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Scattering of Electrons in the Field of the Binomial Potential of a PROTON

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V. K. Gudym [α] & E. V. Andreeva [σ]

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I. Introduction

At the present time, the interaction of an electron with a proton is described with the use of several mechanics, which are fundamentally different from one another: classical, quantum, relativistic, etc. But the absence of a single approach to the description of the electron – proton system indicates that the two-body theory for this system needs an essential additional development. We are sure that such situation in the description of these particles does not allow one to understand the real physical essence of phenomena in the microworld or complicates their interpretation.

Classical mechanics has always dealt with the velocities of bodies, which are smaller than the light velocity. Therefore, all consequences following from the main postulates and laws of the Newton mechanics were confirmed with high accuracy in various experiments. Only in the world of elementary particles moving with velocities close to the light velocity, the relativistic mechanics should be used. In addition, it is commonly considered that the classical mechanics is unfounded for the description of the motion of separate atoms, electrons, and other microscopic (elementary) particles composing the atomic nuclei, atoms, and molecules.

Therefore, in order to overcome the difficulties arising at the application of the classical mechanics to such objects, there appears the necessity to impose the limitations on the energy of interacting particles and on the admissible distances between them. In other words, some limitations on the region of applicability of the classical mechanics were established.

Thus, the opinion has been formed till now that the classical mechanics should be considered as a limiting case of the relativistic mechanics or the special relativity theory (SRT) and as a partial case of the nonrelativistic quantum mechanics.

But, in our opinion, the reason for the neglect of the Newton dynamics and the idea of a localized electron does not lie in the fundamental shortcomings of the classical mechanics. The real reason consists in that the model of microworld commonly accepted now does not completely correspond to the reality. For example, the interaction potential of particles in the macroworld (Coulomb law) was accepted without modifications for the interaction of particles in the microworld. In this case, all arisen difficulties in calculations were referred to the apparent shortcomings of the classical mechanics, rather than to those of the accepted model. But, in essence, the classical mechanics is a mathematical scientific trend. It does not care of a potential to deal with. But the results of calculations will be different for different potentials. At the present time, we have no complete representation of the forces acting between the electron and the proton, as well as between a nucleus and atomic electrons, in the general case. Therefore, the calculations by the classical scheme give erroneous results.

To support this viewpoint, we will solve the classical problem of two bodies interacting by the law of binomial potential. This calculation allowed us to theoretically trace, for the first time, the trajectories of motion of an electron in the atom without the necessity to impose some limitations on the energy of interacting particles and on the admissible distances between them. We note that such results cannot be obtained with the Coulomb law. In other words, the classical mechanics is innocent, since all difficulties follow from the accepted Coulomb-law-based model of microworld.

The study of the processes of scattering is the basic experimental method, with which the character of the interaction of scattered particles is esatblished. The main formula describing the process of scattering is the Rutherford formula [1] based on the assumption that the particles participating in the process of scattering interact by the Coulomb law. It is known that the significant deviation of the distribution of electrons scattered on protons from that calculated by the

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Rutherford formula is observed at medium and high energies [2]. This disagreement of the experimental data and the results of calculations by the Rutherford electrons had led the physicists to the thought that the Coulomb law is not valid at small intraatomic distances [3], including the interaction of an electron and a proton.

In this work, we have shown that the potential of interaction of the electron with the proton at intraatomic distances must be presented in the form of a binomial formula

$$V = -\frac{e^2}{r} + \frac{\Gamma}{r^2}, \qquad (1)$$

where e is the electron charge; Γ is a constant calculated by us and equal to $6.10276 \cdot 10^{-28}$ CGSE units; and r is the distance between the electron and the proton. Then, by solving the classical problem of motion of the electron in the centrally symmetric field of the proton with potential (1), which is represented by the relation

$$E = \frac{m\dot{r}^2}{2} + \frac{M^2}{2mr^2} - \frac{e^2}{r} + \frac{\Gamma}{r^2}, \qquad (2)$$

we obtained the results that cannot be obtained with the Coulomb potential. Such basic notions as the discreteness of energy levels, Balmer formula, Planck's constant and relation, and Bohr's postulates follow in a mathematically strict way from the solution of Eq. (2). In essence, we have shown that, potential (1) give the possibility to study the intraatomic processes by methods of only the classical mechanics without limitations on the energies of interacting particles and on the distance between them [4].

