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Computation of a Beautiful Summation Formula in Association with Contiguous Relation

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GJSFR-F Classification : *MSC 2010: 33C05 , 33C20 , 33D15 , 33D50 , 33D60*

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Notes

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Salahuddin [α] & Shakeeluddin [σ]

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I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ll} a_1, a_2, \cdots, a_A & ; \\ & \\ b_1, b_2, \cdots, b_B & ; \end{array} \quad z \right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers and $\mid z \mid = 1$

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ c & ; \end{array} \quad z\right] = a\ {}_2F_1\left[\begin{array}{ll} a+1,\ b & ; \\ c & ; \end{array} \quad z\right] - b\ {}_2F_1\left[\begin{array}{ll} a,\ b+1 & ; \\ c & ; \end{array} \quad z\right] \tag{2} $$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ \frac{a+b+1}{2} & ; \end{array} \quad \frac{1}{2}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

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$$ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ \frac{a+b-1}{2} & ; \end{array}\ \frac{1}{2}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\,\Gamma(\frac{b+1}{2})} + \frac{2\,\Gamma(\frac{a+b-1}{2})}{\Gamma(a)\,\Gamma(b)}\right] \tag{5} $$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ \frac{a+b-1}{2} & ; \end{array}\ \frac{1}{2}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\,\Gamma(\zeta) \tag{7} $$

II. Main Summation Formula

$$ {}_2F_1\left[\begin{array}{ll} a, & b\ ; \\ \frac{a+b+50}{2} & ; \end{array}\ \frac{1}{2}\right] = \frac{2^b\,\Gamma(\frac{a+b+50}{2})}{(a-b)\,\Gamma(b)\Big[\prod_{\Phi=1}^{24}\{a-b-2\Phi\}\Big]\Big[\prod_{\Omega=1}^{24}\{a-b+2\Omega\}\Big]} $$

$$ \Big[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\Big\{16777216(a^{24}+24a^{23}(-25+49b)+92a^{22}(1850-2450b+2303b^2)+2024a^{21}(-15000+32830b $$
$$ -11515b^2+6909b^3)+3542a^{20}(1078760-1862560b+1862140b^2-296100b^3+127323b^4) $$
$$ +14168a^{19}(-25470000+59043320b-31452400b^2+19575500b^3-1768375b^4+580027b^5) $$
$$ +1748a^{18}(15191158400-29643079200b+30723033320b^2-7949956000b^3+3490772250b^4 $$
$$ -203009450b^5+52782457b^6)+10488a^{17}(-148786880000+357673671040b-232905706320b^2 $$
$$ +148123893400b^3-22677641000b^4+7511349650b^5-304514175b^6+64382997b^7) $$
$$ +7429a^{16}(10023824208640-21069362298880b+22345022210560b^2-7368717121920b^3 $$
$$ +3271593344880b^4-331311422400b^5+86238414360b^6-25753198880b^7+4506809798^8) $$
$$ +6992a^{15}(-417434238672000+1028919146214400b-758185809902080b^2+488413454072320b^3 $$
$$ -178939692136000b^4+32514318327120b^5-2340617754720b^6+491886097080b^7-11267024475b^8) $$
$$ +16524969239^9+1288a^{14}(73498165566150400-162582609590278400b+175048145605518080b^2 $$
$$ -67216907266790400b^3+29982931333152000b^4-4051969321377600b^5+1050250387890480b^6 $$
$$ -56524592337600b^7+9770119794750b^8-1770532417509^9+219546019771^{10}) $$
$$ +2576a^{13}(-990767756776320000+2486449587901233920b-1996078715212677760b^2 $$
$$ +1294802759583674880b^3-308151278611904000b^4+102121179551686400b^5 $$
$$ -9953696252152800b^6+2070088475301840b^7-864663649710008^8+124527446697509^9 $$
$$ -18295501647^5b^{10}+1929343810^1b^{11})+4a^{12}(14326727885044845429760 $$
$$ -3290588518751035127808080b+35774313688848091302400b^2-15286300400532994327040b^3 $$

