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Establishment of a Summation Formula Associated to Hypergeometric Function

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Establishment of a Summation Formula Associated to Hypergeometric Function

Salahuddin^α & Shakeeluddin^σ

Abstract- The main objective of present paper is the development of a summation formulae linked with the Contiguous relation and Hypergeometric function.

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I. INTRODUCTION

Generalized Gaussian Hypergeometric function of one variable :

$${}_A F_B \left[\begin{matrix} a_1, a_2, \dots, a_A ; \\ b_1, b_2, \dots, b_B ; \end{matrix} z \right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!} \quad (1)$$

or

$${}_A F_B \left[\begin{matrix} (a_A) ; \\ (b_B) ; \end{matrix} z \right] \equiv {}_A F_B \left[\begin{matrix} (a_j)_{j=1}^A ; \\ (b_j)_{j=1}^B ; \end{matrix} z \right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \quad (2)$$

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relation :

[Abramowitz p.558(15.2.19)]

$$(a-b) (1-z) {}_2 F_1 \left[\begin{matrix} a, b ; \\ c ; \end{matrix} z \right] = (c-b) {}_2 F_1 \left[\begin{matrix} a, b-1 ; \\ c ; \end{matrix} z \right] + (a-c) {}_2 F_1 \left[\begin{matrix} a-1, b ; \\ c ; \end{matrix} z \right] \quad (3)$$

Recurrence relation :

$$\Gamma(z+1) = z \Gamma(z) \quad (4)$$

Legendre's duplication formula :

$$\sqrt{\pi} \Gamma(2z) = 2^{(2z-1)} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right) \quad (5)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)} \Gamma\left(\frac{b}{2}\right) \Gamma\left(\frac{b+1}{2}\right)}{\Gamma(b)} \quad (6)$$

$$= \frac{2^{(a-1)} \Gamma(\frac{a}{2}) \Gamma(\frac{a+1}{2})}{\Gamma(a)} \quad (7)$$

In the monograph of Prudnikov et al. , a summation formula is given in the form [Prudnikov,491,equation(7.3.7.8)]

$${}_2F_1 \left[\begin{matrix} a, b ; \\ \frac{a+b-1}{2} ; \end{matrix} \frac{1}{2} \right] = \sqrt{\pi} \left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\Gamma(\frac{b+1}{2})} + \frac{2 \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\Gamma(b)} \right] \quad (8)$$

Now using Legendre's duplication formula and Recurrence relation for Gamma function , the above formula can be written in the form

$${}_2F_1 \left[\begin{matrix} a, b ; \\ \frac{a+b-1}{2} ; \end{matrix} \frac{1}{2} \right] = \frac{2^{(b-1)} \Gamma(\frac{a+b-1}{2})}{\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)} \Gamma(\frac{a}{2}) \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})} \right] \quad (9)$$

It is noted that the above formula [Prudnikov,491,equation(7.3.7.8)], i.e. equation(8) or (9) is not correct.The correct form of equation(8) or (9) is obtained by [Asish et. al(2008), p.337(10)]

$${}_2F_1 \left[\begin{matrix} a, b ; \\ \frac{a+b-1}{2} ; \end{matrix} \frac{1}{2} \right] = \frac{2^{(b-1)} \Gamma(\frac{a+b-1}{2})}{\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} \left\{ \frac{(b+a-1)}{(a-1)} \right\} + \frac{2 \Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})} \right] \quad (10)$$

Involving the derived formula obtained by [Asish et. al(2008), p.337(10)], we establish the main formula.

