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A Summation Formula in the Light of Special Functions

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GJSFR-F Classification : *MSC 2010: 40A25, 11B37*

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Notes

A Summation Formula in the Light of Special Functions

Salahuddin [α], M. P. Chaudhary [σ] & Sangeeta Chaudhary [ρ]

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2010 MSC NO : 33C05 , 33C20 , 33D15 , 33D50 , 33D60

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I. Introduction

a) *Generalized Hypergeometric Functions*

A generalized hypergeometric function ${}_pF_q(a_1,...,a_p;b_1,...,b_q;z)$ is a function which can be defined in the form of a hypergeometric series, i.e., a series for which the ratio of successive terms can be written

$$\frac{c_{k+1}}{c_k}=\frac{P(k)}{Q(k)}=\frac{(k+a_1)(k+a_2)...(k+a_p)}{(k+b_1)(K+b_2)...(k+b_q)(k+1)}\ z. \tag{1}$$

Where $k+1$ in the denominator is present for historical reasons of notation[Koepf p.12(2.9)], and the resulting generalized hypergeometric function is written

$${}_pF_q\left[\begin{array}{ccc} a_1,a_2,\cdots,a_p & ; & \\ & & z \\ b_1,b_2,\cdots,b_q & ; & \end{array}\right]=\sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\cdots(a_p)_kz^k}{(b_1)_k(b_2)_k\cdots(b_q)_kk!} \tag{2}$$

or

$${}_pF_q\left[\begin{array}{ccc} (a_p) & ; & \\ & & z \\ (b_q) & ; & \end{array}\right]=\sum_{k=0}^{\infty}\frac{((a_p))_kz^k}{((b_q))_kk!} \tag{3}$$

where the parameters $b_1,b_2,\cdots,b_q$ are positive integers.

The ${}_pF_q$ series converges for all finite z if $p\leq q$, converges for $\mid z\mid<1$ if $p=q+1$, diverges for all z , $z\neq 0$ if $p>q+1$[Luke p.156(3)].

The function ${}_2F_1(a,b;c;z)$ corresponding to $p=2,q=1$, is the first hypergeometric function to be studied (and, in general, arises the most frequently in physical problems), and so is frequently known as "the" hypergeometric equation or, more explicitly, Gauss's hypergeometric function

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[Gauss p.123-162]. To confuse matters even more, the term "hypergeometric function" is less commonly used to mean closed form, and "hypergeometric series" is sometimes used to mean hypergeometric function.

The hypergeometric functions are solutions of Gaussian hypergeometric linear differential equation of second order

$$z(1-z)y'' + [c-(a+b+1)z]y' - aby = 0 \tag{4}$$

The solution of this equation is

$$y = A_0\left[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\right] \tag{5}$$

This is the so-called regular solution, denoted

$$_2F_1(a,b;c;z) = \left[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\right] = \sum_{k=0}^{\infty} \frac{(a)_k\ (b)_k z^k}{(c)_k k!} \tag{6}$$

which converges if c is not a negative integer for all $\mid z \mid < 1$ and on the unit circle $\mid z \mid = 1$ if $R(c-a-b) > 0$.

It is known as Gauss hypergeometric function in terms of Pochhammer symbol $(a)_k$ or generalized factorial function.

Many of the common mathematical functions can be expressed in terms of the hypergeometric function. Some typical examples are

$$(1-z)^{-a} = z\ {}_2F_1(1,1;2;-z) \tag{7}$$

$$\sin^{-1} z = z\ {}_2F_1\left(\frac{1}{2},\frac{1}{2};\frac{3}{2};z^2\right) \tag{8}$$

Gauss' Relations for Contiguous Functions:

The six functions $F(a\pm1,b;c;z)$, $F(a,b\pm1;c;z)$, $F(a,b;c\pm1;z)$ are called contiguous to $F(a,b;c;z)$. Relation between $F(a,b;c;z)$ and any two contiguous functions have been given by Gauss. [Abramowitz p.558(15.2.14)]

$$(a-b)\ {}_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ c\ \ ; & z\end{array}\right] = a\ {}_2F_1\left[\begin{array}{ll} a+1,\ b\ ; & \\ c\ \ ; & z\end{array}\right] - b\ {}_2F_1\left[\begin{array}{ll} a,\ b+1\ ; & \\ c\ ; & z\end{array}\right] \tag{9}$$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$$_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ \frac{a+b+1}{2}\ ; & \frac{1}{2}\end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{10}$$

$$= \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{11}$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$$_2F_1\left[\begin{array}{ll} a,\ b\ ; & \\ \frac{a+b-1}{2}\ ; & \frac{1}{2}\end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{12}$$

