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I. INTRODUCTION

The production and detection of free neutron clusters (an entirely new form of nuclear matter) have been seriously studied, initially through the channel $\text{Be} = \text{Be} + n$ [1]. Other supporting experiments involve the reactions ${}^{12}\text{Be} = {}^8\text{Be} + {}^4n$ [2] and ${}^8\text{He} = {}^4\text{He} + {}^4n$ [3]. In [4] a review is made and the far-reaching implications of 4n are discussed.

Neutron nuclei heavier than 4n have been studied, specifically 8n in the decay of ${}^{252}\text{Cf}$ [5] and ${}^{5-13}n$ in induced fission of ${}^{235}\text{U}$ [6]. All these efforts lasting for decades have set the question: Can a nucleus be made up of neutrons alone?

From the theoretical point of view [7-8], it does not seem possible to bind 4n without destroying many other successful predictions by applying the same forces, e.g., to light nuclei. However, simulations in progress are used to clarify the origin of 4n by employing the Generator Coordinates Method and locating the neutrons at the vertices of a tetrahedron.

A recent publication [9] favours the possible stability of 4n and 6n in the framework of the Isomorphic Shell Model (ISM). In the present work we employ the same model to study the possibility of stability of neutron nuclei in the next 1d2s shell. In order to correct a small numerical mistake in the results of [9], the present research repeats the study of 1s and 1p neutron nuclei. The privilege of the present approach is that while the model has been successfully applied throughout the periodic table [10-19], here the model is employed without any modification, constituting the model unique in the relevant research.

II. THE ISOMORPHIC SHELL MODEL (ISM)

The model follows the sequence of reasoning based on well documented quantum mechanical

principles and mathematical theorems

- The nucleus is composed of two different kinds of fermions (neutrons and protons).
- The wave function describing neutrons or protons or both is anti-symmetric.
- Anti-symmetric wave function of a set of particles (e.g., nucleons or protons) has maxima at positions which are identical to those positions if the particles interact among each other via a repulsive force of unknown nature [20].
- Repulsive particles on a sphere are at equilibrium only for specific numbers of particles which are identical to the number of vertices, or to the number of faces, or to the number of middles of edges or combinations of these numbers related to the regular or semi-regular polyhedra [21].
- Two kinds of repulsive particles (here, neutrons and protons) are at equilibrium on a sphere if the neutrons by themselves are at equilibrium and if the protons by themselves are at equilibrium, and if all these particles taken together are at equilibrium as well [21].
- If the number of repulsive particles is larger than the aforementioned numbers [21], then the extra particles could be at equilibrium on a different sphere which does not disturb the equilibria of particles on other spheres [21].

The first three of the aforementioned cornerstones of the model come directly from basic quantum mechanics and the other three are rigorously proved mathematical theorems [21]. *No ad hoc assumption is introduced anywhere.* This is the outstanding, unique privilege of the present model.

By rigorous application of the above principles and theorems, the most probable forms of nuclear shells are derived for the whole periodic table of nuclei, i.e., up to $Z=126$ and $N=184$ [22]. If in addition the nucleons are considered with finite size (specifically, $r_p = 0.860$ fm and $r_n = 0.974$ fm), then the average size of all shells are derived by considering packing of the shells assumed superimposed with a common center and the most symmetric relative orientation [22] (packing means that the bags of a polyhedron come in contact with the bags of a previous polyhedron). Thus, in the ISM the most probable forms and the average sizes of the nuclear shells, and thus of all nuclei, are determined without reference to nuclear forces.

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Thorough study of regular and quasi-regular polyhedra employed by the present model shows that the symmetries of these polyhedra identically possess the quantization of orbital angular momentum, of spin, and of total angular momentum [23], a fact which permits one to assign quantum states at the vertices of these polyhedra assumed as the average particle positions. Each occupied vertex configuration corresponds to a quantum state configuration with definite quantum state and energy.

In general, the ISM is a microscopic nuclear structure model that incorporates into a hybrid model the prominent features of single-particle and collective approaches in conjunction with the nucleon finite size [24]. The model consists of two parts, namely, the complete quantum mechanical part [24] and the semiclassical part [24, 25].

