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Multi-Objective Geometric Programming in Multiple-Response Stratified Sample Surveys with Quadratic Cost Function

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Abstract- In this paper, the problem of multiple-response in stratified sample surveys has been formulated as a multi-objective geometric programming problem (MOGPP). The fuzzy programming is described for solving the formulated MOGPP. The formulated MOGPP has been solved by Lingo software and the dual solution is obtained. Subsequently with the help of dual solution of formulated MOGPP and the primal-dual relationship theorem the optimum allocations of multiple-response are obtained. A numerical example is given to illustrate the procedure.

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Notes

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I. Introduction

Sampling consists of several characteristics that are to be measured on every selected units of the sample. Such type of sampling are called "Multivariate or Multiple Response Sampling". Ghosh (1958) has given a note on stratified random sampling with multiple characters. Kokan and Khan (1967) proposed an optimum allocation in multivariate surveys and obtained an analytical solution. Ahsan and Khan (1977) have obtained an optimum allocation in multivariate stratified random sampling using prior information. Bethel (1985, 1989) has discussed an optimum allocation algorithm and sample allocation for multivariate surveys. Jahan *et al.* (1994, 2001) have discussed generalized compromise allocation and optimum compromise allocation. Jahan and Ahsan (1995) have obtained an optimum allocation using separable programming. Recently many authors have worked in the field of multivariate stratified sample surveys and obtained optimum allocations with the help of different techniques. Some of them are: Khan *et al.* (2003, 2008), Kozak (2006), Díaz-García and Ulloa (2006, 2008), Khan *et al.* (2010), Khowaja *et al.* (2011), Ansari *et al.* (2011), Ghufran *et al.* (2011), Varshney *et al.* (2011), Khan *et al.* (2012), Iftekhar *et al.* (2013), Gupta *et al.* (2013), Raghav *et al.* (2014) and many others have discussed the problem of optimum allocation in multivariate stratified sample surveys as a multi-objective programming problem and suggested techniques for solving problems.

The engineering design problem was firstly solved by Duffin and Zener in the early 1960s with the help of geometric programming (GP) and further extended by Duffin, Peterson and Zener (1967). Geometric programs are not (in general) convex optimization problems, but they can be transformed to convex problems by a change of variables and a transformation of the objective and constraints functions. The convex programming problems occurring in GP are generally represented by an exponential or power function. GP is a mathematical programming technique for optimizing positive polynomials, which are called posynomials. The degree of difficulty (DD) plays a

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significant role for solving a non-linear programming problem by GP method. If the degree of difficulty of primal problem is zero, then unique dual feasible solution exists. If the problem has positive degree of difficulty, then the objective function can be maximized by finding the dual feasible region, and if there is negative degree of difficulty then inconsistency of the dual constraints may occur. GP method was used by many authors such as: Ahmad and Charles (1987), Jitka Dupačová (2010), Maqbool *et al.* (2011), Ghosh and Roy (2013), Shafiullah *et al.* (2013). Multi-objective geometric programming problem was discussed by Ojha and Biswal (2010), Ojha and Das in (2010) and Islam, S. (2010) in different fields. Shafiullah *et al* (2014) have discussed the fuzzy geometric programming in multivariate stratified sample surveys in presence of non-response with quadratic cost function.

A system with vague and ambiguous information can neither be formulated nor solved effectively by traditional mathematics-based on optimization techniques nor probability-based stochastic optimization approaches. However, fuzzy set theory, which was developed by Zadeh in 1960's and fuzzy programming techniques provide a useful and efficient tool for modeling and optimizing such systems. Zimmermann, H. J. (1978) has discussed fuzzy programming and linear programming. Fuzzy multi-objective programming is given by many authors such as: Sakawa and Yano (1989, 1994), Kanaya (2010), fuzzy non-linear programming is given by many authors such as: Tang and Wang (1997), Tang and Richard (1998), Trappey et al. (1998), Nasseri (2008), Rehana and Mujumdar (2009), Mesbah et al. (2010), Maleki (2002), Kheirfam (2010), Shankar et al. (2010). Nikoo et al. (2013) have described optimal water and waste-load allocations in rivers using a fuzzy transformation technique and many others.

In this paper, we have formulated the problem of multiple-response sample surveys as a multi-objective geometric programming problem (MOGPP). The fuzzy programming approach has described for solving the formulated MOGPP and optimum allocation of sample sizes are obtained. A numerical example is presented to illustrate the procedure.

II. Formulation of The Problem

In stratified sampling the population of N units is first divided into L non-overlapping subpopulation called strata, of sizes $N_1, N_2, ..., N_h, ..., N_L$ with $\sum_{h=1}^{L} N_h = N$ and the respective sample of sizes within strata are drawn to construct the estimators of the unknown parameters which are denoted by $n_1, n_2, ..., n_h, ..., n_L$ with $\sum_{h=1}^{L} n_h = n$. The total cost C incurred in a sample survey is a function of sample allocations $n_h, h = 1,2,\cdots,L$

The problem of determining sample sizes $n_h, h = 1,2,\cdots,L$ is called the problem of allocation in stratified sampling literature. Usually, the total cost C incurred in a sample survey is a function of sample allocations $n_h, h = 1,2,\cdots,L$. The simplest form of the cost function used in a stratified sample survey is a linear function of sample sizes n_h given as:

$$C = c_0 + \sum_{h=1}^{L} c_h n_h \tag{1}$$

Where $c_h, h = 1,2,\cdots,L$ denote per unit cost of measurement in the h^{th} stratum and c_0 is the overhead cost.

If the cost of travelling between the selected units within a stratum is significant, and then the linear cost function may not be a good approximation to the actual cost incurred. Beardwood *et al.* (1959) suggested that the cost of visiting the c_h selected

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units in the h^{th} stratum may be taken as $t_h\sqrt{n_h}$, $h=1,2,\cdots,L,$ approximately, where t_h is the travel cost per unit in the h^{th} stratum. This conjecture is based on the fact that the distance between k randomly scattered points are proportional to $\sqrt{k}$. Under the above situation, the total cost of a stratified sample survey will be the sum of the overhead cost, the measurement cost and the travel cost.

