Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 14 Issue 7 Version 1.0 Year 2014
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Correspondence between Fuzzy Multisets and Sequences

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Abstract- Although fuzzy multisets or fuzzy bags are very interesting in terms of applications, it is less study or explore by researchers. Fuzzy multiset is applicable as a model of information retrieval because it has the mathematical structure or framework which expresses the number and the degree of attribution of an element simultaneously. We present a concise note on fuzzy multisets as the fuzzification or extension of multisets and the generalization of fuzzy sets. We showed the correspondence between fuzzy multisets and sequences. The Bolzano-Weierstrass property of fuzzy multisets is proposed and some theorems stated and proved.

GJSFR-F Classification : FOR Code : MSC 2010: 94D05

Strictly as per the compliance and regulations of :

Ref

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Abstract- Although fuzzy multisets or fuzzy bags are very interesting in terms of applications, it is less study or explore by researchers. Fuzzy multiset is applicable as a model of information retrieval because it has the mathematical structure or framework which expresses the number and the degree of attribution of an element simultaneously. We present a concise note on fuzzy multisets as the fuzzification or extension of multisets and the generalization of fuzzy sets. We showed the correspondence between fuzzy multisets and sequences. The Bolzano-Weierstrass property of fuzzy multisets is proposed and some theorems stated and proved.

I. Introduction

The theory of fuzzy multisets or fuzzy bags was introduced by Yager [16] as an attempt to fuzzify multisets proposed by Knuth [2]. In terms of similarity, fuzzy multisets are the generalized fuzzy sets introduced by Zadeh [18]. The theory of fuzzy multisets is a mathematical framework which can represent multiple occurrences of a subject item with degrees of relevance and it has been studied in relation to a variety of information systems including relational database. Fuzzy multiset is a multiset of pairs, where the first part of each pair is an element of and the second part is the degree to which the first part belongs to fuzzy multiset. An element of a fuzzy multiset can occur more than once with possibly the same or different membership values. Miyamoto [6] gave an up-to-date presentation of the theory of fuzzy multisets, which, however, does not differ significantlyfrom [5]. Since Yager [16] proposed fuzzy bags, a number of studies have been done on the theory and applications [3, 4, 11, 12, 17]. In particular, Miyamoto [5, 6, 9, 10] redefined the basic operations such as union and intersection for fuzzy bags. Singh *et al.* [14] gave an outline on the development of the concept of fuzzy multisets. Syropoulos [15] proposed the generalized fuzzy multisets and their application in computation. Bedregal *et al.* [1] generalized Atanassov's operators to higher dimensions using the concept of fuzzy sets which are the special kind of fuzzy multisets, to define a generalization of Atanassov's operator for n-dimensional fuzzy value (called n-dimensional interval).

In this paper, we propose the inter-relationship between fuzzy multisets and sequences, introduce the Bolzano-Weierstrass property of fuzzy multisets, state and prove some important theorems.

II. Precise Note on Fuzzy Multisets

Definition 1: Let X be a nonempty set. A fuzzy multiset(FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A such that $CM_A\colon X \to Q$ where Q is the set of all crisp multisets (i.e. non- fuzzy multisets) drawn from the unit interval $[0,1]$. Then for any $x \epsilon X$, the value $CM_A(x)$ is a crisp multiset drawn from $[0,1]$. For each $x \epsilon X$, the membership sequence is defined as a decreasingly

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ordered sequence of elements in $CM_A(x)$. It is denoted by $\mu_A^1(x), \mu_A^2(x)$,…,$\mu_A^n(x)$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^n(x)$.

Definition 2(alternative definition): A fuzzy multiset A in X is a set of ordered sequence given as $A = \{\langle x, \mu_1(x), \mu_2(x), \mu_3(x), \ldots, \mu_n(x), \ldots \rangle : x \in X\}$, where $\mu_n(x): X \to [0,1]$is the membership function of A.

