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Rethinking the Double Slit Experiment

By Ke Xiao

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I. Introduction

The interference of Young's double-slit experiments is the "central paradox" of Quantum Mechanics. [3] The quanta exhibit strange behavior after passing through the double-slits: (a) There is a definite symmetric interference pattern for the multi-quanta (photon or electron), regardless of whether they all come together or one at a time; (b) The individual quanta has a random path and target point; (c) There is a white background noise, a photon can be found even at the node; and (d) No interference for single-slit. There are many controversies surrounding wave-particle duality, determinative, causality, localization. Beyond the Copenhagen interpretation, other interpretations include Path-Integral, Hidden Variable, de Broglie-Bohm, etc, each with its own compromises. The fine-structure constant α is deeply involved in the Quantum theory. [1,2] Pauli considered quantum mechanics to be inconclusive without understanding of the fine structure constant. [2] Feynman also said that nobody understands quantum mechanics. [3] As a new approach, this paper discuss a fine structure constant interpretation of double-slit. [4]

II. Wavefunction and Wave-Particle Duality

A plane wave function $\Psi(\mathbf{r}, t)$ and the Born probability density $|\Psi(\mathbf{r}, t)|^2$ are [6]

$$\begin{aligned} \Psi(\mathbf{r}, t) &= Ae^{-\frac{i}{\hbar}(\mathbf{p}\cdot\mathbf{r}-Et)} = Ae^{-\frac{i}{\hbar}Et}\psi(\mathbf{r}) = f(t)\psi(\mathbf{r}) \\ |\Psi(\mathbf{r}, t)|^2 &= \Psi\Psi^* = [Ae^{-\frac{i}{\hbar}Et}\psi(\mathbf{r})][Ae^{-\frac{i}{\hbar}Et}\psi(\mathbf{r})]^* \\ &= Ae^{-\frac{i}{\hbar}Et}\psi(\mathbf{r}) \cdot Ae^{\frac{i}{\hbar}Et}\psi^*(\mathbf{r}) = A^2|\psi(\mathbf{r})|^2 \end{aligned} \tag{1}$$

The fine structure constant can be defined as the conservation of angular momentum related to the same dimensional $\mathbf{p}\cdot\mathbf{r}-Et$

$$\frac{\mathbf{e}^2}{c} = \pm\alpha\hbar = \mathbf{p}\cdot\mathbf{r}-Et \tag{2}$$

The wavefunction had an entropy format $S = k\ln\Psi$ for $\hat{\boldsymbol{H}}\Psi = \boldsymbol{E}\Psi$ in the first paper of Schrödinger in 1926. [5, 12] The Boltzmann constant k is linked to α by the dimensionless blackbody radiation constant α_R and primes $\alpha_R = \mathbf{e}^2(\frac{4\sigma}{ck^4})^{1/3} = (\prod\frac{\mathbf{p}^2}{\mathbf{p}^2+1})^{1/3}\alpha = 0.86976680\alpha = \frac{1}{157.555}$. [7] The Einstein/de Broglie **wave-particle duality is linked to the reciprocal space-time properties of matter**. [8,9] Note that period $T = 1/\nu$ [T] and wavelength $\lambda = 1/k$ [L], the property of *particle-wave* is defined by the spin over *time-space*.

$$\begin{aligned} E &= \hbar\omega \quad \text{i.e.} \quad E = h\nu = h/T \qquad \text{(spin/time)} \\ \mathbf{p} &= \hbar\mathbf{k} \quad \text{i.e.} \quad \mathbf{p} = hk = h/\lambda \qquad \text{(spin/space)} \end{aligned} \tag{3}$$

Author: *Manhattan Beach, CA 90267, USA. e-mail: XK6771@gmail.com*

where the conservation of angular momentum in the reduced Planck (Dirac) constant $h = ET = \mathbf{p}\lambda$ (i.e., $\hbar = E/\omega = \mathbf{p}/\mathbf{k}$), and the electron spin $\hbar/2$ can only be interpreted by the 4-dimensional space-time of the relativistic Dirac equation. [10, 11]

III. Compton Scattering and Interference

In **Fig. 1** (a), the 2D double-slit plane is illustrated so that the slits 1 and 2 are at $\frac{d}{2}$ and $-\frac{d}{2}$, with the slit width δ, a moving target receiver at the point X, with distance Y between the double-slit and target, $L_1 = [Y^2 + (X + \frac{d}{2})^2]^{1/2}$ and $L_2 = [Y^2 + (X - \frac{d}{2})^2]^{1/2}$. Note that the experimental condition requests $Y \gg X \gg |d| \gg \lambda$, so $\triangle L \cong aX$ is linear.