Notes

$$+6818791881087123508480b^4 - 1110197890899430860800b^5 + 285360756947700062720b^6$$
$$-21034488682299980160b^7 + 35773791502192926000b^8 - 1193565477544440000b^9$$
$$+144512508413273000b^{10} - 1755702867191000b^{11} + 158013258047190b^{12})$$
$$+16a^{11}(-669547155132190072320000 + 1702574806656396147148800b$$
$$-145659775294357351372800b^2 + 947326460348108910218240b^3 - 254696002292718122456320b^4$$
$$+83889876852140073594880b^5 - 9963924613447640908800b^6 + 2043242878716115614720b^7$$
$$-117990425471551083360b^8 + 16627075164075616240b^9 - 4533830217868952000b^{10}$$
$$+464207838085300400b^{11} - 47403977414157000b^{12} + 3646459801089000b^{13})$$
$$+88a^{10}(1889644037260035979264000-44675008443384303953920000b+4884737551157640234956800b^2$$
$$-226166014583862202257408000b^3 + 1005169731023529923371520b^4 - 187039485649250815526400b^5$$
$$+47508833391083179548160b^6 - 4307312586654000035840b^7 + 718320606721103509760b^8$$
$$-33396426082829409600b^9+3934416307396054480b^{10}-893301618826780800b^{11}+77602066729842200b^{12}$$
$$-6752703335350000b^{13} + 443749076323000b^{14}) + 176a^9(-12117413105418513408000000$$
$$+3111036718994734998200320000b-279668498932558257249280000b^2+18173328867690691277649920000b^3$$
$$-535941065625167448529920000b^4 + 17485802915445220254080000b^5 - 2395976050800413029312000b^6$$
$$+482844538455392501606400b^7 - 34564912224580914176000b^8 + 4749672227164504640000b^9$$
$$-181710161270101464000b^{10}+17997915932600749600b^{11}-3455707574457320000b^{12}+254972763869250000b^{13}$$
$$-19127115358750000b^{14} + 1071118460090000b^{15}) + 7a^8(32036520834224384519962624000$$
$$-774954359718187014052904960000b + 84946482206673031294189568000b^2$$
$$-4189347406236124909795737600000b^3 + 184918660522586233199824896000b^4$$
$$-3810957758982312498094080000b^5 + 95348706874079672356761600000b^6$$
$$-1005187175780630307379200000b^7 + 16382012261836246649395200000b^8 - 950616408212436725760000b^9$$
$$+108574252129923740160000b^{10} - 34796729916906397440000b^{11} + 2906032975631673888000b^{12}$$
$$-478072081243680000000b^{13} + 29925033054180000000b^{14} - 1954667788920000000b^{15} + 92846719973700000b^{16})$$
$$+56a^7(-34149383145217497799065600000 + 882744429988659622361169920000b$$
$$-82581902088381774685929472000b^2 + 53456887855304247968707379200000b^3$$
$$-16965654806439543313050009600000b^4 + 5465855310775186912465305600000b^5$$
$$-836198058114892662239232000000b^6+165051253917006528398745600000b^7-13833130317182886015820800000b^8$$
$$+1846835733004646409625600000b^9 - 88722756599754973696000000b^{10} + 8466696085796543795200000000b^{11}$$
$$-230733533612999289600000000b^{12}+16253971752787440000000000b^{13}-2316831344616000000000000b^{14}+122610979486800000000000b^{15}$$
$$-7033842422250000000000b^{16} + 281353696890000000000b^{17}) + 4a^6(32565504577080501524024524800000$$
$$-802472204014901668373594112000000b + 879424435527365512536732467200000b^2$$
$$-45659572447846766118316802048000000b^3 + 19956317714641264124700557312000000b^4$$
$$-4463337907739548156566159360000000b^5 + 10963524136377404871326064640000000b^6$$