Notes

The total cost C which is quadratic in $\sqrt{n_h}$ is given as:

$$C = c_0 + \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} \tag{2}$$

Ignoring finite population correction (fpc) of the overall population mean $\overline{Y}_j;\ j=1,2,\cdots,p$ of the j^{th} characteristic. $\overline{y}_{jh} = 1/n_h \sum_{k=}^{n_h} y_{jhk}$ is the sample mean from h^{th} stratum for j^{th} characteristic and y_{jhk} is the value of k^{th} selected unit of the sample from h^{th} stratum for the j^{th} characteristic $k=1,2,\cdots,n_h;h=1,2,\cdots,L;\ j=1,2,\cdots,p$ The variance will be given as:

$$V(\overline{y}_{jst}) = \sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right) W_h^2 S_{jh}^2 \tag{3}$$

The terms in the above eqn. (3) are independent of n_h and therefore it is sufficient to minimize only

$$V(\overline{y}_{jst}) = \sum_{h=1}^{L} \frac{W_h^2 S_{jh}^2}{n_h},\ j=1,2,\cdots,p \tag{4}$$

Multi-objective nonlinear programming problem (MNLPP) for finding out the optimum compromise allocation for a quadratic cost function is expressed as:

$$\left.\begin{array}{ll} Min\ V(\overline{y}_{jst}) = \sum_{h=1}^{L} \frac{W_h^2 S_{jh}^2}{n_h} & \\ subject\ \ to & \\ & \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} \le C_0 \\ and & n_h \ge 0\ ,\ h=1,2,\cdots,L \end{array}\right\},\ j=1,2,\cdots,p \tag{5}$$

where $C_0 = C - c_0$ is the cost available to meet the travel and measurement expenses, $V(\overline{y}_{jst})$ is the sampling variance and $S_{jh}^2, h=1,2,\cdots,L$ are the known population variances.

III. Geometric Programming Approach

The following multi-objective nonlinear programming problem (MONLPP) the cost function quadratic in $\sqrt{n_h}$ and significant travel cost are given as follows:

$$\left.\begin{array}{l} Min\ \ V\left(\bar{y}_{jst}\right)=\sum_{h=1}^{L}\frac{a_{hj}}{n_h} \\ Subject\ to \\ \qquad \sum_{h=1}^{L}c_h n_h+\sum_{h=1}^{L}t_h\sqrt{n_h}\le C_0, \\ and \qquad n_h\ge 0\ ,\ \mathrm{h}=1,2,\cdots,\mathrm{L} \end{array}\right\}\ j=1,2,\cdots,P \qquad (6)$$

Similarly, the expression (6) can be expressed in the standard primal GPP with cost functionquadratic in $\sqrt{n_h}$ where the travel cost is significant is given as follows:

$$\left.\begin{array}{l} Max\quad f_{0j}\left(n\right) \\ Subject\quad f_q\left(n\right)\le 1 \\ n_h\ge 0,\quad h=1,2,\cdots,L \end{array}\right\}\ j=1,2,...,\ p \qquad (7)$$

where $f_q\left(n\right)=\sum_{i\in j[q]}d_i n_1^{p_{i1}}\ n_2^{p_{i2}}\cdots n_L^{p_{iL}}\ ,q=o,1,2,\cdots,k$

or $f_q\left(n\right)=\sum_{i\in j[q]}d_i\left[\prod_{h=1}^{L}n_h^{p_{ih}}\right],d_i>0,n_h>0,q=o,1,2,\cdots,k,$

p_{ih} :arbitrary real numbers, d_i : positive and $f_q\left(n\right)$:posinomials

Let for simplicity $a_{hj}=\frac{W_h^2S_{jh}^2}{n_h}$ & $d_i=a_{hj}=\frac{c_h}{C_0}=\frac{t_h}{C_0}$

The dual form of the primal GPP which is stated in (6) can be given as:

$$\left.\begin{array}{ll} Max\ v_{0j}(w)=\prod_{q=0}^{k}\prod_{i\in j[q]}\left\{\left(\frac{d_i}{w_i}\right)^{w_i}\right\}\prod_{q=1}^{k}\left(\sum_{i\in j[q]}w_i\right)^{\sum_{i\in j[q]}w_i} & (i) \\ Subject\ \sum_{i\in[0]}w_i=1 & (ii) \\ \qquad \sum_{q=0}^{k}\sum_{i\in j[q]}p_{ih}\ w_i=0 & (iii) \\ \qquad w_i\ge 0,q=o,1,\cdots,k\ and\ \ i=1,2,\ldots,m_k & (iv) \end{array}\right\}\ j=1,\ldots,p. \qquad (8)$$

The above formulated GPP (8) can be solved in the following two-steps:

Step 1: For the Optimum value of the objective function, the objective function always takes the form:

$$C_0(x^*)=\left(\frac{Coeffi.\ of\ \ first\ term}{w_{01}}\right)^{w_{01}}\times\left(\frac{Coeffi.\ of\ \ Second\ \ term}{w_{02}}\right)^{w_{02}}$$

$$\times\ldots\ldots\times\left(\frac{Coeffi.\ of\ last\ term}{w_k}\right)^{w_k}\left(\sum w\text{'s in the first constraints}\right)^{\sum w\text{'s in the first constraints}}$$

Notes

$$\left(\sum w\text{'s in the last constraints}\right)^{\sum w\text{'s in the last constraints}}$$

The Multi-Objective objective function for our problem is:

$$\prod_{q=0}^{k}\prod_{i\in j[q]}\left\{\left(\frac{d_i}{w_i}\right)^{w_i}\right\}\prod_{q=1}^{k}\left(\sum_{i\in j[q]} w_i\right)^{\sum_{i\in j[q]} w_i}$$

Notes

Step 2: The equations that can be used for GPP for the weights are given below:

$\sum_{i\in j[o]} w_i$ in the objective function$=1$ (Normality condition)

and for each primal variable n_h and $\sqrt{n_h}$ having m terms.

$$\sum_{i=1}^{m_k}\left(w_i \text{ for each term}\right)\times\left(\text{exponent on } n_h \text{ and } \sqrt{n'_h} \text{ in that term}\right)=0 \quad \text{(Orthogonality condition)}$$

and $w_i \geq 0$ (Positivity condition).

The above problems (8) has been solved with the help of steps (1-2) discussed in section (3) and the corresponding solutions w_{0i}^{*} is the unique solution to the dual constraints; it will also maximize the objective function for the dual problem. Next, the solution of the primal problem will be obtained using primal-dual relationship theorem which is given below:

a) Primal-dual relationship theorem

If w_{0i}^{*} is a maximizing point for dual problem (9), each optimal values of the multi-Response model which is the minimizing points $\left(n^{*}\right)$ for the Primal GPP's (8) satisfies the system of equations:

$$f_{0j}(n)=\begin{cases} w_{0i}^{*}\, v\left(w^{*}\right), & i\in J[0], \\ \dfrac{w_{ij}}{v_L\left(w_{0i}^{*}\right)}, & i\in J[L], \end{cases} \tag{9}$$

where L ranges over all positive integers for which $v_L\left(w_{0i}^{*}\right)>0$.