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Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a, \ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{13} $$

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Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\ \Gamma(\zeta) \tag{14} $$

II. Main Summation Formula

$$ {}_2F_1\left[\begin{matrix} a, \ b \ ; & \frac{1}{2} \\ \frac{a+b+47}{2} \ ; & \end{matrix}\right] = \frac{2^b\ \Gamma(\frac{a+b+47}{2})}{(a-b)\ \Gamma(b)\Big[\prod_{\Xi=1}^{23}\{a-b-(2\Xi-1)\}\Big]\Big[\prod_{\Upsilon=1}^{23}\{a-b+(2\Upsilon-1)\}\Big]} $$

$$ \Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Big\{4194304(a^{24}+a^{23}(-529+1034b)+23a^{22}(5731-7567b+7050b^2)+253a^{21}(-81305+176908b $$
$$ -61617b^2+36378b^3)+253a^{20}(8912547-153015555b+150871410b^2-2370633b^3+994332b^4) $$
$$ +253a^{19}(-729562323+1675642026b-883014850b^2+537100960b^3-47645075b^4+15080702b^5) $$
$$ +437a^{18}(26795032319-518265029919b+526751872080b^2-133987247500b^3+5687164735b^4 $$
$$ -322080707b^5+79712282b^6)+437a^{17}(-1349398815215+3201026487568b-2051983327545b^2 $$
$$ +1265119609782b^3-188817432225b^4+59627107044b^5-2326985463b^6+459878550b^7) $$
$$ +7429a^{16}(3224471854274-6691057943825b+6910962796281b^2-2224676333517b^3+944086166793b^4 $$
$$ -92136046035b^5+22451270811b^6-634632399b^7+101173281b^8)+874a^{15}(-907619931345281 $$
$$ +2196439780017778b-1583061524809980b^2+978474032814608b^3-189247186989894b^4 $$
$$ +58979505572724b^5-4023569409660b^6+771418674912b^7-16288898241b^8+2090914474b^9) $$
$$ +46a^{14}(468566121314958303-1017824993014405123b+1057739894293651278b^2 $$
$$ -392620567074002300b^3+164887334296881130b^4-21145413880972458b^5+5011088841406536b^6 $$
$$ -248923981301100b^7+37710822893535b^8-601137911275b^9+60636519746b^{10}) $$
$$ +46a^{13}(-10480799815306879695+25650191726385965388b-19959681138199929643b^2 $$
$$ +12250081720114143878b^3-2771196216002273550b^4+842260723772929616b^5 $$
$$ -75877233063158814b^6+13859663123661012b^7-509900689579875b^8+59821063101140b^9 $$
$$ -697319977079b^{10}+49611697974b^{11})-46a^{12}(-193076196366405273361+432242928907257686205b $$
$$ -447986443019768183583b^2+182383159014771602043b^3-74937694261890414058b^4 $$
$$ +11295990811521823810b^5-2557424283239432542b^6+166286552352776310b^7 $$
$$ -23083684967418933b^8+6119322572470250b^9-500554470503230b^{10}+322476036831b^{11}) $$
$$ -46a^{11}(2922906805676735359009-7179158161708177901366b+5874213600004333972050b^2 $$
$$ -3541062392770959869904b^3+883384336805408600591b^4-257296997289990731502b^5 $$

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$$+2700428174859124710 0b^6 - 4532263566070187376b^7 + 2082811939283596695b^8$$

$$-1986737796704661 0b^9 + 2877202861947 70b^{10} - 3224760368831b^{12} + 49611697974b^{13})$$

$$-506a^{10}(-32893231824097797 76423 + 749615912119713450 0799b - 767019980528741041 4916b^2$$

$$+3305843522579583360570b^3 - 1306390804960229398137b^4 + 215279129671865805533b^5$$

$$-449183677699947238 10b^6 + 32469134118669391 00b^7 - 367469311611014041b^8$$

$$+9089528264764233b^9 - 2615638965407 0b^{11} + 4550495186393b^{12} - 633927251 89b^{13} + 5512410886b^{14})$$

$$-506a^9(330293448883879372997 35 - 80716238290121139577160b + 67908675762513429453765b^2$$

$$-39548924300013894311222b^3 + 10359881175427248692175b^4 - 2790368186666261220084b^5$$

