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R.Amiri $^{\alpha}$ & M.Tavakkoli $^{\sigma}$

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PACs: 11.30.Pb

I. Introduction

The supersymmetric quantum mechanics manages a family of exactly soluble potentials, one of them being the trigonometric Rosen-Morse potential. As long as this potential is obtained from the Eckart potential by complexification of the argument and one of the constants,also its solutions have been concluded rfom those of the Eckart potential by same procedure.

Supersymmetric quantum mechanics was originally proposed by Witten as a simple learning ground for the basic concepts of supersymmetric quantum field theories but soon after it evolved to a research field on its own rights. Supersymmetric quantum mechanics starts with the factorization of one-dimensional Hamiltonians,

$$H(y) = -\frac{\hbar^2}{2m}\frac{d^2}{dy^2} + V(y) \qquad (1)$$

According to $H(y) = A^{+}(y)A^{-}(y) + \mathcal{E}$ with $A^{\pm}(y) = (\pm\frac{\hbar}{\sqrt{2m}}\frac{d}{dy} + U(y))$ Where *U(y)* is the superpotential [1-3]. supersymmetry is arguably the most attractive mechanism to stabilize the hierarchy between the fundamental scale (e.g. the Planck scale $M_{*} \sim 10^{18}$ *GeV)* and the electroweak scale ($M_w \sim$ 100*GeV*). However, superpartners of the standard model particles have not been observed up to energies of order *Mw*, so supersymmetry must be broken at or above the weak scale.The phenomenology of supersymmetry depends crucially on the mechanism of supersymmetry breaking and the way that supersymmetry breaking is communicated to the observable sector. The supersymmetric quantum mechanics manages a family of exactly soluble potentials, one of them being the trigonometric Rosen-Morse potential .we make the point that the trigonometric Rosen-Morse potential is of possible interest to quark physics in so far as it captures the essentials of the QCD quark-gluon dynamics [4-6].

II. The Morse Potential Energy Function

Exact solutions of the effective mass of schrodonger equations for the mentioned potentials are interesting in the fields of material science and condense matter physics. There are various methods for exact solutions to energy eigenvalues and corresponding wave function [7,8].

We can quite write the schrodinger equation in three dimensions for a diatomic molecule with the mass of nuclei m_1 and m_2 as the following form:

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right)\psi_{nlm}(r,\theta,\varphi) = E\psi_{nlm}(r,\theta,\varphi) \qquad (2)$$

Where $\hbar$, μ , $V(r)$ and E are Planck's constant, reduced mass, spherical potential and eigenvalue energy of a quantum system. The radial part of the wave function $R_{nl}(r)$ is defined by relation of $\psi_{nlm}(r,\theta,\varphi) = \frac{R_{nl}(r)}{r} Y_{lm}(\theta,\varphi)$ In that case, the radial schrodinger equation is written by,

$$\frac{d^2 R_{nl}(r)}{dr^2} + 2(E - V_{eff})R_{nl}(r) = 0 \qquad (3)$$

Where we simply suppose that $\hbar = \mu = 1$ and V_{eff} equal the Rosen-Morse potential plus centrifugal term [8,9].

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Exactly solvable 1-dimensional Schrodinger equations play an important role in quantum physics. The best known are the harmonic oscillator and the radial equation for the hydrogen atom, which are covered in every course of quantum mechanics.

The Morse potential energy function is o the form

$$V_{eff}(r) = D_e\left(1 - e^{-a(r-r_e)}\right)^2 \tag{4}$$

Here r is the distance between the atoms, r_e is the equilibrium bond distance, D_e is the well depth (defined relative to the dissociated atoms), and a controls the width of the potential) the smaller a is, the larger the well). The dissociation energy of the bond can be calculated by subtracting the zero point energy $E(0)$ from the depth of the well. The force constant of the bond can be found by Taylor expansion of $V(r)$ around $r = r_e$ to the second derivative of the potential energy function, from which it can be shown that the parameter, a, is

$$a = \sqrt{\frac{K_e}{2D_e}} \tag{5}$$

Where K_e is the force constant at the minimum of the well. Since the zero of potential energy is arbitrary, the equation for the Morse potential can be rewritten any number of ways by adding or subtracting a constant value. When it is used to mode the atom- surface interaction, the Morse potential is usually written in the form:

$$V_{eff}(r) - D_e = D_e\left(e^{-2a(r-r_e)} - 2e^{-a(r-r_e)}\right) \tag{6}$$

Where r is now the coordinate perpendicular to the surface. This form approaches zero at infinite r and $-D_e$ at its minimum. It clearly shows that the Morse potential is the combination of a short-range repulsion and a longer-range attractive tail.

