Volume 13 Issue 8
Originally issued as F2

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 2 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

A Common Fixed Point for Eight Mappings in an Intuitionistic M- Fuzzy Metric Space with Property 'E'

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Abstract - The aim of this paper is to introduce the concept of an intuitionistic M - fuzzy metric space with property 'E' and prove common fixed point theorem for eight weakly compatible mappings in intuitionistic M - fuzzy metric space with property 'E'.

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GJSFR-F Classification : *MSC 2010 : 13P25, 30F45*

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Ref.

5. O.Kramosil and J.Michalek: Fuzzy metric and statistical metric spaces, Ky-bernetics,11(1975), 330-334.

A Common Fixed Point for Eight Mappings in an Intuitionistic M-Fuzzy Metric Space with Property 'E'

Ranjeeta Jain [α] & N. Bajaj [σ]

Abstract - The aim of this paper is to introduce the concept of an intuitionistic M - fuzzy metric space with property 'E' and prove common fixed point theorem for eight weakly compatible mappings in intuitionistic M - fuzzy metric space with property 'E'.

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I. Introduction

In 1975, Kramosil and Michalek [5] introduced the concept of fuzzy metric space by generalizing the concept of probabilitic metric space to fuzzy situation. Many authors ([1],[2],[3],[5]) obtained common fixed point theorems involving fuzzy metric spaces.

In 2006, Sedghi and Shobe [6] introduced the concept of M – Fuzzy metric space as follows:

DEFINITION (1) : A 3-tuple (X,M,*) is called a M – Fuzzy metric space if X is an arbitrary (non-empty) set, * is a continuous t- norm, and M is a Fuzzy set on $X^3 x$ (0, ∞), satisfying the following condition : for each x, y, z, a ∈ X and t, s > 0,

(1). M(x, y, z, t) > o, (2). M(x, y, z, t) = 1 if and only if x = y = z,

(3). M(x, y, z, t) = M(p{x, y, z}t, where p is a permutation function,

(4). M(x, y, a t) ∗ M(a, z, z, s) ≤ M (x, y, z, t + s).

(5). M(x, y, z) : (0, ∞) → [0,1] is continuous.

As a generalization of fuzzy sets, Atanassov [4] introduce and studied the concept of intuitionistic fuzzy sets. park [3] and Alaca, Turkoglu and Yiliz [2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t–norm and continuous t–conorm as a generalization of fuzzy metric spaces due to George and Veeramani [1] and kramosil and Michalek [5] respectively.

In 2006, Sedghi and Shobe [6] defined M-Fuzzy metric space and proved a common fixed point theorem for four weakly compatible mappings in this space. In 2009, Seema Mehra and Meenakshi Gugnani [8] defined the notion of an intuitionistic M-Fuzzy metric space due to Sidgi and Shobe [6] and

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proved a common fixed point theorem for six mappings for property (E) in this newly defined space. Our result is an intuitionistic Fuzzy version of the results of Seema Mehra and Meenakshi Gugnani [8] result in M- Fuzzy metric space.

We introduce the concept of an intuitionistic M – Fuzzy metric space as follows.

DEFINITION (2): A binary operation $*:[o,1]\times[0,1]$ is a continuous t- norm of it satisfies the following condition

(1) $*$ is associative and commutative,

(2) $*$ is continuous

(3) $a * l = a$ for all $a \in [0,1]$

(4) $a * l = c * d$ whenever $a \le c$ and $b \le d$, for each $a , b , c , d \in [o,1]$.

Two typical example of a continuous t – norm are $a * b = ab$ and $a * b = \min (a, b)$

DEFINITION (3): A binary operation $\lozenge: [0,1]\times[0,1] \to [0,1]$ is a continuous t–connorm if it satisfies the following conditions:

(1). $\lozenge$ is associative and commutative,

(2). $\lozenge$ is continuous,

(3). $a \lozenge o = a$ for all $a \in [0,I]$,

(4). $a \lozenge b \le c \lozenge d$ whenever $a \le c$ and $b \le d$, for each a, b, c, d $\in$ [0, 1].

Two typical examples of a continuous t–conorm are $a \lozenge b = \min (1,a,+b)$ and $a \lozenge b = \max (a, b)$.

DETINITION (4): A 5–tuple $(X,M,N*,\lozenge)$ is called an intuitionistic M–fuzzy metric apace if X is an arbitrary (non-empty) set, * is a continuous t–norm , $\lozenge$ a continuous t–conorm and M, N are fuzzy sets on $X^3(0, \infty)$, satisfying the following conditions : for each x, y, z, a $\in$ X and t,s > 0,

(a) $M (x,y,z,t)+N(x,y,z,t) \le 1$.

