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Bianchi Type-IX Dark Energy Model in Sen-Dunn Scalar-Tensor Theory of Gravitation

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GJSFR-F Classification : *FOR Code : MSC 2010: 81V17*

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Bianchi Type-IX Dark Energy Model in Sen-Dunn Scalar-Tensor Theory of Gravitation

H. R. Ghate [α] & Atish S. Sontakke [σ]

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I. Introduction

Einstein's theory of general relativity is one of the most beautiful structures of theoretical physics which is also known as the most successful theory of gravitation in terms of geometry. In the last decade, several theories of gravitation have been proposed as alternatives to the theory of general relativity. The most popular amongst them are scalar-tensor theories of gravitation formulated by Brans-Dicke [1], Nordtvedt [2], Sen [3], Sen and Dunn [4], Wagonar [5], Saez-Ballester [6], Barber [7]. Sen and Dunn have proposed scalar tensor theory of gravitation in which a scalar field is characterized by the function $\phi=\phi(x^{i})$, where x^{i} represents co-ordinates in the four-dimensional Lyra manifold and the tensor field is identified with the metric tensor g_{ij} of the manifold. Later on Halford [8], Chatterji and Roy [9], Reddy *et al.* [10, 11] have obtained cosmological models in Sen-Dunn theory of gravitation with different contexts. Recently Venkateswarlu and Satish *et al.* [12, 13] have investigated Kantowski-Sach and Bianchi type-VI string cosmological models in Sen-Dunn theory of gravitation.

In the last decade, one of the most remarkable observational discoveries has shown that our universe is undergoing accelerated expansion [14-22]. The Theory of Dark Energy (DE) with negative pressure is mainly responsible for this scenario. It is estimated that the energy density of our universe consist of about 2/3[rd] DE and 1/3[rd] dark matter (DM) of the whole universe [23, 24]. In modern cosmology the origin of cosmology is still mystery.

Many Authors have proposed different DE models such as quintessence [25-31], Chaplygin gas [32], Phantom energy [33], DE in Brane world [34-36]. The existence of DE fluids is the main candidate responsible for the accelerated expansion of the universe and the cosmological models with isotropic pressure give the best fitting for the existence of DE fluids. Thus the DE models have significant importance in study of the expansion of the universe. Many relativists [37-41] have obtained DE cosmological models in various theories of gravitation with different contexts. Recently Ghate and

Ref

5. Wagoner, R.T.: Physical Review D, 1(12), 3209 (1970).

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Sontakke [42-44] have studied DE cosmological models in Saez-Ballester, Lyra and Brans-Dicke theory of gravitation.

The study of Bianchi type models is important in achieving better understanding of anisotropy in the universe. Moreover, the anisotropic universes have greater generality than FRW isotropic models. The simplicity of the field equations made Bianchi type space-times useful. Bianchi type I-IX cosmological models are homogeneous and anisotropic. Bianchi type-IX universe is studied by the number of cosmologists because of familiar solutions like Robertson–Walker Universe, the de-sitter universe, the Taub-Nut solutions etc. Chakraborty [45], Bali and Dave [46], Bali and Yadav [47] studied Bianchi type-IX string as well as viscous fluid models in general relativity. Pradhan et al. [48] have studied some homogeneous Bianchi type-IX viscous fluid cosmological models with varying Λ. Tyagi and Chhajed [49] have obtained Bianchi type-IX string cosmological models for perfect fluid distribution in general relativity. Ghate and Sontakke [50, 51] have studied Bianchi type-IX cosmological models with different context.

To study cosmological models one of the important observational quantity is the deceleration parameter denoted by q and given by $q=-\frac{R\ddot{R}}{\dot{R}^2}$, where R is the mean scale factor of the model. The Deceleration parameter (q) is useful in studying expansion rate of the universe. In any cosmological model, the Hubble constant H_0 and deceleration parameter q play an important role in describing the nature of evolution of the universe. The former one represents the expansion rate of the universe while the latter one characterizes the accelerating $(q<0)$ or decelerating $(q>0)$ nature of the universe. Number of relativists assumes various physical or mathematical conditions to obtain exact solution of the Einstein's field equations. We know that the universe has decelerating expansion if $q>0$, an expansion with constant rate if $q=0$, accelerating power law expansion if $-1<q<0$, exponential expansion (or de-Sitter expansion) if $q=-1$ and super-exponential expansion if $q<-1$. Many relativists have studied the DE cosmological models with different form of deceleration parameters. Pradhan *et al.* [52-54], Yadav and Sharma [55], Rahman and Ansari [56] have studied cosmological models with time dependent deceleration parameters. Singha and Debnath [57], Adhav *et al.* [58, 59] have obtained DE models with special form of deceleration parameter. Adhav *et al.* [60, 61] and Akarsu *et al.* [62, 63] have investigated cosmological models with linearly varying deceleration parameter. Berman [64], Singh *et al.* [65, 66], Adhav *et al.* [67] have studied cosmological models with constant deceleration parameter.