II. MAIN RESULT OF SUMMATION FORMULA

For the result $a \neq b$

$$\begin{aligned} {}_2F_1 \left[\begin{matrix} a, b ; \\ \frac{a+b-25}{2} ; \end{matrix} \frac{1}{2} \right] &= \frac{2^{(b-1)} \Gamma(\frac{a+b-25}{2})}{(a-b)\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-25}{2})} \left\{ \frac{(-7905853580625a + 17901641997225a^2)}{\prod_{\zeta=1}^{13} \{a - (2\zeta - 1)\}} \right. \right. \\ &+ \frac{(-15467069396610a^3 + 7198061846898a^4 - 2078757113719a^5 + 401014719391a^6)}{\prod_{\zeta=1}^{13} \{a - (2\zeta - 1)\}} + \\ &+ \frac{(-53845005500a^7 + 5141534684a^8 - 351523887a^9 + 17085783a^{10} - 576290a^{11} + 12818a^{12})}{\prod_{\zeta=1}^{13} \{a - (2\zeta - 1)\}} + \\ &+ \frac{(-169a^{13} + a^{14} + 7905853580625b - 24433840638090a^2b + 28220510016972a^3b)}{\prod_{\zeta=1}^{13} \{a - (2\zeta - 1)\}} + \\ &+ \frac{(-13421680355421a^4b + 4353881703444a^5b - 770676882300a^6b + 116726473656a^7b)}{\prod_{\zeta=1}^{13} \{a - (2\zeta - 1)\}} + \\ &+ \left. \frac{(-9516460473a^8b + 760534632a^9b - 28610010a^{10}b + 1206972a^{11}b - 16731a^{12}b + 324a^{13}b)}{\prod_{\zeta=1}^{13} \{a - (2\zeta - 1)\}} \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{(-17901641997225b^2 + 24433840638090ab^2 - 12795899385750a^3b^2 + 9417308230395a^4b^2)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-2653257473100a^5b^2 + 642830764680a^6b^2 - 68974530300a^7b^2 + 8396447085a^8b^2)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-383418750a^9b^2 + 25585560a^{10}b^2 - 410670a^{11}b^2 + 14625a^{12}b^2 + 15467069396610b^3)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-28220510016972ab^3 + 12795899385750a^2b^3 - 2013943951500a^4b^3 + 1048337211480a^5b^3)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-174357155700a^6b^3 + 32962166640a^7b^3 - 1967793750a^8b^3 + 196353300a^9b^3 - 3848130a^{10}b^3)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(215280a^{11}b^3 - 7198061846898b^4 + 13421680355421ab^4 - 9417308230395a^2b^4)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(2013943951500a^3b^4 - 116257631490a^5b^4 + 45584598150a^6b^4 - 4137412500a^7b^4)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(629559450a^8b^4 - 16033875a^9b^4 + 1332045a^{10}b^4 + 2078757113719b^5 - 4353881703444ab^5)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(2653257473100a^2b^5 - 1048337211480a^3b^5 + 116257631490a^4b^5 - 2541111300a^6b^5)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(783296280a^7b^5 - 29910465a^8b^5 + 3749460a^9b^5 - 401014719391b^6 + 770676882300ab^6)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-642830764680a^2b^6 + 174357155700a^3b^6 - 45584598150a^4b^6 + 2541111300a^5b^6)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-17383860a^7b^6 + 4345965a^8b^6 + 53845005500b^7 - 116726473656ab^7 + 68974530300a^2b^7)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-32962166640a^3b^7 + 4137412500a^4b^7 - 783296280a^5b^7 + 17383860a^6b^7 - 5141534684b^8)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(9516460473ab^8 - 8396447085a^2b^8 + 1967793750a^3b^8 - 629559450a^4b^8 + 29910465a^5b^8)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} +
\end{aligned}$$

$$\begin{aligned}
& + \frac{(-4345965a^6b^8 + 351523887b^9 - 760534632ab^9 + 383418750a^2b^9 - 196353300a^3b^9)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(16033875a^4b^9 - 3749460a^5b^9 - 17085783b^{10} + 28610010ab^{10} - 25585560a^2b^{10})}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(3848130a^3b^{10} - 1332045a^4b^{10} + 576290b^{11} - 1206972ab^{11} + 410670a^2b^{11} - 215280a^3b^{11})}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-12818b^{12} + 16731ab^{12} - 14625a^2b^{12} + 169b^{13} - 324ab^{13} - b^{14})}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} \Bigg\} + \\
& + \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-24}{2})} \left\{ \frac{(12722110515450a - 18824036930640a^2 + 13489601849988a^3 - 4546842476656a^4)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \right. \\
& + \frac{(1232640848966a^5 - 171472666400a^6 + 23400155704a^7 - 1531147488a^8 + 114243558a^9)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-3420560a^{10} + 136708a^{11} - 1456a^{12} + 26a^{13} - 12722110515450b + 18472182813972a^2b)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-13994735479584a^3b + 6712650500994a^4b - 1371370492800a^5b + 282600177336a^6b)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-24167385792a^7b + 2655027882a^8b - 98567040a^9b + 6069492a^{10}b - 78624a^{11}b + 2574a^{12}b)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(18824036930640b^2 - 18472182813972ab^2 + 5982053123100a^3b^2 - 2566312221600a^4b^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(895379785560a^5b^2 - 109469828160a^6b^2 + 17803186200a^7b^2 - 854334000a^8b^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(77005500a^9b^2 - 1235520a^{10}b^2 + 63180a^{11}b^2 - 13489601849988b^3 + 13994735479584ab^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-5982053123100a^2b^3 + 656015223000a^4b^3 - 161410253760a^5b^3 + 43189763400a^6b^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-2907936000a^7b^3 + 383467500a^8b^3 - 7893600a^9b^3 + 592020a^{10}b^3 + 4546842476656b^4)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} +
\end{aligned}$$