If the sequence of the membership functions have only $n - terms$ (finite number of terms), n is called the "*dimension*"of A. The collection of all fuzzy multisets in X of dimension n is denoted by $\mathcal{FM}(X)$. When we define an operation between two fuzzy multisets A and B, the lengths of the membership sequences $\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x)$, and $\mu_B^1(x), \mu_B^2(x)$,…, $\mu_B^p(x)$, should be equal, if not, we append some zeros if need be.

III. Cardinality of Fuzzy Multisets

Definition 3: The length of an element x in an FMS A is defined as the cardinality of $CM_A(x)$ denoted by $L(x{:}A)$ i.e. the length of x in A for each $x \in X$ as

$$L(x{:}A) = \left| CM_A(x) \right|$$

Definition 4: If A and B are FMS drawn from X, then

$L\,(x{:}\,A, B) \;= max\,\{\,L(x{:}A), L(x{:}B)\}$ ″ or

$L(x) \;=\; \vee\,[L(x{:}A), L(x{:}B)]$ where $L(x) = L(x{:}\,A, B)$ and $\vee$ denotes maximum .

Example1

Consider the fuzzy multisets,

$A = \{(x_1, 0.2), (x_1, 0.3), (x_2, 1), (x_2, 0.5), (x_2, 0.5)''$ of $X = \{x_1, x_2, x_3, x_4\}$, which means that A has x_1, with the membership of 0.2 and 0.3, x_2 with membership of 1 and 0.5 twice.

We represent A as

$$A = \{(0.2,0.3)/x_1\,, (1, 0.5,0.5)/x_2\}'',$$

in which the bag of membership $\{0.2, 0.3\}$ corresponds to x_1, and $\{1, 0.5, 0.5\}$ corresponds to x_2.

When we handle a finite number of fuzzy multisets in a finite universal set, the length L of the membership sequences is set to be a constant for all members and for all the concerned fuzzy bags, by appending appropriate numbers of 0 at the end of the membership sequences.

For the above example, we can set $L = 3, \mu_A^1(x_1) = 0.2, \mu_A^2(x_1) = 0.3, \mu_A^3(x_1) = 0$, $\mu_A^1(x_2) = 1, \mu_A^2(x_2) = \mu_A^3(x_2) = 0.5, \mu_A^1(x_3) = \mu_A^2(x_3) = \mu_A^3(x_3) = \mu_A^1(x_4) = \mu_A^2(x_4) =$ $\mu_A^3(x_4) = 0$.

By appending the zeros, we get

$$A = \{(0.2,0.3,0)/x_1\,, (1, 0.5,0.5)/x_2, (0,0,0)/x_3\,, (0,0,0)/x_4\}.$$

Example 2

Consider the set $X = \{x_1, x_2, x_3, x_4\}$ with

$$A = \;\{(0.3, 0.2,0.4, 0.5)/x_1\,, (1, 0.5, 0.5,0)/x_2, (0,0.5, 0.5)/x_3, (0.1, 0.1)/x_4\,\}$$

$$B = \{(0.3, 0.2,0.4,)/x_1\,, (1, 0.5, 0.5)/x_2, (0.5, 0.5)/x_3, (0.1, 0.1,0.3)/x_4\,\}$$

Then,

$L(x_1{:}\,A) = 4, L(x_2{:}\,A) = 4,\; L(x_3{:}\,A) = 3,\; L\,(x_4{:}\,A) = 2,\;\; L(x_1{:}\,B) = 3,\; L(x_2{:}\,B) = 3, L(x_3{:}\,B) = 2,\; L(x_4{:}\,B) = 3,\; L(x_1) = 4,\; L(x_2) = 4,\; L(x_3) = 3\,,\; L(x_4) = 3.$