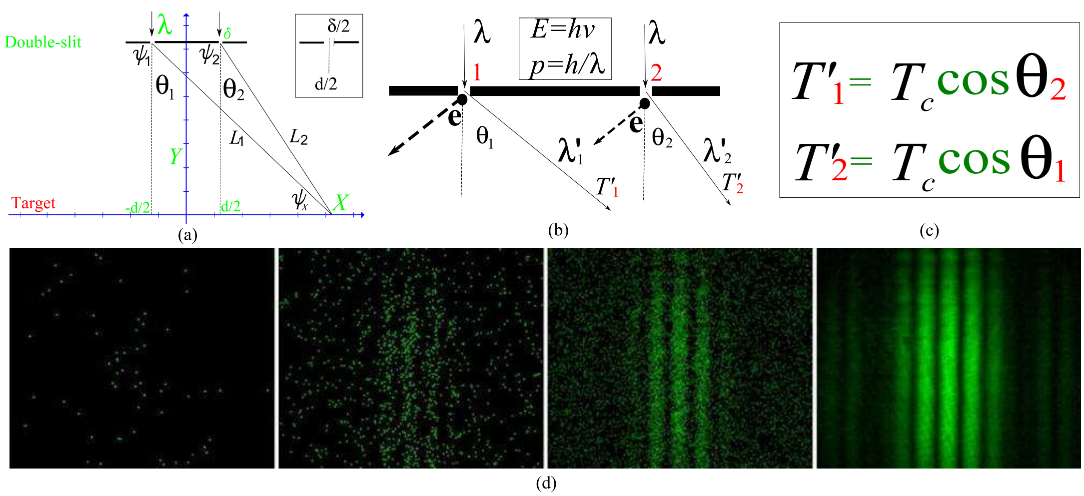

Figure 1 : The 2D illustration of Double-slit (a), the cross-linked photon scattering (b), (c), and (d) the experimental data recorded by Antoine Weis in 2003.

Since the phase velocity $c = \nu\lambda = \omega\lambda\!\!\!^{-} = \omega/k$ and $T = \omega^{-1}$, from Compton scattering $\lambda\!\!\!^{-}{}' - \lambda\!\!\!^{-} = \lambda\!\!\!^{-}_c(1 - \cos\theta)$ (i.e., $\frac{c}{\omega'} - \frac{c}{\omega} = \frac{c}{\omega_c}(1 - \cos\theta)$), we have $T' - T = T_c(1 - \cos\theta)$ in **Fig. 1** (b). [13] For the each slit in **Fig. 1** (c)

$$\begin{aligned} T_1' - T &= T_c[1 - \cos(\theta_1)] \quad (1) \\ T_2' - T &= T_c[1 - \cos(\theta_2)] \quad (2) \end{aligned} \tag{4}$$

Let (4-2) subtract (4-1), and $T_c = \lambda\!\!\!^{-}_c/c = \hbar/m_e c^2 = 1.288 \times 10^{-21}$[sec], then

$$\triangle T = T_2' - T_1' = T_c[\cos(\theta_1) - \cos(\theta_2)] \tag{5}$$

(5) establishes ***the cross-linked angle for the double-slit***

$$\text{If} \quad T_1' = T_c\cos(\theta_2) \quad \text{then} \quad T_2' = T_c\cos(\theta_1) \tag{6}$$

where the T_1' on the slit-1 is related to the scattering angle θ_2 on the slit-2, and *vice versa*. It is a random variable for each particle noted as $\triangle T = \delta T$, and the experimental data shows in **Fig. 1** (d).