$$\begin{aligned}
 & + \frac{(-6712650500994ab^4 + 2566312221600a^2b^4 - 656015223000a^3b^4 + 28280335860a^5b^4)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \frac{(-3724509600a^6b^4 + 793017000a^7b^4 - 22604400a^8b^4 + 2466750a^9b^4 - 1232640848966b^5)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \frac{(1371370492800ab^5 - 895379785560a^2b^5 + 161410253760a^3b^5 - 28280335860a^4b^5)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \frac{(483444360a^6b^5 - 26429760a^7b^5 + 4601610a^8b^5 + 171472666400b^6 - 282600177336ab^6)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \frac{(109469828160a^2b^6 - 43189763400a^3b^6 + 3724509600a^4b^6 - 483444360a^5b^6 + 2674440a^7b^6)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \frac{(-23400155704b^7 + 24167385792ab^7 - 17803186200a^2b^7 + 2907936000a^3b^7 - 793017000a^4b^7)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \frac{(26429760a^5b^7 - 2674440a^6b^7 + 1531147488b^8 - 2655027882ab^8 + 854334000a^2b^8)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \frac{(-383467500a^3b^8 + 22604400a^4b^8 - 4601610a^5b^8 - 114243558b^9 + 98567040ab^9)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \frac{(-77005500a^2b^9 + 7893600a^3b^9 - 2466750a^4b^9 + 3420560b^{10} - 6069492ab^{10} + 1235520a^2b^{10})}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
 & + \left. \frac{(-592020a^3b^{10} - 136708b^{11} + 78624ab^{11} - 63180a^2b^{11} + 1456b^{12} - 2574ab^{12} - 26b^{13})}{\prod_{\eta=1}^{12} \{a - 2\eta\}} \right\} \tag{11}
 \end{aligned}$$

III. DERIVATION OF SUMMATION FORMULA

Substituting $c = \frac{a+b-25}{2}$ and $z = \frac{1}{2}$ in equation (3), we get

$$(a-b) {}_2F_1 \left[\begin{matrix} a, b \\ \frac{a+b-25}{2} \end{matrix}; \frac{1}{2} \right] = (a-b-25) {}_2F_1 \left[\begin{matrix} a, b-1 \\ \frac{a+b-25}{2} \end{matrix}; \frac{1}{2} \right] + (a-b+25) {}_2F_1 \left[\begin{matrix} a-1, b \\ \frac{a+b-25}{2} \end{matrix}; \frac{1}{2} \right]$$

Now involving derived result from (10), we get

$$\begin{aligned}
 L.H.S = & \frac{2^{(b-1)} \Gamma(\frac{a+b-25}{2})}{\Gamma(b)} \left[\frac{(a-b-25)(b-1)}{(a-b+1)} \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-25}{2})} \left\{ \frac{(7905853580625 - 9679554273375a)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \right. \right. \\
 & \left. \left. \frac{(-2821754771550a^2 + 8156137358850a^3 - 4793059238825a^4 + 1486020024775a^5)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} \right\} \right]
 \end{aligned}$$