In the present work, we will demonstrate that if the interaction of the electron with the proton is described by the law of binomial potential, then we can obtain the formulas for the description of the processes of scattering in a wide energy range in good agreement with experimental data without attraction of the auxiliary notions of relativism and quantum mechanics. Thus, we extend the principles of the classical mechanics without limitations onto the intraatomic processes.

II. Substantiation of the BinomialPotential of Interaction of An electron with a Proton

The binomial interaction potential of an electron with a proton is based [4-7] on the indisputable fact that the electrons in atoms are constantly located at some distance from the nucleus. According to the Bohr hypothesis, the electrons are held in such a position as a result of the equlibrium between the force of the Coulomb attraction and the centrifugal force. It is known that the Bohr hypothesis allows one to satisfactorily explain many phenomena occurring in hydrogen atoms. But it met the insurmounable difficulties in the consideration of structures with two and more electrons. In our opinion, these difficulties can be overcome only if we assume the existence of an additional field counteractive to the Coulomb attraction between an electron and a proton.

Then the interaction of the electron with the proton in a hydrogen atom can be described by the potential energy function

$$V = -\frac{e^2}{r} + \frac{\Gamma}{r^x} \qquad (3)$$

where the first term on the right-hand side represents the Coulomb interaction, and the second term is related to the hypothetical interaction, which counteracts the Coulomb attraction.

In (3), the constant Γ and the exponent x can be determined from the system of two algebraic equations corresponding to a certain state of a hydrogen atom, for which the experimental data are available. Consider the ground state of a hydrogen atom. We have [3]

$$-\frac{e^2}{r_0} + \frac{\Gamma}{r_0^x} = E_0, \quad \frac{e^2}{r_0^2} - \frac{x \cdot \Gamma}{r_0^{x+1}} = 0 \qquad (4)$$

where E_0 – the ground-state energy of a hydrogen atom; r_0 – its equilibrium radius, and e – electron charge. The second equation in (4) represents the sum of forces acting on the electron in the ground state.

By transforming system (4) to the form

$$-\frac{e^2}{r_0} + \frac{\Gamma}{r_0^x} = E_0, \quad \frac{e^2}{xr_0} - \frac{\Gamma}{r_0^x} = 0 \qquad (5)$$

and by adding both relations, we obtain

$$-\frac{e^2}{r_0} + \frac{e^2}{xr_0} = E_0 \qquad (6)$$

or

$$\frac{1}{x} - 1 = \frac{E_0 \cdot r_0}{e^2}. \qquad (7)$$

Substituting the numerical tabular values of E_0, r_0, and e^2, we obtain $1/x - 1 = -0.5$. Hence, $x=2$. From the second equation in (2), we obtain

$$\Gamma = \frac{e^2 \cdot r_0}{2}. \qquad (8)$$

Thus, the solution of system (4) relative to Γ and x gives

$$\Gamma = 6.10276 \cdot 10^{-28} \text{ CGSE units, and } x=2. \qquad (9)$$

Let us focus attention on the right-hand side of relation (7)

$$\frac{E_0 \cdot r_0}{e^2} = \frac{1}{2}. \qquad (10)$$

Writing (10) as

$$E_0 = \frac{e^2}{2 \cdot r_0}, \qquad (11)$$

we see that it determines the ground-state energy of a hydrogen atom.

We recall that the Bohr theory yields also an analogous formula[3] as a result of the equality of the centrifugal force and the Coulomb attraction to the nucleus.

It is also known that the solution of the Schrödinger equation gives the same formula, as well as the relation $r_0 = \hbar^2 / me^2$. By substituting the last relation in (8), we obtain $\Gamma = \hbar^2 / 2m$.

We recall these facts in order to show the logical connection of the binomial potential with the Bohr theory and the Schrödinger theory of a hydrogen atom. Indeed, the positive addition to the Coulomb law was included in the previous theories as well. But there, it was a centrifugal force on the basis of the postulate about some motion of electrons around a nucleus. As is known, the postulate of the circular motion of electrons around the nucleus creates insurmountable basic difficulties for the theory of multielectron structures. But our hypothesis about the presence of a repulsive potential removes these obstacles and allowed us to develop a simple model of multielectron atoms.

We will pay attention to some specific features of result (9). First, it is the small value of constant Γ as compared with e^2 ($23.06112 \cdot 10^{-20}$). Therefore, the contribution of the positive addition in (3) to the total interaction energy is insignificant already at a distance of about two equilibrium radii. Hence, at larger distances, the energy of interaction of the electron with the proton has only the electrostatic nature.

