Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 14 Issue 5 Version 1.0 Year 2014
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Study of Viscous Flow through Permeable Walls with Expanding or Contracting Gaps using Laplace Transform and HPM

By Vivek Kumar Sharma & Aisha Rafi
Jagan Nath University, India

Abstract- In the present paper the problem of laminar, incompressible and viscous flow between two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions, has been is solved using Laplace transform and Homotopy Perturbation Method (HPM).the effects of various physical parameter on velocity profile has been studied and presented through graphs and tables.

Keywords: permeation reynolds number, homotopy perturbation method (HPM), differential transformation method (DTM), optimal homotopy analysis method (OHAM).

GJSFR-F Classification : *MSC 2010: 44A10*

STUDYOFVISCOUSFLOWTHROUGHPERMEABLEWALLSWITHEXPANDINGORCONTRACTINGGAPSUSINGLAPLACETRANSFORMANDHPM

Notes

Study of Viscous Flow through Permeable Walls with Expanding or Contracting Gaps using Laplace Transform and HPM

Vivek Kumar Sharma [α] & Aisha Rafi [σ]

Abstract- In the present paper the problem of laminar, incompressible and viscous flow between two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions, has been is solved using Laplace transform and Homotopy Perturbation Method (HPM).the effects of various physical parameter on velocity profile has been studied and presented through graphs and tables.

Keywords: permeation reynolds number, homotopy perturbation method (HPM), differential transformation method (DTM), optimal homotopy analysis method (OHAM).

I. Introduction

Studies of fluid transport in biological organisms often concern the flow of a particular fluid inside an expanding or contracting vessel with permeable walls. For a valve vessel exhibiting deformable boundaries, alternating wall contractions produce the effect of a physiological pump. The possibility of emulating peristaltic motion by successive wall contractions and expansions is described by Uchida and Aoki (1977) and Goto and Uchida (1990). Since fluctuating stresses can influence the responsiveness of endothelial cells (Nerem and Levesque, 1987), their accurate determination in a pulsating environment may be appropriate of a number of investigations concerned with the characterization of atherosclerosis (see Dewey et al., 1981; Levesque and Nerem, 1985; Levesque et al., 1989). Considering that abnormalities in fluctuating stresses have been associated with this disease (Sprague et al., 1987), identification of the role of flow dynamics can be meaningful in predicting pulsatory flow attributes. Such results could be used toward a proper characterization of mechanically assisted respiration (Drazen et al., 1984), hemodialysis in artificial kidneys (Wang, 1971) and peristaltic transport (Fung and Yih, 1968). Majdalani and Zhou studied moderate to large injection and suction driven channel flows with expanding or contracting walls. Using perturbations in cross-flow Reynolds number Re, the resulting equation is solved both numerically and analytically. Boutros *et al.* (2003) studied the solution of the Navier-Stokes equations which described the unsteady incompressible laminar flow in a semi-infinite porous circular pipe with injection or suction through the pipe wall whose radius varies with time. Boutros, Y.Z. and M.B. Abd-el-Malek, N.A.Badran and H.S. Hassan,(2006). Lie-group method for unsteady flows in a semi-infinite expanding or contracting pipe with injection or suction through a porous wall. J. Comput. Appl. Math. Ganji, D.D., H.R. Ashory Nezhad and A. Hasanpour,(2011). Effect of variable viscosity and viscous dissipation on the Hagen-Poiseuille flow and entropy generation. Numerical Methods for Partial Differential Equations.

The equations of continuity, momentum are transformed into ordinary differential equations using Similarity transformation and solved using Laplace transform and Homotopy Perturbation Method.

Author α σ : Department of Mathematics, Jagan Nath University, Jaipur. e-mail: aisharafi.math@udml.ac.in

II. Formulation of the Problem

Consider the laminar, isothermal and incompressible flow between two permeable walls that enable the fluid to enter or exit during successive expansions or contractions. One side of the cross section, representing the distance between the walls is taken to be smaller than the other two. Both walls are assumed to have equal permeability and to expand uniformly at a time dependent rate a^*. Furthermore, the origin $x^*=0$ is assumed to be the center of the channel. This enables us to assume flow symmetry about $x^*=0$. Under these assumptions, the equations for continuity and motion become

Notes

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \quad (1)$$

$$\frac{\partial u^*}{\partial t}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}+\upsilon\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right), \quad (2)$$

$$\frac{\partial v^*}{\partial t}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\upsilon\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right), \quad (3)$$

where u^*=velocity component in x^* direction, v^*=velocity component in y^* direction, p^*=dimensional pressure, ρ =density, υ=kinematic viscosity, t=time. The boundary conditions are:

$$y^*=a(t): u^*=0, v^*=-V_w=-\frac{a^*}{c},$$
$$y^*=0: \quad \frac{\partial u^*}{\partial y^*}=0, \qquad v^*=0, \quad (4)$$
$$x^*=0: \qquad u^*=0.$$

