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Global Stability of Stificrs Impact on Meme Transmission Model

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[1] R. Dawkins, The Selfish Gene, 2d ed., Oxford University Press (1989).

Global Stability of Stificrs Impact on Meme Transmission Model

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Abstract- Spreaders and stiflers play a major role in the persistence or disappearance of memes. In this paper we study the impact of stiflers on the dynamics of memes transmission. We introduce and analyze qualitatively a mathematical model considering stiflers effect. Three equilibrium points of the model are examined: meme free equilibrium, meme cessation equilibrium and meme existence equilibrium. The reproduction number R_0 that generates new memes is found. Local and global stability of the equilibrium points are explored. Finally, we support our results using numerical simulations.

I. Introduction

In recent times, spread of ideas is easily done through the use of fast tech communications. A unit of idea transmission is defined by Dawkins [1] as meme or a unit of imitation. Style of living, eating, clothing and thinking are some examples of memes. Individuals reaction towards memes is either acceptation or rejection. Those who imitate certain memes and is manifested in their life style help in spreading it, they are known as spreaders. On the other hand, those who refuse or not interested in adopting a meme are known as sti ers. Spreaders and sti ers play a significant role in the survival or extinction of memes.

There is a similarity between the propagation of memes and the spread of rumors. Both may be seen as a virus, that is transmitted to another individual by a certain mean. Researchers have applied epidemiological models to study the dynamics of social systems. In particular, the dynamics of rumors spread and transmission of ideas and thoughts. This was based on the fact that both biological diseases and social behavior are a result from interactions between individuals. Daley and kendall are among the earliest researchers to propose a rumor spread model that has some properties in common with epidemic model [2]. Also Cane [3] showed the similarity between the deterministic forms of models for the spread of an epidemic and of a rumor. At the beginning of this century, Thompson et al. [4] explored the dynamics of rumor spreading in chat rooms. Bettencourt et al. [5] applied models similar to epidemiology to the spread of ideas. Kawachi [6] and Kawachi et al. [7] proposed deterministic models for rumor transmission and explored the effects of various contact interactions. Al-Amoudi et al. [8, 9] analyzed qualitatively constant and variable meme propagation models. Piqueira [10] examined an equilibrium study of a rumor spreading model according to propagation parameters and initial conditions. Huang [11] studied the rumor spreading process with denial and skepticism. Wang and Wood [12] adopted an epidemiological approach to model viral meme propagation. Zhao et al. [13, 14] proposed rumor spreading models in social networks considering the forgetting mechanism of spreaders. Huo et al. [15] investigated the psychological effect with rumor spreading in emergency event. Zhao and Wang [16] established a dynamic rumor model considering the medium as a subclass. Recently, Afassinou [17] analyzed the impact of education rate on rumor final size. Finally, Zan et al. [18] examined the dynamics or rumor spread with counterattack mechanism and self-resistance parameter.

In this paper, we investigate the impact of sti ers on the transmission of memes in a population with constant immigration and emigration. We analyze the dynamics of the model qualitatively. The formulation of the model and its equilibria and basic reproduction number are described in section 2. Section 3 analyzes the stability of equilibria both locally and globally using linearization methods and Lyapunov method. Numerical simulations are illustrated in section 4. A brief conclusion is given in Section 5.

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II. Mathematical Model

Let $N(t)$ denote the total population. We assume that the population is divided into three disjoint classes of individuals: $S(t)$, the susceptible class, describing individuals who have not yet been exposed to a particular meme; $I(t)$, the spreader class, referring to individuals who have taken an active interest in the idea or concept that a meme represents, and therefore have a tendency to talk about the meme in social interactions; $Z(t)$, the stifler class, meaning individuals who reject the meme and have no interest to embrace it. Stiflers may play an influential role in memes cessation. We explore this assumption by introducing a new term in the dynamical model given by [8], where we consider that susceptibles become stiflers when contacting them at a certain rate.