$$+305536664589145308709b^6 - 41928657064513848538b^7 + 1600126853018504525b^8$$

$$-9089528264764233b^{10} + 1806125269731510b^{11} - 55630205204275b^{12} + 5438278463740b^{13}$$

$$-546489010 25b^{14} + 36115795 46b^{15}) - 23a^8(-5849633469739333659008547$$

$$+1339441607490051028963 8710b - 1332254741495023876699 6350b^2$$

$$+586252751728962064948 1846b^3 - 215113289249602594053 3226b^4 + 35531905152626510138 6670b^5$$

$$-60814387504205828165478b^6 + 34261981564968596825 10b^7 - 352027907664070995 50b^9$$

$$+80843248554423089 02b^{10} - 416562387856719390b^{11} + 46167369934837866b^{12}$$

$$-1019801379159750b^{13} + 75421645787070b^{14} - 6189781331 58b^{15} + 32678969763b^{16})$$

$$-23a^7(3713410653113709527978 7603 - 8910139947324650080388 5710b$$

$$+7496115881356316366205 9560b^2 - 4075010141095135505307 6768b^3$$

$$+104092347082099935179 24372b^4 - 230846920850769335830 4248b^5$$

$$+189319585005743733067800b^6 - 34261981564968596825 10b^8 + 922430455419304667836b^9$$

$$-714320950610726602 00b^{10} + 9064527132140374752b^{11} - 332573104705552620b^{12}$$

$$+2771932624732202 4b^{13} - 4978479626022 00b^{14} + 29313909646656b^{15} - 204986264877b^{16}$$

$$+8737692450b^{17}) - 23a^6(-1816365075503293031147 86671 + 409524468899085745553 896155b$$

$$-383166283662043789339 491690b^2 + 16117827258867514318577 2280b^3$$

$$-48924876796158281140066 500b^4 + 5890553184666820563182180b^5$$

$$-189319585005743733067800b^7 + 60814387504205828165478b^8 - 6721806620961196791598b^9$$

$$+988204090939883923820b^{10} - 54008563497182494200b^{11} + 5114848566478865084b^{12}$$

$$-151754466126317628b^{13} + 10022177682813072b^{14} - 152895637567080b^{15} + 7251760471953b^{16}$$

$$-44212723797b^{17} + 1514533358b^{18}) - 23a^5(662329119693726610533057375$$

$$-151637102045222658016908 9924b + 1202265485968967736276 728367b^2$$

$$-544218572588615646324600054b^3 + 99386937441957904221828620b^4$$

$$-5890553184666820563182180b^6 + 2308469208507693358304248b^7$$

$$-355319051526265101386670b^8 + 61388100106657746841848b^9 - 4736140852781047721726b^{10}$$

$$+514593994579981463004b^{11} - 22591981623043647620b^{12} + 1684521447545859232b^{13}$$

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$$-4229082776194491 6b^{14} + 2241221211763512b^{15} - 297599428693305b^{16} + 11329150338 36b^{17}$$

$$-61195334 33b^{18} + 165887722b^{19}) - 23a^{4}(-1714166473560132461916610125$$

$$+3618638378432052200750621925b - 2849793884317255245197831595b^{2}$$

$$+8455065768711816796881917 19b^{3} - 993869374419579042218286 20b^{5}$$

$$+4892487679615828114000665 00b^{6} - 104092347082099935179243 72b^{7}$$

$$+215113289249602594053322 6b^{8} - 227917385859399471227850b^{9} + 287405977091250467590 14b^{10}$$

$$-17667686736108172011 82b^{11} + 149875388523780828116b^{12} - 5542392432004547100b^{13}$$

$$+329774668593762260b^{14} - 7191393105615972b^{15} + 30493983187413 9b^{16} - 3587531212275b^{17}$$

$$+108056129965b^{18} - 524095825b^{19} + 10937652b^{20}) - 23a^{3}(2910996690958124489461348125$$

$$-5746357719280004177264775750b + 3191889854580266346156760950b^{2}$$

$$-8455065768711816796881917 19b^{4} + 5442185725886156463246000 54b^{5}$$

$$-16117827258867514318577228 0b^{6} + 40750101410951355053076768b^{7}$$

$$-5862527517289620649481846b^{8} + 870076334600305674846884b^{9} - 7272855749675083393254 0b^{10}$$

$$+70821247855419197398 08b^{11} - 364766318029543204086b^{12} + 24500163440228287756b^{13}$$