The schrodinger equation

$$\left(-\partial_r^2 + V_{eff}(r) - \delta\right)\varphi(r) = 0 \tag{7}$$

Where

$$V_{eff}(r) = \frac{cosr}{sinr} + \left(\frac{\mu^2}{4} - \frac{1}{4}\right)\frac{1}{sin^2 r} \tag{8}$$

This potential is known as the Rosen-Morse potential, also the trigonometric Rosen-Morse potential. to write the stationary states on the Rosen-Morse potential, i.e. solution $\psi(v)$ and E(v) of the following schrodinger equation:

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V_{eff}(r)\right)\psi(v) = E(v)\psi(v) \tag{9}$$

It is convenient to introduce the new variables:

$$x = ar, \quad x_e = ar_e, \ \lambda = \frac{\sqrt{2mD_e}}{a\hbar}, \quad \mathcal{E}_v = \frac{2m}{a^2\hbar^2}E(v) \tag{10}$$

Then, the schrodinger equation takes the simple form:

$$\left(-\frac{\partial^2}{\partial x^2} + V_{eff}(x)\right)\psi_n(x) = \varepsilon_n\psi_n(x) \tag{11}$$

$$V_{eff}(x) = \lambda^2\left(e^{-2(x-x_e)} - 2e^{-(x-x_e)}\right) \tag{12}$$

Its eigenvalues and eigenstates can be written as:

$$\varepsilon_n = -(\lambda - n - \tfrac{1}{2})^2 \tag{13}$$

$$\psi_n(z) = N_n z^{\lambda-n-\frac{1}{2}} e^{-\frac{1}{2}z} L_n^{2\lambda-2n-1}(z) \tag{14}$$

Where $z = 2\lambda e^{-(x-x_e)}$. whereas the energy spacing between vibrational levels in the quantum harmonic oscillator is constant at hv_O, the energy between adjacent levels decreases with increasing v in the Rosen-Morse oscillator.

$$\frac{E_v}{hc} = \omega_e\left(v + \frac{1}{2}\right) - \omega_e\chi_e(v + \tfrac{1}{2})^2 \tag{15}$$

In which the constants ω_e and $\omega_e\, x_e$ can be directly related to the parameters for the Rosen-Morse potential. As is clear from dimensional analysis, for historical reasons the last equation uses spectroscopic notation in which we represents a wave number obeying $E = hc\omega$, and not an angular frequency given by $E = \hbar\omega$ [10-12].

figure 1: the Morse potential (dark) and harmonic oscillator potential (light) unlike the energy levels of the harmonic oscillator potential, which are evenly sapced by $\hbar\omega$, the Morse potential level spacing decreases as the energy approaches the dissociation energy. The dissociation energy D_e is larger than the true energy required for dissociation D_0 due to the zero point energy of the lowest (v=0) vibrational level.[12]

One of the main problems which are involved in many physical processes is the difference of energy state between ground state and first excited state for potential wells. Ground state wave energy E^1 and first excited state energy E^2, as a result:

$$E^1 = \frac{\int_{-\infty}^{+\infty}\varphi(x)(\delta H)\varphi(x)dx}{\int_{-\infty}^{+\infty}\varphi^2(x)dx} = \frac{\int_{-\infty}^{+\infty}\varphi(x)\left[\frac{\left[\psi_0^1(x_0)\right]^2}{I_1}\right]\delta(x-x_0)\varphi(x)dx}{\int_{-\infty}^{+\infty}\varphi^2(x)dx} + \frac{\int_{-\infty}^{+\infty}\varphi(x)\left[\frac{\left[\psi_0^1(-x_0)\right]^2}{I_1}\right]\delta(x+x_0)\varphi(x)dx}{\int_{-\infty}^{+\infty}\varphi^2(x)dx} =$$
$$\frac{1}{4}\left[\frac{1}{I_1} + \frac{1}{I_2}\right]\frac{1}{\int_{-\infty}^{+\infty}\varphi^2(x)dx} \tag{16}$$