In this paper, Bianchi type-IX space-time has considered when universe is filled with DE in Sen-Dunn scalar-tensor theory of gravitation. This work is organized as follows: In section 2, the model and field equations have been presented. The field equations have been solved in section 3 by choosing two different scale factors which renders time-dependent deceleration parameters. The physical and geometrical behavior of the two models have been discussed in section 3.1 and 3.2. In section 4, concluding remarks have been expressed.

II. Metric and Field Equations

Bianchi type-IX metric is considered in the form,

$$ds^2=-dt^2+a^2dx^2+b^2dy^2+\left(b^2\sin^2 y+a^2\cos^2 y\right)dz^2-2a^2\cos y\,dx\,dz\,, \tag{1}$$

where *a, b* are scale factors and are functions of cosmic time t.

The field equations given by Sen and Dunn for the combined scalar and tensor fields (in natural units $c=1, G=1$) are

$$R_{ij}-\frac{1}{2}Rg_{ij}=\omega\phi^{-2}\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)-\phi^{-2}T_{ij}\quad. \tag{2}$$

Ref

48. Pradhan, A., Srivastav, S.K., Yadav, M.K.:Astrophysics and Space Science, 298, 419 (2005)

Notes

Here the function $\omega = \frac{3}{2}$, R_{ij} is the Ricci tensor, R is the Ricci scalar, T_{ij} is the energy stress tensor of the matter. A comma and semicolon denote partial and covariant differentiations and ϕ_i denotes ordinary derivatives with respect to x_i. The scalar field ϕ incorporates the varying nature of Newtonian Gravitational constant. The energy momentum tensor of the fluid which is taken as

$$T_i^j = \left[T_0^0, T_1^1, T_2^2, T_3^3\right] \ . \tag{3}$$

The simplest generalization of EoS parameter of perfect fluid is to determine it separately on each spatial axis by preserving diagonal form of the energy momentum tensor in a consistent way with the considered metric. Hence one can parameterize energy momentum tensor as follows:

$$\begin{aligned} T_i^j &= \left[-\rho, p_x, p_y, p_z\right], \\ T_i^j &= \left[-1, w_x, w_y, w_z\right] \rho \ , \\ T_i^j &= \left[-1, w, w+\delta, w+\gamma\right]\rho . \end{aligned} \tag{4}$$

Here ρ is the energy density of the fluid, p_x, p_y, p_z are the pressures and w_x, w_y and w_z are the directional EoS parameters along *x*, *y* and *z* axes respectively, w is the deviation free EoS parameter of the fluid.

Now parameterizing the deviation from isotropy by setting $w_x = w$ and then introducing skewness parameters δ and γ which are deviations from w on y and z axes respectively. Here δ and γ are not necessarily constants and can be functions of the cosmic time t.

In the co-moving coordinate system the field equation (2) for the metric (1) with the help of equation (4) can be written as

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{a^2}{4b^4} = \phi^{-2}\rho - \frac{3}{4}\frac{\dot{\phi}^2}{\phi^2} , \tag{5}$$

$$2\frac{\ddot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{3a^2}{4b^4} = \frac{3}{4}\frac{\dot{\phi}^2}{\phi^2} - \phi^{-2} w\rho , \tag{6}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{a^2}{4b^4} = \frac{3}{4}\frac{\dot{\phi}^2}{\phi^2} - \phi^{-2}(w+\delta)\rho , \tag{7}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{a^2}{4b^4} = \frac{3}{4}\frac{\dot{\phi}^2}{\phi^2} - \phi^{-2}(w+\gamma)\rho , \tag{8}$$

where the overdot $(\dot{\ })$ denotes the differentiation with respect to t.

From equations (7) and (8) we see that, the deviations from w along y and z axes are same i.e. $\gamma = \delta$.

III. Solutions of Field Equations

The field equations (5)–(7) are three independent equations in six unknowns $a, b, \phi, \rho, w, \delta$. We can introduce more conditions either by an assumption corresponding to some physical situation or an arbitrary mathematical supposition. Three additional conditions relating these unknowns may be used to obtain explicit solutions of the systems.

(i) Firstly, we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$a = b^m, (m \neq 1), \tag{9}$$

where m is proportionality constant.
The motive behind assuming condition is explained with reference to Thorne [68], the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent [69, 70]. To put more precisely, red-shift studies place the limit $\frac{\sigma}{H} \leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy today. Collin *et al.* [71] have pointed out that for spatially homogeneous metric the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

(ii) Secondly, we consider the Guage function ϕ [72] as

$$\phi = \phi_0 R^{\alpha} = \phi_0 V^{\frac{\alpha}{3}}. \tag{10}$$

(iii) Riess *et al.* [73], Amendola [74] and Padmanabhan and Roy-Chowdhary [75] investigated that, for a universe which is decelerating in the past and accelerating in at present time, Deceleration parameter must show signature flipping. From the observations of SNe type Ia, Lima *et al.* [76] agree with the results of Riess and Amendola.

