Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 14 Issue 4 Version 1.0 Year 2014
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

An Amazing Result Involving Contiguous Relation : A Comutational Approach

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GJSFR-F Classification : MSC 2010: 33C05, 33C20, 33D15, 33D50, 33D60

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Strictly as per the compliance and regulations of :

An Amazing Result Involving Contiguous Relation : A Comutational Approach

Salahuddin [α] & Vinesh Kumar [σ]

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I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A, \ B$ are non-negative integers and $\mid z \mid = 1$

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$ (a-b) \ {}_2F_1\left[\begin{array}{ccc} a, \ b & ; & \\ c & ; & z \end{array}\right] = a \ {}_2F_1\left[\begin{array}{ccc} a+1, \ b & ; & \\ c & ; & z \end{array}\right] - b \ {}_2F_1\left[\begin{array}{ccc} a, \ b+1 & ; & \\ c \ ; & & z \end{array}\right] \tag{2} $$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{ccc} a, \ b & ; & \\ \frac{a+b+1}{2} & ; & \frac{1}{2} \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2}) \ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2}) \ \Gamma(\frac{b+1}{2})} \tag{3} $$

$$ = \frac{2^{(b-1)} \ \Gamma(\frac{b}{2}) \ \Gamma(\frac{a+b+1}{2})}{\Gamma(b) \ \Gamma(\frac{a+1}{2})} \tag{4} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$$ {}_2F_1\left[\begin{array}{ccc} a, \ b & ; & \\ \frac{a+b-1}{2} & ; & \frac{1}{2} \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2}) \ \Gamma(\frac{b+1}{2})} + \frac{2 \ \Gamma(\frac{a+b-1}{2})}{\Gamma(a) \ \Gamma(b)}\right] \tag{5} $$

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Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\ \Gamma(\zeta) \tag{7} $$

Notes

II. Main Summation Formula

$$ {}_2F_1\left[\begin{matrix} a,\ \ b & ; & \frac{1}{2} \\ \frac{a+b+49}{2} & ; & \end{matrix}\right] = \frac{2^b\ \Gamma(\frac{a+b+49}{2})}{(a-b)\ \Gamma(b)\Big[\prod_{\mu=1}^{24}\{a-b-(2\mu-1)\}\Big]\Big[\prod_{\xi=1}^{24}\{a-b+(2\xi-1)\}\Big]} $$

$$ \Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Big\{8388608(a^25+a^24(-576+1127b)+4a^{23}(39169-51744b+48363b^2)+92a^{22}(-290928 $$

$$ +634207b-221088b^2+131271b^3)+3542a^{21}(909563-1563264b+1548462b^2-243648b^3 $$

$$ +103071b^4)+3542a^{20}(-82032288+188926633b-99710688b^2+61109118b^3-5432448b^4+1740081b^5) $$

$$ +92a^{19}(221572333763-429292237920b+438974519095b^2-111905902080b^3+48021637825b^4 $$

$$ -2728447008b^5+686171941b^6)+1748a^{18}(-652191333552+1552836538141b-997765722240b^2 $$

$$ +621099121475b^3-93000114480b^4+29813967827b^5-1169334432b^6+236070989b^7) $$

$$ +437a^{17}(118237765837763-245960503305216b+256017026954220b^2-82688287167360b^3 $$

$$ +35577424928010b^4-3489293945088b^5+867694870812b^6-24723070848b^7+4056128811b^8) $$

$$ +437a^{16}(-4391592874255488+10679020370180365b-7723439928047424b^2+4831986595164316b^3 $$

$$ -939198292845120b^4+298335631111518b^5-20512464606144b^6+4042994679612b^7-86530747968b^8 $$

$$ +115674784461b^9)+1288a^{15}(45681097143081937-99574723999503648b+104501659542515343b^2 $$

$$ -38985088377593088b^3+16661320812453658b^4-2153363100178752b^5+523951164774702b^6 $$

$$ -26375984727552b^7+4156630669317b^8-67988444832b^9+7318200659b^{10}) $$

$$ +1288a^{14}(-1160835982753925712+2858915589700984289b-2236018737076611360b^2 $$

$$ +1393738308249650165b^3-317752185259953120b^4+99006207906524274b^5-9037635481929600b^6 $$

$$ +1714993712797890b^7-64723675029840b^8+8093019369325b^9-100363894752b^{10}+8268616329b^{11}) $$