(b) $M (x,y,z,t) > 0$,

(c) $M (x,y,z,t,) = 1$ if and only if $x = y = z$,

(d) $M (x,y,z,t,) = M(p\{x,y,z,\}t)$, where p is a permutation function ,

(e) $M (x,y,a,t,) * M (a,z,z,s,) \le M (x, y, z, t + s)$,

(f) $M (x,y,z,) : (0, \infty) \to [0,1]$ is continuous

(g) $N (x,y,z,t,) > 0$,

(h) $N (x,y,z,t,) = 0$, if and only if $x = y = z$,

(i) $N (x,y,z,t,) = N(p\{x,y,z,\}t)$, where p is a permutation function ,

(j) $N (x, y, a, t,) \lozenge N (a, z, z, s) \ge N (x, y, z, t + s)$,

(k) $N (x, y, z, .) : (0, \infty) \to [0, 1]$ is continuous.

Then (M, N) is called an intuitionistic M – Fuzzy metric on X.

Example(1) : Let X = R and $M (x, y, z, t,) = \dfrac{t}{t + |x - y| + |y + z| + |z - x|}$,

$$N (x, y, z, t,) = \frac{|x - y| + |y - z| + |z - x|}{t + |x - y| + |y - z| + |z - x|},$$

Ref.
8. Mehra S.and Gugnani M.: A common fixed point for six mappings in an intruitionistiM-fuzzy metric space. Indian Journal of Mathematics, Vol. 51 No. 1, (2009) 23-47.

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for every x, y, z and t > 0 Let A and B defined as Ax = 2x +1, Bx = x+2. consider the sequence $x_n = \frac{1}{n}+1$, n = 1, 2 …….. Thus we have $\lim_{n\to\infty} M(Ax_n, 3, 3, t) = \lim_{n\to\infty} M(Bx_n, 3, 3, t) = 1$ and

$\lim_{n\to\infty} N(Ax_n, 3, 3, t) = \lim_{n\to\infty} N(Bx_n, 3, 3, t) = 0$, for every t > 0.Then A and B Satisfying in the property (E).

In 2009, Seema Mehra and Meenakshi Gugnani [8] have proved the following theorem.

THEOREM (A) : Let P, Q, A, B, S and T be self mappings of X Satisfying the following conditions :

(i) $P(X) \subset ST(X)$ and $Q(X) \subset AB(X)$ and ST(X) or AB(X) or AB(X) is complete fuzzy metric subspace of X ,

(ii) AB = BA, ST = TS, PB = BP, TQ = QT,

(iii) The pair (P, AB) and (Q, ST) are weakly compatible and (P, AB) or (Q, ST) Satisfies the property (E),

(iv) If there exists a number K > 1 Such that

M(Px, Qy, Qz, t) ≥ ϕ { (M(ABx, STy, STz, Kt), M(ABx, Qy, STz, Kt), M(ABx, STy, Qz, Kt), M(ABx, Qy, Qy, Kt) M(STy, Qy, Qz, Kt), M(STy, STy Qz, Kt), M(STy, Qy, Qy, Kt), M(STy, Qz, Qz, Kt), M(Qy, STy, STz, Kt), M(Qy, Qy, STz, Kt), M(Qy, STz, STz, Kt), M(STz Qz, Qz,, Kt)} and

N(Px, Qy, Qz, t) ≤ ϕ'{N(ABx, STy, STz, Kt), N(ABx, Qy, STz, Kt), N(ABx, STy, Qz, Kt), N(ABx, Qy, Qy, Kt),N(STy, Qy, Qz, Kt), N(STy, STy Qz, Kt), N(STy, Qy, Qy, Kt), N(STy, Qz, Qz, Kt) N(Qy, STy, STz, Kt), N(Qy, Qy, STz, Kt), N(Qy, STz, STz, Kt), N(STz Qz, Qz,, Kt) , for all x, y, z ∈ X and t > 0,

Then P, Q, A,B,S and T have unique common fixed point in X.

Where A class of implict relation: Let ψ denote a family of mappings and ϕ, ϕ' ∈ ψ, ϕ, ϕ': $[0, 1]^{12} \to [0,1]$, and ϕ, ϕ' are continuous, increasing and decreasing respectively, in each co–ordinate variable. Also $\phi(s, s,\ldots s) > s$, $\phi'(s, s, \ldots, s) < s$ for every $s \in [0,1]$,

Example(3): Let $\phi, \phi' : [0,1]^{12} \to [0,1]$ be define by $\phi(x_1, x_2 \ldots\ldots x_{12}) = (\min\{x_i\})^h$ for some $0 < h < 1$ and $\phi'(x_1,x_2, \ldots.., x_{12}) = (\max\{x_i\})^h$ for some $h > 1$. Then $\phi, \phi' \in \psi$.