The optimal values of the sample sizes of the problems n_h^{*} can be calculated with the help of the primal - dual relationship theorem (9).

IV. Fuzzy Geometric Programming Approach

The solution procedure to solve the problem (9) consists of the following steps:

Step-1: Solve the MOGPP as a single objective problem using only one objective at a time and ignoring the others. These solutions are known as ideal solution.

Step-2: From the results of step-1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

Notes

$$
\begin{array}{c c}
 & \begin{array}{ccccccc} f_{01}(n) & f_{02}(n) & \cdots & f_{0j}(n) & \cdots & f_{0p}(n) \end{array} \\
\begin{array}{c} (n^{(1)}) \\ (n^{(2)}) \\ \vdots \\ (n^{(j)}) \\ \vdots \\ (n^{(p)}) \end{array} &
\begin{bmatrix}
f_{01}^{*}(n^{(1)}) & f_{02}(n^{(1)}) & \cdots & f_{0j}(n^{(1)}) & \cdots & f_{0p}(n^{(1)}) \\
f_{01}(n^{(2)}) & f_{02}^{*}(n^{(2)}) & \cdots & f_{0j}(n^{(2)}) & \cdots & f_{0p}(n^{(2)}) \\
\vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
f_{01}(n^{(j)}) & f_{02}(n^{(j)}) & \cdots & f_{0j}^{*}(n^{(j)}) & \cdots & f_{0p}(n^{(j)}) \\
\vdots & \vdots & & \vdots & & \vdots \\
f_{01}(n^{(p)}) & f_{02}(n^{(p)}) & \cdots & .f_{0j}(n^{(p)}) & \cdots & f_{0p}^{*}(n^{(p)})
\end{bmatrix}
\end{array}
$$

Here $(n^{(1)}),(n^{(2)}),\cdots,(n^{(j)}),\cdots,(n^{(p)})$ are the ideal solutions of the objective functions $f_{01}(n^{(1)}),\ f_{02}(n^{(2)}),\cdots,\ f_{0j}(n^{(j)}),\cdots,\ f_{0p}(n^{(p)})$.

So $U_j = Max\left\{f_{01}(n^{(1)}),\ f_{02}(n^{(2)}),\ \cdots,\ f_{0p}(n^{(p)})\right\}$ and $L_j = f_{0j}^{*}(n^{(j)}),\ j=1,2,...,p$.

$\left[U_j\right.$ and L_j be the upper and lower bonds of the j^{th} objective function $\left.f_{0j}(n),\ j=1,2,..,p.\right]$

Step 3: The membership function for the given problem can be define as:

$$
\mu_j\left(f_{0j}(n)\right) = \begin{cases} 0\,, & if\ \ f_{0j}(n) \geq U_j \\ \dfrac{U_j(n) - \ f_{0j}(n)}{U_j(n) - L_j(n)}, & if\ L_j \leq f_{0j}(n) \geq U_j,\ \ j = 1, 2, \ldots, p \\ 1, & if\ \ f_{0j}(n) \leq L_j \end{cases} \tag{10}
$$

Here $U_j(n)$ is a strictly monotonic decreasing function with respect to $f_{0j}(n)$.

Following figure 3.1 illustrates the graph of the membership function $\mu_j\left(f_{0j}(n)\right)$.

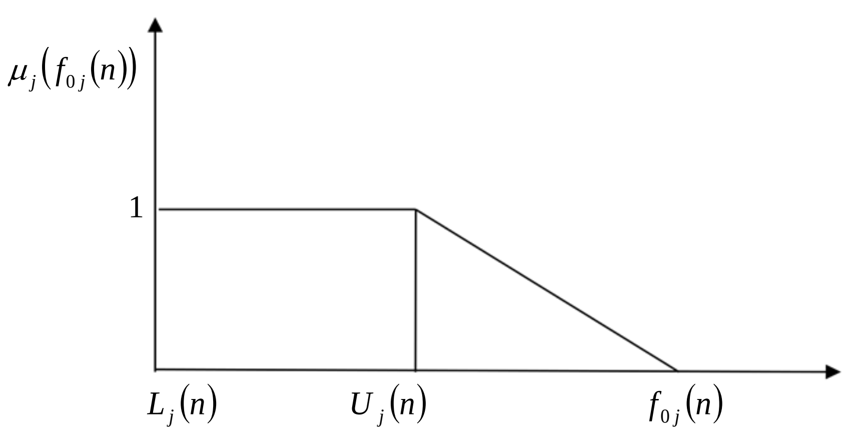

Figure 3.1 : Membership function for minimization variances problem

The membership functions in Eqn. (10)

i.e. $\mu_j\left(f_{0j}(n)\right),\ j = 1,\ 2,\ldots,\ p$, Therefore the general aggregation function can be defined as

$$\mu_{\tilde{D}}(n)=\mu_{\tilde{D}}\{\mu_1(f_{01}(n)),\mu_2(f_{02}(n)),\ldots,\mu_p(f_{0p}(n))\}$$

The fuzzy multi-objective formulation of the problem with cost function quadratic in $\sqrt{n_h}$ and significant travel cost can be defined as:

Notes

$$\left.\begin{array}{l} Max\ \mu_{\tilde{D}}(n) \\ Subject\ to\ \sum_{h=1}^{L}\frac{c_h}{C_0}n'_h+\sum_{h=1}^{L}\frac{t_h}{C_0}\sqrt{n'_h}\le C-c_0=C_0; \\ \qquad n_h\ge 0\ and\ h=1,2,\cdots,L. \end{array}\right\} \tag{11}$$

The problem to find the optimal values of (n^*) for this convex-fuzzy decision based on addition operator (like Tewari *et. al.* (1987)).Therefore the problem (11) is reduced according to max-addition operator as

$$\left.\begin{array}{l} Max\ \mu_D(n^*)=\sum_{j=1}^{p}\mu_j(f_{0j}(n))=\sum_{j=1}^{p}\frac{U_j-(f_{0j}(n))}{U_j-L_j} \\ Subject\ to\ \sum_{h=1}^{L}\frac{c_h}{C_0}n'_h+\sum_{h=1}^{L}\frac{t_h}{C_0}\sqrt{n'_h}\le C_0; \\ \qquad 0\le\mu_j(f_{0j}(n))\le 1, \\ \qquad n_h\ge 0\ and\ h=1,2,\cdots,L. \end{array}\right\} \tag{12}$$

The problem (12) reduces to

$$\left.\begin{array}{l} Max\ \mu_D(n^*)=\sum_{j=1}^{p}\left\{\frac{U_j}{U_j-L_j}-\frac{(f_{0j}(n))}{U_j-L_j}\right\} \\ Subject\ to \\ \qquad f_q(n)\le 1; \\ \qquad n_h\ge 0\ \ and\ \ j=1,2,\ldots,p. \end{array}\right\} \tag{13}$$

$$\text{where } f_q(n)=\sum_{h=1}^{L}\frac{c_h}{C_0}n'_h+\sum_{h=1}^{L}\frac{t_h}{C_0}\sqrt{n'_h}$$

The problem (13) maximizes if the function $F_{oj}(n)=\left\{\frac{(f_{0j}(n))}{U_j-L_j}\right\}$ attain the minimum values.