Second, the value of the exponent x equal exactly to 2 indicates that the dependence of the positive addition in the binomial potential (3) on the distance is the same as that for a centrifugal force. Hence, the meaning of the positive addition to the Coulomb law for a one-electron system (e.g., a hydrogen atom) is not significant: it can be a result of the circular motion of the electron (the centrifugal force) or a result of the action of some constant hypothetic field.

For this reason, the planetary Bohr model for a hydrogen atom gave good results (though it is far from the reality by its essence). Moreover, it is quite regular that this model cannot describe two- and multielectron systems, because the Coulomb attraction of electrons to the nucleus in such systems cannot be compensated by centrifugal forces. The different possibility is given by the binomial potential. Assuming that the positive addition in the binomial potential (3) characterizes some constant field, we showed in[30] that it is easy to model systems with two and more electrons..

Third, the constant

$$\Gamma = \frac{\hbar^2}{2m}, \qquad (12)$$

where $\hbar$ is the Planck's constant, and m is the electron mass, corresponds to the proportionality coefficient postulated by E. Schrödinger for his equation. Just this allowed V. Weisskopf [8] to deduce a potential analogous to (3), by basing on the quantum-mechanical premises of the Schrödinger theory. Hence, two basically different approaches (classical and quantum-mechanical ones) have led to the same function representing the potential energy of interaction of an electron with a proton. This also confirms the logical connection between the binomial potential and quantum theory. Therefore, function (3) can be perceived as a coordinating link between the classical theory and the quantum mechanics of a hydrogen atom. As will be shown below, these two theories with the binomial potential complement each other and are a single unit.

The difference between them consists in the following. The positive addition to the Coulomb law in quantum mechanics is perceived as a minimum kinetic energy [8], whereas it has meaning of a physical field counteracting the Coulomb interaction. The consideration of multielectron systems will allow us to answer the question: Which of two possibilities is more natural? It was mentioned above that the postulated circular motion of an electron around the nucleus had led to basic obstacles in the construction of the theory of multielectron systems. But the postulate introducing some counteracting potential removes these obstacles and opens a possibility to model multielectron systems in the frame of classical mechanics [9,10]. The results of such a modeling should be considered as the experimental corroboration of the validity of the interpretation of a positive addition to the Coulomb law as some counteracting physical potential.

Generally speaking, by introducing the binomial potential, we replace, in essence, the postulate about some motion of an electron around the nucleus, which is extraneous to classical mechanics, by a more natural hypothesis about the presence of some repulsive potential. It is worth noting that the proposed binomial potential (3) does not contradict the foundations of quantum mechanics[7, 11, 12] and opens a way for the construction of the classical mechanics to multielectron systems.

In Fig. 1, we show the dependence of the energy of interaction of an electron with a proton with the binomial potential on the distance between them. It is obvious that the dominating components are the Coulomb interaction at distances more than two equilibrium radii ($\sim 1 \cdot 10^{-8}$ *cm*) and the positive addition at distances less than a half of the equilibrium radius

(~0.25·10^{-8} *cm*), respectively. Hence, the positive addition to the Coulomb component in (3) is a short-range force. This fact explains the circumstance that no deviations from the Coulomb law were measured at macrodistances.

III. Scattering of Electrons by the Field of the Binomial Potential of a Proton

For a trajectory, the solution of Eq. (2) with regard for potential (3) takes the form

$$r = \frac{P}{1-\varepsilon\sin(k\varphi)}, \qquad (14)$$

where $\varepsilon = \sqrt{1+\frac{2E(2m\Gamma+M^2)}{me^4}}$ - the orbit eccentricity; (15)

$$P = \frac{2m\Gamma+M^2}{me^2} \text{ - the orbit parameter;} \qquad (16)$$

$$k = \frac{\sqrt{2m\Gamma+M^2}}{M} \text{ - some coefficient,} \qquad (17)$$

which characterizes the closedness of an orbit; m – electron mass, E – total energy of the system, M – angular momentum, and e - electron charge.

If the total energy in (3) is positive, then the trajectory of motion of an electron in the central field of a proton is an open curve, whose ends tend to infinity. In this case of infinite motion, it is convenient to introduce the so-called impact parameter ρ instead of the angular momentum M:

$$M = \sqrt{2mE\rho^2}\,. \qquad (18)$$

To calculate the trajectory of motion of an electron scattered by a proton, it is necessary to substitute (18) in relations (15)-(17) and to pass to the Cartesian coordinates

$$x = r\cdot\cos\varphi \qquad y = r\cdot\sin\varphi, \qquad (19)$$

where the values of r are determined by formula (14).