To obtain the deterministic solutions of the field equations, we choose two different scale factors which yield time dependent deceleration parameter as follows:

a) Case I: when $R(t) = \left(t^r e^t\right)^{\frac{1}{l}}$

The metric (1) is completely characterized by average scale factor therefore we consider that average scale factor is an integrating function of time [52-56] given by

$$R = (t^r e^t)^{\frac{1}{l}}, \tag{11}$$

where, r and l are positive constants.
The scale factor R is given by

$$R = (a b^2)^{\frac{1}{3}}. \tag{12}$$

Solving field equations (5)-(7) with the help of (1), (11) and (12), we obtain

$$a = (t^r e^t)^{\frac{3m}{(m+2)l}}, \tag{13}$$

$$b = (t^r e^t)^{\frac{3}{(m+2)l}}. \tag{14}$$

With the help of equations (13) and (14), the metric (1) takes the form

$$ds^2 = \left(\begin{array}{l} -dt^2 + (t^r e^t)^{\frac{6m}{(m+2)l}} dx^2 + (t^r e^t)^{\frac{6}{(m+2)l}} dy^2 + \left((t^r e^t)^{\frac{6}{(m+2)l}} \sin^2 y + (t^r e^t)^{\frac{6m}{(m+2)l}} \cos^2 y \right) dz^2 \\ -2(t^r e^t)^{\frac{6m}{(m+2)l}} \cos y dx dz \end{array} \right), \tag{15}$$

Ref

71. Collins, C.B., Glass, E.N., Wilkinson, D.A.: General Relativity and Gravitation, 12(10), 805 (1980)

Notes

Equation (15) represents Bianchi type-IX DE cosmological model in Sen-Dunn scalar tensor theory of gravitation.

i. Some Physical Properties of the Model

For the cosmological model (15), the physical quantities spatial volume V, Gauge function ϕ, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2, energy density ρ, deceleration parameter q, cosmological constant Λ are obtained as follows:

Spatial volume,

$$V = (t^r e^t)^{\frac{3}{l}} . \tag{16}$$

Gauge function,

$$\phi = \phi_0 (t^r e^t)^{\frac{\alpha}{l}} . \tag{17}$$

The rate of expansion H_i,

$$H_x = \frac{3m}{(m+2)l}(rt^{-1}+1), \; H_y = H_z = \frac{3}{(m+2)l}(rt^{-1}+1). \tag{18}$$

Hubble parameter,

$$H = \frac{1}{l}(rt^{-1}+1) . \tag{19}$$

Expansion scalar,

$$\theta = 3H = \frac{3}{l}(rt^{-1}+1). \tag{20}$$

Mean Anisotropy Parameter,

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constant } (\neq 0 \text{ for } m \neq 1) . \tag{21}$$

Shear scalar,

$$\sigma^2 = \frac{3(m-1)^2}{l^2(m+2)^2}(rt^{-1}+1)^2 . \tag{22}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} = constant \; (\neq 0), \quad \text{for } m \neq 1 \; . \tag{23}$$

The energy density,

$$\rho = \phi_0^2 (t^r e^t)^{\frac{2\alpha}{l}} \left(\frac{(72m+36+3\alpha^2 m^2+12\alpha^2 m+12\alpha^2)}{4(m+2)^2 l^2}(rt^{-1}+1)^2 + (t^r e^t)^{\frac{-6}{(m+2)l}} - \frac{1}{4}(t^r e^t)^{\frac{6(m-2)}{(m+2)l}} \right) \tag{24}$$

EoS parameter,

$$w=-\frac{\left(\dfrac{(108-3\alpha^2m^2-12\alpha^2m-12\alpha^2)}{4(m+2)^2l^2}(rt^{-1}+1)^2-\dfrac{6}{(m+2)l}rt^{-2}+(t^re^t)^{\frac{-6}{(m+2)l}}-\dfrac{3}{4}(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)}{\left(\dfrac{(72m+36+3\alpha^2m^2+12\alpha^2m+12\alpha^2)}{4(m+2)^2l^2}(rt^{-1}+1)^2+(t^re^t)^{\frac{-6}{(m+2)l}}-\dfrac{1}{4}(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)}. \qquad (25)$$

Skewness parameter,

$$\delta=-\frac{\left(\dfrac{9(m^2-1)}{(m+2)^2l^2}(rt^{-1}+1)^2-\dfrac{3(m+1)}{(m+2)l}rt^{-2}-(t^re^t)^{\frac{-6}{(m+2)l}}+(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)}{\left(\dfrac{(72m+36+3\alpha^2m^2+12\alpha^2m+12\alpha^2)}{4(m+2)^2l^2}(rt^{-1}+1)^2+(t^re^t)^{\frac{-6}{(m+2)l}}-\dfrac{1}{4}(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)}. \qquad (26)$$

Deceleration parameter,

$$q=\frac{l\,r}{(t+r)^2}-1. \qquad (27)$$

In absence of any curvature, matter energy density (Ω_m) and dark energy (Ω_Λ) are related by the equation

$$\Omega \quad +\Omega_\Lambda=1, \qquad (28)$$

where $\Omega_m=\dfrac{\rho}{3H^2}$ and $\Omega_\Lambda=\dfrac{\Lambda}{3H^2}$.