The fuzzy multi-objective formulation of the standard primal problem with cost function quadratic in $\sqrt{n_h}$ and significant travel cost can be defined as:

$$\left.\begin{aligned} &Min \quad \sum_{j=0}^{p} F_{oj}(n') \\ &Subject\ to \\ &\qquad f_q(n') \le 1; \\ &and \ \ n_h \ge 0\ ,\ j = 1,2,\ldots,p. \end{aligned}\right\} \tag{14}$$

$$\text{where } f_q(n) = \sum_{h=1}^{L} \frac{c_h}{C_0} n'_h + \sum_{h=1}^{L} \frac{t_h}{C_0} \sqrt{n'_h}$$

The dual form of the primal GPP which is stated in (14) can be given as:

$$\left.\begin{aligned} &Max\ v(w) = \prod_{q=0}^{k} \prod_{i\in j[q]} \left\{ \left(\frac{d_i}{w_i} \right)^{w_i} \right\} \prod_{q=1}^{k} \left(\sum_{i\in j[q]} w_i \right)^{\sum_{i\in j[q]} w_i} && (i) \\ &Subject \ \sum_{i\in[0]} w_i = 1 && (ii) \\ &\sum_{q=0}^{k} \sum_{i\in j[q]} p_{ih}\ w_i = 0 && (iii) \\ &w_i \ge 0, q = o,1,\cdots,k \ and \ \ i = 1,2,\ldots,m_k && (iv) \end{aligned}\right\} \tag{15}$$

The optimal values of the sample sizes of the problems n_h^* can be calculated with the help of the primal-dual relationship theorem (9).

V. Numerical Example

In the table below the stratum sizes, stratum weights, stratum standard deviations, measurement costs, and the travel costs within stratum are given for four different characteristics under study in a population stratified in five strata. The data are mainly from Chatterjee (1968) and rest of data from Ghufran *et al.* (2011).

Table 1 : The Values of $N_h, W_h, c_h, t_h \ and\ S_{jh}$ for five Strata and four characteristics

H	N_h	W_h	c_h	t_h	S_{jh}			
					S_{1h}	S_{2h}	S_{3h}	S_{4h}
1	1500	0.25	1	0.5	28	206	38	120
2	1920	0.32	1	0.5	24	133	26	184
3	1260	0.21	1.5	1	32	48	44	173
4	480	0.08	1.5	1	54	37	78	92
5	840	0.14	2	1.5	67	9	76	117

The total budget of the survey is assumed to be 1500 units with an overhead cost $c_0 = 300$ units. Thus $C_0 = C - c_0 = 1500 - 300 = 1200$ units are available for measurement and travel within strata for approaching the selected units for measurement.

For solving MOGPP by using fuzzy programming, we shall first solve the four sub-problems:

Notes

a) *Sub problem1:*

On substituting the table values in sub-problem 1, we have obtained the expressions given below:

Notes

$$\left.\begin{aligned} &Min\ f_{01} = \frac{49}{n_1} + \frac{58.9824}{n_2} + \frac{45.1584}{n_3} + \frac{18.6624}{n_4} + \frac{87.9844}{n_5} \\ &Subject\ to \\ &\quad 0.0008333 n_1 + 0.0008333 n_2 + 0.00125 n_3 + 0.00125 n_4 \\ &\quad + 0.001667 n_5 + 0.0004167\sqrt{n_1} + 0.0004167\sqrt{n_2} \\ &\quad + 0.0008333\sqrt{n_3} + 0.0008333\sqrt{n_4} + 0.00125\sqrt{n_5} \le 1 \\ &\quad and \quad n_h \ge 0, h = 1,2,\dots,L \end{aligned}\right\} \quad (16)$$

The dual of the problem (15) is obtained as:

$$\left.\begin{aligned} &Max\ \ v(w_{0i}^*) = \left((49/w_{01})^{w_{01}}\right)\times\left((58.9824/w_{02})^{w_{02}}\right)\times\left((45.1584/w_{03})^{w_{03}}\right) \\ &\times\left((18.6624/w_{04})^{w_{04}}\right)\times\left((87.9844/w_{05})^{w_{05}}\right)\times\left(\left(\frac{0.0008333}{w_{11}}\right)^{w_{11}}\right) \\ &\times\left(\left(\frac{0.0008333}{w_{12}}\right)^{w_{12}}\right)\times\left(\left(\frac{0.00125}{w_{13}}\right)^{w_{13}}\right)\times\left(\left(\frac{0.00125}{w_{14}}\right)^{w_{14}}\right) \\ &\times\left(\left(\frac{0.001667}{w_{15}}\right)^{w_{15}}\right)\times\left(\left(\frac{0.0004167}{w_{16}}\right)^{w_{16}}\right)\times\left(\left(\frac{0.0004167}{w_{17}}\right)^{w_{17}}\right) \\ &\times\left(\left(\frac{0.0008333}{w_{18}}\right)^{w_{18}}\right)\times\left(\left(\frac{0.0008333}{w_{19}}\right)^{w_{19}}\right)\times\left(\left(\frac{0.00125}{w_{20}}\right)^{w_{20}}\right)\times \\ &((w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})\wedge \\ &(w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})); \qquad (i) \\ &Subject\ to \\ &\qquad w_{01}+w_{02}+w_{03}+w_{04}+w_{05}=1 \ \ \text{(normality condition)} \qquad (ii) \\ &\qquad \left.\begin{aligned} &-w_{01}+w_{11}+(1/2)w_{16}=0 \\ &-w_{02}+w_{12}+(1/2)w_{17}=0 \\ &-w_{03}+w_{13}+(1/2)w_{18}=0 \\ &-w_{04}+w_{14}+(1/2)w_{19}=0 \\ &-w_{05}+w_{15}+(1/2)w_{20}=0 \end{aligned}\right\}\text{(orthogonality condition)} \ \ (iii) \\ &\qquad \left.\begin{aligned} &w_{01},w_{02},w_{03},w_{04},w_{05}>0 \\ &w_{11},w_{12},w_{13},w_{14},w_{15}\ w_{16}, \\ &w_{17},w_{18},w_{19},w_{20}\ge 0 \end{aligned}\right\} \quad \text{(positivity condition)} \qquad (iv) \end{aligned}\right\} \quad (17)$$

For orthogonality condition defined in expression 17(*iii*) are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{19} \\ w_{20} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} + w_{16} = 0 \\ -w_{02} + w_{12} + w_{17} = 0 \\ -w_{03} + w_{13} + w_{18} = 0 \\ -w_{04} + w_{14} + w_{19} = 0 \\ -w_{05} + w_{15} + w_{20} = 0 \end{Bmatrix}$$

Solving the above formulated dual problem (19), we have the corresponding solution as:

$w_{01} = 0.1682412, w_{02} = 0.1845105, w_{03} = 0.1987012, w_{04} = 0.1281561, w_{05} = 0.3203910$ and $v(w^*) = 1.503975$.

