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The Generalized Goldbach's Conjecture: Symmetry of Prime Number

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GJSFR-F Classification : *FOR Code : MSC 2010: 11B25. 11N13*

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The Generalized Goldbach's Conjecture: Symmetry of Prime Number

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I. Introduction

$G(x)$: the number of representatives a large even integer x as a sum of two primes.

$G(x,q,L)$: the number of representatives a large even integer x as a sum of two primes in the former L and tolerance q coprime to arithmetic progression.

p: a prime number.

a: a constant number, $a>0$.

$\mathcal{O}$: mean big O notation describes the limiting behavior of a function when the argument tends towards a particular value or infinity, usually in terms of simpler functions.

$p|k$: p divides k.

$\phi(q)$: $\phi(q)$ is the Euler phi-function.

$Li_2(x)$: express the logarithmic integral function or integral logarithm. $Li_2(x)$ is a special function such as $Li_2(x)=\int_2^x \frac{dt}{\ln^2 t}$.

II. Symmetry of Prime Number

In the former L and tolerance q ($L<q, q\geq 2$) coprime to arithmetic progression, for any one item of this progression such as $\frac{x}{2}$ where $\frac{x}{2}>\phi(q)\cdot q^2$, There are at least a pair of symmetrical primes on $\frac{x}{2}$, namely even integer x can be expressed for a sum of two primes in its series.

Let $G(x,q,L)$ is the number of representatives a large even integer x as a sum of two primes in the former L and tolerance q coprime to arithmetic progression.

1.) If $q=2^m$,

$$G(x,q,L)=\frac{1}{\phi(q)}\cdot 2C\cdot\prod_{p\geq 3}\frac{(p-1)}{(p-2)}\cdot Li_2(x)+\mathcal{O}(x\cdot e^{-a\sqrt{\ln x}}) \qquad (1)$$

where ($p>2$, $p|x$).

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2.) If q is an odd prime number,

$$G(x,q,L) = \frac{1}{q-2}\cdot 2C\cdot\prod_{p\geq 3}\frac{(p-1)}{(p-2)}\cdot Li_2(x)+\mathcal{O}(x\cdot e^{-a\sqrt{\ln x}}) \qquad (2)$$

where ($p>2$, $p|x$).

3.) If q is an odd number, let $\mathrm{Y}(q)= q\cdot\prod(1-\frac{2}{p})$, ($p>2, p|q$),

$$G(x,q,L) = \frac{1}{\mathrm{Y}(q)}\cdot 2C\cdot\prod_{p\geq 3}\frac{(p-1)}{(p-2)}\cdot Li_2(x)+\mathcal{O}(x\cdot e^{-a\sqrt{\ln x}}), \qquad (3)$$

where ($p>2$, $p|x$).

4.) If $q=2^m\cdot j$ (j is an odd number), let $\mathrm{Y}(j)= j\cdot\prod(1-\frac{2}{p})$, ($p>2, p|j$),

$$G(x,q,L) = \frac{1}{\phi(2^m)}\cdot\frac{1}{\mathrm{Y}(j)}\cdot 2C\cdot\prod_{p\geq 3}\frac{(p-1)}{(p-2)}\cdot Li_2(x)+\mathcal{O}(x\cdot e^{-a\sqrt{\ln x}}), \qquad (4)$$

where ($p>2$, $p|x$).

It can be seen, classify two situations.

While $q=2$; $q=3$ or $q=6$ alternative,

$$G(x)=G(x,q,L)=2C\cdot\prod_{p\geq 3}\frac{(p-1)}{(p-2)}\cdot Li_2(x)+\mathcal{O}(x\cdot e^{-a\sqrt{\ln x}}), \qquad (5)$$

where ($p>2$, $p|x$, $a>0$), this formula prompts the expression of Goldbach conjecture.

While $x=2^n$,

$$G(x)=2C\cdot Li_2(x)+\mathcal{O}(x\cdot e^{-a\sqrt{\ln x}}), \qquad (6)$$

The Goldbach's conjecture expression is same with the twin prime conjecture. Since

$$C=\prod_{p\geq 3}(1-\frac{1}{(p-1)^2}), \qquad (7)$$

where $p>2$, p is a prime number.

III. Conclusion

In the former L and tolerance q ($L<q, q\geq 2$) coprime to arithmetic progression, Let $G(x,q,L)$ is the number of representatives a large even integer x as a sum of two primes in its arithmetic progression,

$$G(x,q,L) = \frac{1}{\phi(2^m)}\cdot\frac{1}{\mathrm{Y}(j)}\cdot 2C\cdot\prod_{p\geq 3}\frac{(p-1)}{(p-2)}\cdot Li_2(x)+\mathcal{O}(x\cdot e^{-a\sqrt{\ln x}}).$$

Notes

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where $p > 2$, $p|x$, $q = 2^m \cdot j, \Upsilon(j) = j \cdot \prod(1 - \frac{2}{p})(p > 2, p|j)$,
p is a prime number, j is an odd number.

References

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2. Chengdong Pan and Chengbiao Pan. The Goldbach's Conjecture, section 6.1 and 8.1. Science Press, 2ed.

Notes

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