Here we generalized and extend the results of theorem (A) for eight mappings with property (E) in this newly defined space.

II. Main Result

Theorem 1 : Let P, Q, A, B, F, L, S and T be self mappings of X satisfying the following condition:

(1.2.1) $P(X) \subseteq ST(X) \cup F(X)$ and $Q(X) \subseteq AB(X) \cup L(X)$ and ST(X) or AB(X) and L(X) are complete fuzzy metric subspace of X.

(1.2.2) AB = BA, ST = TS, BP = PB, QT = TQ, FT = TF, LB = BL,

(1.2.3) The pair (P, AB), (P, L) and (Q, ST), (Q, F) are weak compatible and (P, AB) or (Q, ST) and (Q, F) satisfies the property (E)

(1.2.4) If there exists a number k > l such that

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$$M(Px, Qy, Qz, t) \geq \phi\{ M(ABx, STy, Lx, kt), M(Lx, STy, STz, kt), M(ABx, STy, Fz, Kt), M(ABx, Qy, Fz, Kt), M(ABx, Fy, Qz, Kt), M(STz, Qz, Fz, Kt), M(Fy, Qy, Qz, Kt), M(Lx, Qy, Fz, Kt), M(Qy, STy, Fz, Kt), M(ABx, STy, STz, Kt), M(ABx, Qy, STz, Kt), M(ABx, STy, Qz, Kt)\}$$

and

$$N(Px, Qy, Qz, t) \leq \phi'\{N(ABx, STy, Lx, Kt), N(Lx, STy, STz, Kt), N(ABx, STy, Fz, Kt), N(ABx, Qy, Fz, Kt), N(ABx, Fy, Qz, Kt), N(STz, Qz, Fz, Kt), N(Fy, Qy, Qz, Kt), N(Lx, Qy, Fz, Kt), N(Qy, STy, Fz, Kt), N(ABx, Sty, STz, Kt), N(ABx, Qy, STz, Kt), N(ABx, STy, Qz, Kt)\}$$

for all $x, y, z \in X$ and $t > 0$

Then P, Q, A, B, F, L, S and T have a unique common fixed point in X

Proof: Suppose that pair (Q, ST) and (Q, F) Satisfies the property (E). Hence there exists a sequence $\{x_n\}$. Such that

$$\lim_{n\to\infty} M(Qx_n, u, u, t) = \lim_{n\to\infty} M(STx_n, u, u, t) = 1$$

$$\lim_{n\to\infty} N(Qx_n, u, u, t) = \lim_{n\to\infty} N(STx_n, u, u, t) = 0 \text{ and}$$

$$\lim_{n\to\infty} M(Qx_n, u, u, t) = \lim_{n\to\infty} M(Fx_n, u, u, t) = 1$$

$$\lim_{n\to\infty} N(Qx_n, u, u, t) = \lim_{n\to\infty} N(Fx_n, u, u, t) = 0, \text{ for some } u \in X \text{ and every } t > 0. \text{ As } Q(X) \subset AB(X) \cup L(X), \text{ there exists a sequence } \{y_n\} \text{ such that } Qx_n = ABy_n = Ly_n = u.$$

Hence,

$$\lim_{n\to\infty} M(ABy_n, u, u, t) = 1 \text{ and } \lim_{n\to\infty} N(ABy_n, u, u, t) = 0 \text{ and}$$

$$\lim_{n\to\infty} M(Ly_n, u, u, t) = 1 \text{ and } \lim_{n\to\infty} N(Ly_n, u, u, t) = 0$$

We prove that

$$\lim_{n\to\infty} M(Py_n, u, u, t) = 1, \quad \lim_{n\to\infty} N(Py_n, u, u, t) = 0$$