The streams functions and mean flow can be introduced by putting:

$$\psi^*=\frac{\upsilon x^* f^*(y,t)}{a}, u^*=\frac{\upsilon x^* f_y^*}{a^2}, v^*=\frac{-\upsilon f^*(y,t)}{a}, y=\frac{y^*}{a}, f_y^*=\frac{\partial f^*}{\partial y}. \quad (5)$$

Substitution equation (15) into (14):

$$u^*_{y^\cdot t}+u^*u^*_{y^\cdot x^\cdot}+v^*u^*_{y^\cdot y^\cdot}=\upsilon u^*_{y^\cdot y^\cdot y^\cdot} \quad (6)$$

In order to solve equation (16) by chain rule:

$$f^*_{yyyy}+\alpha\left(yf^*_{yyy}+3f^*_{yy}\right)+f^*f^*_{yyy}-f^*_yf^*_{yy}-a^2\upsilon^{-1}f^*_{yyt}=0, \quad (7)$$

where $\alpha(t)=\frac{a^*a}{y}$ is the non dimensional wall dilation rate defined positive for expansion.

From we have

$$f^*_{yyy}+\alpha\left(yf^*_{yy}+2f^*_y\right)+f^*f^*_{yy}-f^*_yf^*_y-a^2\upsilon^{-1}f^*_{yt}=\lambda, \qquad \lambda\neq\lambda(y).$$

Boundary conditions given by equation (14) transformed into

$$f^*_{yy}(0)=0, \qquad f^*(0)=0, \qquad f^*(1)=Re, \qquad f^*_y(1)=0,$$

where Re is the permeation Reynolds number defined by $Re\equiv\frac{av_w}{\upsilon}>0$ for injection. This number happens to be a small quantity in many biological applications

$$\text{at } y=0: f^*=0, \ f^*_{yy}=0,$$
$$\text{at } y=1: f^*=Re, \ f^*_y=0. \quad (8)$$

Equation (16),(17),(18) can be normalized by putting:

Notes

$$\psi = \frac{\varphi^*}{aa^{\cdot}}, u = \frac{u^*}{a^{\cdot}}, v = \frac{v^*}{a}, f = \frac{f^*}{Re}, \tag{9}$$

$$\psi = \frac{xf}{c}, u = \frac{xf'}{c}, v = \frac{-f}{c}, c = \frac{\alpha}{Re}.$$

And so:

$$f'''' + \alpha(yf''' + 3f'') + Reff''' - Ref'f'' = 0 \tag{10}$$

The boundary conditions are:

$$\text{at} \quad y = 0: \quad f = 0, f'' = 0,$$
$$\text{at} \quad y = 1: \quad f = 1, f' = 0. \tag{11}$$

The resulting Equation (20) is the classic Berman's formula , with α = 0 The governing boundary layer and thermal boundary layer equations (15) with the boundary conditions (21) are solved using Laplace transformation & Homotopy Perturbation Method. An equation (20) is non-linear coupled differential equation. To solve this equation we use the following Laplace transformation.

$$L[f(y)] = L(f'''' + \alpha(yf''' + 3f'') + Reff''' - Ref'f''), \tag{12}$$

$$L[f(y)] = \frac{a}{s^2} + \frac{b}{s^4} - \frac{1}{s^4} L[\alpha(yf''' + 3f'') + Re(f'f''' - f'f'')], \tag{13}$$

The inverse Laplace transform

$$f(y) = ay + \frac{by^3}{6} - L^{-1}\left[\frac{1}{s^4} L[\alpha(yf''' + 3f'') + Re(f'f''' - f'f'')]\right]. \tag{14}$$

Now applying the HPM

$$D(f,p) = (1-p)[L(f) - L(f_l)] + p\left[ay + \frac{by^3}{6} - L^{-1}\left[\frac{1}{s^4} L[\alpha(yf''' + 3f'') + Re(f'f''' - f'f'')]\right]\right] = 0, \tag{15}$$

With the following assumption

$$f = f_0 + pf_1 + p^2 f_2 + ... \tag{16}$$

Using equation (26) into equation (25) and comparing the like powers of p, we get the zeoth order equation,

$$f_0 = ay + b\frac{y^3}{6}, \tag{17}$$

with the corresponding boundary conditions are of zeroth order equations are:

$$\text{at} \quad y = 0: \quad f_0 = 0, f_0'' = 0,$$

$$\text{at} \quad y = 1: \quad f_0 = 1, f_0' = 0. \tag{18}$$

And first order equations are:

$$f_1 = -L^{-1}\left[\frac{1}{s^4} L[\alpha(yf_0''' + 3f_0'') + Re(f_0' f_0''' - f_0' f_0'')]\right], \tag{19}$$