$$\begin{aligned}
& + \frac{(-287728912900a^6 + 37194314300a^7 - 3302238225a^8 + 202348575a^9 - 8415550a^{10})}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(226850a^{11} - 3575a^{12} + 25a^{13} - 17901641997225b + 29685290993100ab)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-10954559001990a^2b - 3592186999340a^3b + 4112199672725a^4b - 1448281206600a^5b)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(297494820380a^6b - 37999946520a^7b + 3366572625a^8b - 191148100a^9b + 7617610a^{10}b)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-166140a^{11}b + 2275a^{12}b + 15467069396610b^2 - 28566853405770ab^2 + 16176086383690a^2b^2)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-2441585948170a^3b^2 - 913243080700a^4b^2 + 545677186380a^5b^2 - 117096855260a^6b^2)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(16282228540a^7b^2 - 1285714950a^8b^2 + 76140350a^9b^2 - 2203630a^{10}b^2 + 50830a^{11}b^2)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-7198061846898b^3 + 13970100703252ab^3 - 9041136026470a^2b^3 + 2497465994000a^3b^3)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-147688140500a^4b^3 - 72346451240a^5b^3 + 25649183700a^6b^3 - 3167274800a^7b^3)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(298058150a^8b^3 - 11511500a^9b^3 + 427570a^{10}b^3 + 2078757113719b^4 - 4145753956429ab^4)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(2817304348100a^2b^4 - 907985381900a^3b^4 + 137261041050a^4b^4 - 1392487390a^5b^4)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-1993550300a^6b^4 + 448580500a^7b^4 - 25138425a^8b^4 + 1562275a^9b^4 - 401014719391b^5)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(798461811640ab^5 - 568209672300a^2b^5 + 186672512600a^3b^5 - 33928246450a^4b^5)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(2803879400a^5b^5 + 57203300a^6b^5 - 16343800a^7b^5 + 2414425a^8b^5 + 53845005500b^6)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} +
\end{aligned}$$

$$\begin{aligned}
& + \frac{(-108967904380ab^6 + 73253922980a^2b^6 - 26295630900a^3b^6 + 4437948900a^4b^6)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-471758980a^5b^6 + 17681020a^6b^6 + 742900a^7b^6 - 5141534684b^7 + 9939584616ab^7)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-7206053660a^2b^7 + 2131630800a^3b^7 - 438662900a^4b^7 + 32040840a^5b^7 - 1931540a^6b^7)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(351523887b^8 - 702055497ab^8 + 413863450a^2b^8 - 151787350a^3b^8 + 17678375a^4b^8)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-2187185a^5b^8 - 17085783b^9 + 30147260ab^9 - 21628750a^2b^9 + 4275700a^3b^9 - 904475a^4b^9)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(576290b^{10} - 1108250ab^{10} + 450450a^2b^{10} - 164450a^3b^{10} - 12818b^{11} + 17732ab^{11})}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-12350a^2b^{11} + 169b^{12} - 299ab^{12} - b^{13})}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(a - b - 25)}{(a - b + 1)} \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-24}{2})} \left\{ \frac{(12405876372225 - 6637991183415a - 6934703183742a^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \right. \\
& + \frac{(7758765996242a^3 - 3374923746265a^4 + 826980359391a^5 - 133187685316a^6)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(14182039196a^7 - 1069344705a^8 + 53304823a^9 - 1858142a^{10} + 37362a^{11} - 455a^{12} + a^{13})}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-31933329617025b + 29109857668620ab - 1235086611478a^2b - 7800325758444a^3b)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(4133160125069a^4b - 1097812385480a^5b + 176691959996a^6b - 19086735512a^7b)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(1363905257a^8b - 67092740a^9b + 2055482a^{10}b - 38844a^{11}b + 299a^{12}b + 32426508971490b^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-36042012580570ab^2 + 11400438154410a^2b^2 + 1320226372070a^3b^2 - 1786460663100a^4b^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} +
\end{aligned}$$