Notes

Notes

Basic relations and operations on fuzzy multisets

1. Inclusion $A \subseteq B \leftrightarrow \mu_A^i(x) \le \mu_B^i(x), i = 1, 2, \ldots, n\ \forall x \in X.$
2. Equality $A = B \leftrightarrow \mu_A^i(x) = \mu_B^i(x), i = 1, 2, \ldots, n\ \forall x \in X.$
3. Complement $A' = 1 - A \rightarrow \mu_{A'}^i(x) = 1 - \mu_A^i(x),\ i = 1, 2, \ldots, n\ \forall x \in X.$
4. Union $A \cup B = \{x, \vee(\mu_A^i(x), \mu_B^i(x))\}, i = 1, 2, \ldots, n\ \forall x \in X.$
5. Intersection $A \cap B = \{x, \wedge(\mu_A^i(x), \mu_B^i(x))\}, i = 1, 2, \ldots, n\ \forall x \in X.$
6. Addition $A + B = \{x,\ \mu_A^i(x) + \mu_B^i(x)\}, i = 1, 2, \ldots, n\ \forall x \in X.$

$$A \oplus B = \{x,\ \mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x).\mu_B^i(x)\}, i = 1, 2, \ldots, n\ \forall x \in X.$$

7. Multiplication $A \odot B = \{x,\ \mu_A^i(x).\mu_B^i(x)\}, i = 1, 2, \ldots, n\ \forall x \in X.$
8. Difference $A - B = \{x,\ \mu_A^i(x) - \mu_B^i(x)\}, i = 1, 2, \ldots, n\ \forall x \in X.$
9. Absolute difference $|A - B| = \{x,\ |\mu_A^i(x) - \mu_B^i(x)|\}, i = 1, 2, \ldots, n\ \forall x \in X.$

The following are very important concepts in fuzzy bags or fuzzy multisets:

1. $\mathrm{Supp}(A) = \{x \in X:\ \mu_A^i(x) > 0\}$ where $\mathrm{Supp}(A)$ is the support of A, for $i = 1, 2, \ldots, n\ \forall x \in X$.
2. The crossover point of A is $\{x \in X:\ \mu_A^i(x) = 0.5\}$, for $i = 1, 2, \ldots, n\ \forall x \in X$.
3. $\mathrm{hgt}(A) = \sup \mu_A^i(x)$ where $\mathrm{hgt}(A)$ is the height of A, for $i = 1, 2, \ldots, n\ \forall x \in X$.
4. A is normalized if $\mathrm{hgt}(A) = 1$.

IV. Fuzzy Multisets and Sequences

A sequence is a function, say $\mu_A^i(x)$, from X into some closed unit interval [0, 1]. Then $\mu_A^i(x)$, for $i = 1, 2, \ldots,\ \forall x \in X$, is called the *nth* term of the sequence $\mu_A^1(x), \mu_A^2(x), \ldots$, for $i = 1,2,\ldots$ and this sequence is denoted by $\{\mu_A^i(x)\}$. If A is a fuzzy multiset in X, and every term of $\{\mu_A^i(x)\}$ belongs to A, then $\{\mu_A^i(x)\}$ is said to be a sequence in A or a sequence of elements of A, and we write $\{\mu_A^i(x)\} \subset A\ \forall x \in X$ to indicate this.

Definition 5: A sequence $\{\mu_A^i(x)\}$ is said to be finite if $i = 1,2,\ldots,n$ and infinite if $i = 1,2,\ldots$

Definition 6: A sequence $\{\mu_A^i(x)\}$ is called strictly increasing if each term is larger than the one that precedes it, and it is called strictly decreasing if each term is smaller than the one that precedes it.

Definition 7: If $\mu_A^i(x): X \to [0,1]$ for $x \in X$ and $i = 1,2,\ldots,n$, then $\mu_A^i(x)$ is a sequence in [0, 1], since $A \subset X$.

V. Limit of Fuzzy Multisets

We have established that every fuzzy multiset is a sequence, but the reverse is not true.

Definition 8: Let A be a fuzzy multiset in X s.t. $\mu_A^i(x): X \to [0,1]$ for $x \in X$ and $i = 1,2,\ldots,n$. Then a number l is called the limit of $\mu_A^1(x), \mu_A^2(x), \ldots$, if for any positive number ϵ, $\exists$ a positive number N depending on ϵ s.t. $|\mu_A^i(x) - l| < \epsilon\ \forall i > N$. In such case, we write $\lim_{i\to\infty} \mu_A^i(x) = l$. If l exist, $\{\mu_A^i(x)\}$ is called a convergent sequence, otherwise divergent sequence.