$$E^2 = -\{\int_{-\infty}^{-x_0} dx \left[\frac{E^1 \int_{-\infty}^{x} \varphi^2(x)dx}{\varphi(x)}\right]^2 + \int_{-x_0}^{+x_0} dx \left[\frac{E^1 \int_{-x_0}^{x} \varphi^2(x)dx}{\varphi(x)}\right]^2 + \int_{x_0}^{x} dx \left[\frac{E^1 \int_{x}^{\infty} \varphi^2(x)dx}{\varphi(x)}\right]^2\} \tag{17}$$

$$\Delta E = E^1 + E^2 \tag{18}$$

$$I_+ = \int_{nx_0}^{\infty} \left(\psi_0^1(y)\right)^2 dy, \qquad I_- = \int_{-\infty}^{-nx_0} \left(\psi_0^1(y)\right)^2 dy \tag{19}$$

III. THE SHPE OF THE TRIGONOMETRIC ROSEN-MORSE POTENTIAL

The study of exactly solvable potentials in quantum mechanics for years has received a lot of attention. For solvable models, the simplest generation technique is supersymmetric quantum mechanics, which is equivalent to the factorizatior method, the intertwining technique and the Darboux transformation method.it is well known that the spectrum of the generated Hamiltonian differs little from the initial one. This suggests a way to realize, in practice, the spectral design: (i) one starts from a potential having a spectrum close to the desired one. (ii) then, by appropriately moving creating or deleting a certain set of levels, and iterating the method as many times as needed to achieve the required spectrum, one will arrive at a Hamiltonian (or a set of Hamiltonians),which could model the situation under study [13].

We adopt the following form of the trigonometric Rosen-Morse potential

$$v(z) = -2bcotz + a(a+1)csc^2 z \tag{20}$$

With $a > -\frac{1}{2}$ and displayed in Fig.1. Here, $z = \frac{y}{d}$, $v(z) = V(zd)/(\frac{\hbar^2}{2md^2})$ and $\mathcal{E}_n = E_n/(\frac{\hbar^2}{2md^2})$, with y being the one-dimensional variable, d a properly chosen length scale, *V(y)* the potential in ordinary coordinate space, and E_n the energy level.

Our point here is that *v(z)* interpolates between the Coulomb - and the infinite wall potential going through an intermediary region of linear-*z*-and harmonic-oscillator-z^2 dependences. To see this it is quite instructive to expand the potential in a Taylor series which for appropriately small *z* takes the form of a coulomb like potential with a centrifugal barrier like term, provided by the $csc^2 z$ part,

$$v(z) \approx -\frac{2b}{z} + \frac{a(a+1)}{z^2}, \quad z << 1 \tag{21}$$

For $z < \frac{\pi}{2}$ we can the take Eq. (20) plus a linear like perturbation

$$\Delta v(z) = \frac{a(a+1)}{3} + \frac{2bz}{3} \tag{22}$$

[14-16]

The $\frac{1}{x^2}$ potential has no ground state, and the allowed energies are not quantized. Perhaps some potential are just plain illegal in quantum mechanics. It seems odd, though, that we never encounter such difficulties in classical mechanics.

Well, in the first place there are classical precursors. Moreover there do exist systems represented by a $\frac{1}{x^2}$ potential [17].

IV. THE ROSEN-MORSE POTENTIA AS COMPLEXIFIED ECKART POTENTIAL

Exactly solvable 1-dimensional schrodinger equation play an important role in quantum physics. The best known are the harmonic oscillator and the radial for the hydrogen atom, which are covered in every course of quantum mechanics. A number of other examples were discovered in the 30's of the last century [18].