The mathematical model is governed by the following nonlinear system of ordinary differential equations:

$$\begin{aligned}\frac{dS}{dt} &= B-\alpha SI-\delta SZ-\mu S,\\ \frac{dI}{dt} &= \alpha\theta SI-\beta I^2-\gamma IZ-\mu I,\\ \frac{dZ}{dt} &= \alpha(1-\theta)SI+\beta I^2+\gamma IZ+\delta SZ-\mu Z,\end{aligned} \tag{1}$$

where the positive parameters are defined as follows: B is the sum of the population's birth rate and immigration rate; μ is the sum of the population's death rate and emigration rate; α is the rate at which susceptibles change their meme class, where $\alpha = cq$ such that c is the average number of contact per unit time and q is the probability of transmitting the meme; β is the rate at which spreaders become stiflers by contacting with each other; γ is the rate at which spreaders become stiflers by contacting with stiflers; θ is the fraction of susceptibles who become spreaders (at rate α); $1-\theta$ is the fraction of susceptibles who become stiflers (at rate α), such that $\theta \in (0,1]$; and δ is the rate at which susceptibles become stiflers by contacting with stiflers.

Note that $N(t) = S(t)+I(t)+Z(t)$. It follows from system (1) that $N'(t)+\mu N(t) = B$, which has the solution $N(t) = N_0\exp(-\mu t)+[1-\exp(-\mu t)]B/\mu$, where $N_0 = N(0)$, and therefore, $\lim_{t\to\infty} N(t) = B/\mu$. Thus, the considered region for system (1) is

$$\Gamma = \{(S,I,Z) : S+I+Z \leq \frac{B}{\mu}, S>0, I\geq 0, Z\geq 0\}.$$

The vector field points into the interior of Γ on the part of its boundary when $S+I+Z = B/\mu$. Hence, Γ is positively invariant.

We find the equilibria of the model by equating to zero the right hand side of system (1):

$$\begin{aligned}B-\alpha SI-\delta SZ-\mu S &= 0,\\ \alpha\theta SI-\beta I^2-\gamma IZ-\mu I &= 0,\\ \alpha(1-\theta)SI+\beta I^2+\gamma IZ+\delta SZ-\mu Z &= 0.\end{aligned} \tag{2}$$

Solution to system (2) gives three equilibrium points: the meme free equilibrium $E_0 = (B/\mu, 0, 0)$; the meme cessation equilibrium $E_1 = (\frac{\mu}{\delta}, 0, \frac{B}{\mu}-\frac{\mu}{\delta})$, which exists if and only if $\delta > \mu^2/B$; and meme existence equilibrium $E^* = (S^*, I^*, Z^*)$, where

$$\begin{aligned}S^* &= \frac{B}{\alpha I^*+\delta Z^*+\mu},\\ I^* &= \frac{\alpha\theta S^*-\gamma Z^*-\mu}{\beta}\\ Z^* &= \frac{-\alpha(1-\theta)S^*I^*-\beta {I^*}^2}{\delta S^*+\gamma I^*-\mu}.\end{aligned}$$

The basic reproduction number $\mathcal{R}_0$ may be calculated by the method of next generation matrix [19]. Let $X = (I,Z,S)^T$, then system (1) may be written as: $X' = \mathcal{F}(X) - \mathcal{V}(X)$ where

$$\mathcal{F}(X) = \begin{bmatrix}\alpha\theta SI\\ \alpha(1-\theta)SI\\ 0\end{bmatrix}, \mathcal{V}(X) = \begin{bmatrix}\beta I^2+\gamma IZ+\mu I\\ -\beta I^2-\gamma IZ-\delta SZ+\mu Z\\ -B+\alpha SI+\delta SZ+\mu S\end{bmatrix}.$$

Ref

[8] R. Al-Amoudi, S. Al-Sheikh, and S. Al-Tuwairqi, Qualitative Behavior of Solutions to a Mathematical Model of Memes Transmission, International Journal of Applied Mathematical Research, Vol. 3, No. 1 (2014) 36-44.