$$ +4a^{13}(7885248558387029118799-17740046200259070839424b+18621420718060507752818b^2 $$

$$ -7640510532582764343360b^3+3212288577782897155725b^4-490605934314569676288b^5 $$

$$ +115216473790726873964b^6-7683233847037257600b^7+1135210887503614425b^8 $$

$$ -32003974007617920b^9+3027437157819074b^{10}-25930380807744b^{11}+1215486600363b^{12}) $$

$$ -4a^{12}(137971112401324506880224-341745755847769091259581b+281805523312710309223584b^2 $$

$$ -173404413877307333549010b^3+43827888254763867124416b^4-13217460237988621117867b^5 $$

$$ +1422491466706905381696b^6-253712233399682275036b^7+12395651957628252768b^8 $$

$$-1365375597501222307b^9+2627597019333686 4b^{10}-140105088801841 8b^{11}+1215486600363b^{13})$$

$$-8a^{11}(-9979652555215563016309 37+229088036080537089620841 6b$$

$$-238478386872578945810199 3b^2+104128120970757851644185 6b^3-42510712553734562970259 7b^4$$

$$+71824521873644492752800b^5-1589724846392451112043 7b^6+122136344649151703270 4b^7$$

$$-1593642889524313497 87b^8+52359876882701868 48b^9-3213285571288622 83b^{10}$$

$$+7005254440092 09b^{12}-129651904038 72b^{13}+13312472289 69b^{14})$$

$$-88a^{10}(107885832315447362405179 2-266804359737320688242848 7b$$

$$+22734484231685553621006 72b^2-136387630648581172434198 1b^3+36624984928056164592388 8b^4$$

$$-10441912216416811745423 9b^5+12149335654454468931552b^6-191871957251479857852 5b^7$$

$$+9722301194131572446 4b^8-6964630094652459229b^9+2921168701171475 3b^{11}$$

$$-1194362281515312b^{12}+137610779900867b^{13}-14689624595 52b^{14}+107111846009b^{15})$$

$$-7a^9(-131435064277092371288499193+304470532489986337820912640b$$

$$-310677264213625303638878280b^2+140111784193090489218821376b^3$$

$$-54274554250668426372950732b^4+9523746494510523997822464b^5$$

$$-18719720451893882364077 68b^6+13997598363506317840051 2b^7-11908752799840408611814b^8$$

$$+875553497613452017 36b^{10}-5983985929451642112b^{11}+780214627143555604b^{12}$$

$$-18287985147210240b^{13}+1489115563955800b^{14}-125098738490088b^{15}+7221411553 51b^{16})$$

$$-7a^8(102536260442919612526047916 8-250667925903395213861389151 9b$$

$$+215978490606635743018255296 0b^2-123511544033377719620643536 8b^3$$

$$+33504713744985669818739571 2b^4-8511830124772857828221562 0b^5$$

$$+926119168682541713027366 4b^6-95632641502669134387669 6b^7+11908752799840408611814b^9$$

$$-1222232150119397678976b^{10}+182130615945635828328b^{11}-7083229690073287296b^{12}$$

$$+648691935716351100b^{13}-11909156205490560b^{14}+764820043154328b^{15}-5401990980288b^{16}$$

$$+2532183272 01b^{17})-4a^7(-110799022084541934922726230 69+255220004952143854591345366 08b$$

$$-249888492099538853101569810 95b^2+111553624363161887457710853 12b^3$$

$$-38665756145170113213316875 40b^4+61731310491686005661497305 6b^5$$

$$-795880530198846903647434 36b^6+167357122629670985178421 8b^8-24495797136136056220089 6b^9$$

$$+42211830595325568727550b^{10}-2442726892983034065408b^{11}+253712233399682275036b^{12}$$

$$-7683233847037257600b^{13}+552227975520920580b^{14}-8493067082271744b^{15}+441697168747611b^{16}$$

$$-27009954901 44b^{17}+103163022193b^{18})-4a^6(528956586037154221773361777 44$$

$$-125504499484517913120481096725b+105424690928808184808118256992b^2$$

$$-54248552360062346572187575205b^3+13126541132244000149277247680b^4$$

$$-2195627585159893786910327620b^5+795880530198846903647434 36b^7$$

$$-162070854519444799779789 12b^8+32759510790814294137135 94b^9$$

$$-2672853843979983164941 44b^{10} + 3179449692784902224087 4b^{11} - 1422491466706905381696b^{12}$$