Step 1: Putting $x = y_n$, $y = x_n$, $z = x_{n+1}$ in (1.2.4), we obtain

$$M(Py_n, Qx_n, Qx_{n+1}, t) \geq \phi\{ M(ABy_n, STx_n, Ly_n, Kt), M(Ly_n, STx_n, STx_{n+1}, Kt), M(ABy_n, STx_n, Fx_{n+1}, Kt), M(ABy_n, Qx_n, Fx_{n+1}, Kt), M(ABy_n, Fx_n, Qx_{n+1}, Kt), M(STx_{n+1}, Qx_{n+1}, Fx_{n+1}, Kt), M(Fx_n, Qx_n, Qx_{n+1}, Kt), M(Ly_n, Qx_n, Fx_{n+1}, Kt), M(Qx_n, STx_n, Fx_n, Kt), M(ABy_n, STx_n, STx_{n+1}, Kt), M(ABy_n, Qx_n, STx_{n+1}, Kt), M(ABy_n, STx_n, Qx_{n+1}, Kt), \}$$

Notes

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and

$$N(Py_n, Qx_n, Qx_{n+1}, t) \le \phi' \{ N(ABy_n, STx_n, Ly_n, Kt), N(Ly_n, STx_n, STx_{n+1}, Kt), N(ABy_n, STx_n, Fx_{n+1}, Kt), N(ABy_n, Qx_n, Fx_{n+1}, Kt), N(ABy_n, Fx_n, Qx_{n+1}, Kt), N(STx_{n+1}, Qx_{n+1}, Fx_{n+1}, Kt), N(Fx_n, Qx_n, Qx_{n+1}, Kt), N(Ly_n, Qx_n, Fx_{n+1}, Kt), N(Qx_n, STx_n, Fx_n, Kt), N(ABy_n, STx_n, STx_{n+1}, Kt), N(ABy_n, Qx_n, STx_{n+1}, Kt), N(ABy_n, STx_n, Qx_{n+1}, Kt)\}$$

Letting $n \to \infty$ in the above inequality we get

$$\lim_{n\to\infty} M(Py_n, Qx_n, Qx_{n+1}, t) \ge \phi\{M(u, u, u, Kt), M(u, u, u, Kt), M(u, u, u, Kt), \ldots, M(u, u, u, Kt)\} = 1,$$

$$\lim_{n\to\infty} N(Py_n, Qx_n, Qx_{n+1}, t) \le \phi'\{N(u, u, u, Kt), N(u, u, u, Kt), N(u, u, u, Kt), \ldots\ldots, N(u, u, u, Kt)\} = 0.$$

Therefore,

$$\lim_{n\to\infty} M(Py_n, u, u, t) = 1 , \lim_{n\to\infty} N(Py_n, u, u, t) = 0$$

Hence,

$$\lim_{n\to\infty} Py_n = \lim_{n\to\infty} ABy_n = \lim_{n\to\infty} Ly_n = \lim_{n\to\infty} Qx_n = \lim_{n\to\infty} STx_n = \lim_{n\to\infty} Fx_n = u .$$

Assume that AB(X) and L(X) are complete intutionstic M–Fuzzy metric space, then there exists $x \in X$ s.t $ABx = u$ and $Lx = u$.

Step 2: If $Px \neq u$, putting $y = x_n$, and $z = x_{n+1}$ in (1.2.4) then we have

$$M(Px, Qx_n, Qx_{n+1}, t) \ge \phi\{M(ABx, STx_n, Lx, Kt), M(Lx, STx_n, STx_{n+1}, Kt), M(ABx, STx_n, Fx_{n+1}, Kt), M(ABx, Qx_n, Fx_{n+1}, Kt), M(ABx, Fx_n, Qx_{n+1}, Kt), M(STx_{n+1}, Qx_{n+1}, Fx_{n+1}, Kt), M(Fx_n, Qx_n, Qx_{n+1}, Kt), M(Lx, Qx_n, Fx_{n+1}, Kt), M(Qx_n, STx_n, Fx_{n+1}, Kt), M(ABx, STx_n, STx_{n+1}, Kt), M(ABx, Qx_n, STx_{n+1}, Kt), M(ABx, STx_n, Qx_{n+1}, Kt)\}$$

and

$$N(Px, Qx_{n+1}, t) \le \phi'\{N(ABx, STx_n, Lx, Kt), N(Lx, STx_n, STx_{n+1}, Kt), N(ABx, STx_n, Fx_{n+1}, Kt), N(ABx, Qx_n, Fx_{n+1}, Kt), N(ABx, Fx_n, Qx_{n+1}, Kt), N(STx_{n+1}, Qx_{n+1}, Fx_{n+1}, Kt), N(Fx_n, Qx_n, Qx_{n+1}, Kt), N(Lx, Qx_n, Fx_{n+1}, Kt), N(Qx_n, STx_n, Fx_{n+1}, Kt), N(ABx, STx_n, STx_{n+1}, Kt), N(ABx, Qx_n, STx_{n+1}, Kt), N(ABx, STx_n, Qx_{n+1}, Kt)\}$$

Letting $n \to \infty$ we get

$$M(Px, u, u, t) = 1, N(Px, u, u, t) = 0 .$$

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Hence,

$$Px = u = ABx = Lx.$$

If (P, AB) and (P, L) are weakly compatible , we have

$$P(AB)x = (AB)Px, \text{ so}$$

$$P\,Px = P(AB)x = (AB)Px = AB(AB)x,$$

so we have $Pu = ABu$ and $PLx = LPx$, $Pu = u$ Hence $Pu = ABu = Lu$.