Figure 1 stands for the shell structure for all nuclei up to $N = Z = 20$ according to the ISM [12-14]. Thus, the first three neutron and the first three proton shells are shown. This is a good way to see the relationship between regular nuclei and possible neutron nuclei. Polyhedral vertices, standing for nucleon average positions in definite quantum states (τ, n, ℓ, m_ℓ, s), are numbered as shown. Central axes standing for the quantization of directions of the orbital angular momentum are labelled as τ and pass through the points marked by small solid circles \bullet . At the bottom-left of each block the numbering of a polyhedron proceeded by the letter Z (N) for protons (neutrons) is given. Over this the number of polyhedral vertices and the number of possible unoccupied vertices (holes, h) are also given. At the bottom-right of each block the radius of the polyhedron is listed. Over this the cumulative number of vertices of all previous polyhedra and of this polyhedron is also given and stands as a quantum-geometrical interpretation of magic numbers. Finally, at the bottom-center of each block the distance $P_{n\ell m}$ of the nucleon average position $n\ell m$ from the relevant axis τ is given. The coordinates of nucleon average positions of Fig.1 have been determined [11] and are identically employed in all publications thereafter {e.g., [12-14]}.

At this point it is interesting for one to observe from Fig. 1 that the average structures of a neutron and of the corresponding proton shell on the same line of this figure are presented by reciprocal polyhedra [26]. That is, the average positions of protons (neutrons) are at the directions through the centers of the faces of the corresponding neutron (proton) polyhedron, thus these two polyhedra possess the same rotational symmetry. This relative orientation makes the np distances systematically smaller than the nn and the pp distances. This situation, even using the same r- dependent potential as in Eq.(1) below, leads to a much stronger total average np interaction.

Apparently, if we are concerned with neutron nuclei alone, consideration of only the neutron polyhedra

of Fig.1 is enough. It is important to emphasize that the neutron polyhedra possess *stable* equilibrium for repulsive particles possessing average positions at their vertices, while proton polyhedra possess *unstable* equilibrium [21]. Thus, neutron polyhedra can exist by themselves, as far as their stability is concerned. Even their average sizes are independent from the existence of proton polyhedra. Specifically, the octahedron standing for the 1p neutron shell is closely packed with the neutron zerohedron standing for the 1s neutron shell. Similarly, the icosahedron standing for the 1d_{2s} neutron shell is closely packed with the aforementioned octahedron.

III. SEMI-CLASSICAL VERSION OF THE ISM

Here, we present the semiclassical part of the model, which has been used many times [12-14] in place of the quantum mechanical part of the model [24], in the spirit of the Ehrenfest theorem [27, 28] that for the average values the laws of Classical Mechanics are valid [28].

In the present semiclassical treatment, we employ Eqs. (1-5) as the expression of the two-body (two Yukawa) potential V [16, 29], of the kinetic energy T [11], of the spin-orbit energy V_{LS} [30], and of the binding energy (E_B). Isospin term in Eq.(5) is not needed since the isospin is here taken care of by the different shell structure (forms and sizes) between proton and neutron shells, as apparent from Fig.1.

$$V_{ij} = 1.7 * 10^{17} * e^{-31.8538r_{ij}} / r_{ij} - 241.193 * e^{-1.4546r_{ij}} / r_{ij} \quad (1)$$

$$\langle T \rangle_{n\ell m} = (\hbar^2 / 2M) [1/R_{\max}^2 + \ell(\ell + 1) / \rho_{n\ell m}^2] \quad (2)$$

$$\sum_i V_{LiSi} = \lambda \sum_i (\hbar \omega_i)^2 / (\hbar^2 / m) * \ell_i s_i \quad (3)$$

$$\hbar \omega_i = (\hbar^2 / M)(n + 3/2) / \langle r_i^2 \rangle \quad (4)$$

$$E_B = \sum_{ij} V_{ij} - \sum \langle T \rangle_{n\ell m} - \sum_i V_{LiSi}, \text{ where:} \quad (5)$$

- V_{ij} is the potential energy between a pair of nucleons i, j at a distance r_{ij} .
- $\langle T \rangle_{n\ell m}$ is the average kinetic energy of a nucleon at the quantum state n, ℓ, m and consist of two terms. The first is due to uncertainty and the second to orbital motion of this nucleon.
- n, ℓ, m are the quantum numbers characterizing a polyhedral vertex standing for the average position of a nucleon at the quantum state n, ℓ, m .
- ℓ_i and s_i stand for the orbital angular momentum quantum number ℓ and the intrinsic spin quantum number s of any nucleon i .
- M is the mass of a proton M_p or of a neutron M_n ,
- R_{\max} is the outermost proton or neutron polyhedral radius (R) of a nucleus plus the relevant average nucleon radius r_p for a proton and r_n for a neutron (i.e., R_{\max} is the radius of the nuclear volume in which protons or neutrons are confined),

- $R_{n\ell m}$ is the distance of a nucleon average position at a quantum state (n, ℓ, m) from its orbital angular momentum vector at the direction $n\theta^m \epsilon$.