The Jacobian matrices of $\mathcal{F}(X)$ and $\mathcal{V}(X)$ at the meme free equilibrium point E_0, are

$$D\mathcal{F}(E_0)=\begin{bmatrix} \alpha\theta S_0 & 0 & 0 \\ \alpha(1-\theta)S_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix},$$

$$D\mathcal{V}(E_0)=\begin{bmatrix} \mu & 0 & 0 \\ 0 & -\delta S_0 + \mu & 0 \\ \alpha S_0 & \delta S_0 & \delta S_0 + \mu \end{bmatrix} = \begin{bmatrix} V & 0 \\ J_1 & J_2 \end{bmatrix},$$

where $F = \begin{bmatrix} \alpha\theta S_0 & 0 \\ \alpha(1-\theta)S_0 & 0 \end{bmatrix}$, $V = \begin{bmatrix} \mu & 0 \\ 0 & -\delta S_0 + \mu \end{bmatrix}$, $J_1 = [\ \alpha S_0 \quad \delta S_0\ ]$, $J_2 = [\ \delta S_0 + \mu\ ]$.

Thus the next generation matrix is $FV^{-1} = \begin{bmatrix} \dfrac{\alpha\theta B}{\mu^2} & 0 \\ \dfrac{\alpha(1-\theta)B}{\mu^2} & 0 \end{bmatrix}$. Clearly, the spectral radius of matrix FV^{-1} is $\rho(\ FV^{-1}) = \alpha\theta B/\mu^2$. So, the basic reproduction number of the system is $\mathcal{R}_0 = \alpha\theta B/\mu^2$.

III. Stability of the Equilibria

3.1 Local stability

Here we investigate the local stability of E_0, E_1 and E^*. We state the following theorems:

Theorem 1 ***(Local stability of*** E_0***)*** *If* $\mathcal{R}_0 < 1$*and* $\delta < \mu^2/B$, *the meme free equilibrium point* E_0 *is locally asymptotically stable. If* $\mathcal{R}_0 = 1$ *and* $\delta < \mu^2/B$ *or* $\mathcal{R}_0 < 1$ *and* $\delta = \mu^2/B$, E_0 *is locally stable. If* $\mathcal{R}_0 > 1$ *or* $\delta > \mu^2/B$, E_0 *is unstable.*

Proof. Linearizing system (1) (by linearization method [20]) we obtain the Jacobian matrix evaluated at the equilibrium E_0:

$$J\,(E_0) = \begin{bmatrix} -\mu & \dfrac{-\alpha B}{\mu} & \dfrac{-\delta B}{\mu} \\ 0 & \dfrac{\alpha\theta B}{\mu} - \mu & 0 \\ 0 & \dfrac{\alpha\,(1-\theta)\,B}{\mu} & \dfrac{\delta B}{\mu} - \mu \end{bmatrix}.$$

Clearly the roots of the characteristic equation are: $\lambda_1 = -\mu < 0$; $\lambda_2 = \mu(\dfrac{\delta B}{\mu^2} - 1) < 0$, if $\delta < \mu^2/B$; and $\lambda_3 = \mu(\dfrac{\alpha\theta B}{\mu^2} - 1) = \mu\,(\mathcal{R}_0 - 1) < 0$, if $\mathcal{R}_0 < 1$. Hence, E_0 is locally asymptotically stable if $\mathcal{R}_0 < 1$ and $\delta < \mu^2/B$. If $\mathcal{R}_0 = 1$ and $\delta < \mu^2/B$ then the eigenvalues are λ_1, $\lambda_2 < 0$ and $\lambda_3 = 0$. Also if $\delta = \mu^2/B$ and $\mathcal{R}_0 < 1$ then the eigenvalues are $\lambda_1 < 0$, $\lambda_2 = 0$ and $\lambda_3 < 0$. So, E_0 is locally stable. If $\mathcal{R}_0 > 1$ or $\delta > \mu^2/B$, then the characteristic equation has a positive eigenvalue. So, E_0 is unstable. ■

Theorem 2 ***(Local stability of*** E_1***)*** *Let* $\mathcal{R}_1 = \dfrac{\delta(B\gamma + \mu^2)}{\mu^2(\gamma + \alpha\theta)}$. *If* $\mathcal{R}_1 > 1$, *the meme cessation equilibrium point* E_1 *is locally asymptotically stable. If* $\mathcal{R}_1 = 1$, E_1 *is locally stable. If* $\mathcal{R}_1 < 1$, E_1 *is unstable.*

