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Vertex Semientire Block Graph

By Venkanagouda M Goudar & Rajanna N E
Sri Siddhartha Institute of Technology, India

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Strictly as per the compliance and regulations of :

Ref

5. Sedlacek J., Some properties of interchange graphs. The Graphs and the applications. Academic press, New York (1962).

Vertex Semientire Block Graph

Venkanagouda M Goudar [α] & Rajanna N E [σ]

Abstract- In this communications, the concept of the vertex semientire block graph is introduced. We present characterization of graphs whose vertex semientire block graph is planar, outerplanar and minimally non-outerplanar. Also we establish a characterization of graphs whose vertex semientire block graph is Eulerian.

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I. Introduction

By graph, we mean a finite, undirected graph without loops or multiple edges. We refer the terminology of [5].

The inner vertex number i(G) of a planar graph G is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of G in the plane. A graph G is said to be minimally non-outerplanar if i(G) = 1.

A new concept of a graph valued functions called the pathos vertex semientire graph $Pe_v(G)$ of a plane graph G was introduced [6] and is defined as the graph whose vertex set is V(T) b_i r and the two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the path of pathos and vertices lie on the regions. Since the system of pathos for a tree is not unique, the corresponding vertex semientire block graph is also not unique.

Blockdegree is the number of vertices lies on a block. Blockpath is a path in which each edge in a path becomes a block. Degree of a region is the number of vertices lies on a region.

Now we define the vertex semientire block graph. The vertex semientire block graph denoted by $\mathbf{e}_{vb}(G)$ is the graph whose vertex set is V(T) b_i r and the two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the blocks and vertices lie on the regions.

II. Preliminaries

We need the following results to prove further results.

Theorem 1 [Ref 4]. If G be a connected plane graph then $\mathbf{e}_v(G)$ is planar if and only if G is a tree.

Theorem 2 [Ref 3]. Every maximal outerplanar graph G with p vertices has 2p – 3 edges.

Author α: Department of Mathematics, Sri Siddhartha Institute of Technology, Tumkur, Karnataka.
e-mail: vmgouda@gmail.com
Author σ: Department of Mathematics Adichunchanagiri Institute of Technology, Chikmagalur, Karnataka.
e-mail: ethinamane@gmail.com

III. VERTEX SEMIENTIRE BLOCK GRAPH

We start with a preliminary result.

Remark 1. For any graph G, $G \subseteq e_v(G) \subseteq \mathrm{e}_{vb}(G)$

Remark 2. For any (p, q) graph G the degree of a vertex in vertex semientire block graph e_{vb}(T) is 2p+1.

In the following theorem we obtain the number of vertices and edges in a vertex semientire block graph.

Theorem 3. For any (p, q) graph G with b blocks and r regions vertex semientire block graph $\mathbf{e}_{vb}$(G) has (p + b +r) vertices and $q + \sum_{i=1}^{k} d(b_i) + \sum_{j}^{l} d(r_j)$ edges, where $d(b_i)$ is the block degree of a block bi and $d(r_j)$ is the degree of a region r_j.

Proof. By the definition of vertex semientire block graph $\mathbf{e}_{ev}$(G), the number of vertices is the union of the vertices, blocks and the regions of G. Hence the number of vertices of vertex semientire block graph $\mathbf{e}_{vb}$(G)is (p + k + 1).

Further, by remark 1, the graph G is a sub graph of $\mathbf{e}_{vb}$(G), hence all the edges of G are present $\mathbf{e}_{vb}$(G). Also by the Remark 2, the number of vertices is the degree of a block. Lastly, the degree of regionvertex is the number of vertices lies on the region and is the number of edges in $\mathbf{e}_{vb}$(G). Hence the number of edges vertex semientire block graph $\mathbf{e}_{vb}$(G) is $q + \sum_{i=1}^{k} d(b_i) + \sum_{j}^{l} d(r_j)$

Theorem 4. For any tree T, vertex semientire block graph $\mathbf{e}_{vb}$(G) is always nonseparable.

Proof. Consider a graph G. We have the following cases

Case 1. Suppose G be a tree. All internal vertices of G are the cut vertices C_i. These cut vertices lies on the region as well as on two blocks. Clearly C_i is not a cut vertex in $\mathbf{e}_{vb}$(G). Hence $\mathbf{e}_{vb}$(G)is nonseparable.

Case 2. Suppose G be any graph with at least one cut vertex. Since cut vertex C_i lies on at least two blocks and one region. Hence in $\mathbf{e}_{vb}$(G), C_i becomes non-cut vertex. Hence $\mathbf{e}_{vb}$(G) is nonseparable.

Theorem 5. For any graph G, vertex semientire block graph e_{vb}(G) is planar if and only if G is a tree.

Proof. Suppose $\mathbf{e}_{vb}$(G) is planar. Assume that G is a graph other than a tree, say a cycle C_n, for Without loss of generality we take n=3. Clearly all vertices of C3 lies on a block b_i. By the definition of vertex semientire block graph $\mathbf{e}_{vb}$ (G), C_3 along with bi form a graph K_4. Further all vertices of C3lies on both regions vertices r_1 and r_2. Clearly r_1 and r_2 are adjacent to all vertices of C_3 to form a graph which is homeomorphic to K_5 and is non planar, a contradiction. Hence G must be a tree.