When only binding energies (and not scattering properties) are required as here, just the second term of the above two-body potential of Eq.(1) is sufficient. Thus, for non-scattering

properties, the parameters of the model are the following five: the two-size parameters R_p and R_n , the two parameters from the second term of Eq.(1), and the one parameter, λ , from Eq.(3). With the help of these *universal* (i.e., they are not adjustable and thus they maintain the same values for all properties in all nuclei) parameters all quantities R_{max} , $P_{n\ell m}$, and $\Gamma_{\omega i}$, in Eqs.(1) – (5) are obtainable by employing the coordinates of the nucleon average positions derived from Fig.1 [12-14] and are given in [11].

IV. APPLICATION TO NEUTRON NUCLEI

If only neutrons are considered, the relevant shell structure is derived from Fig.1 by disregarding the proton shell structure.

Application of Eqs. (1-5) for neutron nuclei leads to the results shown in Table 1. Specifically, in its columns 1-9 we give the notation of a nucleus with even number of neutrons, the average positions of Fig. 1 occupied, the relevant state configuration, the quantities ΣV_{ij} , $\Sigma \langle T \rangle_{n\ell m}$, ΣV_{Lis_i} , E_B , the notations stable or unstable, and the average radius of each nucleus, respectively. From column 8 of Table 1 we see that the nuclei 4n - ${}^{16}n$ have at least one state with positive E_B , a fact which implies that they are possible stable neutron nuclei. It is noticeable that several of these nuclei, besides their ground state, show stability for one or more excited states. It is of interest that when 8n is a closed shell nucleus its E_B is negative, while if we consider 2p-2h (i.e., their core is ${}^{12}C$ and not ${}^{16}O$) the 8n has positive E_B . This, of course, is consistent with the structure of ${}^{16}O$ where for its ground state we have 4p-4h structure [24]. From preliminary calculations the same situation occurs for ${}^{18}n$ and ${}^{20}n$. That is, while for these two nuclei their E_B in Table 1 have a negative sign, after considering p-h structure with the next shell their E_B becomes positive. That is, this situation implies that neutron nuclei could be possible even for the next 1f2p shell. Another interesting comment from the results of Table 1 is that the configurations possessing 2s states have larger positive E_B than the other configurations of the same nucleus without 2s states.

Figure 2 shows the space arrangement of neutrons for all neutron nuclei examined and listed in column 1 of Table 1 following the average positions Nos from column 2 of the table.

Table 2 shows the same quantities like Table 1 for the regular nuclei 4He and ${}^{40}Ca$, i.e., the nuclei with $N = Z$ corresponding to the first and the last possible

neutron nuclei of Table1 by employing identically as above the same equations and parameters. In addition this table deals with charge and point neutron – point proton rms radii. Here, the existence of experimental values for binding energies and radii and their impressive closeness to the present predictions give necessary credits to the model employed and to predicted possible neutron nuclei.

The necessary formulae for the radii are

$$\langle r^2 \rangle_p = \frac{\sum_1^Z r_i^2}{Z} + \langle r^2 \rangle_p - 0.116 \frac{N}{Z} \quad \text{and} \quad (6)$$

$$\langle r^2 \rangle_n = \frac{\sum_1^N r_i^2}{N} + \langle r^2 \rangle_n, \quad (7)$$

Where the first is for the calculation of proton rms radii and the second for the estimation of neutron radii. The radii r_i are the radii R_i from Fig.1. The quantity $\langle r^2 \rangle_p$ is taken as 0.8^2 fm^2 and presents the square of the average size of a proton, while the proton bag radius is already given above equal to 0.860 fm . In correspondence for the neutron we take 0.91 fm as the average size of a neutron, while the neutron bag radius, as given above, is 0.974 fm . The 5 quantities 0.8 fm and 0.91 fm have some minimum contribution to the radii only to the results of protons or neutrons rms radii of very light nuclei.

The values of neutron radii given in column 9 of Table 1 come as results of applying Eq.(7) above to the average positions of neutrons given in column 2 for all nuclei of column 2 of Table 1.