Assuming $\psi_1 = A_1 e^{-i\omega T_1'}$ and $\psi_2 = A_2 e^{-i\omega T_2'}$ after the photon scattering for slits 1 and 2, where $|A| \simeq |A_1| \simeq |A_2|$ is real number. The target wavefunction at X is $\psi_X = Ae^{-i\omega T_1'}e^{ikL_1} + Ae^{-i\omega T_2'}e^{ikL_2} = \psi_1 e^{i2\pi L_1/\lambda} + \psi_2 e^{i2\pi L_2/\lambda}$, where λ is the wavelength of the quanta (photon or electron). The probability given for the quanta at the target point X is the same as (1) in the exponential form

$$|\psi_X|^2 = (Ae^{-i\omega T_1'}e^{ikL_1} + Ae^{-i\omega T_2'}e^{ikL_2})(Ae^{i\omega T_1'}e^{-ikL_1} + Ae^{i\omega T_2'}e^{-ikL_2})$$

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$$
\begin{aligned}
&= |A|^2 \cdot [1 + 1 + e^{i(kL_1 - \omega T_1' - kL_2 + \omega T_2')} + e^{-i(kL_1 - \omega T_1' - kL_2 + \omega T_2')}] \\
&= |A|^2 \{ \underset{\text{White noise}}{2} + \underset{\text{Wave interference}}{2\cos[k(L_1 - L_2)} - \underset{\text{Particle scattering}}{\omega(T_1' - T_2')]} \} \\
&= 2|A|^2 [1 + \cos(\underset{\text{space}}{k\triangle L} - \underset{\text{time}}{\omega \delta T'})]
\end{aligned}
\tag{7}
$$

The angle term in (7) is same as $\frac{1}{\hbar}(\mathbf{p}\cdot\mathbf{r} - Et) = (\mathbf{k}\cdot\mathbf{r} - \omega t) = \mathbf{k}(\mathbf{r} - ct)$ in (1), i.e., related to the fine structure constant in (2). It is $\lambda\!\!\!^{-}_c = \alpha a_0 = \frac{\alpha^2}{4\pi R_\infty} = \frac{r_e}{\alpha}$ for the free electron and $\frac{\hbar}{m_i c} = \frac{m_p}{m_i}\frac{\alpha}{\beta}a_0 = \frac{m_p}{m_i}\frac{\alpha^2}{\beta}\frac{1}{4\pi R_\infty}$ for the atomic bonding electron in the Compton scattering. (7) clearly show that the wave-particle duality by the wave-vector k and the photon-frequency ω linked separately to the space $\triangle L$ and time $\delta T'$.

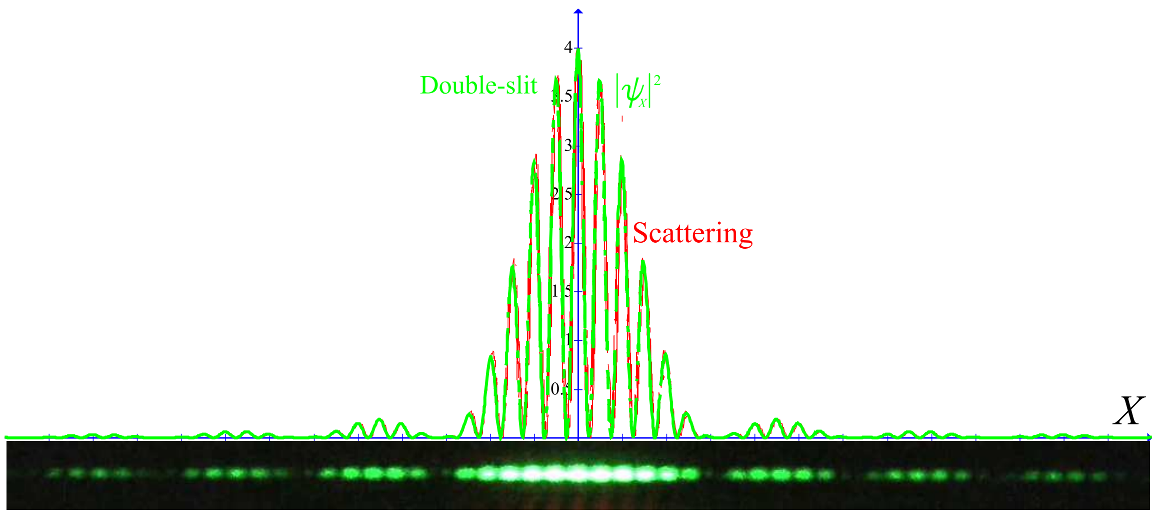

Figure 2 : The scatter graph of the probability density $|\psi_X|^2_{DS}$ in (7) by author (top); and the Laser double-slit experimental data for the *far* target from MIT (bottom).