Step 3: As $p(X) \subset ST(X) \cup F(x)$,

there exists $v \in X$ such that $Px = STv = Fv$. We prove that $STv = Qv$

Case I: If $STv \neq Qv$, Putting $y = v$, and $z = v$ in (1.2.4) then we have

$$M(Px, Qv, Qv, t) \geq \phi\{M(ABx, STv, Lx, Kt), M(Lx, STv, STv, Kt), M(ABx, STv, Fv, Kt), M(ABx, Qv, Fv, Kt), M(ABx, Fv, Qv, Kt), M(STv, Qv, Fv, Kt), M(Fv, Qv, Qv, Kt), M(Lx, Qv, Fv, Kt), M(Qv, STv, Fv, Kt), M(ABx, STv, STv, Kt), M(ABx, Qv, STv, Kt), M(ABx, STv, Qv, Kt)\}$$

and

$$N(Px, Qv, Qv, t) \leq \phi'\{N(ABx, STv, Lx, Kt), N(Lx, STv, STv, Kt), N(ABx, STv, Fv, Kt), N(ABx, Qv, Fv, Kt), N(ABx, Fv, Qv, Kt), N(STv, Qv, Fv, Kt), N(Fv, Qv, Qv, Kt), N(Lx, Qv, Fv, Kt), N(Qv, STv, Fv, Kt), N(ABx, STv, STv, Kt), N(ABx, Qv, STv, Kt), N(ABx, STv, Qv, Kt)\}$$

Case II: If $Qv \neq u$, then we have

$$M(Px, Qv, Qv, t) > M(Px, Qv, Qv, kt),$$

$$N(Px, Qv, Qv, t) < M(Px, Qv, Qv, kt),$$

which is a contradiction,

Thus $STv = Qv = Px = Fv = u$.

Step 4: If (Q, ST) and (Q, F) is weakly compatible mappings then we get

$$Q(ST)v = (ST)Qv \text{ so, } (ST)(ST)v = (ST)Qv. = Q.Qv,$$

so $STu = Qu.$ and $QFv = FQv$

we prove $Pu = u$, for $Qu = Fu$

$$M(Pu, u\,u, t) = M(Pu, Qv, Qv, t)$$

$$\geq \phi\{M(ABu, STv, Lu, Kt), M(Lu, STv, STv, Kt), M(ABu, STv, Fv, Kt), M(ABu, Qv, Fv, Kt), M(ABu, Fv, Qv, Kt), M(STv, Qv, Fv, Kt), M(Fv, Qv, Qv\,Kt), M(Lu\,Qv, Fv, Kt), M(Qv, STv, Fv, Kt), M(ABu, STv, STv, Kt), M(ABu, Qv, STv, Kt), M(ABu, STv, Qv, Kt)\}$$

and

$$N(Pu, u, u\,t) = N(Pu, Qv, Qv, t)$$

$$\leq \phi'\{N(ABu, STv, Lu, Kt), N(Lu, STv, STv, Kt), N(ABu, STv, Fv, Kt), N(ABu, Qv, Fv, Kt), N(ABu, Fv, Qv, Kt), N(STv, Qv, Fv, Kt), N(Fv, Qv, Qv\,Kt), N(Lu\,Qv, Fv, Kt), N(Qv, STv, Fv, Kt), N(ABu, STv, STv, Kt), N(ABu, Qv, STv, Kt), N(ABu, STv, Qv, Kt)\}$$

