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Some Exact Solutions of Non-Newtonian Fluid in Porous Medium with hall Effect Having Prescribed Vorticity Distribution Function

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Abstract- **Two dimensional motion of an incompressible second grade fluid in a porous medium with Hall effects has been considered. Exact solutions are obtained via inverse method when vorticity distribution is proportional to stream function ψ, perturbed by a quadratic term.**

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I. Introduction

The study of magneto hydrodynamic (MHD) fluid flows has been a subject of great interest because of their applications in widespread fields like magneto hydrodynamic generators, designing cooling systems with liquid metals, geothermal energy extraction, handling of biological fluids, flow of nuclear fuel slurries, electromagnetic propulsion and flow of blood. A number of researchers (Noreen Sher Akbar, S. Nadeem, R. Ul Haq and Z. H. Khan[1], Manoj Kumar and C.Thakur[2]) worked on some interesting problems in these directions.

But the above studies cannot be implemented in the case of ionized gases because in ionized gases (where the density is low and the magnetic field is strong), there is a conductivity normal to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions; also, a current is induced in a direction normal to both electric and magnetic fields. This is what we call the Hall Effects. This study has interesting features in problems of MHD generators, Hall accelerators and flight magneto hydrodynamics. N. Ahmad and K. Kr. Das[3], R. K. Deka[4], M.A.M. Abdeen et.al.[5], Haider Zaman et.al.[6] discussed the Hall effects in different situations.

The flows of non-newtonian fluid through porous media have gained a lot of importance in recent years. The flow through porous medium has lot of applications in engineering and science such as groundwater hydrology, petroleum engineering, reservoir engineering, chemical reactors, agricultural irrigation and drainage and recovery of crude oil from the pores of reservoir rocks.

Navier-Stokes equations are inherently non-linear partial differential equations has non general solution and only a small number of exact solutions have been found because the nonlinear inertial terms do not disappear automatically. Exact solutions are very important not only because they are solutions of some fundamental flows but also they serve as accuracy checks for experimental, numerical and asymptotic methods. So in order to perform this task one adopt transformations, inverse or semi-inverse method for the reformulation of equations in solvable form. Some researchers have used hodograph transformation[7,8] in order to linearize the system of governing equations and got some exact solutions. Some authors have used inverse method where some a priory condition is assumed about the flow variables and have found some exact solutions. This method has been extensively used by many researchers for the first grade fluid such as Chandna[9], M. Jamil et.al.[10,11] and others. In case of second grade fluid T. Hayat et.al.[12] applied this method to find some exact solutions. Benharbit and Siddiqui[13] used this method to study steady and unsteady second grade fluid flow by taking vorticity function of the form $\nabla^2\psi=k\,(\psi-Uy)$. Islam, Mohyuddin and Zhou[14] taking the same form of vorticity function studied the non-newtonian fluid in porous medium with Hall Effect. Further this method was also used by Chandna and Ukpong[15], A.M. Siddiqui et.al.[16], B.Singh and C. Thakur[17], Rana Khalid Naeem[18] , Manoj Kumar et.al.[19] in the study of second grade fluid flow.

In this paper we have studied second grade electrically conducting fluid flow in porous media with Hall Effect. The equations are modeled and solved by assuming the vorticity function proportional to the stream function perturbed by a quadratic stream $B(Cx+Dy+Ey^2)$. We have also found exact solution for finitely conducting steady and unsteady fluid flow.