Then equation (28) reduces to

$$\frac{\rho}{3H^2}+\frac{\Lambda}{3H^2}=1. \qquad (29)$$

Using equations (19) and (24), the cosmological constant in equation (29) is obtained as

$$\Lambda=\left[\begin{array}{l}\dfrac{3}{l^2}(rt^{-1}+1)^2\\ -\phi_0^2(t^re^t)^{\frac{2\alpha}{l}}\left(\dfrac{(72m+36+3\alpha^2m^2+12\alpha^2m+12\alpha^2)}{4(m+2)^2l^2}(rt^{-1}+1)^2+(t^re^t)^{\frac{-6}{(m+2)l}}-\dfrac{1}{4}(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)\end{array}\right] \qquad (30)$$

Notes

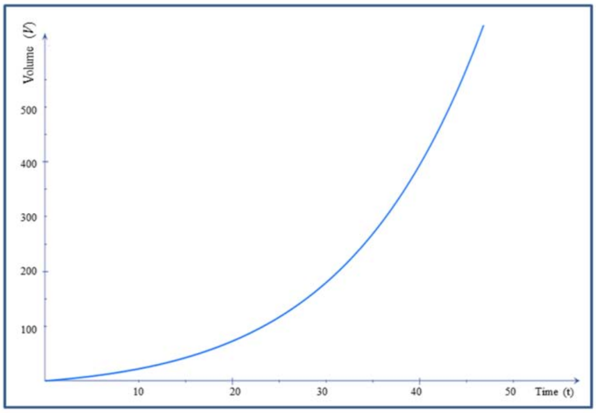

Figure 1. The Plot of Volume verses time (t)

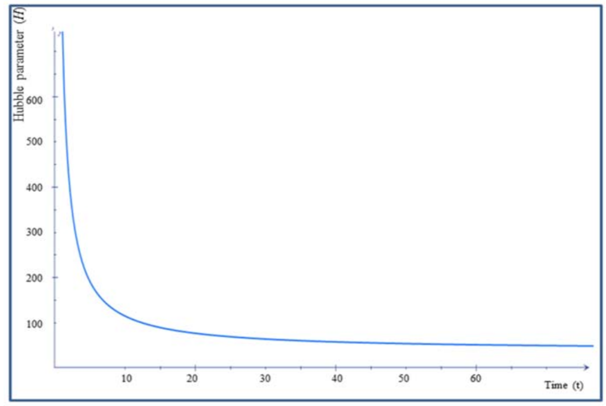

Figure 2. The Plot of Hubble's Parameter (H) verses time (t)

It is observed from equations (13) and (14) that the spatial scale factors are zero at the initial epoch $t=0$, hence the model has a point type singularity [77]. From equations (16) and (20), the spatial volume is zero and expansion scalar is infinite at $t=0$ which show that the universe starts evolving with zero volume at $t=0$ which is big bang scenario. In figures (1) & (2), plots of Spatial volume and Hubble's parameter against time are shown for better understanding.

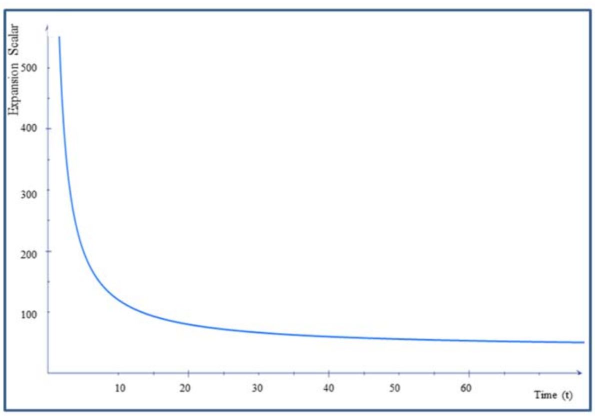

Figure 3. The Plot of Expansion scalar verses time

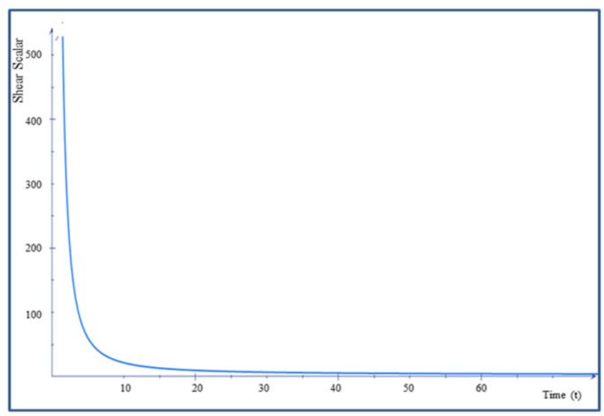

Figure 4. The Plot of Shear scalar verses time

In Fig. 3, the plots of expansion scalar verses time is given, which indicate that the expansion scalar (θ) starts from infinity at $t=0$ and tends to zero for large values of cosmic time t, showing that the universe is expanding with increase of time.

In Fig. 4, the plot of shear scalar (σ^2) verses time (t) is given. This shows that at $t=0$, the Shear scalar (σ^2) tends to infinity *i.e.* $\sigma^2 \to \infty$ and it decreases as time (t) increases. It approaches to zero after infinite time for all cosmological models.

From equations (21) and (23), the mean anisotropy parameter A_m is constant and $\dfrac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe (*i.e.* the model does not approach isotropy) except at $m=1$.

