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Fish Growth Evaluation using a One Step Numerical Algorithm for a Sustainable Development in the Third World Nations

By Enoch, O. O. & Ajenifuja, O. A.
Federal University, Nigeria

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Enoch, O. O. [α] & Ajenifuja, O. A.[σ]

Abstract- We examined fishery as a supplementary source of protein for bridging the gaps created by the birds 'saga in meeting the need for protein in less developed countries. A one step algorithm is proposed, implemented on von Bertalanffy and on seasonal growth models for the evaluation of the desired enhancement in fish growth. Precautionary measure is presented against the use of poultry wastes as a component commonly used in the formulation of fishery feeds. The one-step method gives an enhanced projection in fish growth and also predicts the appropriate proportion of all other constant parameters that will be needed.

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I. Introduction

The recent pandemic on avian-influenza has brought about a great challenge to a major source of protein supply, since poultry birds have served as means of protein provider over the decades. The gap created by this pandemic must be bridged by sourcing for protein from other sources like fishery, etc. Our concern is that if fishery would serve as an alternative source of protein, what considerable input must we give into it? And at what rate must some constant conditions and variable conditions be observed and maintained for the gap in protein need to be quickly and remarkably bridged at an optimal economic state?

II. One-Step Methods Based on Non-Linear Polynomial Interpolant

In this paper, we shall examine the initial value problem of the form;

$$y^1(x) = (x,y), y(a) = y_o, \quad x \in [a,b], y \in R \tag{1}$$

Here we present some one-step methods for the solution of equation (I).

This type of construction was first reported in Fatunla (1976). The resulting method is particularly well ution. For our construction, Firstly, we assume that over the interval $\{x_t,\ x_{t+1}\}$, the theoretical solution, y(x), to the initial value problem (1) is given by the non-polynomial;

Author α: Department of Mathematics, Faculty of Science, Federal University Oye-Ekiti, Ekiti State, Nigeria. e-mail: ope_taiwo3216@yahoo
Author σ: Department of Science Technology, School of Science and Computer Studies, Federal Polytechnic, Ado-Ekiti, Nigeria.
e-mail: joseyajenifuja@yahoo.com

$$f(x) = a_1 e^x + x_t^2 \tag{2}$$

Where a, is an undetermined co-efficient. Let us consider the non-polynomial interpolant at the points $x = x_t$ and $x = x_{t-1}$ and also take the numerical estimate to the theoretical solution $y(x_t)$ to be y_t. We shall by this assumptions have;

$$f(x_t) = a_1 e^{x_t} + x_t^2 \tag{3}$$

Let $y^1(x_t) = f_t(x_t,\ y_t)$. For us to be able to determine the undetermined co-efficient a_1, we are to impose the assumptions that the non-linear polynomial interpolant (4) coincides with y_t and (3) coincides with y_{t-1} and y_n is a numerical estimate to the theoretical solution $y(x_n)$ and $f_n = f(x_n,\ y_n)$ with the mesh-point defined by $x_n = a + nh, \quad n = 0, 1, 2, 3, \ldots$

III. Constraints

The interpolating function must coincide with the theoretical solution at $x = x_n$ and $x = x_{n+1}$, for $n \geq 0$.This condition implies that

$$f(x_t) = y_t = a_1 e^{x_t} + x_t^2 \tag{5}$$

and

$$f(x_{t+1}) = y_{t+1} = a_1 e^{x_{t+1}} + x_{t+1}^2 \tag{6}$$

We also require that the first and the second derivatives of the Interpolating function respectively coincide with the differential equation as well as its first and second derivatives with respect to *x* at $x = x_n$, where $f^{(i)}$ denotes the ith total derivatives off *(x, y)* with respect to *x*

$$f^{(1)} = f(x_t,\ y_t) = f_t \tag{7}$$

and

$$f^{(2)}(x_t = f(x_t,\ y_t) = f_t^{(1)} \tag{8}$$

Differentiating (3) with respect to *x*, we have

$$f^{(1)}(x_t) = a_1 e^{x_t} + 2x_t = f_t \tag{9}$$

and

$$f^{(2)}(x_t) = a_1 e^{x_t} + 2 = f_t^{(1)} \tag{10}$$

Solving for a_1 from equation (10), we have

$$a_1 e^{x_t} + 2 = f_t^{(1)}$$

$$a_1 e^{x_t} + 2 = f_t^{(1)} - 2$$

$$a_1 = \frac{f_t^{(1)} - 2}{e^{x_t}}$$

From (6) and (5), we have

$$f(x_{t+1}) - f(x_t) = y_{t+1} - y_t \quad (12)$$

This implies that

$$y_{t+1} - y_t = a_1 e^{x_{t+1}} - a_1 e^{x_t} + x_{t+1}^2 - x_t^2$$
$$= a_1(e^{x_{t+1}} - e) + (x_{t+1}^2 - x_t^2) \quad (13)$$