Using the primal dual- relationship theorem (9), we have the optimal solution of primal problem: *i.e.*, the optimal sample sizes are computed as follows:

$$f_{0j}(n) = w_{0i}^* \, v(w_{0i}^*)$$

In expression (16), we calculate the values of n as:

$$f_{01}(n) = w_{01}^* \, v(w_{0i}^*)$$
$$\frac{49}{n_1} = 0.1682412 \times 1.503975$$
$$\Rightarrow n_1 = 193.6524$$

$$f_{01}(n) = w_{02}^* \, v(w_{0i}^*)$$
$$\frac{58.9824}{n_2} = 0.1845105 \times 1.503975$$
$$\Rightarrow n_2 \cong 212.5498$$

$$f_{01}(n) = w_{03}^* \, v(w_{0i}^*)$$
$$\frac{45.1584}{n_3} = 0.1987012 \times 1.503975$$
$$\Rightarrow n_3 = 151.1115$$

$$f_{01}(n) = w_{04}^* \, v(w_{0i}^*)$$
$$\frac{18.6624}{n_4} = 0.1281561 \times 1.503975$$
$$\Rightarrow n_4 = 96.8250$$

$$f_{01}(n) = w_{05}^* \, v(w_{0i}^*)$$
$$\frac{87.9844}{n_5} = 0.3203910 \times 1.503975$$
$$\Rightarrow n_5 = 182.5932$$

Notes

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The optimal values and the objective function value are given below:

$n_1^* = 193.6524, n_2^* = 212.5498, n_3^* = 151.1115$, $n_4^* = 96.8250, n_5^* = 182.5932$;
and the objective value of the primal problem is 1.503975.

$$\left.\begin{aligned} & Min\ \ f_{02} = \frac{2652.25}{n_1} + \frac{1811.3536}{n_2} + \frac{101.6064}{n_3} + \frac{8.7616}{n_4} + \frac{1.5876}{n_5} \\ & Subject\ \ to \\ & \quad 0.0008333 n_1 + 0.0008333 n_2 + 0.00125 n_3 + 0.00125 n_4 \\ & \quad + 0.001667 n_5 + 0.0004167\sqrt{n_1} + 0.0004167\sqrt{n_2} \\ & \quad + 0.0008333\sqrt{n_3} + 0.0008333\sqrt{n_4} + 0.00125\sqrt{n_5} \le 1 \\ & and \qquad n_h \ge 0\ ,\ h = 1,2,\dots,L \end{aligned}\right\} \tag{18}$$

$$\left.\begin{aligned} Max\ \ v(w_{0i}^*) = & \left((2652.25/w_{01})^{w_{01}}\right) \times \left((1811.3536/w_{02})^{w_{02}}\right) \times \left((101.6064/w_{03})^{w_{03}}\right) \\ & \times \left((8.7616/w_{04})^{w_{04}}\right) \times \left((1.5876/w_{05})^{w_{05}}\right) \times \left(\left(\frac{0.0008333}{w_{11}}\right)^{w_{11}}\right) \\ & \times \left(\left(\frac{0.0008333}{w_{12}}\right)^{w_{12}}\right) \times \left(\left(\frac{0.00125}{w_{13}}\right)^{w_{13}}\right) \times \left(\left(\frac{0.00125}{w_{14}}\right)^{w_{14}}\right) \\ & \times \left(\left(\frac{0.001667}{w_{15}}\right)^{w_{15}}\right) \times \left(\left(\frac{0.0004167}{w_{16}}\right)^{w_{16}}\right) \times \left(\left(\frac{0.0004167}{w_{17}}\right)^{w_{17}}\right) \\ & \times \left(\left(\frac{0.0008333}{w_{18}}\right)^{w_{18}}\right) \times \left(\left(\frac{0.0008333}{w_{19}}\right)^{w_{19}}\right) \times \left(\left(\frac{0.00125}{w_{20}}\right)^{w_{20}}\right) \times \\ & ((w_{11} + w_{12} + w_{13} + w_{14} + w_{15} + w_{16} + w_{17} + w_{18} + w_{19} + w_{20})\wedge \\ & (w_{11} + w_{12} + w_{13} + w_{14} + w_{15} + w_{16} + w_{17} + w_{18} + w_{19} + w_{20})); \qquad (i) \\ Subject\ to & \\ & w_{01} + w_{02} + w_{03} + w_{04} + w_{05} = 1 \ \ \text{(normality condition)} \qquad (ii) \\ & \left.\begin{aligned} & -w_{01} + w_{11} + (1/2)\, w_{16} = 0 \\ & -w_{02} + w_{12} + (1/2)\, w_{17} = 0 \\ & -w_{03} + w_{13} + (1/2)\, w_{18} = 0 \\ & -w_{04} + w_{14} + (1/2)\, w_{19} = 0 \\ & -w_{05} + w_{15} + (1/2)\, w_{20} = 0 \end{aligned}\right\} \text{(orthogonality condition)} \quad (iii) \\ & \left.\begin{aligned} & w_{01}, w_{02}, w_{03}, w_{04}, w_{05} > 0 \\ & w_{11}, w_{12}, w_{13}, w_{14}, w_{15}\ w_{16}, \\ & w_{17}, w_{18}, w_{19}, w_{20} \ge 0 \end{aligned}\right\} \ \ \text{(positivity condition)} \qquad (iv) \end{aligned}\right\} \tag{19}$$

For orthogonality condition defined in expression 19(iii) are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{19} \\ w_{20} \end{pmatrix} = \begin{Bmatrix} -w_{01}+w_{11}+w_{16}=0 \\ -w_{02}+w_{12}+w_{17}=0 \\ -w_{03}+w_{13}+w_{18}=0 \\ -w_{04}+w_{14}+w_{19}=0 \\ -w_{05}+w_{15}+w_{20}=0 \end{Bmatrix}$$

Solving the above formulated dual problems, we have the corresponding solution as:

$w_{01}=0.4590439, w_{02}=0.3795607, w_{03}=0.1114193, w_{04}=0.3320634, w_{05}=0.01676970$ and $v(w^*)=10.78444$.