$$\begin{aligned}
& + \frac{(553332436780a^5b^2 - 92422616860a^6b^2 + 9931560860a^7b^2 - 678656550a^8b^2 + 31123950a^9b^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-807950a^{10}b^2 + 12350a^{11}b^2 - 17710198832290b^3 + 21701551850580ab^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-9177122387830a^2b^3 + 1292748368400a^3b^3 + 274928487500a^4b^3 - 130437209000a^5b^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(25589291700a^6b^3 - 2554775600a^7b^3 + 180730550a^8b^3 - 6249100a^9b^3 + 164450a^{10}b^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(5971205695175b^4 - 7660435075925ab^4 + 3662292647300a^2b^4 - 775479934700a^3b^4)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(49625392250a^4b^4 + 15829012850a^5b^4 - 3366300700a^6b^4 + 431486900a^7b^4 - 20474025a^8b^4)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(904475a^9b^4 - 1344672714375b^5 + 1740955966520ab^5 - 881151572620a^2b^5)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(214647357400a^3b^5 - 24445050210a^4b^5 + 597483880a^5b^5 + 318074820a^6b^5 - 26132600a^7b^5)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(2187185a^8b^5 + 206381978620b^6 - 276634482140ab^6 + 135560573540a^2b^6 - 35921756500a^3b^6)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(4653044900a^4b^6 - 273535780a^5b^6 + 1040060a^6b^6 + 1931540a^7b^6 - 23443530620b^7)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(29006686440ab^7 - 15427953340a^2b^7 + 3535477200a^3b^7 - 538777300a^4b^7 + 31201800a^5b^7)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-742900a^6b^7 + 1777480575b^8 - 2419163825ab^8 + 1026870650a^2b^8 - 288250950a^3b^8)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(27126775a^4b^8 - 2414425a^5b^8 - 109570175b^9 + 116130300ab^9 - 62984350a^2b^9 + 9538100a^3b^9)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-1562275a^4b^9 + 3893890b^{10} - 5352490ab^{10} + 1480050a^2b^{10} - 427570a^3b^{10} - 129090b^{11})}{\prod_{\eta=1}^{12} \{a - 2\eta\}} +
\end{aligned}$$

$$\begin{aligned}
& + \frac{(91780ab^{11} - 50830a^2b^{11} + 1625b^{12} - 2275ab^{12} - 25b^{13})}{\prod_{\eta=1}^{12} \{a - 2\eta\}} \Bigg] + \frac{2^{(b-1)} \Gamma(\frac{a+b-25}{2})}{\Gamma(b)} \left[\frac{(a-b+25)}{(a-b-1)} \times \right. \\
& \times \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-24}{2})} \left\{ \frac{(-12405876372225 + 31933329617025a - 32426508971490a^2 + 17710198832290a^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \right. \\
& + \frac{(-5971205695175a^4 + 1344672714375a^5 - 206381978620a^6 + 23443530620a^7 - 1777480575a^8)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(109570175a^9 - 3893890a^{10} + 129090a^{11} - 1625a^{12} + 25a^{13} + 6637991183415b)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-29109857668620ab + 36042012580570a^2b - 21701551850580a^3b + 7660435075925a^4b)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-1740955966520a^5b + 276634482140a^6b - 29006686440a^7b + 2419163825a^8b)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-116130300a^9b + 5352490a^{10}b - 91780a^{11}b + 2275a^{12}b + 6934703183742b^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(1235086611478ab^2 - 11400438154410a^2b^2 + 9177122387830a^3b^2 - 3662292647300a^4b^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(881151572620a^5b^2 - 135560573540a^6b^2 + 15427953340a^7b^2 - 1026870650a^8b^2)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(62984350a^9b^2 - 1480050a^{10}b^2 + 50830a^{11}b^2 - 7758765996242b^3 + 7800325758444ab^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-1320226372070a^2b^3 - 1292748368400a^3b^3 + 775479934700a^4b^3 - 214647357400a^5b^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(35921756500a^6b^3 - 3535477200a^7b^3 + 288250950a^8b^3 - 9538100a^9b^3 + 427570a^{10}b^3)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(3374923746265b^4 - 4133160125069ab^4 + 1786460663100a^2b^4 - 274928487500a^3b^4)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-49625392250a^4b^4 + 24445050210a^5b^4 - 4653044900a^6b^4 + 538777300a^7b^4 - 27126775a^8b^4)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} +
\end{aligned}$$