With the corresponding boundary conditions are of first order equations are:

$$\text{at} \quad y = 0: \quad f_1 = 0, f_1'' = 0,$$
$$\text{at} \quad y = 1: \quad f_1 = 0, f_1' = 0. \tag{20}$$

And second order equations are:

$$f_2 = -L^{-1}\left[\frac{1}{s^4} L[\alpha(yf_1''' + 3f_1'') + Re(f_1' f_1''' - f_1' f_1'')]\right], \tag{21}$$

With the corresponding boundary conditions are of first order equations are:

$$\text{at} \quad y = 0: \quad f_2 = 0, f_2^{''} = 0, \qquad \text{at} \quad y = 1: \quad f_2 = 0, f_2^{'} = 0. \tag{22}$$

Solving equations with corresponding boundary conditions, the following functions can be obtained successively, by summing up the results, and $p \to 1$ we write the f($\boldsymbol{\eta}$) profile as:

$$f(y) = ay + b\frac{y^3}{6} - \left(\frac{b\alpha y^5}{120} + \frac{Re\, ab y^4}{24} + \frac{Re\, b^2 y^6}{720} - \frac{ab y^5}{120} - \frac{b^2 y^7}{1680}\right) + Re\, ab\alpha\left(\frac{y^6}{144} - \frac{y^7}{840}\right) + Re\, b^2\alpha\left(\frac{y^8}{5760} - \frac{y^9}{15120}\right) + \frac{b\alpha^2 y^7}{210} + Re\left[Re^2 a^2 b^2\left(\frac{y^{11}}{1140480} - \frac{y^{10}}{72576} + \frac{5y^9}{72576} - \frac{y^8}{10080}\right) + Re^2 ab^3\left(\frac{y^{13}}{9884160} - \frac{y^{12}}{712800} + \frac{29y^{11}}{57024} - \frac{y^{10}}{604800}\right) - Re\, ab^2\alpha\left(\frac{y^{11}}{142560} - \frac{y^{10}}{181440} - \frac{y^9}{6048}\right) + Re^2 b^4\left(\frac{y^{15}}{314496000} - \frac{y^{14}}{5000000} + \frac{y^{13}}{8236800} - \frac{y^{12}}{8553600}\right) - Re\, b^3\alpha\left(\frac{y^{13}}{2471040} + \frac{y^{12}}{2851200} - \frac{y^{11}}{178200}\right) + b^2\alpha^2\left(\frac{y^{11}}{71280} - \frac{y^{10}}{15120}\right) - \frac{Re\, ab^3 y^{10}}{181440}\right]. \tag{23}$$

III. Numerical Discussion and Conclusion

It is observed from Table 1 that the numerical values of f ′(y) in the present paper when α = 0, Re = 1 are in good agreement with results obtained by HPM, DTM and OHAM method. It is noted from Table 2 that the numerical values of f ′(y) in the present paper when α = 1, Re = 2 are in good agreement with results obtained by HPM, DTM and OHAM method.

From figure 1, 2 and 3, we observe that as α and Re increases, value of f(y) also increases. From figure 4,5 and 6 it is observed that when α and Re increase simultaneously, numerical value of f ′(y) decreases.

In this research, the HPM, and Laplace transform were successfully applied to find the analytical solution for two-dimensional viscous flow in a rectangular domain bounded by two moving porous walls. The accuracy of the methods is shown by the figures and tables clearly.

Table 1 : The HPM, DTM, OHAM and present paper solution results for f(y) when α = 0, Re = 1

Y	HPM	DTM	OHAM	Present Paper
0.0	0	0	0	0
0.2	0.337421	0.332018	00.378868	0.327289
0.4	0.628838	00.621875	0.692581	0.652286
0.5	0.746307	0.740259	0.809906	0.7897253
1	1	1	1	1

Table 2 : The HPM, DTM, OHAM and present paper solution results for f(y) when α = 1, Re = 2

Y	HPM	DTM	OHAM	Present Paper
0.0	0	0	0	0
0.2	0.314954	0.311636	0.337174	0.3699881
0.4	0.596412	0.591678	0.630506	0.657786
0.5	0.715375	0.710901	0.749366	0.766599
1	1	1	1	1