The optimal values and the objective function value are given below:

$n_1^*=535.7506, n_2^*=442.5113, n_3^*=84.5596, n_4^*=24.4661, n_5^*=8.77846;$

and the objective value of the primal problem is 10.78444.

$$\left.\begin{aligned} & Min\ \ f_{03}=\frac{90.25}{n_1}+\frac{69.2224}{n_2}+\frac{85.3776}{n_3}+\frac{38.9376}{n_4}+\frac{113.2096}{n_5} \\ & Subject\ \ to \\ & \qquad 0.0008333n_1+0.0008333n_2+0.00125n_3+0.00125n_4 \\ & \qquad +0.001667n_5+0.0004167\sqrt{n_1}+0.0004167\sqrt{n_2} \\ & \qquad +0.0008333\sqrt{n_3}+0.0008333\sqrt{n_4}+0.00125\sqrt{n_5}\le 1 \\ & and \qquad n_h\ge 0, h=1,2,\dots,L \end{aligned}\right\} \tag{20}$$

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$$\left.\begin{aligned}
&Max \quad v(w_{0i}^{*}) = \left(\left(90.25/w_{01}\right)^{w_{01}}\right)\times\left(\left(69.2224/w_{02}\right)^{w_{02}}\right)\times\left(\left(85.3776/w_{03}\right)^{w_{03}}\right)\\
&\qquad\times\left(\left(38.9376/w_{04}\right)^{w_{04}}\right)\times\left(\left(113.2096/w_{05}\right)^{w_{05}}\right)\times\left(\left(\frac{0.0008333}{w_{11}}\right)^{w_{11}}\right)\\
&\qquad\times\left(\left(\frac{0.0008333}{w_{12}}\right)^{w_{12}}\right)\times\left(\left(\frac{0.00125}{w_{13}}\right)^{w_{13}}\right)\times\left(\left(\frac{0.00125}{w_{14}}\right)^{w_{14}}\right)\\
&\qquad\times\left(\left(\frac{0.001667}{w_{15}}\right)^{w_{15}}\right)\times\left(\left(\frac{0.0004167}{w_{16}}\right)^{w_{16}}\right)\times\left(\left(\frac{0.0004167}{w_{17}}\right)^{w_{17}}\right)\\
&\qquad\times\left(\left(\frac{0.0008333}{w_{18}}\right)^{w_{18}}\right)\times\left(\left(\frac{0.0008333}{w_{19}}\right)^{w_{19}}\right)\times\left(\left(\frac{0.00125}{w_{20}}\right)^{w_{20}}\right)\times\\
&\quad ((w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})\wedge\\
&\quad (w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})); \qquad (i)\\
&\textit{Subject to}\\
&\qquad w_{01}+w_{02}+w_{03}+w_{04}+w_{05}=1 \quad \text{(normality condition)} \qquad (ii)\\
&\qquad \left.\begin{aligned}
-w_{01}+w_{11}+(1/2)\,w_{16}&=0\\
-w_{02}+w_{12}+(1/2)\,w_{17}&=0\\
-w_{03}+w_{13}+(1/2)\,w_{18}&=0\\
-w_{04}+w_{14}+(1/2)\,w_{19}&=0\\
-w_{05}+w_{15}+(1/2)\,w_{20}&=0
\end{aligned}\right\}\text{(orthogonality condition)} \quad (iii)\\
&\qquad \left.\begin{aligned}
&w_{01},w_{02},w_{03},w_{04},w_{05}>0\\
&w_{11},w_{12},w_{13},w_{14},w_{15}\ w_{16},\\
&w_{17},w_{18},w_{19},w_{20}\geq 0
\end{aligned}\right\}\quad \text{(positivity condition)} \quad (iv)
\end{aligned}\right\} \qquad (21)$$

For orthogonality condition defined in expression 20(*iii*) are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{19} \\ w_{20} \end{pmatrix} = \begin{Bmatrix} -w_{01}+w_{11}+w_{16} = 0 \\ -w_{02}+w_{12}+w_{17} = 0 \\ -w_{03}+w_{13}+w_{18} = 0 \\ -w_{04}+w_{14}+w_{19} = 0 \\ -w_{05}+w_{15}+w_{20} = 0 \end{Bmatrix}$$

Solving the above formulated dual problems, we have the corresponding solution as:

$w_{01} = 0.1826144, w_{02} = 0.1600251, w_{03} = 0.2184575, w_{04} = 0.1479353, w_{05} = 0.2909677$ and $v(w^*) = 2.349672$.

The optimal values and the objective function value are given below:

$n_1^* = 210.3318, n_2^* = 184.0990, n_3^* = 166.3297, n_4^* = 112.0186, n_5^* = 165.5889;$

and the objective value of the primal problem is 2.349672.

$$\left.\begin{aligned} & Min\ \ f_{04} = \frac{900}{n_1} + \frac{3466.8544}{n_2} + \frac{1319.8689}{n_3} + \frac{54.1696}{n_4} + \frac{268.3044}{n_5} \\ & Subject\ \ to \\ & \qquad 0.0008333n_1 + 0.0008333n_2 + 0.00125n_3 + 0.00125n_4 \\ & \qquad + 0.001667n_5 + 0.0004167\sqrt{n_1} + 0.0004167\sqrt{n_2} \\ & \qquad + 0.0008333\sqrt{n_3} + 0.0008333\sqrt{n_4} + 0.00125\sqrt{n_5} \le 1 \\ & and \qquad\qquad n_h \ge 0\ \ , h = 1,2,\ldots,L \end{aligned}\right\} \tag{22}$$