Conversely suppose a graph G is a tree. By definition, for each edge of a tree G, there is a $K_4 - e$ in $\mathbf{e}_{vb}$(G). Clearly $\mathbf{e}_{vb}$(G) is planar.

Theorem 6. For any tree T the vertex semientire block graph $\mathbf{e}_{vb}$(G)outerplanar if and only if T is a path P_n.

Proof. Suppose vertex semientire block graph $\mathbf{e}_{vb}$(G) is outerplanar. Assume that G is a tree which is not a path P_n. Since each edge is a block and both end vertices lies on a block. These end vertices and a blockvertex form a graph K3 in $\mathbf{e}_{vb}$(G). Further the regionvertex is adjacent to all vertices of G to form a graph such that it has at least two inner verteices, which is non outerplanar, a contradiction.

Notes

Conversely, suppose a graph G is a path P_n. By definition of $\mathbf{e}_{vb}(G)$, the regionvertex r is adjacent to two vertices v_1 ,v_2 to form K_3 and a pathosvertex Pi is adjacent to two vertices v1,v_2 fo K_3 to form $K_4 - x$, which is outerplanar.

Theorem 7. For any graph G, vertex semientire block graph $\mathbf{e}_{vb}(G)$ is not minimally non outerplanar .

Proof. Proof follows from the Theorem 6.

Notes

Theorem 8. For any graph G, vertex semientire block graph $\mathbf{e}_{vb}(G)$ is maximal outerplanar if G is a path P_n.

Proof. Suppose vertex semientire block graph $\mathbf{e}_{vb}(G)$ is maximal outerplanar, then $\mathbf{e}_{vb}(G)$ is connected. Let G be a path P_n, it contains p vertices and p-1 edges. Given that $\mathbf{e}_{vb}(G)$ is maximal outerplanar, by Theorem 2, it has 2p – 3 edges. We know that $V[\mathbf{e}_{eb}(G)] = 2p$ and $E[e_{vb}(G)] = 4p - 3$.

$\Rightarrow 2(2p) - 3 = q = 4p - 3$

$\Rightarrow 2p - 3 = 4p - 3$ *is satisfied*.

Clearly, G=P_n is a nonempty path. Hence necessity is proved.

Theorem 9. For any graph G vertex semientire block graph $\mathbf{e}_{vb}(G)$ is Eulerian if and only if following conditions hold:

i. G is a graph without tree
ii. Each region contains even number of vertices.
iii. number of vertices in each block is even and
iv. the number of vertices in a graph G is even.

Proof. Suppose $\mathbf{e}_{vb}(G)$ is Eulerian. We have the following cases.

Case 1. Assume that G is a tree. Clearly a tree contains at least two vertices vi and vj of odd degree and at least one vertex v_k of even degree. By the Remark 2, in evb(G), deg(v_i) and deg(v_j) becomes even and deg(v_k) becomes odd. Clearly $\mathbf{e}_{vb}(G)$ is non Eulerian, a contradiction.

Case 2. Assume that degree each region contains odd number of vertices. By definition of $\mathbf{e}_{vb}(G)$, the degree of regionvertex in $\mathbf{e}_{vb}(G)$ is becomes odd. Clearly $\mathbf{e}_{vb}(G)$ is non Eulerian, a contradiction.

Case 3. Assume that the number of vertices in each block is odd. By definition of $\mathbf{e}_{vb}(G)$, the degree of blockvertex in $\mathbf{e}_{vb}(G)$ is becomes odd. Clearly $\mathbf{e}_{vb}(G)$ is non Eulerian, a contradiction.

Case 4. Assume that the number of vertices in a graph G is odd. Clearly it follows that G has either at least two vertices of even degree and at least two vertices of odd degree or all vertices of even degree. If all vertices of odd degree then G must be a complete graph. Without loss of generality G is either K_3 or K_4. If G is K_3, then inner regionvertex is of odd degree. By the case 2, $\mathbf{e}_{vb}(G)$ is non Eulerian. Also if G is K4, then each region have odd degree. By the case 2, $\mathbf{e}_{vb}(G)$ is non Eulerian, which is a contradiction.

Conversely suppose G satisfies all the conditions of the Theorem. For a graph G with degree of regionvertex, degree of blockvertex and all vertices of G is even, then the corresponding vertices in $\mathbf{e}_{vb}(G)$ is even. Hence $\mathbf{e}_{vb}(G)$ is Eulerian.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Haray F., Annals of New York, Academy of Sciences, (1977)175, 198.
2. Harrary, F.,Graph Theory, Addison- Wesley Reading Mass, (1969)., p.72, 107.

3. Kulli. V R., On minimally nonouterplanar graphs, proceedings of the Indian National Science Academy, (1975)., 41A.
4. Kulli. V. R. and Akka. D. G., Journal mathematical Science, Vol. 14, No. 6, (1980)., P. 585- 588.
5. Sedlacek J., Some properties of interchange graphs. The Graphs and the applications. Academic press, New York (1962).
6. Venkanagouda. M. Goudar, "On Pathos vertex semientire graph of a tree", international journal of applied mathematical research, 1 (4),. 666- 670 (2012).

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