In Fig. 2, we present) the trajectories of motion of scattered electrons with energies of 400, 188, and 40 *MeV* calculated by (19).

The deviation angle χ for the flight of a particle past the scattering center takes the form

$$\chi = \pi - 2\varphi_0, \qquad (20)$$

where the angle φ_0 is determined in terms of the integral

$$\varphi_0 = \int_{r_{\min}}^{\infty} \frac{M\cdot dr}{r^2\sqrt{2m\left[E-U(r)\right]-\frac{M^2}{r^2}}} \qquad (21)$$

taken between the positions of the particle, which are nearest to the center and infinitely remote[1]. Here, $r_{\min}$ is a root of the radicand.

With regard for relation (18), formula (21) takes the form

$$\varphi_0 = \int_{r_{\min}}^{\infty} \frac{dr}{r^2\sqrt{\frac{1}{\rho^2}\left[1-\frac{U(r)}{E}\right]-\frac{1}{r^2}}}. \qquad (22)$$

Substituting potential (1) in (22) and integrating, we obtain

$$\varphi_{0b} = \sqrt{\frac{E\rho^2}{E\rho^2+\Gamma}}\cdot\arccos\left[\frac{1}{\sqrt{1+\frac{4E(E\rho^2+\Gamma)}{e^4}}}\right]. \qquad (23)$$

Thus, we obtain the analytic formula for the deviation angle χ for an electron scattered by a proton as a function of the energy E and the impact parameter ρ under the assumption that they interact by the binomial potential law (3). If we set Γ=0 in (23), which corresponds to the interaction of the electron with the proton by the Coulomb law[1], we obtain

$$\varphi_{0k} = \arccos\left[\frac{1}{\sqrt{1+\frac{4E^2\rho^2}{e^4}}}\right] \qquad (24)$$

Hence, the distinction of the scattering formulas obtained with the help of the Coulomb and binomial potentials consists only in the presence of the coefficient

$$k = \sqrt{\frac{E\rho^2+\Gamma}{E\rho^2}} \qquad (25)$$

in (23).

In the general case, formulas (23) and (24) together with (20) allow one to calculate the deviation angles χ of an individually scattered electron as a function of its kinetic energy and the impact parameter in the cases where the electron interacts with the proton

by the binomial potential law (1) and by the Coulomb law, respectively.

In Fig. 3, we present the results of calculations by formulas (23) and (24).

It is worth noting that the scatterings of electrons in the fields of the Coulomb and binomial potentials are almost identical al low energies. The noticeable difference of the scattering angles χ is observed only at energies of hundreds and more *eV*. In the relativistic region of energies, the scattering in the Coulomb potential is not observed at all (Fig. 4b). Here, the Rutherford theory of scattering by the Coulomb potential contradicts the experiment. In due time, this contradiction was one of the main reasons to think that the laws of classical mechanics must be corrected for atoms at small distances at high energies. Then the theory of scattering was developed with the attraction of the main positions of the special relativity theory [2]. However, our calculations with the binomial potential testify that the contradiction between the Rutherford theory and the experiment is only due to our insufficient knowledge of the forces between an electron and a proton.

The experimental data on the process of scattering are presented in terms of the so-called effective scattering cross-section $d\sigma$. In view of the one-to-one connection (23) between the scattering angle χ and the impact parameter ρ, the efficient cross-section of the scattering into the solid angle $d\Theta$ can be presented as [1]

$$d\sigma = \frac{\rho(\chi)}{\sin(\chi)} \cdot \frac{d\rho}{d\chi} \cdot d\theta . \qquad (26)$$

Let us denote $k\varphi_0 = (\pi - \chi)/2$. Then formula (23) yields

$$\rho^2 = \frac{e^4}{4E^2} \cdot ctg^2\left(\frac{\chi}{2}\right) - \frac{\Gamma}{E} \qquad (27)$$

Differentiating relation (27) with respect to χ and substituting in the formula for the efficient scattering cross-section (26), we obtain

$$d\sigma = \frac{e^4}{16E^2} \cdot \frac{1}{\sin^4\left(\frac{\chi}{2}\right)} \cdot d\theta . \qquad (28)$$

Formula (28) coincides with the Rutherford formula by its external form[3]. But formula (28) and the Rutherford formula are basically different by their essence. The difference consists in the definition of χ with regard for coefficient (25):

$$\chi = \pi - 2k\varphi_0 . \qquad (29)$$

Generally speaking, this result is surprising, because it is commonly accepted[1,2] that the Rutherford formula is not valid at the relativistic energies. However, we have shown, by performing successively all mathematical operations made by E. Rutherford, that the same formula for the efficient scattering cross-section can be obtained with the binomial potential.