Ref

77. MacCallum, M.H.H.: Communication in Mathematical Physics, 20, 57 (1971)

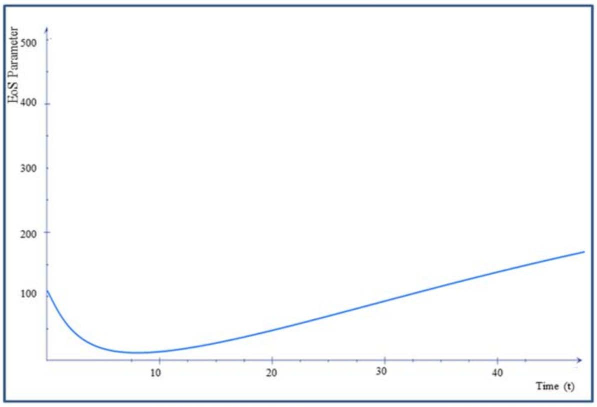

Figure 6. The Plot of Equation of State parameter verses time

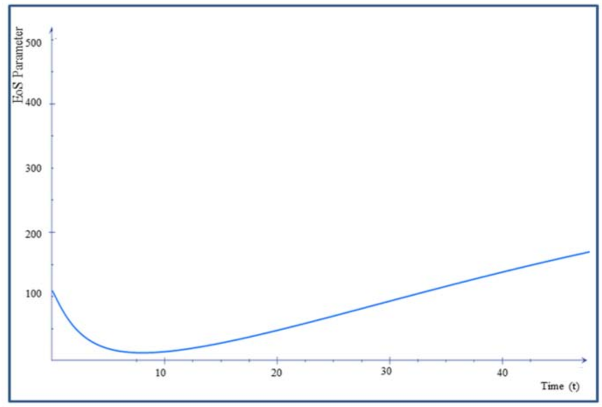

Figure 6. The Plot of Equation of State parameter verses time

From equation (24), we note that the energy density of the fluid $\rho(t)$ is a decreasing function of time. Fig. 5 is the plot of energy density of the fluid versus time. We observed that ρ is positive decreasing function of time and it approaches to zero as $t \to \infty$.

Fig. 6 depicts the variation of EoS parameter (w) versus cosmic time (t) in which we observed that it is positive throughout the evolution of the universe hence the universe is matter dominated.

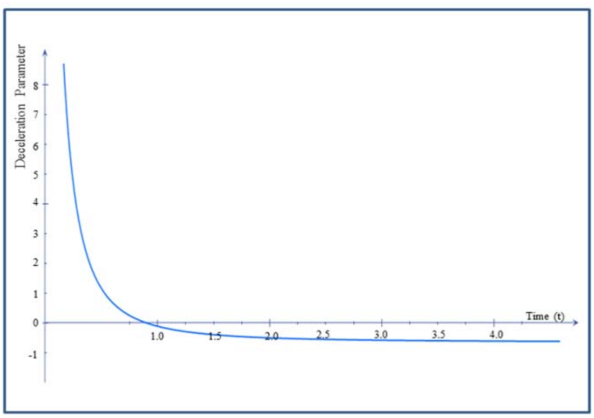

Figure 7. The Plot of Deceleration parameter verses time

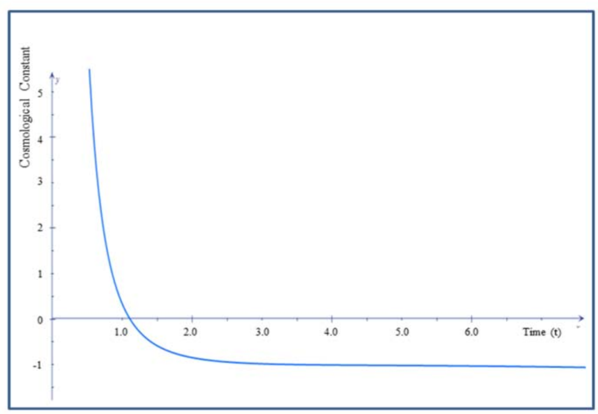

Figure 8. The Plot of Cosmological Constant verses time

We observed that the cosmological model is in accelerating phase when $q > 0$ and it evolves from decelerating phase to accelerating phase when $-1 \le q < 0$. For the model (15), it is seen that the model is in accelerating phase for $t < \sqrt{lr} - r$ and evolves from decelerating phase to accelerating phase for $t > \sqrt{lr} - r$. Our model is evolving from decelerating phase to accelerating phase for $t \ge 3$ and $r = 1$. For the model (15), one can choose the value of DP consistent with the observations. [*i.e.* $-1 < q < 0$]. Fig. 7 represents the plot of deceleration (q) parameter versus cosmic time which gives the behavior of q as in accelerating phase at present epoch which is consistent with recent observations of Type Ia Supernaovae [14-18, 78, 79].

For illustrative purposes, evolutionary behaviors of some cosmological parameters are shown graphically (Figure 1-8).