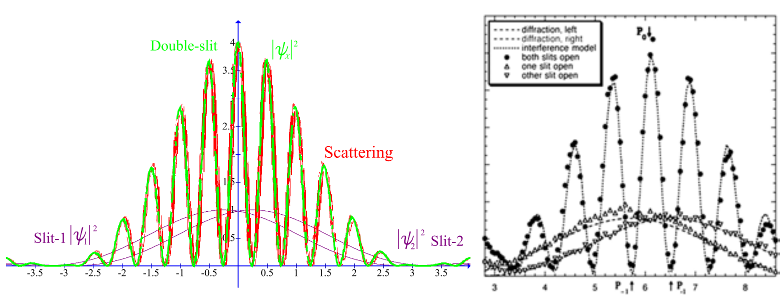

Figure 3 : The scatter graph of $|\psi_X|^2_{DS}, |\psi_1|^2$ and $|\psi_2|^2$ in (7) by author (left); and the "one photon at a time" double-slit experimental data for the *near* target from Teachspin (right)

The visible photon-frequency is about $\omega \sim 10^{14}[Hz]$. The term of $\omega\delta T'$ are random and small effect on the visible-light scattering by the tightly bound electron ($m_i \gg m_e$). Therefore, the term $\frac{2\pi}{\lambda}(L_1 - L_2) = akX$ is the major-variable in control. The cosine term becomes $\{-1, 0, 1\}$ when $\frac{2\pi}{\lambda}(L_1 - L_2) - \omega(T_1 - T_2) = akX - \omega\delta T' = \{(2n \pm 1)\pi, (n + \frac{1}{2})\pi, 2n\pi\}$ where $k = \frac{2\pi}{\lambda}$, $n = 0, \pm1, \pm2, \cdots$, and let $|\psi_X|^2 = \{0, 2, 4)\}|A|^2$. The continued distribution is changes into a quantum interference pattern during coherence and cross-correlation. [14] Assuming $A = \text{sinc}[k\triangle L]$ and $A_{1,2} = \text{sinc}[k(\triangle L \pm \frac{d}{2})]$, the graph of (7) matches perfectly with experimental data from MIT for the *far* target using lasers (**Fig.**

2),[1] and Teachspin for the *near* target with "one photon at a time" (**Fig. 3**).[2] The scatter graph in **Fig. 2** and **Fig. 3** clearly show that the interference pattern with the random scattering effect as the individual particle of "one photon at a time."

(7) indicates that the double-slit experiment has the interference pattern (a) with a random quantum scatting (b) and a background white noise (c). Notice that the cosine function is an even function responsible for making the symmetric interference pattern appearing on the target plate. It also shows that there is no interference pattern for the *narrow* single-slit (d), since $|\psi_j e^{i2\pi L_j/\lambda}|^2 = (A_j e^{-i\omega t_j} e^{ikL_j})(A_j e^{i\omega t_j} e^{-ikL_j}) = |A_j|^2$ where $j = 1, 2$ (**Fig. 3**).

IV. Diffraction and Interference of N-Slits

As an exception, a slit which is wider than a wavelength produces Fraunhofer diffraction similar to the *weak* interference, due to the slit-edge electron-photon scattering effect. The probability for the diffraction of a *wide* single-slit at the target point X can be derived as a Taylor-Maclaurin series or Euler-Viete infinite product,

$$\begin{aligned} |\psi_X|^2_{SS} &= |A|^2 = |\mathrm{sinc}(akX)|^2 = [\sum_{n=1}^{\infty}(-1)^n \tfrac{(akX)^{2n}}{(2n+1)!}]^2 \\ &= \{\prod_{n=1}^{\infty}[1-(\tfrac{akX}{n\pi})^2]\}^2 = [\prod_{n=1}^{\infty}\cos(\tfrac{akX}{2^n})]^2 \end{aligned} \quad (8)$$

where $akX = \frac{\pi d}{\lambda}\sin\theta$ and slit width d. Unlike θ in the Fraunhofer approximation for the *far* field limitation, X can be measured directly regardless *far* or *near* target. (8) looks like but is not Gaussian distribution. **Fig. 4** shows the graph of (8) compared with the single-slit diffraction experimental data using laser and adjustable slit-width.