Thus, the Rutherford theory of scattering with the use of the binomial potential is in good agreement with the experiment in the whole ranges of energies and impact parameters. Hence, there is no need to introduce the relativistic corrections and the form-factor into the Rutherford formula, as was made by the author of work [2].

In Fig. 4, we compare the curve calculated by formula (28) for the efficient scattering cross-section of electrons with energies of 400 and 188 *MeV* with the experimental data given in [2].

In view of Eq. (2), the quite rightful questions about the energy of the particle, its velocity, and its mass can arise. Moreover, which velocity will an ultrarelativistic electron have?

We start the discussion of these questions by the citation of work [13]: "Now the time comes, when it is necessary to stop the fraud of a new generation …, which is suggested that the increase of the mass with the velocity is the experimental fact". We note that no direct experiment that will show the dependence of the mass on the velocity was realized till now [14]. But we should like to attract the attention of the reader to formula (2).

Relation (2) is basic in the two-body problem. The result of calculations will depend on the choice of the interaction potential or, in other words, on the model. In macroworld, the Newton model is accepted. It corresponds to the reality, and, therefore, the results of calculations correspond always to experimental data.

As for the microworld, it was assumed that the interaction of two particles occurs by the law Coulomb. We note that the calculations involving the Coulomb law give sometimes the positive result, for example, for the scattering of α-particles in the Rutherford experiment. However, the model of microworld with the Coulomb law give very frequently a negative result. This is demonstrated by results of work [2]. This and analogous works forced the physicists to come to the thought that the classical mechanics should be modified at the description of the microworld and to accept the relativistic mechanics or SRT. However, our studies have shown that the introduction of the binomial potential in the two-body problem removes all difficulties related to the Coulomb law.

But if this classical formula (2) is proper and restricts neither the energy of interacting particles nor the distance between them, then it cannot, apparently, restrict the velocities of interacting particles. Therefore, the introduction of formulas restricting the velocity of particles into the classical mechanics has no physical sense and does not correspond to the reality.

It is worth considering the rightfulness of the application of the Lorentz formulas (see work [2]) to the model of scattering of electrons, as well as the preconditions of the derivation of the Lorentz formulas. The formulas for the Lorentz transformation and a number of consequences following from them (notions "body's size", "time interval between two events", "dependence of body's mass on the velocity", and "limitation of a velocity") were based on the model of two reference systems that move "straightforwardly" and "uniformly" relative of each other. .

What do we see in our model of the scattering of electrons on protons? First of all, we have one reference system, and the "straightforwardness" and "uniformity" are not present. Then we may ask: Why did the authors of work [2] introduce formulas of the Lorentz theory, which limit the velocities and were obtained under other preconditions, in Eq. (2)? We think that the reason for a disagreement of the scattering theory [2] based on the Coulomb law with experiments lies not in "the inability of the classical mechanics to describe the physical phenomena in the microworld", but because the model of the microworld based on the Coulomb law is far from the reality.

Since few persons believe now that body's mass is independent of its velocity and, hence, of its kinetic energy, we will make a brief digression to the history and see how A. Einstein understood this question

A. Einstein introduced the notion of rest energy and showed that body's mass is a measure of body's energy, but he never asserted that the mass depends on body's velocity.

Generally speaking, the idea of the velocity-dependent mass arose in the years before the creation of the relativity theory and for the first years of its existence. This idea was formed in articles, where the authors tried to conform Maxwell's equations of electromagnetism with the Newton equations. Later on, those works stimulated the experiments performed by W. Kaufmann and A. Bucherer, whose data were processed with the help of formulas of the Newton nonrelativistic mechanics with the use of the Coulomb law. In our opinion, such processing of experimental data was incomplete and did not correspond to the reality. But it led to the conclusion that the mass increases with the velocity. We are sure that if those calculations were based on the more suitable binomial potential, then the results would be different.

