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Contact Normal Generic Submanifolds of a Nearly Hyperbolic Cosymplectic Manifold

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Keywords: contact normal generic submanifolds, nearly hyperbolic cosymplectic manifold, integrability condition, leaves of distributions, totally geodesic.

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Contact Normal Generic Submanifolds of a Nearly Hyperbolic Cosymplectic Manifold

Saadet DOĞAN [α] & Müge KARADAĞ [σ]

Abstract- We introduce and study contact normal generic submanifolds of a nearly hyperbolic cosymplectic manifold. We deal with the integrability conditions of the distributions of such manifolds. In addition to these, we study geometry of the leaves of distributions.

Keywords: contact normal generic submanifolds, nearly hyperbolic cosymplectic manifold, integrability condition, leaves of distributions, totally geodesic.

MSC 2010: 53C15, 53C25, 53D15

I. Introduction

Almost hyperbolic (φ,ξ,η,g)-structure was defined and studied by Upadhyay and Dube [8]. Uddin S, Wong BR and Mustafa AA studied warped product pseudo slant submanifolds of a nearly cosymplectic manifold [7]. The notion of semi invariant submanifolds and CR- submanifolds of nearly hyperbolic cosymplectic manifold was introduced by Ahmad M and Ali K [1,2]. In addition to these, Dogan S and Karadag M studied slant submanifolds of an almost hyperbolic contact metric manifolds[4] and pseudo-slant submanifolds of nearly hyperbolic cosymplectic manifolds[5]. M. Kobayashi study contact normal submanifolds and contact generic normal submanifolds in Kenmotsu manifolds [6]. U.C. De and A.K. Sengupta deal with generic submanifolds of Lorentzian Para-Sasakian manifold [3].

In this paper, we introduce contact generic normal submanifolds of a nearly hyperbolic cosymplectic manifold.

II. Preliminaries

Let M be an n-dimensional almost hyperbolic contact metric manifold with almost hyperbolic contact metric structure(φ,ξ,η,g), where a tensor φ of type (1,1), a vector field η called structure vector field and ξ , the dual 1-form of ξ satisfying the followings

$$\varphi^2 X = X + \eta(X)\xi \tag{2.1}$$

$$g(X,\xi) = \eta(X), \quad \eta(\xi) = -1 \tag{2.2}$$

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7. Uddin S, Wong BR,Mustafa AA. : Warped Product Pseudo Slant Submanifolds of a Nearly Cosymplectic Manifold. Hindawi Publ Corp 1-13, (2012).

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$$\varphi\xi = 0, \quad \eta \circ \varphi = 0 \tag{2.3}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.4}$$

for any vector fields *X* and *Y* in *TM* [4]. In this case,

$$g(\varphi X, Y) = -g(X, \varphi Y) \tag{2.5}$$

X and *Y* in *TM*

An almost hyperbolic contact metric manifold with almost hyperbolic contact metric structure (φ, ξ, η, g) is said to be nearly hyperbolic cosymplectic manifold [4] if

$$(\overline{\nabla}_X \varphi)Y + \varphi(\overline{\nabla}_Y X) = 0 \tag{2.6}$$

$$\overline{\nabla}_X \xi = 0 \tag{2.7}$$

for all *X,Y* tangent to *M*.

Let *M* be submanifold of a nearly hyperbolic cosymplectic manifold *M* with induced metric *g* and if ∇ and $\nabla^{\perp}$ are the induced connections on the tangent bundle *TM* and the normal bundle $TM^{\perp}$ of *M*, respectively, then Gauss and Weingarten formulae are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y) \tag{2.8}$$

$$\overline{\nabla}_X N = -A_N X + \nabla_X^{\perp} N \tag{2.9}$$

for each *X,Y* in *TM* and $N \in TM^{\perp}$, where *h* and A_N are the second fundamental form and the shape operator, respectively, for the immersion of *M* into $\overline{M}$. They are related as

$$g(h(X, Y), N) = g(A_N X, Y) \tag{2.10}$$

where *g* denotes the Riemannian metric on $\overline{M}$ as well as induced on *M*. For any vector field *X* in *TM*,

$$\varphi X = PX + FX \tag{2.11}$$

where *PX* is the tangential component and *FX* is the normal component of φX. Similarly for any $N \in TM^{\perp}$,

$$\varphi N = tN + fN \tag{2.12}$$

where *tN* is the tangential component and *fN* is the normal component of φN.

A submanifold M of a nearly hyperbolic cosymplectic manifold $\overline{M}$ is said to be a contact generic normal submanifold if the structure vector field ξ is normal to *M* and if there exists a differentiable distribution *D* on *M* such that:

$$TM = D \oplus D^{\perp}, \quad \varphi D = D, \quad \varphi D^{\perp} \subseteq TM^{\perp} \tag{2.13}$$

where $D^{\perp}$ is the complementary distribution of *D* in *TM* [6].