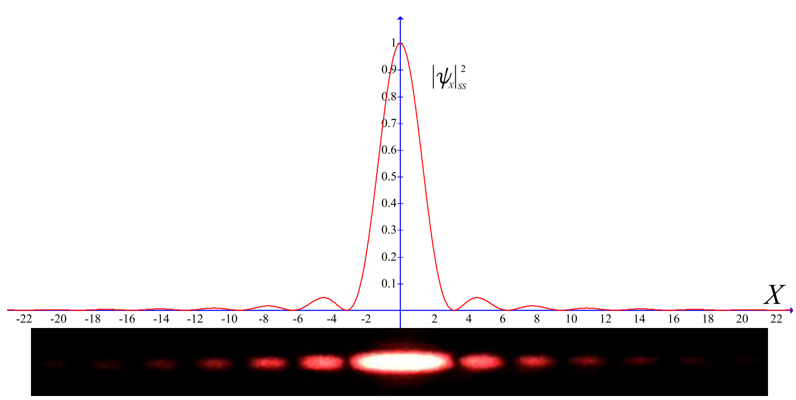

Figure 4: The graph of the probability density $|\psi_X|^2_{SS}$ in (8) by author (top); and the single-slit diffraction experimental data from MIT (bottom)

Therefore, the double-slit equation is simply the diffraction of two *narrow* single-slits with wave interference and photon scattering. This is supported by the experimental result of very narrow but widely separated double-slit, i.e., moving two single-slits closer, in the double-slit experiment of electron. [15] From (7) and (8), the double-slit equation is

$$\begin{aligned} |\psi_X|^2_{DS} &= \underbrace{2}_{\text{two}} \cdot \underbrace{|\mathrm{sinc}(a'kX)|^2}_{\text{diffraction}}[1+\cos(\underbrace{akX}_{\text{interference}} - \underbrace{\omega\delta T'}_{\text{scattering}})] \\ &= \{2\cdot\mathrm{sinc}(a'kX)\cdot\cos[(akX-\omega\delta T')/2]\}^2 \end{aligned} \quad (9)$$

[1] http://scripts.mit.edu/_tsg/www/demo.php?letnum=P%2010 (2011)
[2] http://www.teachspin.com/instruments/two_slit/experiments.shtml (2011)

where $a'kX = \frac{\pi\delta}{\lambda}\sin\theta$ and δ is the narrow slit width; $akX = \frac{\pi d}{\lambda}\sin\theta$ ($d > \delta$ in **Fig. 2-3**). The total number of fringes are $N = 2m - 1$, where $m = d/\delta = a/a'$ equal to the ratio of the width of two-slits d divides the slit width δ. The double-slit experiment is a combination of the diffraction, scattering, and interference processes.

The same principle can be applied to *N-narrow* slit experiments (**Fig. 5**). [15]

$$\begin{aligned}\psi_X &= \sum_{n=0}^{N/2}\psi_{\pm n} = A\sum_{n=1}^{N/2}\cos(akX - \omega\delta T') \\ &= A\frac{\sin[N(akX+\omega\delta T')/2]}{\sin[(akX+\omega\delta T')/2]}\end{aligned} \tag{10}$$

The probability for the diffraction grating of *N-narrow* slits at the target point X is

$$\begin{aligned}|\psi_X|^2_{NS} &= \mathrm{sinc}^2(a'kX)\{\frac{\sin[N(akX-\omega\delta T')/2]}{\sin[(akX-\omega\delta T')/2]}\}^2 \\ &= \frac{1-\cos(2a'kX)}{2(a'kX)^2}\cdot\frac{1-\cos[N(akX-\omega\delta T')]}{1-\cos(akX-\omega\delta T')]}\end{aligned} \tag{11}$$