II. Basic Flow Equation

The basic equations governing the motion of second grade electrically electrically conducting fluid flow in porous media with Hall Effect are given by :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

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$$\frac{\partial p^*}{\partial x}+\rho\left[\frac{\partial u}{\partial t}-v\omega\right]=\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2 u-\frac{1}{K}\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)u-\alpha_1 v\nabla^2\omega-\frac{\sigma B_0^{\ 2}}{1+\varphi^2}(u-\varphi v), \quad (2)$$

$$\frac{\partial p^*}{\partial y}+\rho\left[\frac{\partial v}{\partial t}+u\omega\right]=\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2 v-\frac{1}{K}\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)v+\alpha_1 u\nabla^2\omega-\frac{\sigma B_0^{\ 2}}{1+\varphi^2}(v+\varphi u) \quad (3)$$

where $u=u(x, y, t)$, $v=v(x, y, t)$ are the velocity components, $p^*=p^*(x, y, t)$ is the pressure field, μ is the viscosity of the fluid, α_1 is the normal stress moduli, K is the permeability, $\varphi = \sigma B_0/en_e$ is the Hall parameter, σ is the electrical field conductivity, B_0 is the magnetic field, e is the electric charge and n_e is the number density of electrons.

In equations (2) – (3), the vorticity and the modified pressure are given respectively as

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}, \quad (4)$$

$$p^*=\frac{\rho}{2}|\mathbf{v}|^2+p-\alpha_1\left[u\nabla^2 u+v\nabla^2 v+\frac{1}{4}|A_1|^2\right], \quad (5)$$

where,

$$|A_1|^2=4\left(\frac{\partial u}{\partial x}\right)^2+4\left(\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 \quad (6)$$

These systems of equations have three unknowns u, v, and p. Once the velocity field is determined, the pressure field (5) can be found by integrating equations (2) and (3). The continuity equation (1) implies the existence of stream function $\psi(x, y, t)$ such that

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}. \quad (7)$$

Using integrability condition $\partial^2 p^*/\partial x\partial y = \partial^2 p^*/\partial y\partial x$, equation (2) and (3) reduces to

$$\rho\left[\frac{\partial}{\partial t}(\nabla^2\psi)-\{\psi,\nabla^2\psi\}\right]=\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)\nabla^4\psi-\frac{1}{K}\left(\mu+\alpha_1\frac{\partial}{\partial t}\right)\nabla^2\psi-\alpha_1\{\psi,\nabla^4\psi\}-\frac{\sigma B_0^2}{1+\varphi^2}\nabla^2\psi . \quad (8)$$

In equation (8) if $K\to\infty$, and neglecting the Hall effects, we obtain the Benharbit and Siddiqui [13] case. If $K\to\infty$, $\alpha_1=0$, and neglecting Hall effects in equation (8), we recover the viscous case equations of Hui[20].

III. Exact Solutions

We shall investigate fluid motion for which vorticity distribution is proportional to the stream function perturbed by the quadratic term in x and y as given by

$$\nabla^2\psi=A\left[\psi-B\left(Cx+Dy+Ey^2\right)\right], \quad (9)$$

where A,B,C,D and E are constants but $A\neq 0$.The special case of A= 0 corresponds to an irrotational flow.

Introducing this value equation (8) reduces to

$$A\left(\rho-\alpha_1 A-\frac{\alpha_1}{K}\right)\psi_t+\rho BA\{(D+2Ey)\psi_x-\psi_y C\}-\alpha_1 A^2 B\{(D+2Ey)\psi_x-C\psi_y\}=$$

$$\mu A^2\left\{\psi-B\left(Cx+Dy+Ey^2\right)-\frac{2BE}{A}\right\}-\frac{\sigma B_0^2}{1+\varphi^2}A\left[\psi-B\left(Cx+Dy+Ey^2\right)\right]-\frac{\mu A}{K}\left\{\psi-B\left(Cx+Dy+Ey^2\right)\right\}$$

Letting $\Psi=\psi-B\left(Cx+Dy+Dy^2\right)$, the equation reduces to

$$A\left(\rho-\alpha_1 A-\frac{\alpha_1}{K}\right)\Psi_t+\left(\rho BA-\alpha_1 A^2 B\right)\{(D+2Ey)\Psi_x-C\Psi_y\}=\mu A^2\left\{\Psi-\frac{2BE}{A}\right\}-\frac{\mu A}{K}\Psi-\frac{\sigma B_0^2}{1+\varphi^2}A\Psi \quad (10)$$

Also,

$$\nabla^2\Psi=A\Psi-2BE. \quad (11)$$

In equation (10) if B=0, $\alpha_1=0$ & $K\to\infty$ and neglecting the Hall effects we obtain the Taylor [20] case. If $\alpha_1=0$, D=E=0, $K\to\infty$ and in the case of Hall effects equation (10) reduces to Hui[21] case. Also if $K\to\infty$ and neglecting the Hall effects we obtain the Benharbit and Siddiqui [13] case. Putting D=E=0, we reproduce the S. Islam et.al.[14] case.