Proof. The Jacobian matrix at the equilibrium E_1 gives:

$$J\,(E_1) = \begin{bmatrix} -\dfrac{\delta B}{\mu} & -\dfrac{\alpha\mu}{\delta} & -\mu \\ 0 & \dfrac{\alpha\theta\mu}{\delta} - \gamma(\dfrac{B}{\mu} - \dfrac{\mu}{\delta}) - \mu & 0 \\ \dfrac{\delta B}{\mu} - \mu & \dfrac{\alpha(1-\theta)\mu}{\delta} - \gamma(\dfrac{B}{\mu} - \dfrac{\mu}{\delta}) & 0 \end{bmatrix}.$$

The roots of the characteristic equation are: $\lambda_1 = -\mu < 0$; $\lambda_2 = \dfrac{1}{\mu\delta}\big(\mu^2(\gamma + \alpha\theta) - \delta(B\gamma + \mu^2)\big) < 0$, if $\mathcal{R}_1 > 1$; and $\lambda_3 = -\dfrac{1}{\mu}(B\delta - \mu^2) < 0$. Hence, E_1 is locally asymptotically stable if $\mathcal{R}_1 > 1$. If $\mathcal{R}_1 = 1$ then the eigenvalues are λ_1, $\lambda_3 < 0$ and $\lambda_2 = 0$. So, E_1 is locally stable. If $\mathcal{R}_1 < 1$ then the characteristic equation has a positive eigenvalue. So, E_1 is unstable. ■

Ref

[20] L.Perko, Differential Equations and Dynamic Systems, Springer Verlag (1991).

Theorem 3 ***(Local stability of*** E^****)*** *If* $a_1 > 0$, $a_3 > 0$ *and* $a_1 a_2 > a_3$, *the meme existence equilibrium* E^* *is locally asymptotically stable.*

Proof. The Jacobian matrix at the equilibrium E^* gives:

$$J(E^*) = \begin{bmatrix} -\alpha I^* - \delta Z^* - \mu & -\alpha S^* & -\delta S^* \\ \alpha\theta I^* & \alpha\theta S^* - 2\beta I^* - \gamma Z^* - \mu & -\gamma I^* \\ \alpha(1-\theta) I^* + \delta Z^* & \alpha(1-\theta) S^* + 2\beta I^* + \gamma Z^* & \gamma I^* + \delta S^* - \mu \end{bmatrix}.$$

This may be simplified using system (2) to be

$$J(E^*) = \begin{bmatrix} -\dfrac{B}{S^*} & -\alpha S^* & -\delta S^* \\ \alpha\theta I^* & -\beta I^* & -\gamma I^* \\ \alpha(1-\theta)I^* + \delta Z^* & \beta I^* + \dfrac{\mu Z^*}{I^*} - \dfrac{\delta S^* Z^*}{I^*} & \delta S^* + \gamma I^* - \mu \end{bmatrix}.$$

The characteristic equation about E^* is $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$, where

$$\begin{aligned} a_1 &= \beta I^* + \mu + \frac{B}{S^*} - \delta S^* - \delta S^*, \\ a_2 &= \alpha\delta(1-\theta)S^* I^* + \delta^2 S^* Z^* + \gamma\mu Z^* - \gamma\delta S^* Z^* - \delta B - \delta\beta S^* I^* \\ &\quad -\frac{\gamma B I^*}{S^*} + \frac{B\mu}{S^*} + \mu\beta I^* + \frac{\beta B I^*}{S^*} + \alpha^2\theta S^* I^*, \\ a_3 &= \alpha\theta\delta\mu S^* Z^* - \delta^2\alpha\theta {S^*}^2 Z^* + \alpha\beta\delta S^* {I^*}^2 + \beta\delta^2 S^* I^* Z^* + \frac{\gamma B\mu Z^*}{S^*} \\ &\quad -B\gamma\delta Z^* - \alpha^2\gamma S^* {I^*}^2 - \alpha\gamma\delta S^* I^* Z^* - \delta B\beta I^* - \alpha^2\theta\delta {S^*}^2 I^* \\ &\quad +\frac{\beta B\mu I^*}{S^*} + \alpha^2\theta\mu S^* I^*. \end{aligned}$$

If $a_1 > 0$, $a_3 > 0$ and $a_1 a_2 > a_3$, then by using Routh-Herwitz Criteria [21] all eigenvalues of $J(E^*)$ have negative real parts. Thus, E^* is locally asymptotically stable. ■