Notes

Notes

b) Case II: when $R(t) = (\sinh(\xi t))^{\frac{1}{n}}$

The average scale factor in terms of cosmic time is considered as [80-84]

$$R = (\sinh(\xi t))^{\frac{1}{n}}. \tag{31}$$

Solving field equations (5)-(7) with the help of (1), (12) and (31), we obtain

$$a = (\sinh(\xi t))^{\frac{3m}{(m+2)n}}, \tag{32}$$

$$b = (\sinh(\xi t))^{\frac{3}{(m+2)n}}. \tag{33}$$

With the help ofequations (32) and (33), the metric (1) takes the form

$$ds^2 = \left(\begin{array}{l} -dt^2 + (\sinh(\xi t))^{\frac{6m}{(m+2)n}} dx^2 + (\sinh(\xi t))^{\frac{6}{(m+2)n}} dy^2 \\ + \left((\sinh(\xi t))^{\frac{6}{(m+2)n}} \sin^2 y + (\sinh(\xi t))^{\frac{6m}{(m+2)n}} \cos^2 y \right) dz^2 - 2(\sinh(\xi t))^{\frac{6m}{(m+2)n}} \cos y dx dz \end{array} \right). \tag{34}$$

Equation (34) represents Bianchi type-IX DE cosmological model in Sen-Dunn scalar tensor theory of gravitation.

i. Some Physical Properties of the Model

For the cosmological model (34), the physical quantities spatial volume V ,Gauge function ϕ, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2, energy density ρ ,deceleration parameter q, cosmological constant Λ are obtained as follows:

Spatial volume,

$$V = (\sinh(\xi t))^{\frac{3}{n}}. \tag{35}$$

Gauge function,

$$\phi = \phi_0 (\sinh(\xi t))^{\frac{\alpha}{n}}. \tag{36}$$

The rate of expansion H_i,

$$H_x = \frac{3m\xi}{(m+2)n} \coth(\xi t), \quad H_y = H_z = \frac{3\xi}{(m+2)n} \coth(\xi t). \tag{37}$$

Hubble parameter,

$$H = \frac{\xi}{n} \coth(\xi t). \tag{38}$$

Expansion scalar,

$$\theta = 3H = 3\frac{\xi}{n} \coth(\xi t). \tag{39}$$

Mean Anisotropy Parameter,

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constant} \ (\neq 0 \ \text{for} \ \ m \neq 1). \tag{40}$$

Shear scalar,

$$\sigma^2 = \frac{3\xi^2 (m-1)^2}{(m+2)^2 n^2} (\coth(\xi t))^2. \tag{41}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} = constant(\neq 0) \ (m \neq 1). \tag{42}$$

The energy density,

$$\rho = \phi_0^2 (\sinh(\xi t))^{\frac{2\alpha}{n}} \left(\frac{3\xi^2 (m^2\alpha^2 + 4m\alpha^2 + 4\alpha^2 + 24m + 12)}{4(m+2)^2 n^2} (\coth(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} - \frac{1}{4} (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \right). \tag{43}$$

EoS parameter,

$$w = -\frac{\left[\frac{3\xi^2 (36 - m^2\alpha^2 - 4m\alpha^2 - 4\alpha^2)}{4(m+2)^2 n^2} (\coth(\xi t))^2 - \frac{6\xi^2}{(m+2)n} (\operatorname{cosech}(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{-6}{(m+2)n}} - \frac{3}{4} (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \right]}{\left[\frac{3\xi^2 (m^2\alpha^2 + 4m\alpha^2 + 4\alpha^2 + 24m + 12)}{4(m+2)^2 n^2} (\coth(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} - \frac{1}{4} (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \right]}. \tag{44}$$

Skewness parameter,

$$\delta = -\frac{\left[\frac{9\xi^2 m(m+1)}{(m+2)^2 n^2} (\coth(\xi t))^2 - \frac{3\xi^2 (m+1)}{(m+2)n} (\operatorname{cosech}(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} - (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \right]}{\left[\frac{3\xi^2 (m^2\alpha^2 + 4m\alpha^2 + 4\alpha^2 + 24m + 12)}{4(m+2)^2 n^2} (\coth(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} - \frac{1}{4} (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \right]}. \tag{45}$$

Deceleration paremeter,

$$q = n\left(1 - \tanh^2(\xi t)\right) - 1 . \tag{46}$$

Using equations (38) and (43), the cosmological constant in equation (29) is obtained as

Notes

$$\Lambda = \frac{3\xi^2}{n^2}(\coth(\xi t))^2 - \phi_0^2(\sinh(\xi t))^{\frac{2\alpha}{n}} \left(\begin{array}{l} \dfrac{3\xi^2(m^2\alpha^2 + 4m\alpha^2 + 4\alpha^2 + 24m + 12)}{4(m+2)^2 n^2}(\coth(\xi t))^2 \\ + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} - \dfrac{1}{4}(\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \end{array} \right) . \quad (47)$$