V. CONCLUSIONS

From the ten even neutron nuclei examined in Table 1, seven show the possibility of having at least one state with positive E_B . From the remaining three nuclei of this table the 2n definitely has negative E_B , while the other two, namely ${}^{18}n$ and ${}^{20}n$, from preliminary calculations are expected to obtain positive E_B through a p-h structure with the next 1f2p shell. From the nuclei with positive E_B , namely, 4n - ${}^{16}n$, the 4n , 6n , and ${}^{16}n$ have only one state with positive E_B . The nuclei 8n , ${}^{12}n$, and ${}^{14}n$ have two states with positive E_B , while the nucleus ${}^{10}n$ has four states with positive E_B .

It is noticeable that 8n and ${}^{20}n$, even though closed shell nuclei, do not exhibit positive E_B . This is here understood as a result of the structure of 2n (i.e., the neutron zerohedron) which favours prolate structures. Thus, the states 1p1/2 and 1d3/2 with average positions towards the z axis, which is perpendicular to the neutron zerohedron, are less favoured. The same explanation is valid for ${}^{18}n$ which also possesses 1d3/2 states in its structure.

It is important to emphasize that the present calculations have the following characteristics:

- They employ the same model already successfully applied to many calculations of regular nuclei with very good results [10-16, 19, 22-25], a model based

on fundamental quantum mechanics [20] and mathematical theorems [21] *without any ad hoc assumption*.

- b) While the two-body potential employed here [Eq.(1)] has been strictly derived from nuclear physics [16, 29], it is almost identical to potentials derived from particle physics via chromodynamics.
- c) The radii in column 9 of the table for possible neutron nuclei are identical to the neutron radii of regular nuclei with neutrons at the same quantum states [23].

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Table 1 : Energy components and rms charge and point neutron - proton radii (in fm) of 4He and 40Ca

Nuc.	Pos.	Config.	ΣV	$\Sigma <T>$	E_c	E_R	$E_{B,m}$	$E_{B,e}$	$_{ch} \langle r^2 \rangle^{1/2}_m$	$_{ch} \langle r^2 \rangle^{1/2}_e$	$_{n} \langle r^2 \rangle^{1/2}$	$_{p} \langle r^2 \rangle^{1/2}$
4He	1-4	(1s) ⁴	44.6	14.2	0.5	1.7	28.2	28.3	1.71	1.68		
40Ca	1-40	(1s) ⁴ (1p) ¹² (1d) ²⁰ (2s) ⁴	771.7	363.0	64.8	1.7	342.2	342.1	3.47	3.48	-0.29	-0.30

Table 2 : Calculations of binding energies (in MeV) and radii (in fm) for the nuclei listed in the first column of the table. 8

Nuc	Average positions Nos.	State configurations	ΣV	$\Sigma <T>$	ΣE_s	E_B	st. un.	Rad.
² n	1-2	(1s1/2) ²	7.27	10.93	0.00	-3.66	un.	1.33
⁴ n	1-2, 7-8	(1s1/2) ² (1p3/2) ²	23.13	-19.98	0.20	3.35	st.	2.11
⁶ n	1-2, 5-8	(1s1/2) ² (1p3/2) ⁴	40.53	-36.55	0.39	4.37	st.	2.31
⁸ n	1-2, 5-10	(1s1/2) ² (1p3/2) ⁴ (1p1/2) ²	50.79	-53.12	0.00	-2.33	un.	2.41
	1-2, 5-8, 25, 27	(1s1/2) ² (1p3/2) ⁴ (1d5/2) ²	61.02	-54.60	0.58	7.00	st.	2.72
		(1s1/2) ² (1p3/2) ⁴ (2s1/2) ²	61.02	-48,13	0.39	13.28	st.	2.72
¹⁰ n	1-2, 5-10, 25, 27	(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (1d5/2) ²	71.67	-69.77	0.19	2.09	st.	2.71
		(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (2s1/2) ²	71.67	-63.30	0.00	8.37	st.	2.71
	1-2, 5-8, 25-28	(1s1/2) ² (1p3/2) ⁴ (1d5/2) ⁴	79.22	-76.87	0.77	3.12	st.	2.94
		(1s1/2) ² (1p3/2) ⁴ (1d5/2) ² (2s1/2) ²	79.22	-70.40	0.58	9.40	st.	2.94
¹² n	1-2, 5-10, 18, 20, 25, 27	(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (1d5/2) ⁴	90.96	-92.03	0.38	-0.69	un.	2-90
		(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (1d5/2) ² (2s1/2) ²	90.96	-85.56	0.19	5.59	st.	2.90
	1-2, 5-8, 18, 20, 25-28	(1s1/2) ² (1p3/2) ⁴ (1d5/2) ⁶	97.32	-99.13	0.72	-1.09	un.	3.08
		(1s1/2) ² (1p3/2) ⁴ (1d5/2) ⁴ (2s1/2) ²	97.32	-92.66	0.77	5.43	st.	3.08
¹⁴ n	1-2, 5-10, 17-20, 25, 27	(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (1d5/2) ⁶	110,82	-114.29	0.56	-2.91	un.	3.02
		(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (1d5/2) ⁴ (2s1/2) ²	110.82	-107.82	0.38	3.38	st.	3.02
	1-2, 5-8, 17-20, 25-28	(1s1/2) ² (1p3/2) ⁴ (1d5/2) ⁶ (2s1/2) ²	115.99	-114,92	0.96	2.03	st.	3.17
¹⁶ n	1-2, 5-10, 17-20, 25-28	(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (1d5/2) ⁶ (2s1/2) ²	130.52	-130.09	0.56	0.99	st.	3.11
¹⁸ n	1-2,5-10,17-20, 21,23,25-28	(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (1d5/2) ⁶ (2s1/2) ² (1d3/2) ²	150.96	-152.35	0.28	-1.11	Un	3.18
²⁰ n	1-2, 5-10, 17-28	(1s1/2) ² (1p3/2) ⁴ (1p1/2) ² (1d5/2) ⁶ (2s1/2) ² (1d3/2) ⁴	171.96	-174.61	0.00	-2.65	Un	3.23