Here, it is worth noting the very foundation of the nonrelativistic mechanics, where the mass is a measure of body's inertia. At that time, it was not realized yet that the notion "body's inertia" has a more general character, and the measure of body's inertia is its total energy equal to the sum of the rest energy and the kinetic energy, rather than body's mass. But the subsequent development of the theory showed that the notion of mass should be referred only with the rest energy, rather than with the total energy.

VI. Conclusion

Thus, we have solved the problem of two bodies interacting by the law of binomial potential, by considering the interaction of the electron with the proton. Moreover, the solution of this problem met no difficulties concerning the energy of interacting particles. The interacting particles can possess any energy from several eV to ultrarelativistic values. In this case, we have first obtained the possibility to theoretically observe the trajectories of motion of ultrarelativistic particles at their any approach down to intraatomic distances.

It is known that nothing similar to this result can be obtained, if the two-body problem is solved on the basis of the Coulomb law, where the problem of limitation of the energies of interacting bodies and the distances between these particles arises immediately.

In the past, these problems generated the opinion of physicists that the classical mechanics cannot describe the motion of particles with high and superhigh energies and their motion at intraatomic distances. The modification of the classical mechanics by positions of the relativistic mechanics only strengthened this viewpoint. We should like to notice two following aspects of this modification. On the one hand, this confirms supposedly that the classical mechanics cannot describe the intraatomic processes. On the other hand, this gives a practical conformation to the conclusions of the relativistic theory. Therefore, we are faced now with the steady dominating opinion that the classical mechanics is the theory of sufficiently slow motions of macroscopic bodies consisting of a great number of atoms and molecules and can be considered as a limiting case of the relativistic mechanics and as a partial case of the nonrelativistic quantum mechanics.

However, our studies of the scattering of electrons on protons have shown that if the binomial potential is taken instead of the Coulomb potential, then all difficulties of the scattering theory for these particles disappear, and no limitations are required for the calculations performed on the purely classical principles.

Hence, the present work indicates that the classical mechanics with the binomial potential (1) can quite properly represent all aspects of states of the electron – proton system without limitations of the energy of particles starting from a hydrogen atom up to the scattering of ultrarelativistic electrons.

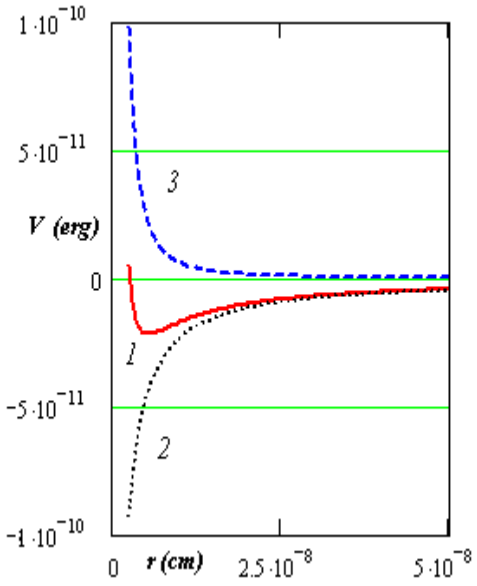

Figure captions

Fig. 1 : Interaction energy of an electron and a proton *vs* the distance between them (1), the Coulomb component (2), the hypothetical component Figure (3)