$$\begin{aligned}
& + \frac{(1562275a^9b^4 - 826980359391b^5 + 1097812385480ab^5 - 553332436780a^2b^5)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(130437209000a^3b^5 - 15829012850a^4b^5 - 597483880a^5b^5 + 273535780a^6b^5 - 31201800a^7b^5)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(2414425a^8b^5 + 133187685316b^6 - 176691959996ab^6 + 92422616860a^2b^6 - 25589291700a^3b^6)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(3366300700a^4b^6 - 318074820a^5b^6 - 1040060a^6b^6 + 742900a^7b^6 - 14182039196b^7)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(19086735512ab^7 - 9931560860a^2b^7 + 2554775600a^3b^7 - 431486900a^4b^7 + 26132600a^5b^7)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-1931540a^6b^7 + 1069344705b^8 - 1363905257ab^8 + 678656550a^2b^8 - 180730550a^3b^8)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(20474025a^4b^8 - 2187185a^5b^8 - 53304823b^9 + 67092740ab^9 - 31123950a^2b^9 + 6249100a^3b^9)}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(-904475a^4b^9 + 1858142b^{10} - 2055482ab^{10} + 807950a^2b^{10} - 164450a^3b^{10} - 37362b^{11})}{\prod_{\eta=1}^{12} \{a - 2\eta\}} + \\
& + \frac{(38844ab^{11} - 12350a^2b^{11} + 455b^{12} - 299ab^{12} - b^{13})}{\prod_{\eta=1}^{12} \{a - 2\eta\}} \left. \right\} + \frac{(a - b + 25)}{(a - b - 1)} \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-25}{2})} \times \\
& \times \left\{ \frac{(-7905853580625 + 17901641997225a - 15467069396610a^2 + 7198061846898a^3)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} \right. \\
& + \frac{(-2078757113719a^4 + 401014719391a^5 - 53845005500a^6 + 5141534684a^7 - 351523887a^8)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(17085783a^9 - 576290a^{10} + 12818a^{11} - 169a^{12} + a^{13} + 9679554273375b)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-29685290993100ab + 28566853405770a^2b - 13970100703252a^3b + 4145753956429a^4b)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-798461811640a^5b + 108967904380a^6b - 9939584616a^7b + 702055497a^8b - 30147260a^9b)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} +
\end{aligned}$$

$$\begin{aligned}
& + \frac{(1108250a^{10}b - 17732a^{11}b + 299a^{12}b + 2821754771550b^2 + 10954559001990ab^2)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-16176086383690a^2b^2 + 9041136026470a^3b^2 - 2817304348100a^4b^2 + 568209672300a^5b^2)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-73253922980a^6b^2 + 7206053660a^7b^2 - 413863450a^8b^2 + 21628750a^9b^2 - 450450a^{10}b^2)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(12350a^{11}b^2 - 8156137358850b^3 + 3592186999340ab^3 + 2441585948170a^2b^3)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-2497465994000a^3b^3 + 907985381900a^4b^3 - 186672512600a^5b^3 + 26295630900a^6b^3)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-2131630800a^7b^3 + 151787350a^8b^3 - 4275700a^9b^3 + 164450a^{10}b^3 + 4793059238825b^4)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-4112199672725ab^4 + 913243080700a^2b^4 + 147688140500a^3b^4 - 137261041050a^4b^4)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(33928246450a^5b^4 - 4437948900a^6b^4 + 438662900a^7b^4 - 17678375a^8b^4 + 904475a^9b^4)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-1486020024775b^5 + 1448281206600ab^5 - 545677186380a^2b^5 + 72346451240a^3b^5)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(1392487390a^4b^5 - 2803879400a^5b^5 + 471758980a^6b^5 - 32040840a^7b^5 + 2187185a^8b^5)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(287728912900b^6 - 297494820380ab^6 + 117096855260a^2b^6 - 25649183700a^3b^6)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(1993550300a^4b^6 - 57203300a^5b^6 - 17681020a^6b^6 + 1931540a^7b^6 - 37194314300b^7)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(37999946520ab^7 - 16282228540a^2b^7 + 3167274800a^3b^7 - 448580500a^4b^7 + 16343800a^5b^7)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-742900a^6b^7 + 3302238225b^8 - 3366572625ab^8 + 1285714950a^2b^8 - 298058150a^3b^8)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} +
\end{aligned}$$

$$\begin{aligned}
& + \frac{(25138425a^4b^8 - 2414425a^5b^8 - 202348575b^9 + 191148100ab^9 - 76140350a^2b^9)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(11511500a^3b^9 - 1562275a^4b^9 + 8415550b^{10} - 7617610ab^{10} + 2203630a^2b^{10} - 427570a^3b^{10})}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} + \\
& + \frac{(-226850b^{11} + 166140ab^{11} - 50830a^2b^{11} + 3575b^{12} - 2275ab^{12} - 25b^{13})}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}} \left. \vphantom{\frac{(25138425a^4b^8 - 2414425a^5b^8 - 202348575b^9 + 191148100ab^9 - 76140350a^2b^9)}{\prod_{\varsigma=1}^{13} \{a - (2\varsigma - 1)\}}} \right\}
\end{aligned}$$

On simplification, we get the summation formula.

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