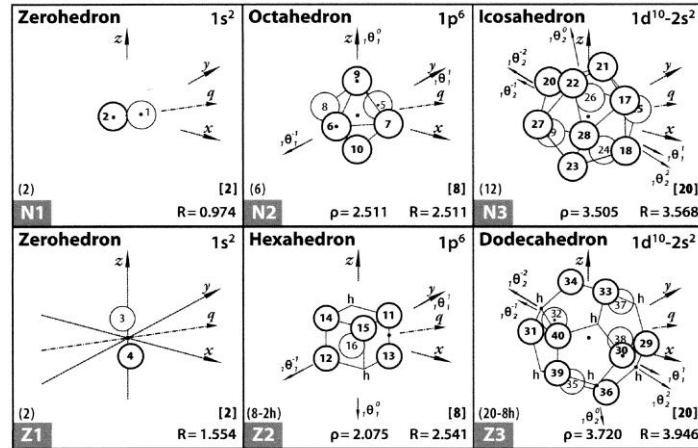


Fig.1

Figure 1: Most probable forms and average sizes of the first three neutron and the first three proton shells up to $N=Z=20$

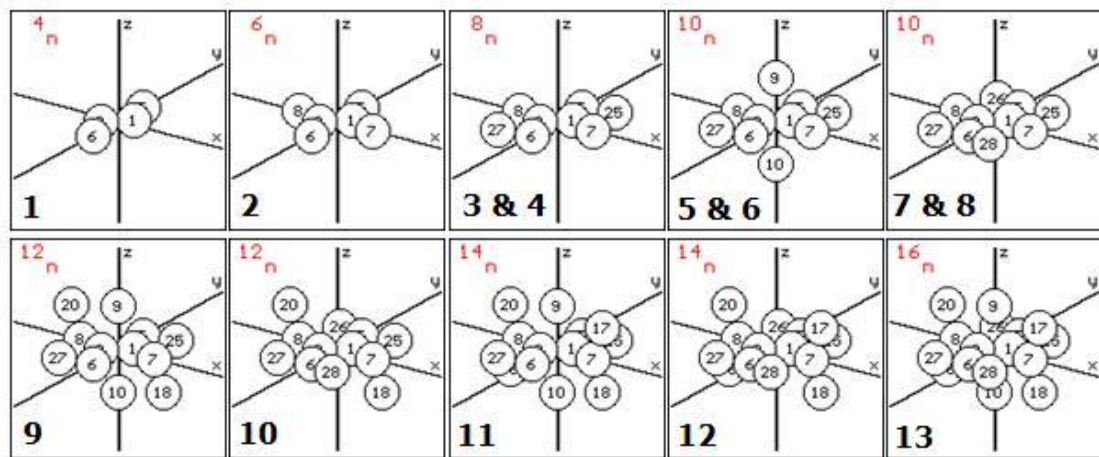


Figure 2: Most probable forms and average sizes of possible neutron nuclei according to Table 1 following the numbering of column 4. From this column we can see that the same number may correspond to more than one state configurations shown in column 3 of the table. The numbering of bags in this figure corresponds to the numbering of bags in Fig. 1. That is, it specifies the same point in space.