Theorem 1: Let $\lim_{i\to\infty} \mu_A^i(x) = A$ and $\lim_{i\to\infty} \mu_B^i(x) = B$, $i = 1,2,\ldots,$ for FMS A and B in X, then

(a) $\lim_{i\to\infty}(\mu_A^i(x) + \mu_B^i(x)) = \lim_{i\to\infty} \mu_A^i(x) + \lim_{i\to\infty} \mu_B^i(x) = A + B$

(b) $\lim_{i\to\infty}(\mu_A^i(x). \mu_B^i(x)) = (\lim_{i\to\infty} \mu_A^i(x)) (\lim_{i\to\infty} \mu_B^i(x)) = A.B$

(c) $\lim_{i\to\infty}(\mu_A^i(x) - \mu_B^i(x)) = \lim_{i\to\infty} \mu_A^i(x) - \lim_{i\to\infty} \mu_B^i(x) = A - B$

(d) $\lim_{i\to\infty} \mu_A^i(x)' = (\lim_{i\to\infty} \mu_A^i(x))' = A'$

Proof

(a) If we can show that for any given positive number ϵ, $\exists$ a positive number N depending on ϵ s.t. $\left| \mu_A^i(x) + \mu_B^i(x) - (A+B) \right| \le \epsilon \; \forall \; i > N$, we are done. This implies that,

$$\left| (\mu_A^i(x) - A) + (\mu_B^i(x) - B) \right| = \left| \mu_A^i(x) - A \right| + \left| \left|\mu_B^i(x) - B \right| \le \epsilon \qquad (1)$$

By hypothesis, given $\epsilon > 0, \exists \; N_1 \text{and} N_2$ (where N is the largest) s.t.

$$\left|\mu_A^i(x) - A \right| \le \frac{\epsilon}{2} \text{and} \left| \mu_B^i(x) - B \right| \le \frac{\epsilon}{2} \forall \; i > N_1 \text{ and} i > N_2 \qquad (2)$$

Substituting 2 into 1, we get; $\left| \mu_A^i(x) - A \right| + \left| \left|\mu_B^i(x) - B \right| =. \quad \right| \mu_A^i(x) + \mu_B^i(x) - (A+B) \Big| \le \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$

(b) We must show that for any $\epsilon > 0, \exists$ a positive number N depending on ϵ s.t. $\left| \mu_A^i(x). \mu_B^i(x) - (A.B) \right| \le \epsilon \; \forall \; i > N$. This means,

$$\begin{aligned} &\left| \mu_A^i(x). \mu_B^i(x) - \mu_A^i(x)B + \mu_A^i(x)B - (A.B) \right| \\ &= \left|\mu_A^i(x)(\mu_B^i(x) - B) + B(\mu_A^i(x) - A)\right| \\ &= \left|\mu_A^i(x)\right| \left|(\mu_B^i(x) - B)\right| \\ &+ \left|B\right| \left|(\mu_A^i(x) - A) \right| \end{aligned} \qquad (3)$$

Again, by hypothesis, given $\epsilon > 0, \exists \; N_1 \text{and} N_2$ (where N is the largest) s.t.

$$\left| \mu_A^i(x) - A \right| \le \frac{\epsilon}{2\left|B\right|} \text{and} \left|\mu_B^i(x) - B \right| \le \frac{\epsilon}{2\left|\mu_A^i(x)\right|} \forall \; i > N_1, N_2 \qquad (4)$$

Substituting 4 into 3, the result follows as

$$\left|\mu_A^i(x). \mu_B^i(x) - (A.B) \right| = \frac{\left|\mu_A^i(x)\right|\epsilon}{2\left|\mu_A^i(x)\right|} + \frac{\left|B\right|\epsilon}{2\left|B\right|} \le \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

(c) We are done if for any given $\epsilon > 0, \exists$ a positive number N depending on ϵ s.t. $\left| \mu_A^i(x) - \mu_B^i(x) - (A-B) \right| \le \epsilon \; \forall \; i > N$. Then, $\left| \mu_A^i(x) - \mu_B^i(x) - (A-B) \right| = \left| \mu_A^i(x) - \mu_B^i(x) - A + B \right| = \left|\mu_A^i(x) - A + \left(-\mu_B^i(x) + B\right) \right| = \left|\mu_A^i(x) - A \right| + \left| - (\mu_B^i(x) - B) \right| = \left|\mu_A^i(x) - A \right| + \left|\mu_B^i(x) - B \right|$ and the result follows from (a).