$$-785241134148004600b^{14} + 37182013246955104b^{15} - 718570455725991b^{16} + 24037272585858b^{17}$$

$$-254575770250b^{18} + 5908110560b^{19} - 26076963b^{20} + 400158b^{21})$$

$$+a^{2}(6469153378001424769439 2179375 - 9683302758361323062557071 9375b$$

$$+7341346665534612596160550 1850b^{3} - 6554525933929687063955012 6685b^{4}$$

$$+2765210617728625793436475 2441b^{5} - 8812824524227007154808308 870b^{6}$$

$$+1724106652711952764227369 880b^{7} - 30641859054385549164091605 0b^{8}$$

$$+34361789935831795303605090b^{9} - 3881121101475429669947496b^{10}$$

$$+270213825600199362714300b^{11} - 206073763789093364448 18b^{12} + 918145332357196763578b^{13}$$

$$-4865603513750795878 8b^{14} + 1383595772683922520b^{15} - 51341542613571549b^{16}$$

$$+896716714137165b^{17} - 23019056809896b^{18} + 223402757050b^{19} - 3817046673b^{20} + 15589101b^{21}$$

$$-162150b^{22}) + a(-2537379133562625794765760 9375 + 9683302758361323062557071 9375b^{2}$$

$$-1321662275434400960770898 42250b^{3} + 8322868270393720061726430 4275b^{4}$$

$$-3487653347040121134388906 8252b^{5} + 9419062784678972147739611 565b^{6}$$

$$-2049332187884669518489371 330b^{7} + 308071569722711736661690330b^{8}$$

$$-40842416574801296626042960b^{9} + 3793056515325750057404294b^{10}$$

$$-330241275438576183462836b^{11} + 19883174729733853565430b^{12} - 1179908819413754407848b^{13}$$

$$+4681994967866263565 8b^{14} - 1919688367735537972b^{15} + 49707869464675925b^{16}$$

$$-1398848575067216b^{17} + 22648181807067b^{18} - 423937432578b^{19} + 3871293415b^{20} - 447577 24b^{21}$$

$$+174041b^{22} - 1034b^{23}) - b(-2537379133562625794765760 9375 + 6469153378001424769439 2179375b$$

$$-6695292389203686325761100 6875b^{2} + 3942582889188304662408203 2875b^{3}$$

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$$-15233569752955712042260319625b^4+417763967365757397164009343 3b^5$$

$$-85408445021615319143511486 9b^6+13454156980400467415719658 1b^7$$

$$-16712848513524296273665910b^8+16643975302993485668700038b^9$$

$$-1344537130611298265144 14b^{10}+88815050328546425746 06b^{11}-48211679150411646597 0b^{12}$$

$$+21554041580488081938b^{13}-793259819995775594b^{14}+23954601405401546b^{15}$$

$$-589687282248955b^{16}+11709429123403b^{17}-184579267719b^{18}+2254874391b^{19}-20570165b^{20}$$

$$+131813b^{21}-529b^{22}+b^{23}))\Big\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Big\{16777216(23a^{23}+23a^{22}(-176+329b)$$

$$+253a^{21}(5215-3008b+2679b^2)+253a^{20}(-452960+879605b-182736b^2+103071b^3)$$

$$+253a^{19}(60311301-52865600b+47979950b^2-5173760b^3+2071525b^4)+437a^{18}(-1866667344$$

$$+3685773417b-1241401600b^2+703813250b^3-46402160b^4+14003509b^5)+437a^{17}(136919748025$$

$$-147120185088b+134215406415b^2-24378110400b^3+9700951575b^4-429551424b^5+101173281b^6)$$

$$+7429a^{16}(-288016551040+571859066695b-244761992496b^2+138065017179b^3-15710445600b^4$$

$$+4662174165b^5-147161136b^6+27592713b^7)+874a^{15}(110646186438247-134786297605760b$$

$$+122504471439780b^2-28930053259008b^3+11338806793578b^4-879944046720b^5+201426804900b^6$$

$$-4709156352b^7+708212967b^8)+46a^{14}(-52099555819934768+103225068855282101b$$

$$-51223393194057600b^2+28510225961592100b^3-4278910526304480b^4+1236471420894246b^5$$

$$-68930276102400b^6+12367973723700b^7-218534286960b^8+261364309 25b^9)$$

$$+46a^{13}(1523275998062398905-2020689131217706688b+1815921091026075341b^2$$

$$-504115076937750400b^3+192797827726790850b^4-19872507249052416b^5+4362411218030418b^6$$