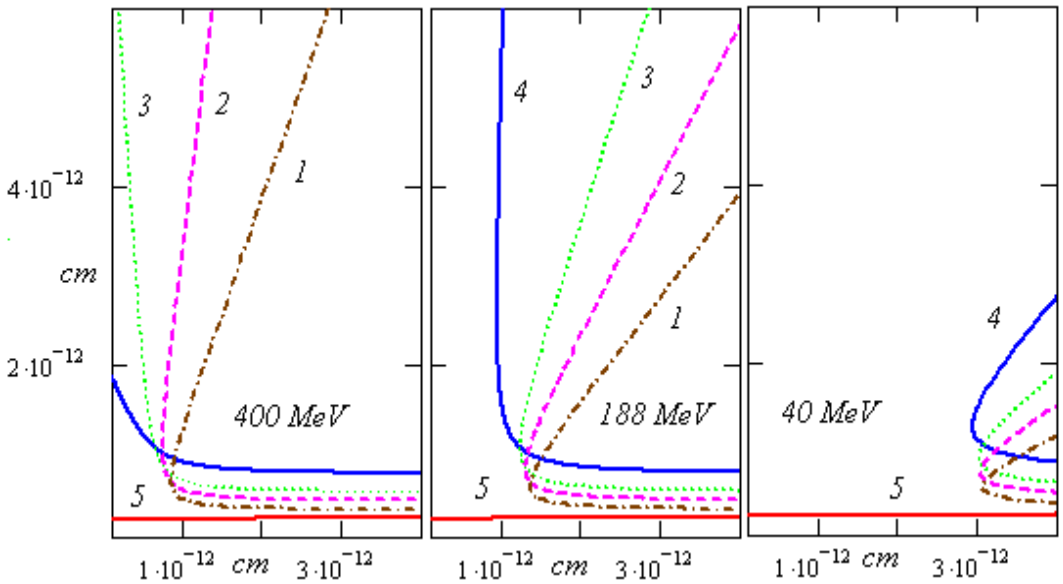

Fig. 2 : Trajectories of motion of electrons with energies of 400 (a), 188 (b) and 400 (c) MeV scattered by a proton. Calculations are performed by formula (8) for impact parameters ρ : 1 - $4\cdot10^{-13}$; 2 - $5\cdot10^{-13}$; 3 - $6\cdot10^{-13}$; 4 - $8\cdot10^{-13}$ cm; 5 – a trajectory of motion of an electron in the Coulomb potential

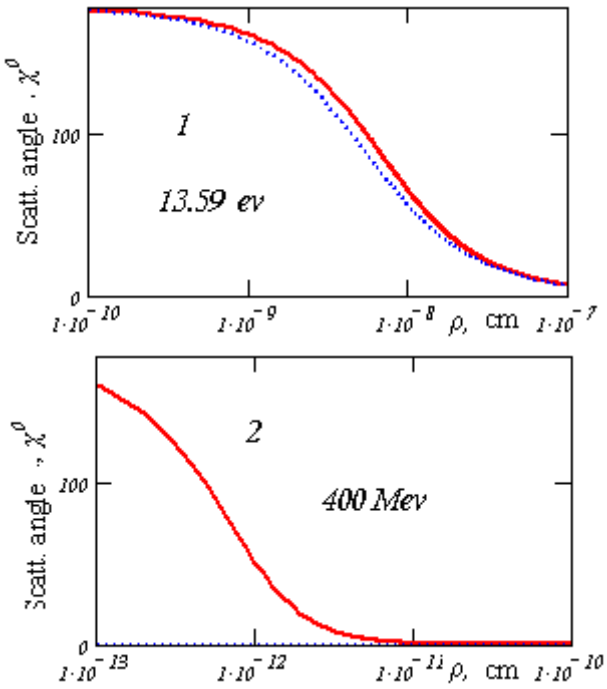

Fig. 3 : Dependence of the scattering angle | on the impact parameter ⟩ for the interaction according to the binomial law (—) and the Coulomb law (---) with electron energies of (a) 13.59 eV and (b) 400 MeV

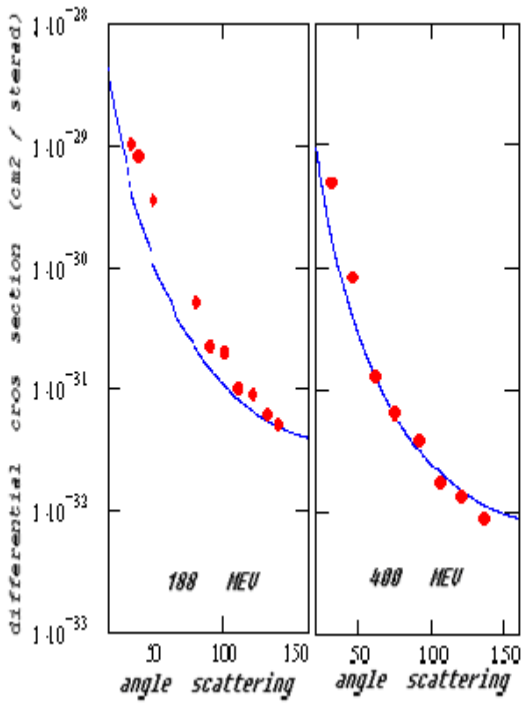

Fig. 4 : Comparison of the calculated effective cross-sections for electrons with energies of 400 and 188 MeV with experimental values: points – experiment [2]; curves – calculated data

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