$$-1299462831292472909083607040b^{7}+20620912320523394927663616b^{8}$$
$$-140956235440970447097600b^{9}+1554394910423122214487040b^{10}-6290455808806015694848b^{11}$$
$$+502668779829351629312b^{12}-11797461132837932160b^{13}+699536271369260640b^{14}$$
$$-87237415743072000b^{15}+387986748011310b^{16}-19694758782300b^{17}+656491959410b^{18})$$
$$+8a^{5}(-86750926705733524374159360000+22525346551434294730501914624 0b$$
$$-21779131117221050908754313216 0b^{2}+140039902905405576335376515072b^{3}$$
$$-47213295693633229440107806720b^{4}+14971510271375150107101921280b^{5}$$
$$-25036370588875264227900211200b^{6}+4822484423113274122084884480b^{7}$$
$$-457274210060784346849382400b^{8}+590662191033282868654336 0b^{9}$$
$$-336165591923013490077440b^{10}+30757829800511451782144b^{11}-1064027984710141301760b^{12}$$
$$+71156051885803261440b^{13}-1454165064083793600b^{14}+72230760361109280b^{15}$$
$$-7948165895602800b^{16}+294356800178700b^{17}-1330726944750b^{18}+36419895330b^{19})$$
$$+2a^{4}(140378287973490249294813541171 2-35121367245418730144765942169 60b$$
$$+38381074103219751653610199449 60b^{2}-20806384420292820197913604915 20b^{3}$$
$$+89761460613542207131930119372 8b^{4}-21488892577826560048791552000 0b^{5}$$
$$+51630321577757365038691450880b^{6}-6732703340931905058752593920b^{7}$$
$$+1035802801652608895414433792b^{8}-80575753601204966699212800b^{9}$$
$$+8535593768666229096832000b^{10}-411469014402202290293760b^{11}+31273116443390308870656b^{12}$$
$$-936555486456029081600b^{13}+52229705189648000000b^{14}-938501032983494400b^{15}$$
$$+38739909899250840b^{16}-378991033886480 0b^{17}+11532966854500b^{18}-466921735000b^{19}$$
$$+10272278170b^{20})+8a^{3}(-103194595403724570443938529280 0$$
$$+268590691527016263466206678220 8b-26705673273574744776916756070 40b^{2}$$
$$+17005869655952719577924008673 28b^{3}-60358810798804009830804514406 4b^{4}$$
$$+18768571354706702445951896780 8b^{5}-33825211518162119650051358720b^{6}$$
$$+6329583335998288659875692544b^{7}-664372083123351160351567872b^{8}$$
$$+82584438684866658992162816b^{9}-5379663985242038662410240b^{10}$$
$$+468923751059587357334528b^{11}-19430022031286625950464b^{12}+1224066602288528015360b^{13}$$
$$-32091266652796288000b^{14}+1482193431840486400b^{15}-23623272446432400b^{16}$$
$$+800622453849960b^{17}-7010659026000b^{18}+171963858500b^{19}-626358425b^{20}+10737573b^{21})$$
$$+28a^{2}(588025748075779227729788928000-148986884159129598296567316480 0b$$
$$+16191343734973529693986939207 68b^{2}-91115363198987317869896584396 8b^{3}$$
$$+38654200346211213173702472499 2b^{4}-98135025937179261422775828480b^{5}$$
$$+22954186835008845998638235648b^{6}-3246914570925961085847076864b^{7}$$
$$+481497860946850591303254016b^{8}-41716419188349098383319040b^{9}$$