M is called to be a contact generic normal submanifold in $\overline{M}$ if the structure vector field ξ is normal to *M* and

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4. Dogan S, Karadag M. : Slant Submanifolds of an Almost Hyperbolic Contact Metric Manifolds.J Math Syst Sci 4 , 285-288, (2014).

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$$\varphi\left(TM^{\perp}\right)\subset TM \tag{2.14}$$

holds [6].

The leaves of a distribution D on a manifold M are totally geodesic in M if and only if $\nabla_X Y \in D$ for all X, Y in D. Which is equivalent to the conditions

$$\nabla_X W \in D^{\perp} \tag{2.15}$$

for all $X \in D$ and $W \in D^{\perp}$. Similarly for the totally geodesicness of the leaves of $D^{\perp}$, the conditions

$$\nabla_Z W \in D^{\perp} \tag{2.16}$$

and

$$\nabla_Z X \in D \tag{2.17}$$

for all $X \in D$ and $Z, W \in D^{\perp}$ are equivalent [3].

III. Integrability of Distributions

a) ***Theorem:*** *Let M be a contact generic normal submanifolds of a nearly hyperbolic cosymplectic manifold $\overline{M}$. Then $D^{\perp}$ is integrable if and only if M is mixed geodesic.*

Proof: Let M be a mixed geodesic contact generic normal submanifolds of a nearly hyperbolic cosymplectic manifold $\overline{M}$. From (2.6), we get

$$\begin{aligned}
&\left(\overline{\nabla}_X\varphi\right)Y + \varphi\overline{\nabla}_Y X = 0\\
&\overline{\nabla}_X\varphi Y - \varphi\overline{\nabla}_X Y + \varphi\overline{\nabla}_Y X = 0\\
&\overline{\nabla}_X\varphi Y = \varphi[X, Y]
\end{aligned} \tag{3.1}$$

for any vector fields X, Y in $D^{\perp}$. For all X, Y in $D^{\perp}$ and Z in D. From (3.1), we get

$$\begin{aligned}
g([X,Y],\varphi Z) &= -g\left(\overline{\nabla}_X\varphi Y, Z\right)\\
&= g\left(A_{\varphi Y}X, Z\right) - g\left(\nabla_X^{\perp}\varphi Y, Z\right)\\
&= g\left(h(X,Z), \varphi Y\right) = 0
\end{aligned}$$

for any X, Y in $D^{\perp}$ and Z in D. Then $D^{\perp}$ is integrable.

Contrary to this, let $D^{\perp}$ be integrable. That is ; $[X,Y] \in D^{\perp}$ for all X,Y in $D^{\perp}$. Then $\varphi[X,Y]$ in $TM^{\perp}$. In this case, from (3.1)

$$\begin{aligned}
\varphi[X,Y] &= \overline{\nabla}_X\varphi Y\\
\varphi[X,Y] &= -A_{\varphi Y}X + \nabla_X{}^{\perp}\varphi Y
\end{aligned} \tag{3.2}$$

for all vector fields X,Y in $D^{\perp}$. If we take the inner product of (3.2) with Z in D, we find

$$g(\varphi[X,Y], Z) = -g\left(A_{\varphi Y}X, Z\right) + g\left(\nabla_X^{\perp}\varphi Y, Z\right)$$

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6. Kobayashi M. : Contact Normal Submanifolds and Contact Generic Submanifolds in Kenmotsu Manifolds, Revista Mathematica 4, (1991).

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$$g\left(A_{\varphi Y}X,Z\right) = 0$$
$$g\left(h(X,Z),\varphi Y\right) = 0$$

for all X,Y in $D^{\perp}$ and Z in D. Then M is mixed geodesic.

b) ***Theorem:*** *Let M be a contact generic normal submanifolds of a nearly hyperbolic cosymplectic manifold* $\overline{M}$. *Then D is integrable if and only if M is D-geodesic.*

Proof: Let M be a mixed geodesic contact generic normal submanifolds of a nearly hyperbolic cosymplectic manifold $\overline{M}$. From (2.6), we get

$$\left(\overline{\nabla}_X\varphi\right)Y + \varphi\overline{\nabla}_Y X = 0$$
$$\overline{\nabla}_X\varphi Y - \varphi\overline{\nabla}_X Y + \varphi\overline{\nabla}_Y X = 0 \tag{3.3}$$
$$\overline{\nabla}_X PY + \overline{\nabla}_X FY - \varphi\nabla_X Y - \varphi h(X,Y) + \varphi\nabla_Y X + \varphi h(Y,X) = 0$$

for all vector fields X,Y in TM. If we use Gauss and Weingarten equations in (3.3) and we consider tangential and normal component, we have

$$(\nabla_X P)Y = A_{FY}X - P\nabla_Y X \tag{3.4}$$
$$(\nabla_X F)Y = -h(X,PY) - F\nabla_Y X \tag{3.5}$$

for all vector fields X,Y in TM. If we take X,Y in D, we find

$$(\nabla_X F)Y = -F\nabla_X Y \tag{3.6}$$

and

$$(\nabla_X F)Y = -h(X,PY) - F\nabla_Y X \tag{3.7}$$

for all vector fields X,Y in TM. From (3.6) and (3.7), we get that D is integrable if and only if M is D-geodesic.