3.2 Global stability

First, we explore the global stability of E_0 Consider the Lyapunov function [22]:

$$L = I + Z.$$
$$\frac{dL}{dt} = (-\mu + \alpha S) I + (-\mu + \delta S) Z.$$

Since $E_0 \in \Gamma$ then $S \le \dfrac{B}{\mu}$ and we have

$$\frac{dL}{dt} \le (-\mu + \frac{\alpha B}{\mu}) I + (-\mu + \frac{\delta B}{\mu}) Z \le 0 \text{ if } \alpha B < \mu^2 \text{ and } \delta < \mu^2/B.$$

It follows that $\frac{dL}{dt} < 0$ if $\alpha B \le \mu^2$ and $\delta < \mu^2/B$; with $\frac{dL}{dt} = 0$ if and only if $I = Z = 0$. Hence, the only solution of system (1) in Γ on which $\frac{dL}{dt} = 0$ is E_0 . Therefore, by LaSalle's Invariance Principle [22], every solution of system (1), with initial conditions in Γ, approaches E_0 as $t \to \infty$. Thus, E_0 is globally asymptotically stable and we may state the following theorem.

Theorem 4 ***(Global stability of*** E_0***)*** *If* $\alpha B \le \mu^2$ *and* $\delta < \mu^2/B$ *then* E_0 *is globally asymptotically stable in* Γ.

Next, we examine the global stability of E_1 and E^*. We may define the same Lyapunov function for both equilibriums. Consider the Lyapunov function:

$$L = \frac{1}{2}\left[(S - S^*) + (I - I^*) + (Z - Z^*)\right]^2.$$
$$\frac{dL}{dt} = \left[(S - S^*) + (I - I^*) + (Z - Z^*)\right]\left[B - \mu S - \mu I - \mu Z\right].$$

Using $B = \mu S^* + \mu I^* + \mu Z^*$, we have

Ref

[22] M.W.Hirsch, S.Smale, and R.L.Devaney, Differential equations, dynamical systems and an introduction to chaos, Elsevier Academic press (1974).

$$\frac{dL}{dt} = -\mu\left[(S - S^*) + (I - I^*) + (Z - Z^*)\right]^2 \leq 0.$$

Hence, E_1 and E^* are globally stable and we may state the following theorem.

Theorem 5 ***(Global stability of*** E_1 *and* E^****)*** *The equilibrium points* E_1 *and* E^* *are globally stable whenever they exist.*

Notes

We summarize the result of this section as follows:

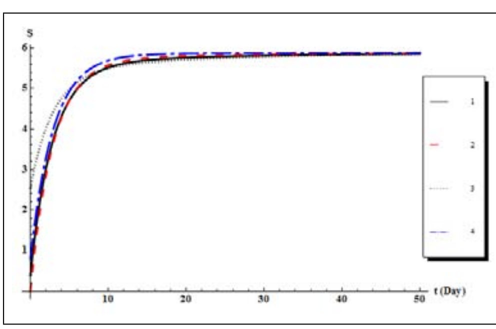
- If $\mathcal{R}_0 < 1$ and $\delta < \mu^2/B$, then E_0 is locally asymptotically stable. If $\alpha B \leq \mu^2$ and $\delta < \mu^2/B$ then E_0 is globally asymptotically stable.
- If $\mathcal{R}_1 > 1$ then E_1 is locally asymptotically stable. But E_1 is globally stable unconditionally whenever it exists.
- If $a_1 > 0$, $a_3 > 0$ and $a_1 a_2 > a_3$ then E^* is locally asymptotically stable. But E^* is globally stable unconditionally whenever it exists.