Notes

Notes

$$+42155863053774336159068 16b^{10}-233981535209296474349568b^{11}+167721446039900266142 72b^{12}$$
$$-60516229282180924672 0b^{13}+31416744401697149440b^{14}-728446051233638400b^{15}$$
$$+27555842441473920b^{16}-3927341684145 60b^{17}+107352136094 00b^{18}-84659596000b^{19}$$
$$+1627283350b^{20}-5350950b^{21}+68103b^{22})+56a(-350941492603214674211635200000$$
$$+9136781741367615504865296384 00b-9309152567649968772029507174 40b^2$$
$$+5848911132157624907331073474 56b^3-2172469294932871285568647987 20b^4$$
$$+6590998697795561475683647488 0b^5-1268577774911503518631723008 0b^6$$
$$+2291538654085596716282675200b^7-2626486808826365075387187 20b^8$$
$$+3117361521382020743471104 0b^9-2276521650775048577331200b^{10}$$
$$+1872477457468170306252 80b^{11}-8992280885059851724800b^{12}+5274865130755744384 00b^{13}$$
$$-1677386722830932 0960b^{14}+7100927009870387 20b^{15}-147048124096025 60b^{16}$$
$$+447872724172160b^{17}-5753553921360b^{18}+123515535640b^{19}-882312200b^{20}+12723370b^{21}$$
$$-37835b^{22}+329b^{23})+7(1487056693247618921980231680000$$
$$-3807945612886480201004875776000b+4101124076932069592394419404800b^2$$
$$-2387147616741179381388410880000b^3+9904868702221308856158262394 88b^4$$
$$-2652228413822552777514654105 60b^5+5996050556945672809400500224 0b^6$$
$$-9125628368819773305677441440 0b^7+1292587838978295446849454080b^8$$
$$-1232038867469925870705049 60b^9+1174336127054962551128064 0b^{10}$$
$$-7363444742395662491648 00b^{11}+4909448578700566190080 0b^{12}-2069618165579821301760b^{13}$$
$$+9832113141370655744 0b^{14}-27887407233511936 00b^{15}+946242612455078 40b^{16}$$
$$-1768023172751360b^{17}+42226385098240b^{18}-493112860800b^{19}+7974742160b^{20}-51895360b^{21}$$
$$+521640b^{22}-1400b^{23}+7b^{24}))\Big\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Big\{16777216(10409396852733332453861621760000$$
$$+49a^{24}-1965272358578002175585 1571200000b+1646472094612181837643408998 4000b^2$$
$$-8255567632297965635515082342 400b^3+2807565759469804985896270823424b^4$$
$$-6940074136458681949932748800 00b^5+1302620183083220060960980992 00b^6$$
$$-1912365456132179876747673600 0b^7+2242556458395706916397383680b^8$$
$$-2132664706553658359808000000b^9+1662886752788831661752320 0b^{10}$$
$$-1071275448211504115712000 0b^{11}+5730691154017938171904 0b^{12}-2552217741455800320000b^{13}$$
$$+9466563724920171520 0b^{14}-2918700196794624000b^{15}+744669900459865 60b^{16}$$
$$-15604767974400 00b^{17}+26554144883200b^{18}-3608589600 00b^{19}+3820967920b^{20}$$
$$-30360000b^{21}+170200b^{22}-600b^{23}+b^{24}+392a^{23}(-25+47b)+4508a^{22}(810-470b+423b^2)$$
$$+14168a^{21}(-25640+50290b-10575b^2+6063b^3)+24794a^{20}(2251480-1992800b+1837700b^2$$
$$-202100b^3+82861b^4)+9016a^{19}(-382851600+767177240b-262918000b^2+152585500b^3$$

$$-10357625b^4+3231579b^5)+85652a^{18}(3450995840-3761722080b+3509386600b^2-654804000b^3$$
$$+269298250b^4-12429150b^5+3065857b^6)+171304a^{17}(-72246778880+146411482240b$$
$$-64193227920b^2+37389550920b^3-4424777400b^4+1374663990b^5-45987855b^6+9197571b^7)$$
$$+21413a^{16}(30933070037760-384565215027 20b+360324843955 20b^2-88257684384 00b^3$$
$$+36183542613 60b^4-29694730848 0b^5+72476859480b^6-1839514200b^7+303519843b^8)$$
$$+2576a^{15}(-7578099791715200+154367978475443 20b-791789186123 5200b^2+460308519205 1200b^3$$
$$-72864987032 8800b^4+22431913155 6240b^5-1354618256 8800b^6+266545607 5800b^7-53115972525b^8$$
$$+7318200659b^9)+1288a^{14}(534353975074492160-729298575143883520b+68297270438472 0640b^2$$
$$-19932463759500 80000b^3+81102026692000000b^4-9032081143377600b^5+217247289245 1120b^6$$
$$-10073179759200 0b^7+16263604920750b^8-261364309250b^9+30318259873b^{10})$$
$$+784a^{13}(-184787336212484044 80+376776080768267456 00b-216129390293503302 40b^2$$
$$+124904755335564083 20b^3-23891721593266048 00b^4+72608216210003328 0b^5$$
$$-601911282287649 60b^6+116099798234196 00b^7-426850072539000b^8+57238783725750b^9$$
$$-7579564968 25b^{10}+74417546961b^{11})+196a^{12}(175337449239305935 3600$$
$$-256922311001710049 2800b+239602065771286094 4896b^2-793062123725984732672b^3$$
$$+319113433095819478272b^4-434297136616384204 80b^5+102585465271296250 88b^6$$
$$-65923866746571 2256b^7+103786891986845496b^8-3103084352573920b^9+348417442460516b^{10}$$
$$-3869712441972b^{11}+322476036831b^{12})+784a^{11}(-65745042342818415 10400$$
$$+13374838981915502187520b-83564834003320169410 56b^2+47849362353019118095 36b^3$$
$$-10496658530668425772 80b^4+31385540612766787532 8b^5-32094162289826610 688b^6$$
$$+60476400612832455 68b^7-3106850885438071 20b^8+4040348474665474 4b^9-100268549051985 6b^{10}$$
$$+947362934867 96b^{11}-895766768975b^{12}+63392725189b^{13})+4312a^{10}(190638981664766647 91040$$
$$-29565216243831799705600b+27373937047905413090304b^2-99808237203006283161 60b^3$$
$$+39589952544834086720 00b^4-62368384401301204096 0b^5+14419247777579983436 8b^6$$
$$-11522435922046100 480b^7+17625690280831776 00b^8-741674127633067 20b^9$$
$$+8029421035502152b^{10}-168231176915360b^{11}+13405613025350b^{12}-109297802050b^{13}$$
$$+65578681 23b^{14})+112a^9(-77002429216870366919065 60+15586807606910103717 3555 20b$$
$$-10429104797087274595829760b^2+58988884774904756422973 44b^3$$
$$-14388527428786601196288 00b^4+42190156502377347761 0240b^5-50341512657489445392000b^6$$
$$+92341786650232320481 28b^7-59413525513277295360 0b^8+74637706426870787200b^9$$
$$-26240049065080250 40b^{10}+23752964520108023 2b^{11}-4262733848373000b^{12}+286413127404250b^{13}$$
$$-20361122801 25b^{14}+10316302219 3b^{15})+7a^8(12925878389782955446849454080$$
$$-21011894470610920603097497 60b+19259914437874023652130160 64b^2$$
$$-75928238071240132611607756 8b^3+29594365761503111297555251 2b^4$$
$$-52259909721232496782786560b^5+11783378468870511387236352b^6$$