$$+115216473790726873964b^{13} - 29101186251813312 00b^{14} + 1687122750574540 44b^{15}$$

$$-2240986758221232b^{16} + 947956646362 11b^{17} - 510999146784b^{18} + 157819546 43b^{19})$$

$$-2a^5(-37770180364040075630515852 7625 + 840006312597078894057676620672b$$

$$-746504836196868708716896610562b^2 + 29298202272555074183977815 6096b^3$$

$$-671910208246569975921135206 89b^4 + 439125517031978757382065 5240b^6$$

$$-12346262098337201132299461 12b^7 + 297914054367050023987754670b^8$$

$$-3333311273078683399237862 4b^9 + 45944413752233971679865 16b^{10}$$

$$-2872980874945779710112 00b^{11} + 264349204759772422357 34b^{12} - 9812118686291393525 76b^{13}$$

$$+637599978918016324 56b^{14} - 138676583651511 6288b^{15} + 651863353978666 83b^{16}$$

$$-762410727001728b^{17} + 260574078807 98b^{18} - 1255085623 68b^{19} + 3081683451b^{20})$$

$$-2a^4(959983440905270027294733276000 - 210382454131306280288973454 7475b$$

$$+153928890506468787818469027 4080b^2 - 527901057497272758919053580 818b^3$$

$$+671910208246569975921135206 89b^5 - 262530822644880002985544953 60b^6$$

$$+773315122903402264266337508 0b^7 - 117266498107449844365588499 2b^8$$

$$+189960939877339492305327562b^9 - 161149933683447124206510 72b^{10}$$

$$+170042850214938251881038 8b^{11} - 876557765095277342488 32b^{12} + 6424577155565794311450b^{13}$$

$$-204632407307409809280b^{14} + 107298906032201557 52b^{15} - 205214826986658720b^{16}$$

$$+7773667346770185b^{17} - 812821000555 20b^{18} + 2208995339950b^{19} - 96208654 08b^{20} + 182538741b^{21})$$

$$-28a^3(-114695677025205243600075339375 + 2225153458030773577304777400 00b$$

$$-133545478363571496889383360075b^2 + 37707218392662339922789541487b^4$$

$$-2092728733753933870284129686 4b^5 + 774979319429462093888393931 5b^6$$

$$-159362320518802696368158361 6b^7 + 30877886008344429905160884 2b^8$$

$$-3502794604827262230470534 4b^9 + 42864683918125511336462 26b^{10}$$

$$-2975089170593081475548 16b^{11} + 247720591253296190784 30b^{12} - 1091501504654680620480b^{13}$$

$$+64111962179483907590b^{14} - 17933140653692820 48b^{15} + 75413505074528789b^{16}$$

$$-1290527910433440b^{17} + 387743308692 25b^{18} - 3676908211 20b^{19} + 7730303427b^{20} - 30821472b^{21}$$

$$+431319b^{22}) - 28a^2(10949556710701056784228893 0000 - 16773787981580146179319176 1875b$$

$$+133545478363571496889383360075b^3 - 1099492075046205627274778767 20b^4$$

$$+533217740140620506226354721 83b^5 - 150606701326868835440168938 56b^6$$

$$+356983560142198361573671158 5b^7 - 53994622651658935754563824 0b^8$$

$$+7766931605340632590971957 0b^9 - 71451236156726025666021 12b^{10}$$

$$+6813668196359398451719 98b^{11} - 402579319018157584605 12b^{12} + 26602029597229296789 74b^{13}$$

$$-1028568619055241225 60b^{14} + 480707633895570577 8b^{15} - 1205408303055972 96b^{16}$$