IV. Numerical Simulation

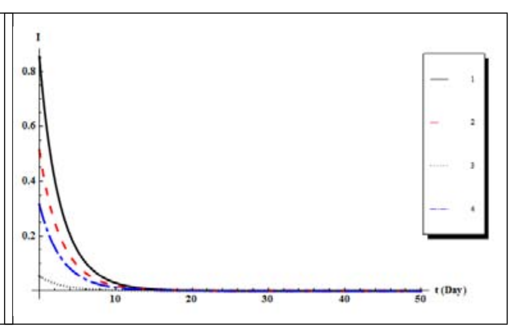
In this section, we illustrate numerical simulations of system (1) using different values of the parameters to support our results. Four different initial values are chosen such that $S + I + Z \leq B/\mu$:

1. S(0) =0.3890, I(0) =0.8540, Z(0) = 0.5360,
2. S(0) = 0.0010, I(0) = 0.5140, Z(0)= 0.3600,
3. S(0) = 2.5485, I(0) = 0.0540, Z(0) = 0.8675,
4. S(0) = 0.8000, I(0) = 0.3154, Z(0) = 0.0250.

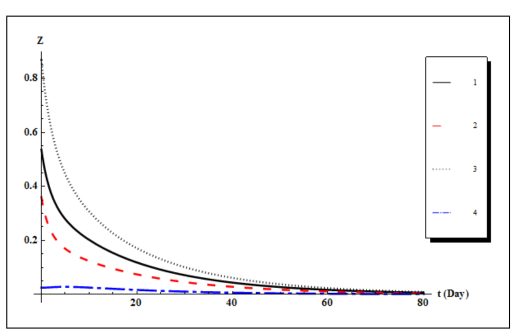
(a) Using the parameters: $\beta = 0.05, \mu = 0.34, \quad = 0.015, \alpha = 0.0125, \theta = 0.333, B = 2, \delta = 0.05$. Here $\mathcal{R}_0 = 0.0720156 < 1$. We see from Fig. 1(a) that the number of susceptibles to the meme increases as a function of time to approach the value of S_0 for the four sets of initial conditions. While Fig. 1(b,c) show that the number of spreaders and sti ers decreases as a function of time and approaches zero. Thus, for all sets of initial conditions the solution curves tend to the meme free equilibrium E_0. Hence, system (1) is locally asymptotically stable about E_0 for the above set of parameters.



(a)

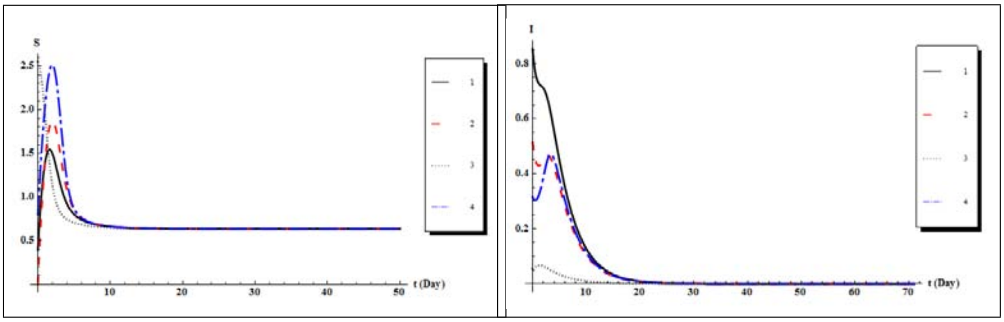


(b)

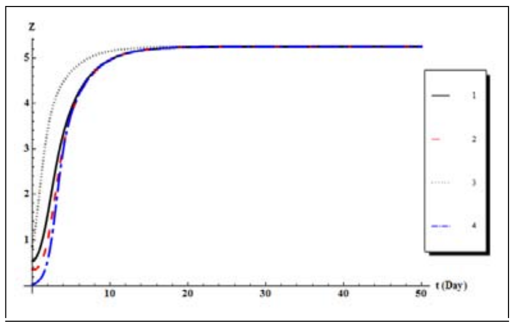


(c)

Figure 1 : Time plots of systems (1) with different initial conditions for $R_0 < 1$: (a) Susceptibles; (b) Spreaders; (c) Stiflers.