Dividing equation (10) by A, we have

$$\left(\rho-\alpha_1 A-\frac{\alpha_1}{K}\right)\Psi_t+\left(\rho B-\alpha_1 AB\right)\{(D+2Ey)\Psi_x-C\Psi_y\}=\delta\left(\Psi-\frac{2\mu BE}{\delta}\right), \quad (12)$$

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where,

$$\delta = \left\{ \frac{\mu(1+\varphi^2)(KA-1) - \sigma B_0^2 K}{K(1+\varphi^2)} \right\}. \quad (13)$$

a) Creeping Flow

For creeping flow, we have from equation (12),

$$\delta\left(\Psi - \frac{2\mu BE}{\delta}\right) = 0.$$

Since $\delta \neq 0$, we must have $\Psi = \frac{2\mu BE}{\delta}$.

Hence, $\psi = B(Cx + Dy + Ey^2) + \frac{2\mu BE}{\delta}$. (14)

The stream function (14) gives the exact solutions

u = B(D+2Ey),

v = -BC,

$$p = p_0 - \frac{\rho}{2}B^2(C^2+D^2) - 2\alpha_1 B^2E^2 + B\left[2\rho BCE - \frac{\mu D}{K} - \frac{\sigma B_0^2}{1+\varphi^2}(D+\varphi C)\right]x - 2BE\left[\frac{\mu}{K} + \frac{\sigma B_0^2}{1+\varphi^2}\right]xy + B\left[\frac{\mu C}{K} - \frac{\sigma B_0^2}{1+\varphi^2}(\varphi D - C)\right]y - BE\left[\frac{\sigma B_0^2}{1+\varphi^2}\varphi\right]y^2.$$

b) Steady Flow

For steady flow we have $\Psi_t = 0$, so the system of equations (12) and (11) can be written as

$$(\rho B - \alpha_1 AB)\{(D+2Ey)\Psi_x - C\Psi_y\} = \delta\left(\Psi - \frac{2\mu BE}{\delta}\right), \quad (15)$$

&

$$\nabla^2\Psi = A\Psi - 2BE, \quad (16)$$

We now introduce the following co-ordinate transformations

$$\Psi = \psi - B(Cx + Dy + Ey^2)$$
$$\xi = Cx + Dy + Ey^2$$
$$\eta = y$$

Equation (15) and (16) respectively reduces to

$$(\rho - \alpha_1 A)BC\Psi_\eta = \delta\left(\Psi - \frac{2\mu BE}{\delta}\right), \quad (17)$$

$$\{C^2 + (D+2E\eta)^2\}\Psi_{\xi\xi} + 2(D+2E\eta)\Psi_{\xi\eta} + 2E\Psi_\xi + \Psi_{\eta\eta} = \delta\left(\Psi - \frac{2\mu BE}{\delta}\right). \quad (18)$$

Case I :

One of the solution of equation (17), when, $(\rho - \alpha_1 A) = 0$, is

$$\Psi = \frac{2\mu BE}{\delta}.$$

Hence, $\psi = B(Cx + Dy + Ey^2) + \frac{2\mu BE}{\delta}$.