Notes

$$
\begin{aligned}
&+3995694313535505b^{17}-622890886598 40b^{18}+1442344848455b^{19}-126134020 32b^{20}+195880443b^{21}\\
&-726432b^{22}+6909b^{23})-b(1192568192774434123539907640625\\
&-306587587899629589958409004 0000b+32114789567057468208021095025 00b^{2}\\
&-1919966881810540054589466552000b^{3}+755403607280801512610317055250b^{4}\\
&-211582634414861688709344710976b^{5}+44319608833816773969090492276b^{6}\\
&-7177538231004372876823354176b^{7}+920045449939646599019494351b^{8}\\
&-94939532437593678916557696b^{9}+7983722044172450413047496b^{10}\\
&-551884449605298027520896b^{11}+31540994233548116475196b^{12}-1495156745787056317056b^{13}\\
&+58837253120289534856b^{14}-1919126086049648256b^{15}+51669903671102431b^{16}\\
&-1140030451048896b^{17}+20384654706196b^{18}-290558364096b^{19}+3221672146b^{20}-26765376b^{21}\\
&+156676b^{22}-576b^{23}+b^{24})-7a(-170366884682062017648558234375\\
&+670951519263205847172767047500b^{2}-890061383212309430921910960000b^{3}\\
&+601092726089446515111352727850b^{4}-240001803599165398302193320192b^{5}\\
&+71716856848295950354560626700b^{6}-14584000282979648833791163776b^{7}\\
&+2506679259033952138613891519b^{8}-304470532489986337820912640b^{9}\\
&+33541119509834600807672408b^{10}-2618148983777566738523904b^{11}\\
&+195283289055868052148332b^{12}-10137169257290897622528b^{13}+526040468504981109176b^{14}\\
&-18321749215908671232b^{15}+666675985966974215b^{16}-15354962849197056b^{17}\\
&+387765466952924b^{18}-5642126555520b^{19}+95596876298b^{20}-791011584b^{21}+8335292b^{22}\\
&-29568b^{23}+161b^{24}))\Big\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Big\{268435456(3a^{2}4+a^{23}(-575+1078b)+23a^{22}(8921-5145b+4606b^{2})\\
&+253a^{21}(-77095+150388b-31255b^{2}+17766b^{3})+1771a^{20}(1616303-1417115b+1295273b^{2}\\
&-139825b^{3}+56588b^{4})+161a^{19}(-1046829225+2079583870b-701082550b^{2}+401485280b^{3}\\
&-265256 25b^{4}+8120378b^{5})+437a^{18}(31431018189-337989625 05b+31115280280b^{2}-5662501250b^{3}\\
&+22841038885b^{4}-101504725b^{5}+243611 34b^{6})+437a^{17}(-1247763838705+2496774883568b\\
&-1070526853935b^{2}+611526437130b^{3}-69822872175b^{4}+210967420 44b^{5}-669931185b^{6})\\
&+1287659 94b^{7}+437a^{16}(63279202335014-77204535727935b+70972718319307b^{2}-16814438930835b^{3}\\
&+6703353718635b^{4}-523216255485b^{5}+122670490257b^{6}-2897234865b^{7}+450680979b^{8})\\
&+46a^{15}(-1670772770171612 5+3342798324748628 2b-1663783210911262 0b^{2}+940791938783995 2b^{3}\\
&-1419804931510350b^{4}+419788673696868b^{5}-23634441372780b^{6}+4382164307808b^{7}\\
&-788691713 25b^{8}+99149815 38b^{9})+322a^{14}(792950221948980 59-10547396303966955 5b\\
&+9614974168385207 0b^{2}-2683390220093990 0b^{3}+1048757469145669 0b^{4}-109143144065025 0b^{5}\\
&+247315972449000b^{6}-103757798451 00b^{7}+1527762530955b^{8}-21077766875b^{9}+2090914474b^{10})\\
&+14a^{13}(-35950100218471660065+71474117681249818612b-39360973797993711325b^{2}\\
&+21863665400825735130b^{3}-3919212579014104050b^{4}+1122192015900766384b^{5}
\end{aligned}
$$