If $N = 2$, $\left[\frac{\sin(2\phi/2)}{\sin(\phi/2)}\right]^2 = [2\cos(\phi/2)]^2 = 2(1+\cos(\phi)) = 4\cos^2(\phi/2)$, (11) go back the double-slit equation (9). Unfortunately, the wave pattern is so attractive in the classical grating equation, and the particle scattering is ignored. (11) contains a particle scattering term $\omega\delta T'$, and its scatter graph compare with experiment from UTPD is shown in **Fig. 5** [3]

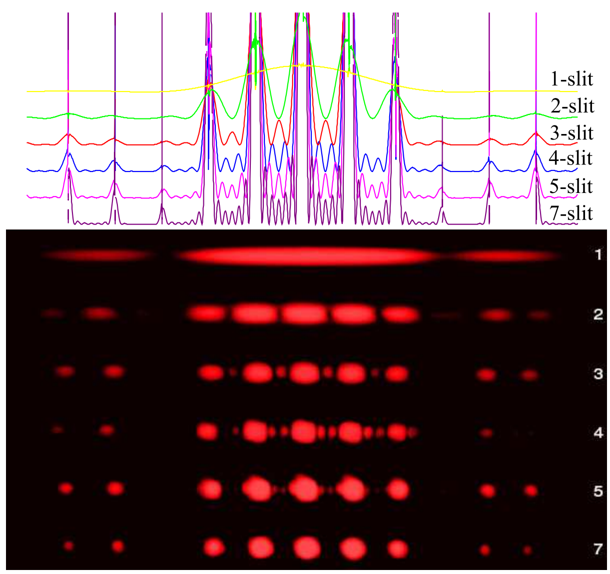

Figure 5: The scatter graph of the probability density $|\psi_X|^2_{1,2,3,4,5,7}$ in (11) by author (top); and the *N-narrow* slits (1, 2, 3, 4, 5, 7) diffraction data from UTPD (bottom)

V. Combination of Quantum and Classical Slit

In this model, the slit-edge electron-photon scattering plays a hidden role (each slit has two edges), which is linked to the fine structure constant. [7] Compton scattering creating redshift has been confirmed by sunlight. The fine structure constant must also play critical rule in the double-slit experiment of electron, neutron, He-atoms, and C-60 molecules.

[3]http://electron9.phys.utk.edu/phys136d/modules/m9/diff.htm (2011)

Fig. 6 shows that there is a classical distribution (or de-coherent) if the quanta-slit interaction is weak (e.g., the double-slit experiment for neutron, C-60 and other heavy atoms). Other types of quantum scattering (e.g., Møller) may be involved. The Neutron double-slit data can be described as [16]

$$|\psi_X|^2_{NDS} = [2\mathrm{sinc}(a'kX)\cos(akX - \omega\delta T")]^2 + \sum|\mathrm{sinc}[a'k(X \pm x))]|^2 \qquad (12)$$

Fig. 6 shows that many quanta have passed the slit without interaction (leaking), they are not taking place in the interference. This is also true for the photon-recoil atomic interferometry, and the light-quanta passing a widely separated double-slit. [16] **Fig. 7** shows the Young's interference patterns of synchrotron radiation. The photon energy is fixed at 180 eV, while the spacing of the double slit is changed from 30 to 200 µm. [17]

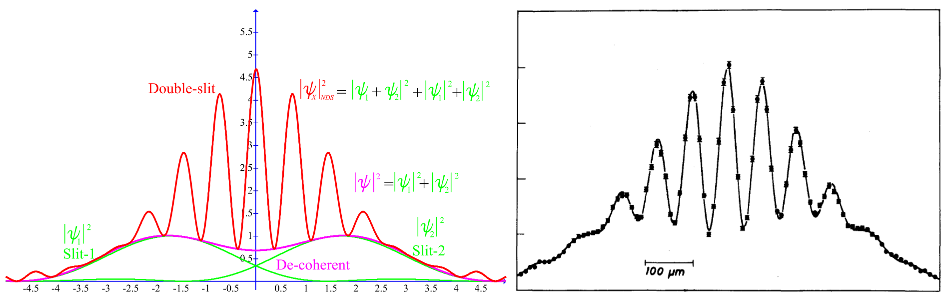

Figure 6 : The scatter graph of the probability density $|\psi_X|^2_{NDS} = |\psi_1+\psi_2|^2 + |\psi_1|^2 + |\psi_2|^2$ in (12) by author (left); and the Neutron double-slit data (right) [16]