The stream function gives the exact solutions

u = B(D+2Ey),

v = -BC,

$$p = p_0 - \frac{\rho}{2}B^2(C^2+D^2) - 2\alpha_1 B^2E^2 + B\left[2\rho BCE - \frac{\mu D}{K} - \frac{\sigma B_0^2}{1+\varphi^2}(D+\varphi C)\right]x - 2BE\left[\frac{\mu}{K} + \frac{\sigma B_0^2}{1+\varphi^2}\right]xy + B\left[\frac{\mu C}{K} - \frac{\sigma B_0^2}{1+\varphi^2}(\varphi D - C)\right]y - BE\left[\frac{\sigma B_0^2}{1+\varphi^2}\varphi\right]y^2.$$

Case II :

When $(\rho - \alpha_1 A) \neq 0$, solving equation (17) (By variable separable method) we get

$$\Psi = \frac{2\mu BE}{\delta} + g(\xi)e^{\lambda\eta}, \quad (19)$$

where, $\lambda = \left\{ \frac{\mu(1+\varphi^2)(KA-1) - \sigma B_0^2 K}{K(1+\varphi^2)(\rho - \alpha_1 A)BC} \right\}$. (20)

To find g(ξ), we substitute equation (19) in to equation (18) and get

$$e^{\lambda\eta}\left[\{C^2 + (D+2E\eta)^2\}g''(\xi) + \{2\lambda(D+2E\eta) + 2E\}g'(\xi) + (\lambda^2 - \delta)g(\xi)\right] = 0. \quad (21)$$

Since ξ, η are independent variables, we must have two cases $E = 0$, $g'(\xi) = 0$.

Case IIa: In this case $E = 0$ and equation (21) becomes

$$(C^2 + D^2)g''(\xi) + 2\lambda D g'(\xi) + (\lambda^2 - \delta)g(\xi) = 0 \quad (22)$$

The solution of (22) combined with equation (19) and also using $\Psi = \psi - B\left(Cx + Dy + Ey^2\right)$, $\xi = Cx + Dy + Ey^2$, $\eta = y$ and taking $E = 0$, we obtain the stream function as :

$$\psi(x, y) = B(Cx + Dy) + A_1 e^{m_1(Cx+Dy)+\lambda y} + A_2 e^{m_2(Cx+Dy)+\lambda y} \text{ for } M > 0, \tag{23.1}$$

$$= B(Cx + Dy) + (B_1 + B_2(Cx + Dy))e^{m_3(Cx+Dy)+\lambda y} \text{ for } M = 0, \tag{23.2}$$

$$= B(Cx + Dy) + C_1 e^{\alpha(Cx+Dy)+\lambda y}\cos(\beta(Cx + Dy) + C_2) \text{ for } M < 0, \tag{23.3}$$

where,

$$M = \lambda^2 D^2 - (C^2 + D^2)(\lambda^2 - \delta^2),$$

$$m_1 = \frac{-\lambda D + \sqrt{M}}{(C^2 + D^2)}, m_2 = \frac{-\lambda D - \sqrt{M}}{(C^2 + D^2)}, m_3 = \frac{-\lambda D}{(C^2 + D^2)},$$

$$\alpha = \frac{-\lambda D}{(C^2 + D^2)}, \beta = \frac{\sqrt{M}}{(C^2 + D^2)}.$$

A_1, A_2, B_1, B_2, C_1 and C_2 are arbitrary constants.

It is noted that the results of S. Islam et.al. (2008) can be recovered as special case by taking C=0 and appropriately choosing the value of constants in the present result.

The exact solution given by equation (23.1) when $M > 0$

$$u = \left[BD + A_1(m_1 D + \lambda)e^{m_1 Cx+(m_1 D+\lambda)y} + A_2(m_2 D + \lambda)e^{m_2 Cx+(m_2 D+\lambda)y}\right], \tag{24.1}$$

$$v = -\left[BC + A_1 m_1 C e^{m_1 Cx+(m_1 D+\lambda)y} + A_2 m_2 C e^{m_2 Cx+(m_2 D+\lambda)y}\right]. \tag{24.2}$$