Notes

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Step 5: If $Pu \neq u$, then we have

$$M(Pu, u, u, t) > M(Pu, u, u, Kt)$$

$$N(Pu, u, u, t) < N(Pu, u, u, Kt),$$

which is contradiction. Thus

$$Pu = u = ABu = Lu. \quad (1)$$

Step 6: Now we prove $Qu = u$. For

$$M(u, Qu, Qu, t) = M(Pu, Qu, Qu, t)$$

$$\geq \phi\{M(ABu,STu,Lu, Kt), M(Lu, STu, STu, Kt), M(ABu, STu, Fu, Kt), M(ABu, Qu, Fu, Kt), M(ABu,Fu, Qu, Kt), M(STu, Qu, Fu, Kt), M(Fu, Qu, Qu Kt), M(Lu Qu, Fu, Kt), M(Qu, STu, Fu, Kt), M(ABu,STu,STu, Kt), M(ABu,Qu,STu, Kt), M(ABu,STu,Qu, Kt)\}$$

and

$$N(u, Qu, Qu, t) = N(Pu, Qu, Qu, t)$$

$$\leq \phi'\{N(ABu,STu,Lu, Kt), N(Lu, STu, STu, Kt), N(ABu, STu, Fu, Kt), N(ABu, Qu, Fu, Kt), N(ABu, Fu, Qu, Kt), N(STu, Qu, Fu, Kt), N(Fu, Qu, Qu Kt), N(Lu Qu, Fu, Kt), N(Qu, STu, Fu, Kt), N(ABu,STu,STu,Kt), N(ABu,Qu,STu,Kt), N(ABu, STu, Qu, Kt)\}$$

Step 7: If $Qu \neq u$ then we have,

$$M(u, Qu, Qu, t) > M(u, Qu, Qu, Kt)$$

$$N(u, Qu, Qu, t) < N(u, Qu, Qu, Kt),$$

which is contradiction. Thus

$$Pu = Qu = ABu = STu = Fu=Lu = u. \quad (2)$$

Step 8: Now we show that $Bu = u$ by putting $x = Bu$, $y = x_{2n+1}$ and $z = x_{2n}$ in (1.2.4)

If $Bu \neq u$ then

$$M(P(Bu), Qx_{2n+1}, Qx_{2n}, t) \geq \phi\{M(AB(Bu), STx_{2n+1}, L(Bu), Kt), M(L(Bu), STx_{2n+1}, STx_{2n}, Kt), M(AB(Bu), STx_{2n+1}, Fx_{2n}, Kt), M(AB(Bu), Qx_{2n+1}, Fx_{2n}, Kt), M(AB(Bu), Fx_{2n+1}, Qx_{2n}, Kt), M(STx_{2n}, Qx_{2n}, Fx_{2n}, Kt), M(Fx_{2n+1}, Qx_{2n+1}, Qx_{2n}, Kt), M(L(Bu), Qx_{2n+1}, Fx_{2n}, Kt), M(Qx_{2n+1}, STx_{2n+1}, Fx_{2n}, Kt), M(AB(Bu), STx_{2n+1}, STx_{2n}, Kt), M(AB(Bu), Qx_{2n+1}, STx_{2n}, Kt), M(AB(Bu), STx_{2n+1}, Qx_{2n}, Kt)\}$$

and

$$N(P(Bu), Qx_{2n+1}, Qx_{2n}, t) \leq \phi'\{N(AB(Bu), STx_{2n+1}, L(Bu), Kt), N(L(Bu), STx_{2n-1}, STx_{2n}, Kt), N(AB(Bu), STx_{2n+1}, Fx_{2n}, Kt), N(AB(Bu), Qx_{2n+1}, Fx_{2n}, Kt), N(AB(Bu), Fx_{2n+1}, Qx_{2n}, Kt), N(STx_{2n}, Qx_{2n}, Fx_{2n}, Kt), N(Fx_{2n+1}, Qx_{2n+1}, Qx_{2n}, Kt), N(L(Bu), Qx_{2n+1}, Fx_{2n}, Kt), N(Qx_{2n+1} STx_{2n+1}, Fx_{2n}, Kt), N(AB(Bu), STx_{2n+1}, STx_{2n}, Kt), N(AB(Bu), Qx_{2n+1}, STx_{2n}, Kt), N(AB(Bu), STx_{2n+1}, Qx_{2n}, Kt)\}$$

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since AB=BA, BP=PB and LB=BL ,

we have

P(Bu) = B(Pu) = Bu, AB (Bu) = BA (Bu) = Bu

L(Bu) = B(Lu) = Bu.

Letting $n \to \infty$, we have

M (Bu, u, u, t) > M (Bu, u, u, kt)

N (Bu, u, u, t) < N (Bu, u, u, kt),

which is contradiction.

Thus Bu = u.