Recall that;

$$x_t = a + th, \; t = 0, 1, 2, 3, \quad (14)$$

and

$$x_{t+1} = a + (t+1)h \Rightarrow a + th + h \Rightarrow x_t + h$$

therefore

$$x_{t+1} = a + (t+1)h \Rightarrow a + th + h \Rightarrow x_t + h,$$

and

$$x_{t+1}^2 = (x_t + h)^2 \Rightarrow (x_t + h)(x_t + h) \quad (15)$$
$$= x_t^2 + 2x_t h + h^2 \quad (16)$$

Let us put (11) and (16) into (13) to obtain

$$y_{t+1} - y_t = \frac{f_t^{(1)} - 2}{e^{x_t}}(e^{x_{t+1}} - e^{x_t}) + (x_t^2 + 2x_t h + h^2 - x_t^2$$

$$y_{t+1} - y_t = \frac{f_t^{(1)} - 2}{e^{x_t}}(e^{x_{t+1}} - e^{x_t}) + (2x_t h + h^2)$$

Thus the above numerical scheme can be written in the form

$$y_{t+1} = y_t + [f_t^{(1)} - 2](e^h - 1) + h(2x_t + h) \quad (17)$$

Equation (17) is a one-step method that can be used to solve equation of the form (1) .Equation (17) can be regarded as a numerical integration scheme which is particularly well suited to initial value problems havingoscillatory and exponential solutions and it was first reported and implemented in Fatunla (1976).

IV. Prove of Convergence for the New Scheme

According to Henrici (1962): we define any algorithm for solving a differential equation in which the approximation y_{t+1} to the x_{t+1} solution at the x_{t+1} can be calculated if only x_t, y_t and h are known as a ONE-STEP METHOD. We proceed to establish that our numerical algorithms are one step methods. From $F(x_t) = a_1 e^{x_t} + x_t^2$; the numerical integrator generated is given by

$$y_{t+1} = y_t + (f_t^{(1)} - 2)(e^h - 1) + h(2x_t + h),$$

If we expand eh, we shall have

$$e^h = \sum_{r=0}^{\infty} \frac{(-1)^r h^r}{r!} = 1 - h + h^2/2! - h^3/3! + \ldots \quad (18)$$

This implies

$$y_{t+1} = y_t + (f_t^{(1)} - 2)(x - h + h^2/2! - h^3/3! + \ldots - 1) + h(2x_t + h) \quad (19)$$

$$y_{t+1} = y_t + (f_t^{(1)} - 2)(-h + h^2/2! - h^3/3! + \ldots) + h(2x_t + h) \quad (20)$$

$$y_{t+1} = y_t + h(f_t^{(1)} - 2)(-h + h^2/2! - h^3/3! + \ldots) + h(2x_t + h) \quad (21)$$

Let A $= (-1 + h^2/2! - h^3/3! + \ldots)$ and B $= (2x_t + h)$, We shall have, $y_{t+1} = y_t + h[(f_t^{(1)} - 2)A + B]$

This is the convergence of the first scheme;

$$y_{t+1} = y_t + h\{(f_t^{(1)} - 2)c - 1 + h/2! - h^2/3! + \ldots) + (2x_t + h)\} \quad (22)$$

which can be written as $y_{t+1} = y_t + h(f_t^{(1)} - 2A + B)$, we have been able to write it in the form $y_{t+1} = y_t + h\phi(x_t, y_t; h)$, for which $h\phi(x_t, y_t; h) = (f_t^{(1)} - 2A + B)$.