The exact solution given by equation (23.2) when $M = 0$

$$u = \left[BD + B_2 D e^{m_3 Cx+(m_3 D+\lambda)y} + \{B_1 + B_2(Cx + Dy)\}(m_3 D + \lambda)e^{m_2 Cx+(m_2 D+\lambda)y}\right]. \tag{25.1}$$

$$v = -\left[BC + B_2 C e^{m_3 Cx+(m_3 D+\lambda)y} + \{B_1 + B_2(Cx + Dy)\}m_3 C e^{m_2 Cx+(m_2 D+\lambda)y}\right], \tag{25.2}$$

And equation (23.3) gives the exact solution for $M < 0$

$$u = \left[BD + C_1(\alpha D + \lambda)e^{\alpha Cx+(\alpha D+\lambda)y}\cos(\beta(Cx + Dy) + C_2) + C_1\beta e^{\alpha Cx+(\alpha D+\lambda)y}\sin(\beta(Cx + Dy) + C_2)\right]. \tag{26.1}$$

$$v = -\left[BC + C_1\alpha C e^{\alpha Cx+(\alpha D+\lambda)y}\cos(\beta(Cx + Dy) + C_2) - C_1\beta e^{\alpha Cx+(\alpha D+\lambda)y}\sin(\beta(Cx + Dy) + C_2)\right]. \tag{26.2}$$

In all the above cases p can be calculated by putting the value of u and v in equation (5).

Case IIb: In this case we take $g'(\xi) = 0$, which implies $g(\xi) = a_\circ$, where $a_\circ \neq 0$ is an arbitrary constant and $\delta = \lambda^2$, using this in equation (19), $\Psi = \psi - B\left(Cx + Dy + Ey^2\right)$ and $\eta = y$ we get stream function $\psi = \dfrac{2\mu BE}{\lambda^2} + B(Cx + Dy + Ey^2) + a_0 e^{\lambda}$, (27)

Hence, we get the exact solution as:

$$u = \left[B(D + 2Ey) + a_0\lambda e^{\lambda y}\right],$$

$$v = -BC, \tag{28}$$

Also p can be calculated by putting the value of u and v in equation (5).

Discussion

Solution (23.1) represents a uniform flow in (x, y) plane, in the region $y > 0$, perturbed by a part which grows and decays exponentially as y increases if $\lambda \rangle 0$ and $\lambda \langle 0$, respectively. The reverse holds true for the region $y < 0$, and the flow is exponential in x and y in both cases. The solution (23.2) can be used to describe, in $y > 0$, a uniform flow plus a different type of perturbation which again grows and decays as y increases in the same as in (23.2). Solution (23.3) represents a uniform flow with a perturbation part which is periodic in x and y and grows and decays exponentially as y increases, respectively, when $\lambda \rangle 0$ and $\lambda \langle 0$ for the region $y > 0$. A similar description can be given for a flow in $y < 0$.

c) Unsteady Flow

By rewriting equation (12) in the form

$$\frac{\partial \Psi}{\partial t} + U_1\left\{(D + 2Ey)\frac{\partial \Psi}{\partial x} - C\frac{\partial \Psi}{\partial y}\right\} = \gamma\left(\Psi - \frac{2\mu BE}{\delta}\right), \tag{29}$$

where,

$$\gamma=\left\{\frac{\mu\left(1+\varphi^2\right)(KA-1)-\sigma B_0^2K}{K\left(1+\varphi^2\right)(\rho-\alpha_1A-\alpha_1/K)}\right\},$$

$$U_1=\frac{(\rho B-\alpha_1AB)}{(\rho-\alpha_1A-\alpha_1/K)} \quad . \qquad (30)$$

Solving equation (29) by variable separable method we get

$$\Psi=F(X,Y)e^{\gamma t}+\frac{2\mu BE}{\delta}, \qquad (31)$$

where F(X, Y) is an unknown function to be determined and

$$X=x-U_1(D+2Ey)t,$$
$$Y=y+U_1Ct. \qquad (32)$$

Putting the value of Ψ in equation (16)

$$\nabla^2\left[F(X,Y)e^{\gamma t}+\frac{2\mu BE}{\delta}\right]=A\left[F(X,Y)e^{\gamma t}+\frac{2\mu BE}{\delta}\right]-2BE,$$

$$F_{XX}+F_{YY}=AF(X,Y)+2EB\left(\frac{\mu A}{\rho}-1\right)e^{-\gamma t}. \qquad (33)$$