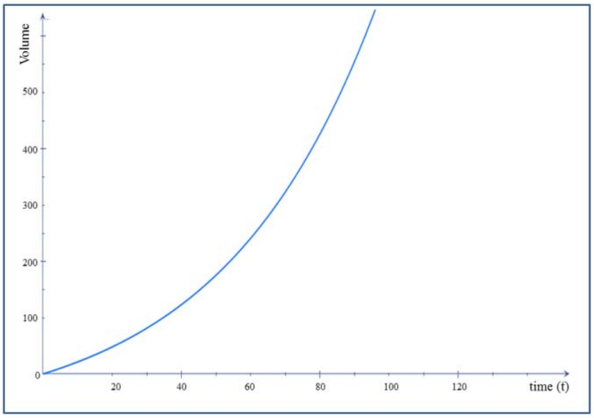

Figure 9. The Plot of Volume verses time

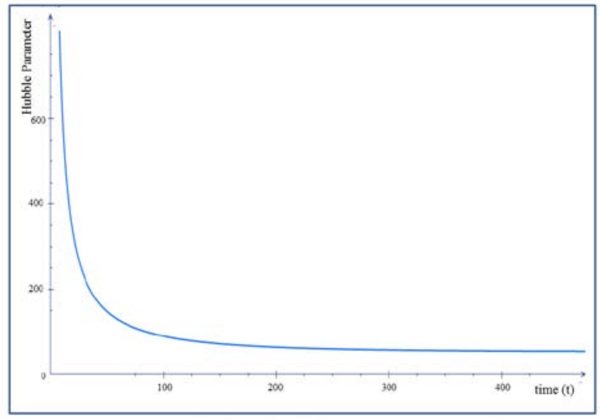

Figure 10. The Plot of Hubble Parameter verses time

It is observed that from equations (32) and (33), the spatial scale factors are zero at the initial epoch $t = 0$, hence the model has a point type singularity [77].

In figures (9) & (10), plots of spatial volume and Hubble's parameter against time are shown for better understanding. From Equation (35) and (39), we observed that the spatial volume is zero at $t = 0$ and the expansion scalar θ is infinite, which show that the universe starts evolving with zero volume at $t = 0$, which is big bang scenario.

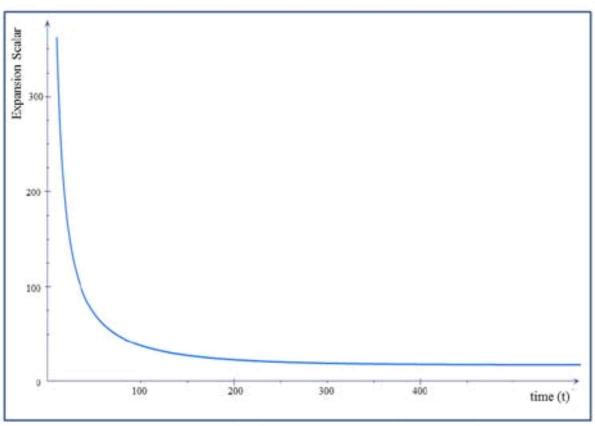

Figure 11. The Plot of Expansion Scalar verses time

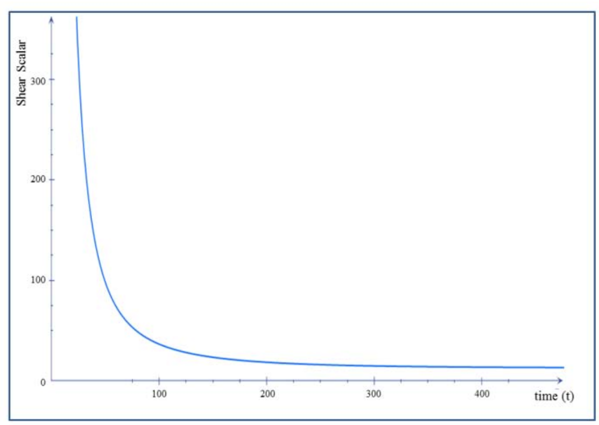

Figure 12. The Plot of Shear Scalar verses time

From equations (39) and (41), the mean anisotropy parameter A_m is constant and $\frac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe (*i.e.* the model does not approach isotropy) except at $m = 1$. The plots of expansion scalar and shear scalar against time are shown in figures (11) & (12).

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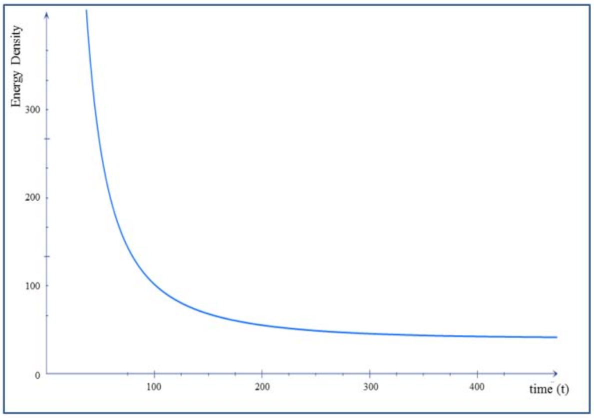

Figure 13. The Plot of Energy Density verses time

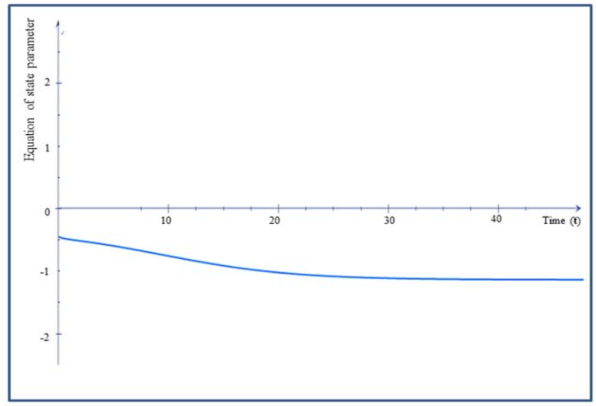

Figure 14. The Plot of Equation of State parameter verses time

From equation (43), we note that the energy density of the fluid $\rho(t)$ is a decreasing function of time. Figure 13 is the plot of energy density of the fluid versus time. We observed that ρ is positive decreasing function of time and it approaches to zero as $t \to \infty$.