(a) (b)

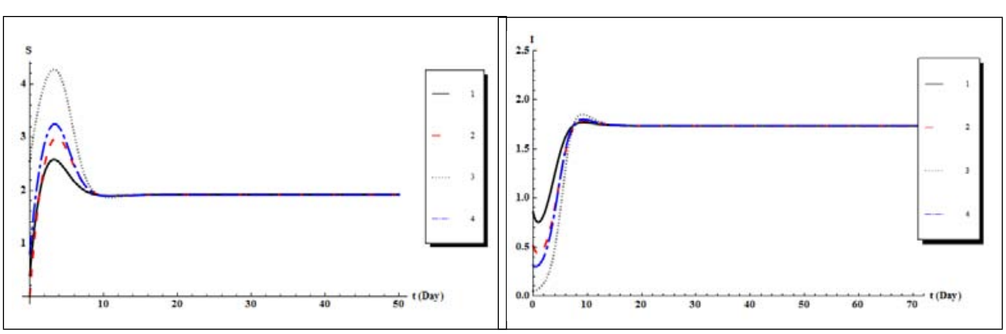


(c)

Figure 2 : Time plots of systems (1) with different initial conditions for R_0, $R_1 > 1$: (a) Susceptibles; (b) Spreaders; (c) Stiflers.

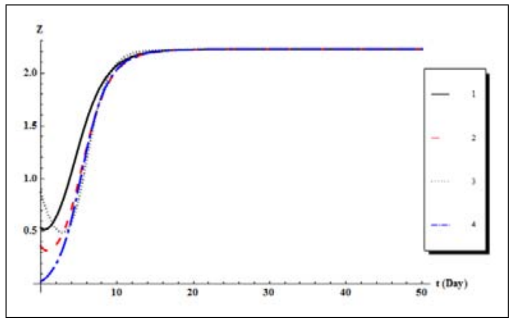
(b) Using the same initial conditions and parameters as in (a) except for: $\alpha = 0.4, \theta = 0.6, \delta = 0.535$. Here $\mathcal{R}_0 = 4.15225 > 1$ and $\mathcal{R}_1 = 2.6425 > 1$. We see from Fig. 2(a) that for the second value of parameters the number of susceptibles increases first then it starts to decrease and all solutions approach a certain value S_1. Fig. 2(b) shows that the number of spreaders at the beginning of the meme increases slightly, then it starts to decrease and finally approaches the value $I_1 = 0$. Fig. 2(c) shows a different behavior for the sti ers, they grow at first then they approach a certain value Z_1. Thus, for all sets of initial conditions the solution curves tend to the meme cessation equilibrium E_1. Hence, system (1) is locally asymptotically stable about E_1 for the above set of parameters.

(c) Using the same initial conditions and parameters as in (b) except for: $\delta = 0.0035$. Here $\mathcal{R}_0 = 4.15225 > 1$ and $\mathcal{R}_1 = 0.002 < 1$. We see from Fig. 3(a) that when the parameter δ is very small the number of susceptibles increases first then it starts to decrease and all solutions approach a certain value S^*. Fig. 3(b) shows that the number of spreaders at the beginning of the meme decreases slightly, then it starts to increase and finally approaches the value I^*. Fig. 3(c) shows a similar behavior for the sti ers, they decrease slightly at first then they increase to approach a certain value Z^*. Thus, for all sets of initial conditions the solution curves tend to the meme existence equilibrium E^*. Hence, system (1) is locally asymptotically stable about E^* for the above set of parameters.



(a) (b)

Notes



(c)

Figure 3 : Time plots of systems (1) with different initial conditions for R_0, > 1 and $R_1 < 1$: (a) Susceptibles; (b) Spreaders; (c) Stiflers.

V. Conclusions

In this paper, a nonlinear memes transmission model with sti ers effect on susceptibles in is analyzed. Sufficient conditions have been given ensuring local and global stability of the three equilibrium points. First, the meme free equilibrium point E_0 is shown to be locally asymptotically stable whenever the basic reproduction number R_0 of the model is less than unity and when the condition $\delta < \mu^2/B$ is satisfied. In addition, if $\alpha B \leq \mu^2$ is satisfied, then E_0 becomes globally asymptotically stable and the meme will disappear. Second, the meme cessation equilibrium point E_1, which exists only if $\delta > \mu^2/B$, is shown to be locally asymptotically stable if $\mathcal{R}_1 > 1$. Moreover, it is globally stable with no conditions. Thus, if E_1 exists the meme will eventually end. Lastly, the meme existence equilibrium E^*, if it exists, is shown to be locally asymptotically stable if the conditions: $a_1 > 0$, $a_3 > 0$ and $a_1 a_2 > a_3$ are satisfied and it is globally stable with no conditions also. Therefore, if E^* exists the meme will persists. Finally, some numerical simulations are used to support the qualitative results. In conclusion, sti ers in uence produces a new equilibrium point which eventually ceases the meme from spreading if the analytical conditions are satisfied.