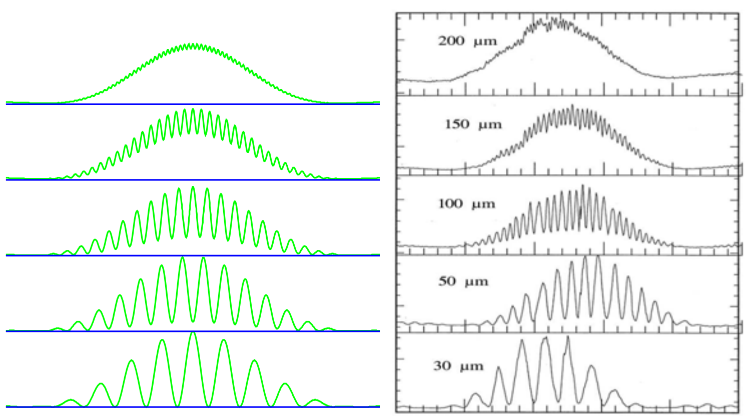

Figure 7 : The graphic $|\Psi|^2$ in (12) by author (left) compare to the Young's interference patterns of synchrotron radiation (right), $h\nu = 180eV$, the spacing of the double slit d is changed from 30 to 200 µm. [17]

VI. Space-Time 3D Model of Double-Slit

Applying the time evolution $e^{-kct} = e^{-bY}$ to the diffraction, interference and scattering in (12), the 3D expression of the double-slit is

$$\begin{aligned}|\psi_X|^2 &= (1 - e^{-kct}) \cdot |\psi_X|^2_{DS} + e^{-kct} \cdot \left\{|\psi_{X+d/2}|^2_{SS} + |\psi_{X-d/2}|^2_{SS}\right\} \\ &= (1 - e^{-bY})[2\mathrm{sinc}(a'kX)\cos(akX - \omega\delta T")]^2 + e^{-bY}\sum|\mathrm{sinc}[a'k(X \pm \tfrac{d}{2})]|^2\end{aligned} \qquad (13)$$

where $aX = [Y^2 + (X+\frac{d}{2})^2]^{0.5} - [Y^2 + (X-\frac{d}{2})^2]^{0.5}$ and $\delta T" = T_c\{[1+(\frac{X+d/2}{Y})^2]^{-0.5} - [1+(\frac{X-d/2}{Y})^2]^{-0.5}\}$. The contour map of (13) compare with the He-atom, and the 3D graph of (13) with the weak measurement are shown in **Fig. 8**. [18]