(d)If for any given $\epsilon > 0, \exists$ a positive number N_1 (i.e. $N > N_1$) depending on ϵ s.t. $\left| \mu_A^i(x) - A \right| \le \frac{\epsilon}{2} \; \forall \; i > N$, then the theorem will follow. But $A' = 1 - A \to A' = 1 -$

Notes

Notes

$\mu_A^i(x)$ since $\mu_A^i(x)$ is the membership function of A. Then $\lim_{i\to\infty} \mu_A^i(x)' = (\lim_{i\to\infty} \mu_A^i(x))' = A' \to \left| \mu_A^i(x)' - A' \right| \leq \epsilon, i.e. \left|1 - \mu_A^i(x) - A'\right| = \left| - \mu_A^i(x) + 1 - A' \right| = \left| - \mu_A^i(x) + A \right| = \left| - (\mu_A^i(x) - A) \right| = \left| \mu_A^i(x) - A \right| \leq \frac{\epsilon}{2}$, hence the result.

Theorem 2: If $\lim_{i\to\infty} \mu_A^i(x)$ exists, then it must be unique.

Proof: If $lim_{i\to\infty} \mu_A^i(x) = l_1$ *and* $lim_{i\to\infty} \mu_A^i(x) = l_2$, *then* $l_1 = l_2$. *By hypothesis, given any* $\epsilon > 0, \exists$ *a positive number* N *depending on* ϵ *s.t.* $\left|\mu_A^i(x) - l_1\right| \leq \frac{\epsilon}{2}$ *and* $\left|\mu_A^i(x) - l_2\right| \leq \frac{\epsilon}{2}\ \forall\, i > N$. *Then* $\left|l_1 - l_2\right| = \left|l_1 - \mu_A^i(x) + \mu_A^i(x) - l_2\right| \leq \left| -\left(-l_1 + \mu_A^i(x)\right) \right| + \left| \mu_A^i(x) - l_2 \right| \leq \left| \mu_A^i(x) - l_1 \right| + \left|\mu_A^i(x) - l_2\right| \leq \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$ *i.e.* $\left|l_1 - l_2\right| < \epsilon \to \left|l_1 - l_2\right| = 0$ *and as such,* $l_1 = l_2$. W^5

VI. Bolzano-Weierstrass (B-W)Property of Fuzzy Multisets

Definition 9: Let A be a fuzzy multiset in X. A fuzzy multiset A is compact if every sequence in A has a subsequence that converges to an element x∈X.

Theorem 3: Every fuzzy multiset has a convergent subsequence.

Proof: Every fuzzy multiset is actually a sequence and is contained in the finite interval $[a, b]$ i.e. $a = 0, b = 1$. If we divide the interval into two equal intervals, then at least one of these, say, $[a_1, b_1]$ contains infinitely many points. Dividing $[a_1, b_1]$again, yields $[a_2, b_2]$ which also contains infinitely many points. Continuing the same process again and again, we obtain a set of intervals $[a_i, b_i]$, $i = 1,2, \ldots$, each interval contained in the preceding one and hence;

$$b_1 - a_1 = \frac{(b-a)}{2},\ b_2 - a_2 = \frac{(b_1 - a_1)}{2} = \frac{(b-a)}{2^2}, \ldots,\ b_i - a_i = \frac{(b-a)}{2^i}\ ;$$

from which we see that ,

$$\lim_{i\to\infty}(\, b_i - a_i) = 0.$$

This set of nested intervals corresponds to [0, 1] which represents a limit point and so proves theorem. Q.E.D.

Corollary: A fuzzy multiset is compact if it has the B-W property.

The proof is straightforward.

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Notes