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$$
\left.\begin{aligned}
&Max\quad v(w_{0i}^{*}) = \left((900/w_{01})^{w_{01}}\right)\times\left((3466.8544/w_{02})^{w_{02}}\right)\times\left((1319.8689/w_{03})^{w_{03}}\right)\\
&\qquad \times\left((54.1696/w_{04})^{w_{04}}\right)\times\left((268.3044/w_{05})^{w_{05}}\right)\times\left(\left(\frac{0.0008333}{w_{11}}\right)^{w_{11}}\right)\\
&\qquad \times\left(\left(\frac{0.0008333}{w_{12}}\right)^{w_{12}}\right)\times\left(\left(\frac{0.00125}{w_{13}}\right)^{w_{13}}\right)\times\left(\left(\frac{0.00125}{w_{14}}\right)^{w_{14}}\right)\\
&\qquad \times\left(\left(\frac{0.001667}{w_{15}}\right)^{w_{15}}\right)\times\left(\left(\frac{0.0004167}{w_{16}}\right)^{w_{16}}\right)\times\left(\left(\frac{0.0004167}{w_{17}}\right)^{w_{17}}\right)\\
&\qquad \times\left(\left(\frac{0.0008333}{w_{18}}\right)^{w_{18}}\right)\times\left(\left(\frac{0.0008333}{w_{19}}\right)^{w_{19}}\right)\times\left(\left(\frac{0.00125}{w_{20}}\right)^{w_{20}}\right)\times\\
&\qquad ((w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})\wedge\\
&\qquad (w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20}));\qquad (i)\\
&Subject\ to\\
&\qquad w_{01}+w_{02}+w_{03}+w_{04}+w_{05}=1\quad \text{(normality condition)}\qquad (ii)\\
&\qquad \left.\begin{aligned}
&-w_{01}+w_{11}+(1/2)\,w_{16}=0\\
&-w_{02}+w_{12}+(1/2)\,w_{17}=0\\
&-w_{03}+w_{13}+(1/2)\,w_{18}=0\\
&-w_{04}+w_{14}+(1/2)\,w_{19}=0\\
&-w_{05}+w_{15}+(1/2)\,w_{20}=0
\end{aligned}\right\}\text{(orthogonality condition)}\qquad (iii)\\
&\qquad \left.\begin{aligned}
&w_{01},w_{02},w_{03},w_{04},w_{05}>0\\
&w_{11},w_{12},w_{13},w_{14},w_{15}\ w_{16},\\
&w_{17},w_{18},w_{19},w_{20}\geq 0
\end{aligned}\right\}\quad \text{(positivity condition)}\qquad (iv)
\end{aligned}\right\}\qquad (23)
$$

For orthogonality condition defined in expression 23(*iii)* are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{19} \\ w_{20} \end{pmatrix} = \left\{ \begin{array}{l} -w_{01} + w_{11} + w_{16} = 0 \\ -w_{02} + w_{12} + w_{17} = 0 \\ -w_{03} + w_{13} + w_{18} = 0 \\ -w_{04} + w_{14} + w_{19} = 0 \\ -w_{05} + w_{15} + w_{20} = 0 \end{array} \right\}$$

Solving the above formulated dual problems, we have the corresponding solution as:

$w_{01} = 0.1807496, w_{02} = 0.3538838, w_{03} = 0.2688674, w_{04} = 0.05522913, w_{05} = 0.1412701$

and $v(w^*) = 23.86496$

The optimal values and the objective function value are given below:

$n_1^* = 208.6433, n_2^* = 410.5009, n_3^* = 205.6989, n_4^* = 41.09857, n_5^* = 79.5823;$

and the objective value of the primal problem is 23.86496.

VI. Conclusions

This paper constitutes a reflective study of fuzzy programming for solving the multi-objective geometric programming problem (MOGPP). The problem of multiple-response in stratified sample survey has been formulated as MOGPP and the dual solution is obtained with the help of Lingo software. The optimum allocations are obtained with the help of primal-dual relationship theorem along with corresponding dual solution. A numerical example is illustrated to ascertain the practical utility of the given method in multiple-response stratified sample surveys.

References Références Referencias

1. Ahmed, J. and. Bonham Charles D. (1987): Application of Geometric Programming to Optimum Allocation Problems in Multivariate Double Sampling. *Appl. Maths. and Comm.*21 (2), pp. 157-169.
2. Ahsan, M.J. and Khan, S.U. (1977). Optimum allocation in multivariate stratified random sampling using prior information. *Journal of Indian Statistical Association* 15, 57-67.
3. Ansari, A.H., Varshney, R., Najmussehar and Ahsan, M.J. (2011): An optimum multivariate multi-objective stratified sampling design. *Metron*, LXIX(3), 227- 250.

Notes

Notes

4. Aoyama, H. (1963). Stratified random sampling with optimum allocation for multivariate populations. *Annals of the Institute of Statistical Mathematics* 14, 251-258.
5. Beardwood, J., Halton, J.H. and Hammersley, J.M. (1959). The shortest path through many points, *Proc. Cambridge Phil. Soc.*, 55, 299-327.
6. Bethel, J. (1985). An optimum allocation algorithm for multivariate surveys. *Proceedings of the Survey Research Methods section*, American Statistical Association 209-212.
7. Bethel, J. (1989). Sample allocation in multivariate surveys. *Survey Methodology* 15, 47-57.
8. Davis, M. and Rudolph, E.S. (1987): Geometric programming for optimal allocation of integrated samples in quality control. *Comm.Stat.Theo.Meth.,* 16 (11), 3235-3254.
9. Díaz-García, J.A. & Cortez, L.U. (2006). Optimum allocation in multivariate stratified sampling: multi-objective programming, *Comunicación Técnica No. I-06-07/28-03-2006 (PE/CIMAT)*, México.
10. Díaz-García, J.A. and Cortez, L.U. (2008). Multi-objective optimisation for optimum allocation in multivariate stratified sampling, *Survey Methodology* 34, 215-222.
11. Díaz-García, J.A. and Cortez, L.U. 2008. Multi-objective optimisation for optimum allocation in multivariate stratified sampling, *Survey Methodology* 34, 215-222.
12. Duffin, R.J., Peterson, E.L., Zener, C. (1967): Geometric programming: Theory & applications. *New York: John Wiley & Sons*.
13. Ghufran, S., Khowaja, S. Najmussehar, and Ahsan, M.J. (2011). A multiple response stratified sampling design with travel cost. The South Pacific Journal of Natural and Applied Sciences, 29, 31-39.
14. Gupta, N., Ali, I., Bari, A. (2013) An Optimal Chance Constraint Multivariate Stratified Sampling Design Using Auxiliary Information, *Journal of Mathematical Modelling and Algorithms in Operations Research*, DOI: 10.1007/s10852-013-9237-5 (*JMMA, Springer*).
15. Gupta, N., Shafiullah, Iftekhar, S. and Bari, A. (2012). Fuzzy Goal Programming Approach to solve Non-Linear Bi-Level Programming Problem in stratified double sampling design in presence of non-response. *International Journal of Scientific & Engineering research,* 3(10), 1-9.
16. Iftekhar, S., Haseen, S., Ali, Q.M. and Bari, A. (2013). A Compromise Solution in Multivariate Surveys with stochastic random cost function, *International Journal of Engineering Science and Technology,* 4(6), 2400-2406.
17. Islam, S. (2010): Multi-Objective Geometric Programming Problem and its Applications. *Yugoslav Journal of Operations Research*, 20 (2), pp. 213-227.
18. Jahan, N. & Ahsan, M.J. 1995. Optimum allocation using separable programming. *Dhaka University Journal of Science* 43, 157-164.
19. Jahan, N., Khan, M.G.M. and Ahsan, M.J. 1994. A generalized compromise allocation. *Journal of the Indian Statistical Association* 32, 95-101.
20. Jahan, N., Khan, M.G.M. and Ahsan, M.J. 2001. Optimum compromise allocation using dynamic programming. *Dhaka University Journal of Science* 49, 197-202.
21. Kanaya , Z. A. (2010). An Interactive Method for Fuzzy Multi objective Nonlinear Programming Problems, JKAU *Sci.*, 22(1), pp. 103-112.
22. Khan, M., Ali, I., Raghav, Y. S. and Bari A. (2012). Allocation in Multivariate Stratified Surveys with Non-Linear Random Cost Function. *American Journal of Operations Research,* 2 (1), 100-105.
23. Khan, M.G.M., Khan, E.A. and Ahsan, M.J. 2003. An optimal multivariate stratified sampling design using dynamic programming. *Australian & New Zealand J. Statist* 45, 107-113.
24. Khan, M.G.M., Khan, E.A. and Ahsan, M.J. 2008. Optimum allocation in multivariate stratified sampling in presence of non-response. *Journal of Indian Society of Agricultural Statistics* 62, 42-48.