Plane wave solution of Helmholtz equation (33) exist in the form

$$F(X,Y)=g(\xi),\ \xi=X\cos\theta+Y\sin\theta,\ -\pi\le\theta\le\pi \qquad (34)$$

Using equation (34) in equation (33) we obtain

$$g''(\xi)-Ag(\xi)=2EB\left(\frac{\mu A}{\delta}-1\right)e^{-\gamma t} \quad . \qquad (35)$$

Following cases arises:

Case I: When A=0

$$g(\xi)=-2EBe^{-\gamma t}\frac{\xi^2}{2}+C_1\xi+C_2, \qquad (36)$$

where C_1 and C_2 are arbitrary constants depending on θ.

A combination of (36) with (34) gives

.

This relation with (32),(31) and $\Psi=\psi-B\left(Cx+Dy+Ey^2\right)$ gives stream function as

$$\psi=-EB\xi^2+(C_1\xi+C_2)e^{\gamma t}+\frac{2\mu BE}{\delta}+B\left(Cx+Dy+Ey^2\right), \qquad (37)$$

where,

$$\xi=\{x-U_1(D+2Ey)t\}\cos\theta+\{y+U_1Ct\}\sin\theta \ ,$$

$$\gamma=\frac{-\mu\left(1+\varphi^2\right)-\sigma B_0^2K}{K\left(1+\varphi^2\right)\left(\rho-\alpha_1/K\right)},$$

$$U_1=\frac{\rho B}{\left(\rho-\alpha_1/K\right)}.$$

Hence we get exact solutions as

$$u=-2EB\xi(-U_1 2Et\cos\theta+\sin\theta)+C_1(-U_1 2Et\cos\theta+\sin\theta)+B(D+2Ey),$$

$$v=-2EB\xi\cos\theta-C_1e^{\gamma t}\cos\theta-BC.$$

P can be calculated by putting the value of u and v in equation (5).

Case II: If $A=\Omega^2\rangle 0$ then solution of equation (35) is

$$g(\xi)=D_1e^{\Omega\xi}+D_2e^{-\Omega\xi}+2EB(\Omega^2-1)e^{-\gamma t}\left(\frac{\xi^2}{2}-\frac{\Omega^2\xi^4}{24}\right), \qquad (38)$$

where D_1 and D_2 are arbitrary constants depending on θ.

Combining (38) with equation (34)

$$F(X,Y)=D_1e^{\Omega\xi}+D_2e^{-\Omega\xi}+2EB(\Omega^2-1)e^{-\gamma t}\left(\frac{\xi^2}{2}-\frac{\Omega^2\xi^4}{24}\right).$$

Putting this value in (31) and using $\Psi=\psi-B\left(Cx+Dy+Ey^2\right)$ we get stream function as

$$\psi=\left(D_1e^{\Omega\xi}+D_2e^{-\Omega\xi}\right)e^{\gamma t}+\frac{1}{12\delta}EB\left(\Omega^2-1\right)\left(12\xi^2-\Omega^2\xi^4\right)+\frac{2\mu BE}{}+B\left(Cx+Dy+Ey^2\right),$$

where,

$$\xi=\left\{x-U_1\left(D+2Ey\right)t\right\}\cos\theta+\left\{y+U_1Ct\right\}\sin\theta,$$

$$\gamma=\frac{\mu\left(1+\varphi^2\right)\left(K\Omega^2-1\right)-\sigma B_0^2K}{K\left(1+\varphi^2\right)\left(\rho-\alpha_1\Omega^2-\alpha_1/K\right)},$$

$$U_1=\frac{\rho B-\alpha_1\Omega^2B}{\left(\rho-\alpha_1\Omega^2-\alpha_1/K\right)}.$$