Before proceeding further we first introduce a properly chosen length scale d and change variables in the one-dimensional equation

$$H(y) = -\frac{\hbar^2}{2m}\frac{d^2}{dy^2} + V(y) \tag{23}$$

To dimensionless ones according to

$$z = \frac{y}{d}, \; v(z) = \frac{V(dz)}{({\hbar^2}/{2md^2})}, \quad \epsilon_n = \frac{E_n}{({\hbar^2}/{2md^2})} \tag{24}$$

Next we employ the Eckart potential,

$$v(z) = -2bcothz + a(a-1)csch^2 z \tag{25}$$

Where $b > a^2$. The exact solutions to the Eckart potential read:

$$\psi_n(x) = c_n(x-1)^{(\beta_n-n-a)/2}(x+1)^{-(\beta_n+n+a)/2}P_n^{\beta_n-n-a\,,-(\beta_n+n+a)}(x)$$

$$x = \coth z\,, \qquad \beta_n = \frac{b}{n+a} \tag{26}$$

Here, $P_n^{(\beta_n-n-a,-(\beta_n+n+a))}(x)$ are the well known Jacobi polynomials with $n \leq (b^{1/2} - a)$ and c_n is a constant. Equation {26} equivalently rewrites to

$$\psi_n(x) = c_n(x^2-1)^{-(n+a)/2}e^{-\beta_n \ \mathrm{arccoth}\ x}P_n^{(\beta_n-n-a\ ,-(\beta_n+n+a))}(x) \tag{27}$$

The corresponding energy value spectrum is determined by

$$\epsilon_n = -(n+a)^2 - \frac{b^2}{(n+a)^2} \qquad (28)$$

Complexify the argument of the Eckart potential and one of its constants according to

$$z \to -iz\,,\ or\ ,equivalently,\ x \to ix\,,\ \ b \to ib \qquad (29)$$

Substitution of Eq.{29} into {25} results in

$$v(z) = -2b\cot z + a(a-1)\csc^2 z \qquad (30)$$

Morse potential is concluded from Eq.{27} through complexification of b and x leading to

$$\psi_n(ix) = c_n((ix)^2-1)^{-(n+a)/2} e^{-\beta_n}\ \ \mathrm{arccoth}\ \ ix\ \ P_n^{((i\beta_n-n-a),(i\beta_n+n+a))}(ix) \qquad (31)$$

[19]

V. SPECTRUM AND WAVE FUNCTIONS OF THE TRIGONOMETRIC ROSEN-MORSE POTENTIAL

Solving schrodinger' equation for the trigonometric Rosen-Morse potential as obtained in Ref [20,21] All in all, the trigonometric Rosen-Morse potential and its real orthogonal polynomial solutions open new venues in the calculation of interesting observables in both supersymmetric quantum mechanics and particle spectroscopy [20,21].

The one- dimensional schrodinger equation with the trigonometric Rosen-Morse potential reads:

$$\frac{d^2}{dz^2}R_m(z) + (2b\cot z - a(a+1)\csc^2 z + \varepsilon_n)R_m(z) = 0 \qquad (32)$$

Our pursued strategy in solving it will be to first reshape it to the particular case of a Sturm-Liouville equation {32} of the form

$$s(x)\frac{d^2}{dx^2}F_m(x) + \frac{1}{w(x)}\left(\frac{d}{dx}s(x)w(x)\right)\frac{d}{dx}F_m(x) + \lambda_m F_m(x) = 0 \qquad (33)$$

And they try to solve it by means of the so called Rodrigues representation

$$F_m(x) = \frac{1}{K_m w(x)}\frac{d^m}{dx^m}(w(x)s(x)) \qquad (34)$$

Where Km is the normalization constant. And

$$\lambda_m = -m(K_1\frac{d}{dx}F_1(x) + \frac{1}{2}(m-1)\frac{d^2}{dx^2}s(x) \qquad (35)$$

[21].

VI. CONCLUSION

In this paper we have discussed on the solution of the schrodinger equation with Rosen-Morse potential. In that case we have written radial part of schrodinger equatin with centrifugal term. Helping the factorization method and comparing it with associated Jacobi differential equation, we have obtained energy eigenvalue and wave function. We also have considered two different cases such as s-wave and Eckart potential for the Rosen-Morse.