25. Khan, M.G.M., Maiti, T. and Ahsan, M.J. 2010. An optimal multivariate stratified sampling design using auxiliary information: an integer solution using goal programming approach. *Journal of Official Statistics* 26, 695-708.
26. Khan, M.G.M., Maiti, T. and Ahsan, M.J. 2010. An optimal multivariate stratified sampling design using auxiliary information: an integer solution using goal programming approach. *Journal of Official Statistics* 26, 695-708.
27. Kheirfam, B. F. H. (2010). Sensitivity analysis for fuzzy linear Programming problems with Fuzzy variables, *Advanced Model and Optimization*, 12(2).
28. Kokan, A.R. and& Khan, S.U. 1967. Optimum allocation n multivariate surveys: An analytical solution. *Journal of Royal Statistical Society* B 29, 115-125.
29. Kozak, M. 2006. On sample allocation in multivariate surveys. Communication in *Statistics-Simulation and Computation* 35, 901-910.
30. LINGO User's Guide (2013). Published by Lindo Systems Inc., 1415 North Dayton Street, hicago, Illinois-60622, USA.
31. Maleki, H. R. (2002) .Ranking functions and their applications to fuzzy linear programming, Far East. *Appl. Math.* 4, pp. 283–301.
32. Maqbool, S., Mir, A. H. and Mir, S. A. (2011). Geometric Programming Approach to Optimum Allocation in Multivariate Two-Stage Sampling Design. Electronic Journal of Applied Statistical Analysis, 4(1), pp. 71 – 82.
33. Mesbah, S.M., Kerachian, R. and Torabian, A. (2010). Trading Pollutant Discharge Permits in Rivers Using Fuzzy Nonlinear Cost Functions. *Desalinatio*n 250(1), pp. 313-317.
34. Nasseri, S.H. (2008).Fuzzy nonlinear optimization, *The Journal of Nonlinear Analysis and its Applications*. 1(4) pp. 230-235.
35. Nikoo, M.R., Kerachian, R., Karimi, A. and Azadnia, A.A. (2013). Optimal Water and Waste-Load Allocations in Rivers Using a Fuzzy Transformation Technique: A Case Study. *Environmental Monitoring and Assessment* 185(3), pp. 2483-2502.
36. Ojha, A.K. and Das, A.K. (2010). Multi-Objective Geometric Programming Problem being cost coefficients as continuous function with weighted mean. *Jour. of comp.* 2(2), pp. 2151-9617.
37. Pramanik, S. and Banerjee, D. (2012): Multi-Objective Chance Constrained Capacitated Transportation problem based on Fuzzy goal programming, *International Journal of Computer Applications (0975-8887) Vol. 44-No.20.*
38. Raghav, Y.S., Ali, Irfan and Bari, A. (2014): Allocation in Multivariate Stratified Sampling in Presence of Non-Response: A New Approach, *Journal of Nonlinear Analysis and Optimization Theory & Applications (JNAO).*
39. Rehana, S. and Mujumdar, P.P. (2009). An Imprecise Fuzzy Risk Approach for Water Quality Management of a River System. *Journal of Environmental Management* 90, pp. 3653-3664.
40. Sakawa, M. and Yano, H. (1994). Fuzzy dual decomposition method for large-scale multi-objective non-linear programming problem. *Fuzzy Sets and Systems*, 67, pp. 19-27.
41. Sakawa, M., and Yano, H. (1989). An interactive fuzzy satisfacing method for multi-objective nonlinear programming problems with fuzzy parameters, *Fuzzy Sets and Systems,* 30(10), pp. 221-238.
42. Shafiullah, Ali, I. and Bari, A. (2013): Geometric Programming Approach in Three – Stage Sampling Design, *International Journal of Scientific & Engineering Research (France)*,4(6), pp. 2229-5518.
43. Shafiullah, Khan, M.F. and Ali, I. (2014). Fuzzy geometric programming in multivariate stratified sample surveys in presence of non-response with quadratic cost function. American Journal of Operations Research,4.
44. Shankar R., N., Ananda Rao, G., Madhu Latha, J. and Sireesha , V. (2010). Solving a Fuzzy Nonlinear Optimization Problem by Genetic Algorithm, *Int. J. Contemp. Math. Sciences.* 5(16), pp. 791 -803.

Notes

Notes

45. Tang, J. and Wang, D. (1997). A non-symmetric model for fuzzy nonlinear programming problems with penalty coefficients. *Computers and Operations Research,* 248, pp. 717-725.
46. Tang, J. D. W. and Richard F. Y. K. (1998). Model and method based on GA for non-linear programming problems with fuzzy objective and resources. *International Journal of Systems Science*, 29(8), pp. 907-913.
47. Trappey, L. F. C., Liu, C. R. and Chang, T. C. (1988). Fuzzy non-linear programming: Theory and application in manufacturing, *International Journal of Production Research*, 26(5), pp. 957-985.
48. Varshney, R., Najmussehar, Ahsan, M. J. (2011). An optimum multivariate stratified double sampling design in non-response. Springer, Optimization letters. Verlag.
49. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8, 338-353.
50. Zimmermann, H.J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems* 1, 45-55.

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