Figure 14 depicts the variation of EoS parameter (w) versus cosmic time (t) in which we observed that it is negative throughout the evolution of the universe which shows that the universe is in dark energy era.

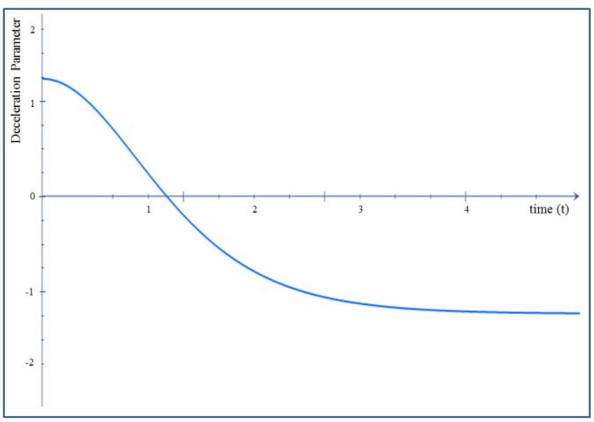

Figure 15. The Plot of Deceleration Parameter verses time

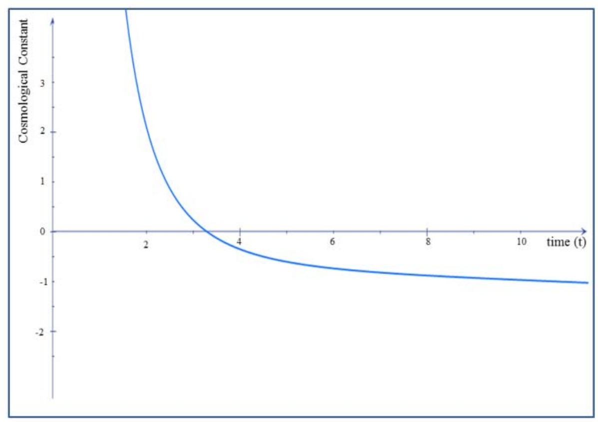

Figure 16. The Plot of Cosmological constant verses time

From equation (46), we observe that the deceleration parameter $q > 0$, for $t < \frac{1}{\xi}\tanh^{-1}\left(1-\frac{1}{n}\right)^{\frac{1}{2}}$ and $q < 0$, for $t > \frac{1}{\xi}\tanh^{-1}\left(1-\frac{1}{n}\right)^{\frac{1}{2}}$. The cosmological model is in accelerating phase when $q > 0$ and it evolves from decelerating phase to accelerating phase when $-1 \le q < 0$. For the model (34), it is seen that the model is in accelerating phase for $t < \frac{1}{\xi}\tanh^{-1}\left(1-\frac{1}{n}\right)^{\frac{1}{2}}$ and evolves from decelerating phase to accelerating phase for $t > \frac{1}{\xi}\tanh^{-1}\left(1-\frac{1}{n}\right)^{\frac{1}{2}}$. For the model (34), one can choose the value of DP consistent with the observations. $[i.e. -1 < q < 0]$. Figure 15 represents the plot of deceleration parameter (q) versus cosmic time. Also recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range

Notes

$-1 \le q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observations [14-18, 78, 79].

For illustrative purposes, evolutionary behaviors of some cosmological parameters are shown graphically (Figure 9-16).

IV. Conclusion

Notes

Bianchi type-IX cosmological models have obtained when universe is filled with DE in Sen-Dunn scalar-tensor theory of gravitation. To find deterministic solution, we have considered two different scale factors which yield time-dependent deceleration parameter.

In Case I, the solution of the field equations has obtained by choosing the scale factor $R = (t^r e^t)^{\frac{1}{l}}$, which yields time dependent deceleration parameter $q = \frac{l\,r}{(t+r)^2} - 1$. It is observed that the universe is matter dominated.

In Case II, we choose the scale factor $R = (\sinh(\xi t))^{\frac{1}{n}}$ which yields time dependent deceleration parameter $q = n\left(1 - \tanh^2(\xi t)\right) - 1$. It is observed that the universe is in dark energy era.It is worth to mention that in both cases, the models obtained are point type singular, expanding, shearing. The models obtained are anisotropic throughout evolution of the universe (*i.e.* not isotropic). In an early phase of universe, the deceleration parameter is positive and decreases with increase in cosmic time t. It remains constant ($q = -1$) for large values of t. Hence the universe had a decelerated expansion in the past and has accelerated expansion at present which are in good agreement with the recent SN Ia observations [14-18, 78, 79].We hope that these models will be useful for a better understanding of dark energy in cosmology to study an accelerating expansion of the Universe.

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