Hence we get velocity components as

$$u=\left(D_1e^{\Omega\xi}-D_2e^{-\Omega\xi}\right)e^{\gamma t}\Omega\left(-U_1 2Et\cos\theta+\sin\theta\right)+\frac{1}{12}EB\left(\Omega^2-1\right)\left(24\xi-4\Omega^2\xi^3\right)\left(-U_1 2Et\cos\theta+\sin\theta\right)+B\left(D+2Ey\right)$$

$$v=-\left(D_1e^{\Omega\xi}-D_2e^{-\Omega\xi}\right)e^{\gamma t}\Omega\cos\theta-\frac{1}{12}EB\left(\Omega^2-1\right)\left(24\xi-4\Omega^2\xi^3\right)\cos\theta-BC.$$

Also p can be calculated using equation (5).

Case III: If $A=-\Omega^2\langle 0$, then the general function for $g(\xi)$ is

$$g(\xi)=B_1\cos\Omega(\xi+B_2(\theta))+\left(\frac{\xi^2}{2}+\frac{\Omega^2\xi^4}{24}\right)2EB\left(-\Omega^2-1\right)e^{-\gamma t}, \tag{39}$$

where B_1 and B_2 are arbitrary constants depending on the parameter θ.

Combining equation (39) with equation (34) we get

$$F(X,Y)=B_1\cos\Omega(\xi+B_2(\theta))+\left(\frac{\xi^2}{2}+\frac{\Omega^2\xi^4}{24}\right)2EB\left(-\Omega^2-1\right)e^{-\gamma t}$$

Putting this value in equation (31) and using $\Psi=\psi-B\left(Cx+Dy+Ey^2\right)$ we get stream function as

$$\psi=\left\{B_1\cos\Omega\left(\xi+B_2(\theta)\right)\right\}e^{\gamma t}+\frac{1}{12\delta}EB\left(12\xi^2+\Omega^2\xi^4\right)\left(-\Omega^2-1\right)+\frac{2\mu BE}{}+B\left(Cx+Dy+Ey^2\right),$$

where,

$$\xi=\left\{x-U_1\left(D+2Ey\right)t\right\}\cos\theta+\left\{y+U_1Ct\right\}\sin\theta,$$

$$\gamma=\frac{\mu\left(1+\varphi^2\right)\left(-K\Omega^2-1\right)-\sigma B_0^2K}{K\left(1+\varphi^2\right)\left(\rho+\alpha_1\Omega^2-\alpha_1/K\right)},$$

$$U_1=\frac{\rho B+\alpha_1\Omega^2B}{\left(\rho+\alpha_1\Omega^2-\alpha_1/K\right)}.$$

Hence we get exact solutions as

$$u=-\Omega\sin\theta\left\{B_1\cos\Omega\left(\xi+B_2(\theta)\right\}e^{\gamma t}+\frac{1}{12}EB\left(24\xi+4\Omega^2\xi^3\right)\left(-\Omega^2-1\right)\left(-U_{12}Et\cos\theta+\sin\theta\right)+B\left(D+2Ey\right),$$

$$v=-\Omega\cos\theta\{B_1\cos\Omega(\xi+B_2(\theta)\}e^{\gamma t}-\frac{1}{12}EB\left(24\xi+4\Omega^2\xi^3\right)\left(-\Omega^2-1\right)\cos\theta+BC$$

P can be calculated by putting the values of u and v in equation (5).

IV. Conclusion

In this paper we have found the exact solutions of the governing equations of incompressible second grade fluid in a porous medium with the Hall currents under the assumption that vorticity distribution is proportional to the stream function perturbed by a quadratic term. We recovered the solutions of Benharbit and Siddiqui[13], Hui[21] and Islam, Mohyuddin and Zhou[14] in limiting cases if the corresponding conditions are applied. Our solutions are compatible in a limiting case with those of Benharbit and Siddiqui, Hui and Islam et.al. Expressions for streamlines, velocity components and pressure fields are defined in each case. Our solutions are more general and several results of various authors can be recovered in a limiting case.

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