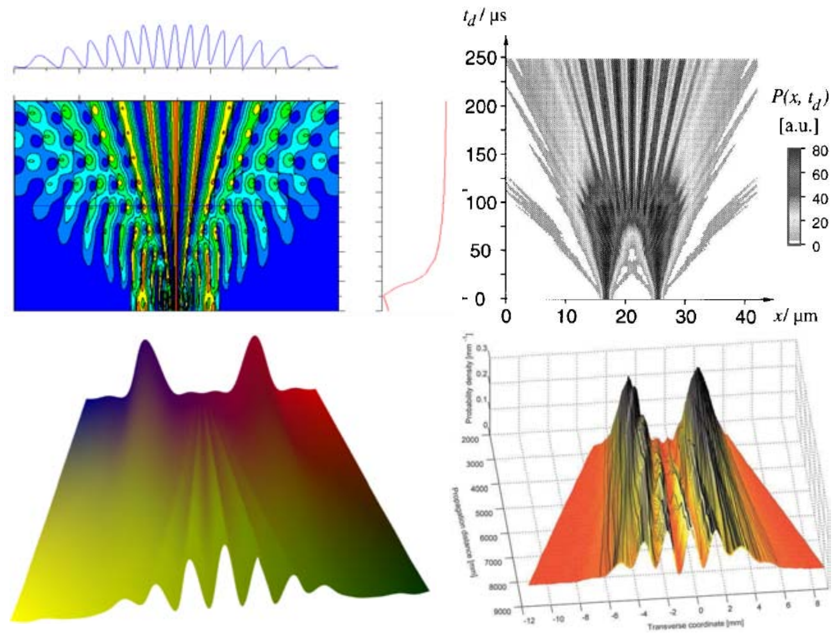

Figure 8 : The contour map and 3D graph of $|\Psi|^2$ in (13) by author (left) to compare with the He-atom and the weak measurement (right), originally explored by Wigner function and Bohmian trajectories. [18]

In (13), the wave-particle $\mathbf{P}\cdot\mathbf{r}-Et=akX-\omega\delta T'=k(aX-c\delta T')$ link to the space-time, and the X-enlarge distorted 3D graph shows in **Fig. 9**.

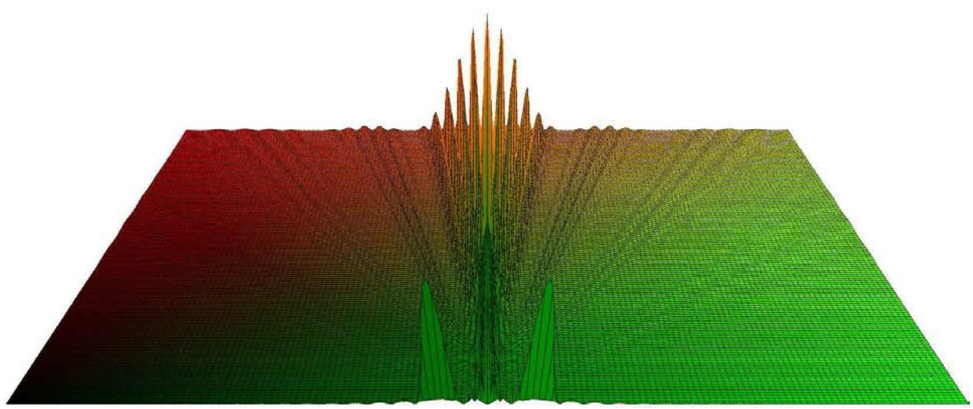

Figure 9 : The double-slit distorted 3D graph of $|\Psi|^2_{3D}$ in (13) by author

The experiment for which-way also show the de-coherent by the secondary scattering after the quanta passed slit. [19] It further prove that the quantum scattering is a critical issue behind the geometric parameters for the wave-pattern. In other word, ***there will be no interference if without relativistic quantum scattering.*** The fine structure constant is the magic hand behind the double-slit experiment. This model clearly displays the particle-wave duality by particle scattering and wave-interference. The quanta can be counted as one *particle* at a *time*, and the multi-quantas are displaced as the cosine-type *wave*-pattern in *space*. The electromagnetic wave frequency is in the region of $\omega=kc=10^0\sim10^{24}$[Hz]. Since $T_c=1.288\times10^{-21}$[sec], the double-slit wave interference can be tested from soft-X-ray (10^{18}[Hz]) to microwave (10^8[Hz]). The visible light ($\sim10^{14}$[Hz]) is a *wave*-like rather than the *particle*-like, and the γ-ray (10^{20-24}[Hz]) is a *particle*-like rather than the *wave*-like. This physical reality was obscured by the trigonometric identities, however, there are many different versions of the classical grating equations. The quantum physical principle is the same and independent of the mathematical expression and coordinate system.

VII. Conclusion

The wave-particle duality relate to the space-time property of matter by Planck constant. The fine structure constant is linked to the double-slit and the uncertainty principle in Quantum Mechanics. Compton scattering and interference of double-slit is established by the cross-linked angle $T_1' = T_c \cos(\theta_2)$, and *vice versa.* The single-slit direction is described by Sinc-function which could combine the classical and quantum interference effect in the same experiment. This space-time model of double-slit explain the experimental mystery of the double-slit.

VIII. Acknowledgments

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