Since u = ABu,

we have u = Au ,

therefore,

u = Au = Bu = Pu = Lu. ……(3)

Step 9: Finally we show that Tu = u. By putting x = u, y = Tu and z = u in (1.2.4) If Tu ≠ u, then

M(Pu, Q(Tu), Qu, t) ≥ ϕ { M(ABu, ST(Tu), Lu, Kt), M(Lu, ST(Tu), STu, Kt),
M(ABu, ST(Tu), Fu, Kt), M(ABu, Q(Tu), Fu, Kt),
M(ABu, F(Tu), Qu,Kt), M(STu, Qu, Fu, Kt), M(F(Tu), Q(Tu), Qu, Kt),
M(Lu,Q(Tu), Fu, Kt), M(Q(Tu), ST(Tu), Fu, Kt), M(ABu, ST(Tu), STu, Kt),
M(ABu, Q(Tu), STu, Kt), M(ABu, ST(Tu), Qu, Kt)} and

N(Pu,Q(Tu), Qu, t) ≤ ϕ'{ N(ABu, ST(Tu), Lu, Kt), N(Lu, ST(Tu), STu, Kt), N(ABu, ST(Tu), Fu, Kt),
N(ABu, Q(Tu), Fu, Kt), N(ABu, F(Tu) ,Qu, Kt), N(STu, Qu , Fu, Kt),
N(F(Tu), Q(Tu), Qu, Kt), N(Lu, Q(Tu), Fu, Kt), N(Q(Tu), ST(Tu)
Fu, Kt), N(ABu, ST(Tu), STu, Kt), N(ABu, Q(Tu), STu, Kt), N(ABu,
ST(Tu), Qu, Kt)}

Since S T = TS, TQ = QT and FT = TF,

We have,

ST(Tu) = T(STu) = Tu,

QTu = TQu = Tu and

FTu = TFu = Tu. Then,

M(u, Tu, u, t) > M(u, Tu, u, Kt)

N(u, Tu, u, t) < N(u, Tu, u, Kt),

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Notes

which is a contradiction,

thus Tu = u ,

since u = STu,

we have u = Su = Tu . .....(4)

By combining the above result (1), (2), (3) and (4) we get

Au = Bu = Su = Tu = Fu = Lu = Pu = Qu = u. So P, Q, A, B, L, F, S and T have a common fixed point u.

Now to prove the uniqueness: suppose that v ≠ u is another common fixed point of P, Q, A, B, L, F, S and T , then

$$M(v, u, u, T) = M(Pv, Qu, Qu, T)$$

$$\geq \phi\{M(ABv, STu, Lv, Kt), M(Lv, STu, STu, STu, Kt), M(ABv, STu, Fu, Kt), M(ABv, Qu, Fu, Kt), M(ABv, Fu, Qu, Kt), M(STu, Qu, Fu, Kt), M(Fu, Qu, Qu, Kt), M(Lv, Qu, Fu, Kt), M(Qu, STu, Fu, Kt), M(ABv, STu, STu, Kt), M(ABv, Qu, STu, Kt), M(ABv, STu, Qu, Kt)\}$$

$$> M(v, u, u, Kt).$$

Similarly

$$N(v, u, u, t) = N(Pv, Q u, Qu, t)$$

$$\leq \phi'\{N(ABv, STu, Lv, Kt), N(Lv, STu, STu, STu, Kt), N(ABv, STu, Fu, Kt), N(ABv, Qu, Fu, Kt), N(ABv, Fu, Qu, Kt), N(STu, Qu, Fu, Kt), N(Fu, Qu, Qu, Kt), N(Lv, Qu, Fu, Kt), N(Qu, STu, Fu, Kt), N(ABv, STu, STu, Kt), N(ABv, Qu, STu, Kt), N(ABv, STu, Qu, Kt)\}$$

$$< N(v, u, u, Kt),$$

which is a contradiction ,

therefore v = u is common fixed point of P, Q, A, B, L, F, S and T.

COROLLARY: Let f, g be self mappings of X satisfying the following conditions

(i) f (X) ⊂ g (X) and g (X) is complete fuzzy metric subspace of X .

(ii) The pair (f, g) is weakly compatible and (f, g) satisfies the property (E).