$$-179742611510400b^7+25204793629125b^8-334546315840b^9+303182598 73b^{10})$$

$$+26a^{12}(-46738665698539844640+91784298587795117985b-50292357649927039888b^2$$

$$+27376830740023687911b^3-4861870680816380800b^4+1350325066901821930b^5$$

$$-99722984753260512b^6+16721579297625870b^7-508861134117600b^8+53344455517925b^9$$

$$-485092157968b^{10}+24805848987b^{11})+a^{11}(2333775765553860535641 8-3285539922195769807296 0b$$

$$+28952254818229910029668b^2-8932261695719974004224b^3+3291278251514442174622b^4$$

$$-400249492825308222720b^5+82302941585946050424b^6-4388608849494744576b^7$$

$$+540688988712851550b^8-11125003186943360b^9+651114949032548b^{10}-644952073662b^{12})$$

$$-22a^{10}(1274781008587492428414 4-24623395110033406407199b+14414748449268814208640b^2$$

$$-7580198303956038791706b^3+1491441502941101280816b^4-388981529590737600413b^5$$

$$+33102061417630424960b^6-4888486075362902428b^7+173245521378051408b^8$$

$$-11757702406746153b^9+29596134046934b^{11}-573290732144b^{12}+633927251 89b^{13})$$

$$-22a^9(-159174616402628497304455+233000874791703045269120b-198962130903196818972645b^2$$

$$+65380182773479633225920b^3-22683573168494062454175b^4+2984544052387419084992b^5$$

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$$-541870830159333995749b^6 + 299560510446521209 60b^7 - 2395236539336594525b^8$$

$$+11757702406746153b^{10} - 505681963042880b^{11} + 63043447430275b^{12} - 6995059331 20b^{13}$$

$$+546489010 25b^{14}) - 2a^8(1413553372774907620162336 0 - 2655946442407824474196871 5b$$

$$+1612124389809876002742744 0b^2 - 800692167718156613245084 3b^3$$

$$+1638593601699205190176800b^4 - 3783592680021002122460 55b^5 + 3133994334304517450395 2b^6$$

$$-3011394038219097288615b^7 + 2634760193270253977 5b^9 - 190570073515856548 8b^{10}$$

$$+2703444943564257775b^{11} - 661519474352880 0b^{12} + 5797102534698 75b^{13} - 502628860008 0b^{14}$$

$$+3094890665 79b^{15}) + a^7(2208793196453847886378068 03 - 3288394088828426491835609 60b$$

$$+2661862282876559483733725 84b^2 - 8857380645642457006374758 4b^3$$

$$+2728934142778048237581429 2b^4 - 3359674931126195881854720b^5$$

$$+3979584463542500106343 92b^6 - 6022788076438194577230b^8 + 6590331229823466611 20b^9$$

$$-1075466936579838534 16b^{10} + 43886088494947445 76b^{11} - 434761061738272620b^{12}$$

$$+8268160129478400b^{13} - 568926791290200b^{14} + 41158026516 48b^{15} - 204986264877b^{16})$$

$$+a^6(-1125939521154262717909258128 + 2015056862560521162160964955b$$

$$-1215154200556285221697944320b^2 + 5378488099323801650020522 16b^3$$

$$-1002744790663857544331126 40b^4 + 1515073722316320969292106 0b^5$$

$$-3979584463542500106343 92b^7 + 6267988668609034900790 4b^8 - 1192115826350534790647 8b^9$$

$$+728245351187869349120b^{10} - 8230294158594605042 4b^{11} + 25927976035847733 12b^{12}$$

$$-2006709160293992 28b^{13} + 3170792700710400b^{14} - 176047027482600b^{15} + 1093260079344b^{16}$$

$$-44212723797b^{17}) + a^5(5066968478525294055658370415 - 7393789471350456781405463232b$$

$$+5357661269531597371164089007b^2 - 1593656779593915351419429120b^3$$

$$+3223875454257754131549611 00b^4 - 1515073722316320969292106 0b^6$$

$$+3359674931126195881854720b^7 - 7567185360042004244921 10b^8 + 6565996915252321986982 4b^9$$

$$-8557593650996227209086b^{10} + 400249492825308222720b^{11} - 35108451739447370180b^{12}$$

$$+914135333456411136b^{13} - 56877685361135316b^{14} + 769071096833280b^{15} - 34635291871785b^{16}$$

$$+187713972288b^{17} - 6119533433b^{18}) + a^4(-1441167509289801442673939760 0$$

