c^{18}\\
&+1020779300145901406781440 0a^{3}c^{18}-438862071949706762649 60a^{4}c^{18}\\
&-257199814553074663424 0a^{5}c^{18}-330479890561079705 6a^{6}c^{18}+1860646091358208 00a^{7}c^{18}\\
&+723933740400640a^{8}c^{18}-1476539187200a^{9}c^{18}-5369233408a^{10}c^{18}\\
&+4149488493819913465579438080c^{19}-259998190380059335852032000ac^{19}\\
&-664939755295456886784 00a^{2}c^{19}+143449614767893354905600a^{3}c^{19}\\
&+152929002840611880960a^{4}c^{19}-234811509052538880 00a^{5}c^{19}-92763567371059200a^{6}c^{19}\\
&+772152282316800a^{7}c^{19}+3509783101440a^{8}c^{19}+7836529885800809782837248 0c^{20}\\
&-36984974259472889701990 40ac^{20}-213529839712155237089 28a^{2}c^{20}\\
&+1519703045075202211840a^{3}c^{20}+70971228485126717 44a^{4}c^{20}-143554548575764480a^{5}c^{20}\\
&-778274903949312a^{6}c^{20}+1429911633920a^{7}c^{20}+6499598336a^{8}c^{20}\\
&+123013658112111578972160 0c^{21}-42210468546500477583360ac^{21}\\
&-4287838365614100971 52a^{2}c^{21}+1178172013852753920 0a^{3}c^{21}+828389035553587 20a^{4}c^{21}\\
&-529475850731520a^{5}c^{21}-3208944549888a^{6}c^{21}+1584991840599873159168 0c^{22}\\
&-376931892360107786240ac^{22}-509181941930314956 8a^{2}c^{22}+631552971112448 00a^{3}c^{22}\\
&+533284446535680a^{4}c^{22}-891373486080a^{5}c^{22}-5402263552a^{6}c^{22}\\
&+164683127651366338560c^{23}-2535414992732160000ac^{23}-40343282122752000a^{2}c^{23}\\
&+209278992384000a^{3}c^{23}+1902536294400a^{4}c^{23}+134504276406829056 0c^{24}\\
&-12074558160896000ac^{24}-210655982387200a^{2}c^{24}+322961408000a^{3}c^{24}+2936012800a^{4}c^{24}\\
&+8310278533939200c^{25}-36275025346560ac^{25}-659545915392a^{2}c^{25}+36494873985024c^{26}\\
&\left.\left.-51673825280ac^{26}-939524096a^{2}c^{26}+101468602368c^{27}+134217728c^{28}\right\}\right] \qquad (8)
\end{aligned}
$$

Derivation of main result (8):

Substituting $b=-a-55, z=\frac{1}{2}$ in given result (2),we get

$$
(2a+55)\ {}_2F_1\left[\begin{matrix} a & , & -a-55 & ; & \frac{1}{2} \\ & c & & ; & \end{matrix}\right]
$$

$$
= a\ {}_2F_1\left[\begin{matrix} a+1 & , & -a-55 & ; & \frac{1}{2} \\ & c & & ; & \end{matrix}\right] + (a+55)\ {}_2F_1\left[\begin{matrix} a & , & -a-54 & ; & \frac{1}{2} \\ & c & & ; & \end{matrix}\right]
$$

Now involving the result established in Ref[6], we can prove the main result.

Global Journal of Science Frontier Research (F) Volume XIII Issue IX Version I Year 2013

Notes

Notes

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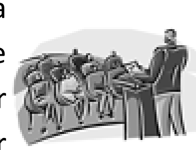

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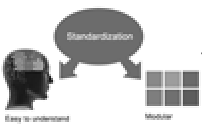

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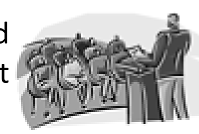

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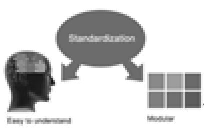

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- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

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Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

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Figure Legends: Self-explanatory legends of all figures should be incorporated separately under the heading 'Legends to Figures'. In the full-text online edition of the journal, figure legends may possibly be truncated in abbreviated links to the full screen version. Therefore, the first 100 characters of any legend should notify the reader, about the key aspects of the figure.

6. AFTER ACCEPTANCE

Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals Inc. (US).

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Before start writing a good quality Computer Science Research Paper, let us first understand what is Computer Science Research Paper? So, Computer Science Research Paper is the paper which is written by professionals or scientists who are associated to Computer Science and Information Technology, or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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- Do not give permission to anyone else to "PROOFREAD" your manuscript.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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