V. Definition: Henrici (1962)

We define any algorithm for solving a differential equation in which the approximation y_{t+1} to the solution at the point X_{t+1} can be calculated, if only X_t, Y_t and h are known, as a ONE-STEP METHOD. It is a common practice to write the functional dependence, y_{t+1}, on the quantities X_t, Y_t and h in the form.

$$y_{t+1} = y_t + h\Phi(x_t, y_t; h.$$

VI. Convergence

THEOREM 1: Given a differential equation of the form $y^1 = f(x, y)$, $y(a) = \ell$, let $f(x, y)$ be defined and continuous for all points (x, y) in the region Dom, defined by $a \leq x \leq y$, $-\infty, \leq y \leq \infty$,a and b finite, and let there exist a constant 1 such that for every x, y^*, y with (x, y) and (x, y^*) both in Dom

$$|f(x, y) - f(x, y^*)| \leq L|y - y^*|, \quad (23)$$

and ℓ is any given number, there exist a unique solution y(x) of the initial value problem. The inequality (23) is known as a Lipschitz condition and the constant L as a Lipschitz constant. This condition can be regarded as being intermediate between differentiability and continuity, in the sense that if $F(x, y)$ in Dom, this implies that $F(x, y)$ satisfies a Lipschitz condition with respect to y for all (x, y) in Dom. (Fatunla, 1988; Lambert, 1973a; and Ibijola, 1998). By the mean value theorem, F(x, y) possessing a continuous derivative with respect to y for all (x, y) in Dom, will imply that;

$$f(x,y)-f(x,y^*)=\frac{\partial f(x,\overline{y})}{\partial y}(y-y^*). \quad (24)$$

it follow that (24) can now be satisfied if we choose

$$L=\sup\frac{\partial f(x,\overline{y}),}{\delta y}\, then,$$

$$y_{t+1}=y_t+h\{Af_t^{(1)}-2A+B\}$$

$$\phi(x_t,y_t;h)=Af^{(1)}_{(x_t,y_t)}-2A+B$$

$$\phi(x_t,y_t^*;h)=Af^{(1)}_{(x_t,y_t^*)}-2A+B$$

Hence

$$\phi(x_t,y_t^*;h)-\phi(x_t,y_t;h)=A(f^{(1)}_{(x_t,y_t^*)}-f^{(1)}_{(x_t,y_t)})-2A+2A-B+B \quad (25)$$

$$\phi(x_t,y_t^*;h)-\phi(x_t,y_t;h)=A(f^{(1)}_{(x_t,y_t^*)}-f^{(1)}_{(x_t,y_t)}) \quad (26)$$

let $\overline{y_t}$ be defined as a point in the interior of the interval whose endpoints are y and y*, by applying the mean value, we have;

$$f^{(1)}_{(x_t,y_t^*)}-f^{(1)}_{(x_t,y_t)}=\frac{\partial f^{(1)}_{(x_t\,,\,\overline{y}_t^*)}}{\partial y_t}(y_t^{(*)}-y_t) \quad (27)$$

let$L_1=sup\frac{\partial f^{(1)}_{(x_t\,,\,y_t)}}{\partial y_t}$. Substitute (27) into (26)

$$\phi(x_t\,,\,y_t^*\,;h)-\phi(x_t\,,y_t\,;h)=A\left[\frac{\partial f^{(1)}_{(x_t\,,\,\overline{y}_t^*)}}{\partial y_t}(y_t^{(*)}-y_t)\right]$$

$$=A\,sup\,\frac{\partial f^{(1)}_{(x_t\,,\,\overline{y}_t)}}{\partial y_t} \quad (28)$$

$$(x_t\,,\,\overline{y}_t)\varepsilon\,Dom$$

$$\phi(x_t\,,\,y_t^*\,;h)-\phi(x_t\,,y_t\,;h)=AL_1(y_t^*-y_t) \quad (29)$$

Taking the absolute value of both sides of (29), we have

$$|\phi(x_t\,,\,y_t^*\,;h)-\phi(x_t\,,y_t\,;h)|\le|Al_1||y_t^*-y_t|$$

$$|\phi(x_t\,,\,y_t^*\,;h)-\phi(x_t\,,y_t\,;h)|\le L|y_t^*-y_t| \quad (30)$$

VII. Numerical Experiments

a) Von Bertalanffy fish growth

The differential equation presented below is the model on the von Bertalanffy fish growth; $dy/dx=\alpha y^{2/3}-\beta y, y(0)=2$ and its theoretical solution is given as$y=1/(\alpha/\beta+Ae^{-\beta x/2})^2$, where y = y(x) is the weight of the fish, α and β are positive constants. We determine and illustrate the predicted growth of a fish, using the new numerical methods.