$$-843581541921218 10b^6 + 14835535781621100b^7 - 469070372518125b^8 + 53762807034860b^9$$
$$-548865049425b^{10} + 38586876202b^{11}) - 2a^{12}(-558336951075055 8630069$$
$$+7896740167166969665365b - 7087705250539547642827b^2 + 2206545302765470623411b^3$$
$$-836963484922477152786b^4 + 103571236559409789170b^5 - 2230714985768072895 0b^6$$
$$+123537468415941379 8b^7 - 1661954915527563 93b^8 + 38370895927213 85b^9 - 308337748365647b^{10}$$
$$+17557028671 91b^{11}) - 2a^{11}(7767954491490205996802 5 - 15246859305917884620042 2b$$
$$+9003262599765487620894 6b^2 - 48651361704472855978832b^3 + 9737398318034636797639b^4$$
$$-265601624628998309379 0b^5 + 23465850863234028594 8b^6 - 37787398450588219888b^7$$
$$+1501966116788209527b^8 - 139758311120734306b^9 + 1780767039157106b^{10} - 17557028671 91b^{12}$$
$$+270108133414b^{13}) - 22a^{10}(-1042470154542718591896 19 + 1538295744077017471466 55b$$
$$-1346866623313093665911 24b^2 + 45003731311168981668954b^3 - 1630106382809589220017 3b^4$$
$$+222583718295503425724 5b^5 - 43997146313607252711 4b^6 + 27267243691424920092b^7$$
$$-298475703581515450 9b^8 + 64599798050692905b^9 - 161887912650646b^{11} + 28030704396877b^{12}$$
$$-3492777587 25b^{13} + 30603384574b^{14}) - 2a^9(110090482688932326463992 05$$
$$-211491307199022851161842 80b + 1304584752873054391030533 5b^2$$
$$-675036718236365209746121 8b^3 + 143301662536697719066876 5b^4 - 3596063748957993318580 12b^5$$
$$+333770463713285871422 15b^6 - 438340022467236518228 6b^7 + 14537771981431388189 5b^8$$
$$-710597778557621955b^{10} + 139758311120734306b^{11} - 38370895927213 85b^{12} + 376339649244020b^{13}$$
$$-3393520466875b^{14} + 228044575374b^{15}) - 7a^8(-30074304640317904031207129$$
$$+454576529014104298333960 10b - 382342977518027030727929 54b^2$$
$$+1318979998232192913626605 8b^3 - 440726856909424105624284 6b^4 + 607683841814605872882 930b^5$$
$$-982029030214541229697 14b^6 + 476693515498674502571 4b^7 - 41536491375518251970b^9$$
$$+938066496970477131 4b^{10} - 429133176225202722b^{11} + 47484426157930398b^{12}$$
$$-938140745036250b^{13} + 70277076423930b^{14} - 5182831258 50b^{15} + 281353696 89b^{16})$$
$$-7a^7(1902354123976999034366261 25 - 352512867635431785373491 218b$$
$$+220189994397267940717904408b^2 - 105433147752212001296291552b^3$$
$$+220006238274939847365691 32b^4 - 452570700374199724565901 6b^5$$
$$+316238775975782866448 424b^6 - 476693515498674502571 4b^8 + 12524000641921043377 96b^9$$
$$-856970516016211774 32b^{10} + 107963995573109199 68b^{11} - 352964195474118228b^{12}$$
$$+296710715632422 00b^{13} - 477285872874600b^{14} + 287970797370 24b^{15} - 18087023371 5b^{16}$$
$$+8038677054b^{17}) + a^6(777047070580415197112919948 3 - 117260423435212802466425578 35b$$
$$+916222376030806638849209323 4b^2 - 304757559717906750436763314 4b^3$$
$$+837301129723740812841854612b^4 - 84609550663464967130901540b^5$$
$$+221367143183048006513896 8b^7 - 687420321150178860787998b^8 + 66754092742657174284430b^9$$

Notes

Notes

$$-96793721889935955596508b^{10}+469317017264680571896b^{11}-446142997153614579006b^{12}$$

$$+1181014158689705340b^{13}-79635743128578000b^{14}+1087184303147880b^{15}-53607004242309b^{16}$$

$$+2927599278845b^{17}-106458155558b^{18})+a^{5}(-296657029788288903750528109 35$$

$$+513357911769117846574258859492b-304888137753347677416106588895b^{2}$$

$$+120479363570298267458855523046b^{3}-180749648168004099657506782 0b^{4}$$

$$+8460955066346496713090154 0b^{6}-316799490261939807196131 12b^{7}$$

$$+425378689270224111018051 0b^{8}-719212749791598663716024b^{9}+489684180250107536593 90b^{10}$$

$$-5312032492579966187580b^{11}+207142473118819578340b^{12}-15710688222610729376b^{13}$$

$$+351440923889380500b^{14}-19310278990055928b^{15}+228645503646945b^{16}-9219276273228b^{17}$$

$$+443575648 25b^{18}-1307380858b^{19})+a^{4}(93481474129316029149694056225$$

$$-1328609778924127729581608335 65b+860883954345156523463099949 51b^{2}$$

$$-203047479320711178817671081 27b^{3}+180749648168004099657506782 0b^{5}$$

$$-837301129723740812841854612b^{6}+154004366792457893155983924b^{7}$$

$$-308508799836596873936999 22b^{8}+286603325073395438133753 0b^{9}$$

$$-358623404218109628403806b^{10}+194747966360692735595278b^{11}-16739269698449543055 72b^{12}$$