IV. Geometry of Leaves of Distributions

a) ***Theorem:*** *Let M be a contact generic normal submanifolds of a nearly hyperbolic cosymplectic manifold* $\overline{M}$. *D is integrable and the leaves of D are totally geodesic in M if and only if* $A_{XP}X = P\nabla_Z X$ *for all X in D and Z in* $D^{\perp}$.

Proof: Let the leaves of D be totally geodesic in M. That is; $\nabla_X Y \in D$ for all X,Y in D. From (3.4), we find

$$(\nabla_X P)Z = A_{FZ}X - P\nabla_Z X$$
$$\nabla_X PZ - P\nabla_X Z = A_{FZ}X - P\nabla_Z X \tag{4.1}$$

for all X in D and Z in $D^{\perp}$. If we take the inner product of (4.1) with Y in D, we get

$$g\left(A_{FZ}X,Y\right) - g\left(P\nabla_Z X,Y\right) = 0$$

for all X in D and Z in $D^{\perp}$. Then we get

$$A_{FZ}X = P\nabla_Z X$$

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Notes

for any vector fields X in D and Z in $D^{\perp}$. Now, we suppose that

$$A_{FZ}X = P\nabla_Z X$$

for any vector fields X in D and Z in $D^{\perp}$. From (3.4), we get

$$\nabla_X PZ - P\nabla_X Z = A_{FZ}X - P\nabla_Z X \tag{4.2}$$

Notes

for all vector fields X,Y in D, Z in $D^{\perp}$. From (4.2), we find $P\nabla_X Z = 0\, X$ for any vector fields X in D and Z in $D^{\perp}$.

Then $\nabla_X Z \in D^{\perp}$. In addition to this, we find

$$(\nabla_X g)(\varphi Y, Z) = \nabla_X g(\varphi Y, Z) - g(\nabla_X \varphi Y, Z) - g(\varphi Y, \nabla_X Z)$$

$$0 = -g(\nabla_X \varphi Y, Z) - g(\varphi Y, \nabla_X Z)$$

$$g(\nabla_X \varphi Y, Z) = -g(\varphi Y, \nabla_X Z) \tag{4.3}$$

for all vector fields X,Y in D, Z in $D^{\perp}$. In this case,

$$g(\varphi[X,Y], Z) = -g(\varphi Y, \nabla_X Z) = 0$$

for all vector fields X,Y in D, Z in $D^{\perp}$. Then $[X,Y] \in D$. That is; D is integrable.

b) ***Theorem:*** *Let M be a contact generic normal submanifolds of a nearly hyperbolic cosymplectic manifold $\overline{M}$ .Let $D^{\perp}$ be integrable and the leaves of $D^{\perp}$ be totally geodesic in M. Then $A_{FV}U = 0$, for all vector fields U,V in $D^{\perp}$.*

Proof: $D^{\perp}$ be integrable and the leaves of $D^{\perp}$ be totally geodesic in M. From (3.4), we get

$$(\nabla_U P)V = A_{FV}U - P\nabla_V U$$

$$\nabla_U PV - P\nabla_U V = A_{FV}U - P\nabla_V U$$

$$0 = A_{FV}U$$

for all vector field U,V in $D^{\perp}$.

Now, we suppose that $A_{FV}U = 0\, FV$ for all vector field U,V in $D^{\perp}$. We will show that $D^{\perp}$ is integrable and the leaves of $D^{\perp}$ are totally geodesic in M. From (3.4), we find

$$(\nabla_U P)V = -P\nabla_V U \tag{4.4}$$

for all vector field U,V in $D^{\perp}$. Then we get

$$\nabla_U PV - P\nabla_U V = -P\nabla_V U$$

$$P[U,V] = 0$$

for all vector fields U,V in $D^{\perp}$. In this case; $[U,V] \in D^{\perp}$. That is; $D^{\perp}$ is integrable. From (3.4), we get

$$P\nabla_U V = P\nabla_V U$$

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for any vector fields U,V in $D^{\perp}$. Then $\nabla_V U \in D^{\perp}$ and $\nabla_U V \in D^{\perp}$. In this case; the proof is complete.

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Notes

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