(iii) If there exists a number K > 1 S.t.

$$M(fx, fy, fz, t) \geq \phi\{M(gx, gy, fx, Kt), M(fx, gy, gz, Kt)), M(gx, gy, gz, Kt), M(gx, fy, gz, Kt), M(gx, gy, fz, Kt), M(gz, fz, gz, Kt), M(gy, fy, fz, Kt), M(fy, fx, gz, Kt), M(fy, gy, gz, Kt), M(gx, gy, gz, Kt), M(gx, fy, gz, Kt), M(gx, gy, fz, Kt)\} \text{ and}$$

$$N(fx, fy, fz, t) \leq \phi'\{N(gx, gy, fx, Kt), N(fx, gy, gz, Kt)), N(gx, gy, gz, Kt), N(gx, fy, gz, Kt), N(gx, gy, fz, Kt), N(gz, fz, gz, Kt), N(gy, fy, fz, Kt), N(fy, fx, gz, Kt), N(fy, gy, gz, Kt), N(gx, gy, gz, Kt), N(gx, fy, gz, Kt), N(gx, gy, fz, Kt)\}$$

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for all x, y, z ∈ X and t > 0 then f, g, have a unique common fixed point in X .

Example 3 : - Let X = [0,1] with the usual generalized metric D .

Define,

$$M(x, y, z, t) = \frac{t}{t+|x-y|+|y-z|+|z-x|},$$

$$N(x, y, z, t) = \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|}, \text{ for all } x, y, z \in X \text{ and } t > 0$$

M(x, y, z, 0) = 0, N(x, y, z, 0) = 1 for all x, y, z ∈ X clearly (X, M, N, ∗, ◊) is a complete intuitionistic M–Fuzzy metric space where ∗ and ◊ are defined b a* b = min (a, b) and a ◊ b = max (a, b). LetA, B, S, T, P, Q, L and F be defined as

$$Sx = x,\ Tx = \frac{x}{2},\ Ax = \frac{x}{5},\ Bx\frac{x}{3},\ Px = \frac{x}{6},\ Qx = 0,\ Fx = \frac{x}{4} \text{ and } Lx = \frac{x}{7} \text{ for all } x, y, z \in X.$$

Then

$$P(x) = \left[0, \frac{1}{6}\right] \subset \left[0, \frac{1}{2}\right] \cup \left[0, \frac{1}{4}\right] = ST(X) \cup F(X) \quad \text{and}$$

$$Q(X) = \{0\} \subset \left[0, \frac{1}{15}\right] \cup \left[0, \frac{1}{7}\right] = ABx(X) \cup L(X)$$

Clearly

AB = BA, ST = TS, PB = BP, TQ = QT, FT = TF, LB = BL.

Moreover, the pairs (P, AB), (Q, ST), (P, L) and (Q, F) are weakly compatible at 0 and the pair (Q, ST) and (Q, F) satisfies the property (E) if $\lim_{n\to\infty} x_n = 0$, where { x_n } is a sequence in X s.t.

$$\lim_{n\to\infty} M(Qx_n, u, u, t) = \lim_{n\to\infty} M(STx_n, u, u, t) = 1,$$

$$\lim_{n\to\infty} N(Qx_n, u, u, t) = \lim_{n\to\infty} N(STx_n, u, u, t) = 0 \text{ and}$$

$$\lim_{n\to\infty} M(Qx_n, u, u, t) = \lim_{n\to\infty} M(Fx_n, u, u, t) = 1, \text{ and}$$

$$\lim_{n\to\infty} N(Qx_n, u, u, t) = \lim_{n\to\infty} N(Fx_n, u, u, t) = 0$$

For u = 0 ∈ X and t > 0 If we take K = 2 and t = 1, then conditions (1.2.4) of the main theorem is satisfied and 0 is the unique common fixed point of P, Q, A, S, L, F, S and T.

Ref.

1 A. George and P. Veeramani: On some results in fuzzy metric spaces, Fuzzy Sets and Systems,64(1994), 395-399.

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Notes

References Références Referencias

1. A. George and P. Veeramani: On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 64(1994), 395-399.
2. C. Alaca , D. Turkoglu and C. Yiliz:Fixed points in intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals, 29(2006), 1073-1078.
3. J.H.Park:Intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals, 22(2004), 1039-11046.
4. K. Atanassov: Intuitionistic fuzzy Sets and Systems, 20(1986), 87-96.
5. O.Kramosil and J.Michalek:Fuzzy metric and statistical metric spaces, Ky-bernetics, 11(1975), 330-334.
6. S. Sedghi and N. Shobe:Fixed point theorem in M-fuzzy metric spaces with property (E), Advances in Fuzzy Mathematics, 1(1) (2006), 55-65.
7. Singh S.L. and Singh:S.P., A fixed point theorem, Indian J.of Pure and App. Math., 11(1980), 1584-1586.
8. Mehra S. and Gugnani M.:A common fixed point for six mappings in an intruitionisti M-fuzzy metric space. Indian Journal of Mathematics, Vol. 51 No. 1, (2009) 23-47.

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