$$-1106650425374630881265664b^7+16382012261836246649395 2b^8-869060650218034413568 0b^9$$
$$+9030316198779586979 84b^{10}-2696924010778310476 8b^{11}+2044216657268167200b^{12}$$
$$-318196223093280000b^{13}+1797702042234000b^{14}-112541478756 00b^{15}+47830128471 3b^{16})$$
$$+56a^7(-1140703546102471663209676800+2291538654085596716282675200b$$
$$-1623457285462980542923538432b^2+9042261908568983799822417 92b^3$$
$$-2404536907475680378125926 40b^4+6889263461590391602978406 4b^5$$
$$-9281877366374806493454336 b^6+1650512539170065283987456b^7-1256483969725787884224 00b^8$$
$$+1517511406574090719334 4b^9-6768634064742000056 32b^{10}+5837836796331758899 2b^{11}$$
$$-1502463477307141440b^{12}+9522406986388464 0b^{13}-1300065623764800b^{14}+614154926925 60b^{15}$$
$$-341643774795b^{16}+12058015581b^{17})+196a^6(2141446627480597431928750080$$
$$-3624507928318581481804922880b+3279169547858406571234033664b^2$$
$$-1380620878292331414287810560b^3+5268400160995649493744025 60b^4$$
$$-1021892677096949560322457 60b^5+2237453905383143851291033 6b^6$$
$$-2389137308899693320683520b^7+3405310959788559727027 20b^8-2151488698677921903872 0b^9$$
$$+2133049662456795816448b^{10}-8133816010977666048 0b^{11}+5823688917300001280b^{12}$$
$$-130820007885436800b^{13}+690164540613744 0b^{14}-834979558214 40b^{15}+3268699899390b^{16}$$
$$-16294615650b^{17}+470733341b^{18})+392a^5(-4736122167540272816990453760$$
$$+9415712425422306795480678 40b-7009644709798518673055416320b^2$$
$$+3830320684634020907337121792b^3-1096372070297273471877120000b^4$$
$$+3055410259464316348388147 20b^5-4554426436468926690373632 0b^6$$
$$+7808364729678838446379008b^7-6805281712468415175168 00b^8+7850768655914180522240 0b^9$$
$$-4198845596207671368960b^{10}+3424076606209798922 24b^{11}-1132854990713704960 0b^{12}$$
$$+6710820370539392 00b^{13}-133136134845264 00b^{14}+579949269753120b^{15}-627885856380 0b^{16}$$
$$+2009669263 50b^{17}-905256425b^{18}+20963833b^{19})+98a^4(7074906215872363468 6844731392$$
$$-1241411025675926448896370278 40b+1104405724177463233534356357 12b^2$$
$$-4927249861126857945371797094 4b^3+1831866543133514431263879987 2b^4$$
$$-3854146587235365668580229120b^5+8145435801894393520285941 76b^6$$
$$-9694659889394024750314291 2b^7+1320847575161330237141606 4b^8$$
$$-9625064035717292953190 40b^9+9026013911231697271091 2b^{10}-4158302078248459142144b^{11}$$
$$+2783180359627397350 40b^{12}-8099976466370048000b^{13}+3940613832357120 00b^{14}$$
$$-6984435027974400b^{15}+248006805705240b^{16}-24269703960 00b^{17}+62263978500b^{18}-255656500b^{19}$$
$$+4601817b^{20})+56a^3(-2983934520926474226735513600 00+5848911132157624907331073474 56b$$
$$-4555768159949365893494829219 84b^2+2429409950850388511132001239 04b^3$$
$$-7430851578676007213540573184 0b^4+2000570041505793947648235929 6b^5$$
$$-3261398031989054722736914432b^6+5345688785530424796870737 92b^7$$