References Références Referencias

[1] R. Dawkins, The Selfish Gene, 2d ed., Oxford University Press (1989).

[2] K. Dietz, Epidemics and Rumors: A survey, Journal of the Royal Statistical Society, Series A (General), Vol. 130, No.4, (1967) 505-528.

[3] V. Cane, A Note on the size of epidemics and the number of people hearing a rumor, Journal of the Royal Statistical Society, Series B (Methodological), Vol. 28, No. 3, (1966) 487-490.

[4] K. Thompson, R. Estrada, D. Daugherty, and A. Cintron-Arias, A deterministic approach to the spread of rumors, Working paper, Washington, DC, USA (2003).

[5] L. Bettencourt, A. Cintron-Arias, D. Kaiser, and C. Castillo-Chavez, The power of a good idea: Quantitative modeling of the spread of ideas from epidemiological models, Physica A, Vol. 364 (2006) 513-536.

[6] K. Kawachi, Deterministic models for rumors transmission, Nonlinear analysis: Real world applications, Vol. 9 (2008) 1989-2028.

[7] K. Kawachi, M. Seki, H. Yoshida, Y. Otake, k. Warashina and H. Ueda, A rumor transmission model with various contact interactions, Journal of theoretical biology, Vol. 253 (2008) 55-60.

[8] R. Al-Amoudi, S. Al-Sheikh, and S. Al-Tuwairqi, Qualitative Behavior of Solutions to a Mathematical Model of Memes Transmission, International Journal of Applied Mathematical Research, Vol. 3, No. 1 (2014) 36-44.

[9] R. Al-Amoudi, S. Al-Sheikh, and S. Al-Tuwairqi, Stability analysis of a variable meme transmission model, International Journal of Advanced Mathematical Sciences, Vol. 2, No. 2 (2014) 107-115.

[10] J. Piqueira, Rumor propagation model: an equilibrium study, Mathematical Problems in Engineering, Vol. (2010), Article ID 631357, doi:10.1155/2010/631357.

[11] W. Huang, On rumor spreading with skepticism and denial, Working paper (2011).

[12] L. Wang, B. Wood, An epidemiological approach to model the viral propagation of memes, Applied Mathematical Modelling, Vol. 35 (2011) 5442–5447.

[13] L. Zhao, Q. Wang, J. Cheng, Y. Chen, J. Wang, and W. Huang, Rumor spreading model with consideration of forgetting mechanism: A case of online blogging LiveJournal, Physica A, Vol. 390 (2011) 2619–2625.

[14] L. Zhao, X. Wang, X. Qiu, and J. Wang, A model for the spread of rumors in Barrat–Barthelemy–Vespignani (BBV) networks, Physica A, Vol. 392 (2013) 5542–5551.

[15] L. Huo, T. Lin, and P. Huang, Dynamical Behavior of a Rumor Transmission Model with Psychological Effect in Emergency Event, Abstract and Applied Analysis, Volume 2013 (2013), Article ID 282394, 9 pages.

[16] X. Zhao and J. Wang, Dynamical Model about Rumor Spreading with Medium, Discrete Dynamics in Nature and Society, Volume 2013 (2013), Article ID 586867, 9 pages.

[17] K. Afassinou, Analysis of the impact of education rate on the rumor spreading mechanism, Physica A 414 (2014) 43–52.

[18] Y. Zan, J. Wua, P. Li, and Q. Yua, SICR rumor spreading model in complex networks: Counterattack and self-resistance,Physica A 405 (2014) 159–170.

[19] F.Brauer, P.van den Driessche , and J.Wu, Mathematical Epidemiology, Springer-Verlag (2008).

[20] L.Perko, Differential Equations and Dynamic Systems, Springer Verlag (1991).

[21] L.Edelstein- Keshet, Mathematical Models in Biology, SIAM (2005).

[22] M.W.Hirsch, S.Smale, and R.L.Devaney, Differential equations, dynamical systems and an introduction to chaos, Elsevier Academic press (1974).

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