$$+548689761061974567 00b^{13}-337699905064905418 0b^{14}+65311026849476100b^{15}$$

$$-2929365575043495b^{16}+305125951404 75b^{17}-998153397745b^{18}+42706256 25b^{19}-100217348b^{20})$$

$$+a^{3}(-1787461689354206481614282735 25+258379684607101604925551403510b$$

$$-1077955383205635876511295993 82b^{2}+203047479320711178817671081 27b^{4}$$

$$-120479363570298267458855523046b^{5}+30475755971790675043676331 44b^{6}$$

$$-738032034265484009074040 8864b^{7}+92328599876253503953862406b^{8}$$

$$-1350073436472730419492243 6b^{9}+990082088845717596716988b^{10}-9730272340894571195766 4b^{11}$$

$$+44130906055309412468 22b^{12}-306091315611560291820b^{13}+8640516508702647800b^{14}$$

$$-432764291840637792b^{15}+7347909812774895b^{16}-267237053025810b^{17}+2474513046250b^{18}$$

$$-646391300080b^{19}+247630075b^{20}-4494798b^{21})-7a^{2}(-31975554579096364765102762875$$

$$+317925260868550393400422785 75b-153993626172233696644470856 26b^{3}$$

$$+122983422049308074780442849 93b^{4}-435554482504782396308723698 5b^{5}$$

$$+1308889108615438055498870462b^{6}-220189994397267940717904408b^{7}$$

$$+38234297751802703072792954b^{8}-3727385008208726831515810b^{9}+42330093875554372357210 4b^{10}$$

$$-257236074279013932025 56b^{11}+20250586430112993265 22b^{12}-78721947595987422650b^{13}$$

$$+4422888117457195220b^{14}-109334325288454360b^{15}+443072541507673 7b^{16}-6683146216708 5b^{17}$$

$$+1942482497480b^{18}-16124898650b^{19}+327704069b^{20}-1129645b^{21}+15134b^{22})$$

$$+b(1155279578871850810789867706 25-22382888205367455335571934012 5b$$

$$+1787461689354206481614282735 25b^{2}-93481474129316029149694056225b^{3}$$

$$+296657029788288903750528109 35b^{4}-7770470705804151971129199483b^{5}$$

$$+133164788678389932405638287 5b^6 - 2105201324822253282184499 03b^7$$

$$+2201809653778646529279841 0b^8 - 22934343399939809021716 18b^9 + 1553590898298041199360 50b^{10}$$

$$-11166739021501117260138b^{11} + 5033014030586032409 10b^{12} - 2553299714675717499 8b^{13}$$

$$+7685554742789417 50b^{14} - 276530114204011 18b^{15} + 545272797514085b^{16} - 13735354948593b^{17}$$

$$+1685395052 25b^{18} - 2862472613b^{19} + 19505035b^{20} - 205183b^{21} + 575b^{22} - 3b^{23})$$

$$-7a(165039939838835830112838243 75 - 3179252608685503934004227857 5b^2$$

$$+369113835153002292750787719 30b^3 - 189801396989161104225944047 95b^4$$

$$+73336844538445406653465513 56b^5 - 167514890621732574952036540 5b^6$$

$$+35251286763543178537349121 8b^7 - 4545765290141042983339601 0b^8$$

$$+604260877711493860462408 0b^9 - 48346437670991977674663 0b^{10} + 4356245515976538462869 2b^{11}$$

$$-2256211476333419904390b^{12} + 142948235362499637224b^{13} - 4851802299824799530b^{14}$$

$$+219669604197766996b^{15} - 4819768873301085b^{16} + 155870089159888b^{17} - 2110020944955b^{18}$$

$$+47830429010b^{19} - 3585300095b^{20} + 5435452b^{21} - 16905b^{22} + 154b^{23}))\Big\}\Bigg] \tag{8}$$

III. Derivation of the Summation Formula

Putting $c = \frac{a+b+49}{2}$ and $z = \frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+49}{2} & ; & \end{matrix}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b & ; & \frac{1}{2} \\ \frac{a+b+49}{2} & ; & \end{matrix}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1 & ; & \frac{1}{2} \\ \frac{a+b+49}{2} & ; & \end{matrix}\right]$$

Now involving the derived formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

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Notes