All in all the Rosen-Morse potential and its real orthogonal polynomial solutions open new venues in the calculation of interesting observables in both supersymmetric quantum mechanics and particle spectroscopy.

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Table 1 : Some Potentials and Relations [22-28].

potential	relation
Rosen-Morse	$V(r) = -V_1 sech^2 \alpha r + V_2 tanh\alpha r^{*}$
Woods-Saxon	$V(r) = -\dfrac{V_0}{1 + exp\left(\dfrac{r-R}{a}\right)}^{**}$
Eckart	$V(r) = -k\dfrac{coshr}{sinhr} + \left(\dfrac{\mu^2}{4} - \dfrac{1}{4}\right)\dfrac{1}{sinh^2 r}^{***}$
Poschl-Teller	$V(r) = \left(\alpha^2 - \dfrac{1}{4}\right)\dfrac{1}{4sin^2\dfrac{r}{2}} + \left(\beta^2 - \dfrac{1}{4}\right)\dfrac{1}{4cos^2\dfrac{r}{2}}^{****}$
Morse	$V(r) = D_e\left(1 - e^{-a(r-r_e)}\right)^{2*****}$
Manning-Rosen	$V(r) = -k\dfrac{sinhr}{coshr} - \left(\dfrac{\mu^2}{4} - \dfrac{1}{4}\right)\dfrac{1}{cosh^2 r}^{******}$
Hulthen	$V(r) = -\dfrac{z}{\alpha}\dfrac{exp\dfrac{-r}{\alpha}}{1 - exp\dfrac{-r}{\alpha}}^{*******}$

1. Rosen-Morse, In this relation V_1 and V_2 denotes the depth of the potential and α is the range of the potential.
2. The Wood-Saxon potential, is a mean field potential for the nucleons (protons and neutrons) inside the atomic nucleus, which is used to approximately describe the forces applied on each nucleon, in the shell model for the structure of the nucleus.in this potential V_0 (having dimension of energy) represents the potential well depth, a is a length representing the surface thichness of the nucleus, and $R = r_0 A^{\frac{1}{3}}$ is the nuclear radius where $r_0 = 1.25$ fm and A is the mass number.

3. **Eckart,** This potential was proposed and solved by C.Eckart. A natural real domain for this potential is $]0,\infty[$
4. **Poschl-Teller,** This potential was proposed and solved by Poschl and Teller.A natural real domain for this potential is $]0,\pi[$.
5. **Morse,** In this potential r is the distance between the atoms, re is the equilibrium bond distance.
6. **Manning-Rosen,** This potential was proposed and solved by M.F.Manning and N.Rosen. A natural real domain for this potential is $]-\infty,\infty[$.
7. **Hulthen,** The Hulthen potential is a short - range potential which behaves like a Coulomb potential for small values for r and decreases exponentially for large values of r. in this potential a is the screening parameter and z is a constant which is identified with the atomic number when the potential is used for atomic phenomena.

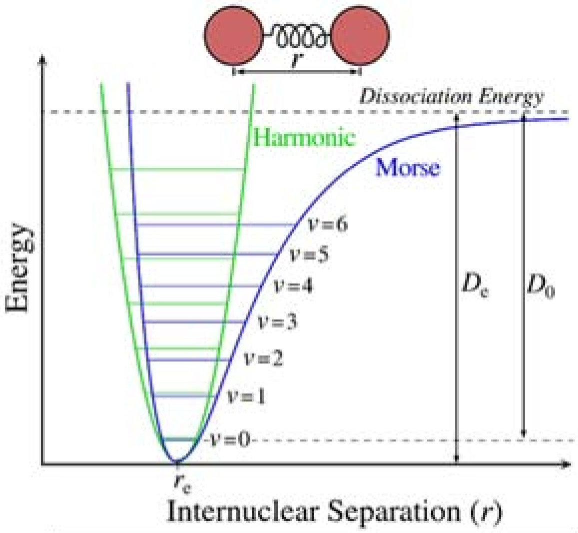

Figure 1: the Morse potential and harmonic oscillator potential

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