(i) H = .01: α = .95: R1 = .5: β = 2

This is the Numerical Result on Model for von Bertalanffy Fish Growth

X(T)	Y(X(T))	FY(T+1)	TFNUME
0.000000	2.000000	1.979900	0.020100
0.010000	2.019090	1.997950	0.021141
0.020000	2.038363	2.015636	0.022727
0.030000	2.057818	2.032856	0.024962
0.040000	2.077457	2.049607	0.027850
0.050000	2.097282	2.065888	0.031394
0.060000	2.117294	2.081698	0.035595
0.070000	2.137493	2.097036	0.040457
0.080000	2.157881	2.111901	0.045981
0.090000	2.178460	2.126292	0.052168
0.100000	2.199231	2.140212	0.059019

For convergence y (0) must be equal to β: convergency is guaranteed for 0.001<h<0.01.

(ii) H = .01: α = .901: Y' = R1*Y(T)^3/2-R2*Y(T), Y(0) = 2: R1 = .72: β = 4

This is the Numerical Result on Model for von Bertalanffy Fish Growth

X(T)	Y(X(T))	Y(T+1)	Truncation Error
0.000000	4.000000	3.979900	0.020100
0.010000	4.036202	4.012372	0.023830
0.020000	4.072731	4.044457	0.028274
0.030000	4.109586	4.076072	0.033514
0.040000	4.146769	4.107234	0.039535
0.050000	4.184280	4.137961	0.046319
0.060000	4.222121	4.168275	0.053846
0.070000	4.260292	4.198195	0.062098
0.080000	4.298794	4.227743	0.071051
0.090000	4.337628	4.256942	0.080686
0.100000	4.376794	4.285816	0.090978

For convergence y(0) must be equal to β: convergency is guaranteed for 0.001<h<0.01.

(iii) H = .001: α = .8: Y' = R1*Y(T)ˆ3/2-R2*Y(T), Y(0) = 2: R1 = .72: β = 4

This is the Numerical Result on Model for von Bertalanffy Fish Growth

X(T)	Y(X(T))	Y(T+1)	Truncation Error
0.000000	4.000000	3.997999	0.002001
0.010000	4.003201	4.000559	0.031569
0.020000	4.006405	4.003116	0.061397
0.030000	4.009612	4.005669	0.091485
0.040000	4.012821	4.008217	0.121834
0.050000	4.016032	4.010762	0.152445
0.060000	4.019246	4.013302	0.183318
0.070000	4.022463	4.015838	0.214452
0.080000	4.025682	4.018370	0.245849
0.090000	4.028904	4.020898	0.277507
0.100000	4.032128	4.023422	0.309427

For convergence y(0) must be equal to β: convergency is guaranteed for 0.001<h<0.01

(iv) H = .001: α = .9: Y' = R1*Y(T)ˆ3/2-R2*Y(T), Y(0) = 2: R1 = .72: β = 2

This is the Numerical Result on Model for von Bertalanffy Fish Growth

X(T)	Y(X(T))	Y(T+1)	Truncation Error
0.000000	2.000000	1.997999	0.002001
0.010000	2.001801	1.999619	0.002182
0.020000	2.003603	2.001237	0.002367
0.030000	2.005407	2.002851	0.002557
0.040000	2.007213	2.004461	0.002752
0.050000	2.009020	2.006068	0.002952
0.060000	2.010829	2.007672	0.003157
0.070000	2.012640	2.009272	0.003368
0.080000	2.014452	2.010869	0.003583
0.090000	2.016266	2.012462	0.003803
0.100000	2.018081	2.014052	0.004029

For convergence y(0) must be equal to β: convergency is guaranteed for 0.001<h<0.01.

b) *Seasonal Growth*

The model on seasonal growth is given by $dy/dx = r\,x\cos(wx), y(0) = 2$ where r and w are constants. In this work, we illustrate the behavior of the numerical solution of this equation. The theoretical solution is $y = Ke^{r\sin(wx)/w}$

(i) H = .01: r = .95: R1 = .5: W = 2

This is the Numerical Result on Model for Seasonal Growth

X(T)	Y(X(T))	Y(T+1)	Truncation Error
0.000000	2.000000	1.980000	0.020000
0.010000	2.019090	1.998250	0.020841
0.020000	2.038363	2.016236	0.022127
0.030000	2.057818	2.033956	0.023862
0.040000	2.077457	2.051407	0.026050
0.050000	2.097282	2.068588	0.028693
0.060000	2.117294	2.085498	0.031795
0.070000	2.137493	2.102136	0.035357
0.080000	2.157881	2.118500	0.039381
0.090000	2.178460	2.134592	0.043868
0.100000	2.199231	2.150412	0.048819

For convergence y(0) must be equal to w: convergency is guaranteed for 0.001<h<0.01.