$$-52366842577951561372446720b^8+57116176441313601158328320b^9$$
$$-35540373720321203211878 4b^{10}+27066470295660254577664b^{11}-10918786000380710233 60b^{12}$$
$$+5956092694084904448 0b^{13}-1545988867136179200b^{14}+609819084084582 40b^{15}$$
$$-9775392767632 80b^{16}+27741489178200b^{17}-248152198000b^{18}+4952601500b^{19}-18728325b^{20}$$
$$+249711b^{21})+28a^2(10252810192330173980986048512 00-18618305135299937544059014348 80b$$
$$+16191343734973529693986939207 68b^2-76301923638784985076905017344 0b^3$$
$$+27415052930871251181150142464 0b^4-62226088906345859739298037760b^5$$
$$+12563206221819507321953332096 0b^6-16516380417676354937185894 40b^7$$
$$+21236620551668257823547392 0b^8-17579162790046519027097600b^9$$
$$+15352032303638297881292 80b^{10}-83234157311061343641600b^{11}+51106162412640130432 00b^{12}$$
$$-18363924179956635392 0b^{13}+80522146978538316 80b^{14}-1893298279584051 20b^{15}$$
$$+5928613214366 0 80b^{16}-872398231387 20b^{17}+19179950801 20b^{18}-159149144 00b^{19}+2355607 10b^{20}$$
$$-832370b^{21}+7567b^{22})+56a(-47599320161081002512560947 2000$$
$$+91367817413676155048652 96638400b-74493442079564799148283 6582400b^2$$
$$+38370098789573751923743 8111744b^3-12543345444792403623130693 6320b^4$$
$$+32179066502048992472145 5923 20b^5-57319443143921547740971008 00b^6$$
$$+88274442998865962236 1169920b^7-96869294964773376756613120b^8$$
$$+97775439739834528514867 20b^9-70203584696746763356160 0b^{10}+48644994475897032775680b^{11}$$
$$-23504203705364536627 20b^{12}+11437668104345676032 0b^{13}-37394000205764032 00b^{14}$$
$$+1284679048273408 00b^{15}-27950766521139 20b^{16}+669871689619 20b^{17}-92528754360 0b^{18}$$
$$+14937959960b^{19}-117806920b^{20}+1186570b^{21}-4025b^{22}+21b^{23}))\Big\}\Bigg] \qquad (8)$$

III. Derivation of the Summation Formula

Substituting $c=\frac{a+b+50}{2}$ and $z=\frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix}a,\ b\\ \frac{a+b+50}{2}\end{matrix};\quad \frac{1}{2}\right]=a\ {}_2F_1\left[\begin{matrix}a+1,\ b\\ \frac{a+b+50}{2}\end{matrix};\quad \frac{1}{2}\right]-b\ {}_2F_1\left[\begin{matrix}a,\ b+1\\ \frac{a+b+50}{2}\end{matrix};\quad \frac{1}{2}\right]$$

Now involving the derived formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

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