$$+2323472231289630313380266536 5b - 1235874724062886664113237704 0b^2$$

$$+3606933776254835213741125479b^3 - 3223875454257754131549611 00b^5$$

$$+1002744790663857544331126 40b^6 - 2728934142778048237581429 2b^7$$

$$+3277187203398410380353600b^8 - 4990386097068693739918 50b^9 + 3281171306470422817795 2b^{10}$$

$$-3291278251514442174622b^{11} + 1264086377012259008 00b^{12} - 886870007543237910 0b^{13}$$

$$+196829884210006080b^{14} - 9910117137587172b^{15} + 116712900362400b^{16} - 4239315838275b^{17}$$

$$+20277743920b^{18} - 5240958 25b^{19}) + a^3(3107723477643850851671872972 5$$

$$-3967288204421512256708630304 0b + 1906606151437531286390544667 8b^2$$

$$
\begin{aligned}
&-36069337762548352137411254 79b^4 + 159365677959391535141942 9120b^5\\
&-537848809932380165002052216b^6 + 885738064564245700637475 84b^7\\
&-160138433543631322649016 86b^8+143836402101655193097024 0b^9-166764362687032853417532b^{10}\\
&+89322616957199740042 24b^{11} - 711797599240615885686b^{12} + 231892935391365184 00b^{13}\\
&-131147039423323660 0b^{14}+252848665483729 92b^{15}-102568501262279 1b^{16}+106532342448 00b^{17}\\
&-3075663902 50b^{18} + 1308961280b^{19} - 26076963b^{20}) + a^2(-36292240882792148263256898000\\
&+39168558458823474157546413225b - 1906606151437531286390544 6678b^3\\
&+12358747240628866641132377040b^4 - 5357661269531597371164089007b^5\\
&+1215154200556285221697944320b^6 - 266186228287655948373372584b^7\\
&+32242487796197520054854880b^8-4377166879870330017398190b^9+317124465883913912590080b^{10}\\
&-28952254818229910029668b^{11} + 1307601298898103037088b^{12} - 83532370187199465686b^{13}\\
&+2356276086926649600b^{14}-107068908038367720b^{15}+1818336842252784b^{16}-58652132603355b^{17}\\
&+542492499200b^{18}-12138927350b^{19}+46232208b^{20}-677787b^{21})+a(20204201158151210778288335625\\
&-39168558458823474157546413225b^2 + 39672882044215122567086303040b^3\\
&-23234722312896303133802665365b^4 + 7393789471350456781405463232b^5\\
&-2015056862560521162160964955b^6 + 328839408882842649183560960b^7\\
&-53118928848156489483937430b^8+5126019245417466995920640b^9-541714692420734940958378b^{10}\\
&+32855399221957698072960b^{11} - 2386391763282673067610b^{12} + 92951700036014507648b^{13}\\
&-4748353167342976646b^{14}+117803224107434240b^{15}-4248341006477155b^{16}+64291520883456b^{17}\\
&-1610682983229b^{18} + 13374996800b^{19} - 222540065b^{20} + 761024b^{21} - 7567b^{22})\\
&+b(-20204201158151210778288335625 + 36292240882792148263256898000b\\
&-31077234776438508516718729725b^2 + 14411675092898014426739397600b^3\\
&-5066968478525294055658370415b^4 + 1125939521154262717909258128b^5\\
&-220879319645384788637806803b^6 + 28271067455498152403246720b^7\\
&-3501841560857826940698010b^8+280451821889248334251168b^9-23337757655538605356418b^{10}\\
&+1215205308162035960640b^{11} - 70070695910870349630b^{12} + 2396579567716999328b^{13}\\
&-96704766947027878b^{14} + 2139674957676160b^{15} - 59833929886925b^{16} + 815733629328b^{17}\\
&-15258759153b^{18} + 114598880b^{19} - 1319395b^{20} + 4048b^{21} - 23b^{22}))\Big\}\Bigg]
\end{aligned}
\tag{15}
$$

III. Derivation of the Summation Formula

Substituting $c = \frac{a+b+47}{2}$ and $z = \frac{1}{2}$ in equation (9), we get

$$
(a-b)\ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+47}{2} & ; & \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ccc} a+1,\ b & ; & \frac{1}{2} \\ \frac{a+b+47}{2} & ; & \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ccc} a,\ b+1 & ; & \frac{1}{2} \\ \frac{a+b+47}{2} & ; & \end{array}\right]
$$

Now involving the derived formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

Notes

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