(ii) H = .01: r = .98: Y' = gycosx'y(0) = 3: R1 = .5: W = 3

This is the Numerical Result on Model for Seasonal Growth

X(T)	Y(X(T))	Y(T+1)	Truncation Error
0.000000	3.000000	2.980000	-0.288753
0.010000	3.029544	3.009012	-0.291186
0.020000	3.059379	3.037595	-0.292856
0.030000	3.089506	3.065744	-0.293754
0.040000	3.119927	3.093454	-0.293875
0.050000	3.150645	3.120721	-0.293209
0.060000	3.181661	3.147542	-0.291753
0.070000	3.212978	3.173913	-0.289501
0.080000	3.244598	3.199832	-0.286447
0.090000	3.276522	3.225298	-0.282590
0.100000	3.308753	3.250311	-0.277925

For convergence y(0) must be equal to w: convergency is guaranteed for 0.001<h<0.01 and .9<r<1.0.

(iii) H = .001: r = .89: R1 = .5: W = 2

This is the Numerical Result on Model for Seasonal Growth

X(T)	Y(X(T))	Y(T+1)	Truncation Error
0.000000	2.000000	2.002002	-0.002002
0.010000	2.001781	2.003588	-0.001807
0.020000	2.003563	2.005172	-0.001609
0.030000	2.005347	2.006754	-0.001407
0.040000	2.007133	2.008334	-0.001201
0.050000	2.008920	2.009911	-0.000991
0.060000	2.010709	2.011486	-0.000777
0.070000	2.012499	2.013059	-0.000560
0.080000	2.014291	2.014629	-0.000338
0.090000	2.016084	2.016197	-0.000113
0.100000	2.017879	2.017764	-0.000116

For convergence y(0) must be equal to w: convergency is guaranteed for 0.001<h<0.01 and .9<r<1.0.

(iv) H = .001: r = .89: R1 = .5: W = 2

This is the Numerical Result on Model for Seasonal Growth

X(T)	Y(X(T))	Y(T+1)	Truncation Error
0.000000	1.000000	0.998000	0.002000
0.010000	1.001001	0.999002	0.001998
0.020000	1.002002	1.000002	0.002000
0.030000	1.003005	1.001000	0.002005
0.040000	1.004008	1.001996	0.002012
0.050000	1.005013	1.002990	0.002022
0.060000	1.006018	1.003982	0.002036
0.070000	1.007025	1.004972	0.002052
0.080000	1.008032	1.005960	0.002072
0.090000	1.009041	1.006946	0.002094
0.100000	1.010050	1.007930	0.002120

For convergence y (0) must be equal to w: convergency is guaranteed for 0.001<h<0.01.

(v) H = .01: r = .95: R1 = .5: W = 1

This is the Numerical Result on Model for Seasonal Growth

X(T)	Y(X(T))	Y(T+1)	Truncation Error
0.000000	1.000000	0.980000	0.020000
0.010000	1.009545	0.989225	0.020321
0.020000	1.019181	1.998318	0.020864
0.030000	1.028909	1.007278	0.021631
0.040000	1.038729	1.016103	0.022625
0.050000	1.048641	1.024794	0.023847
0.060000	1.058647	1.033349	0.025298
0.070000	1.068746	1.041768	0.026979
0.080000	1.078941	1.050050	0.028891
0.090000	1.089230	1.058196	0.031034
0.100000	1.099615	1.066206	0.033410

For convergence y(0) must be equal to w: convergency is guaranteed for 0.001<h<0.01.

VIII. Data Interpretation and Conclusion

This presents to us that the fishes grow in sizes, with H being the interval of feed input and r, R1 and w being the aeration, rate of picking and feed quality.

These days it is not scientifically right to use poultry waste in the composition of feed in fishery since this could further enhance the spread of avian-influenza through fish to men. Economically, if the figures presented above are measured in hundreds, the growth rate will favour the third world countries and help in increasing the quantity of fish supply in these countries. This will also serve as a means of bridging the gaps in protein deficiency created by the bird flu saga in the less developing nations.

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