Notes

Notes

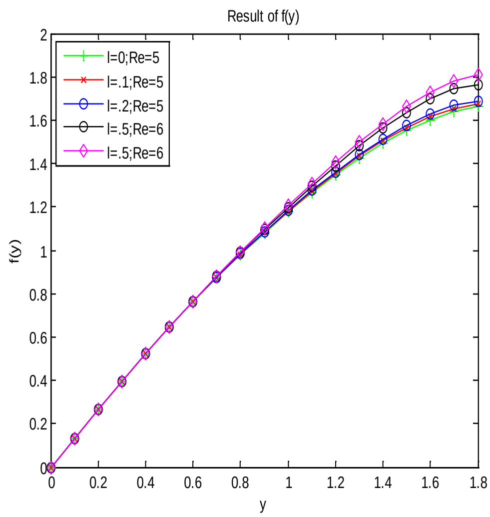

Figure 1

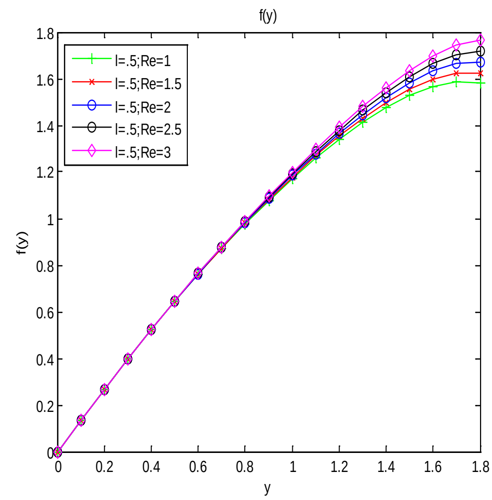

Figure 2

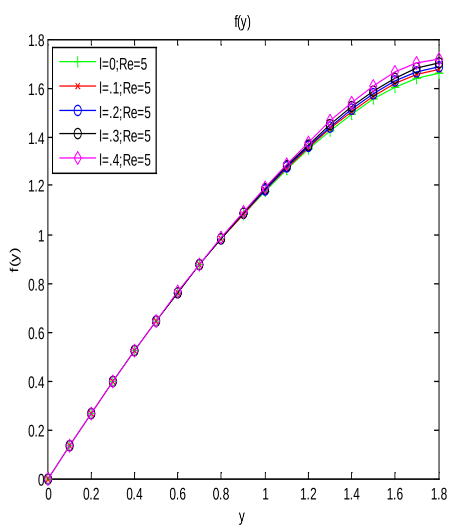

Figure 3

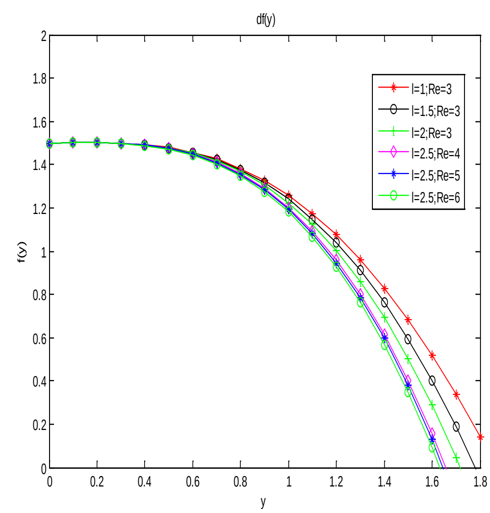

Figure 4

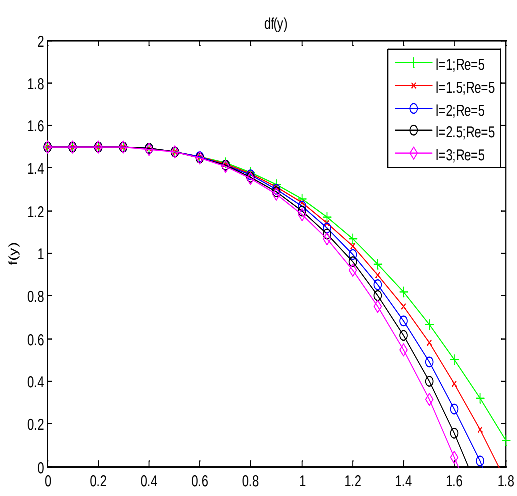

Figure 5

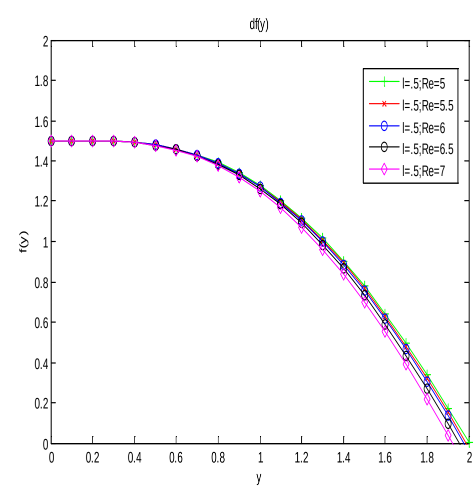

Figure 6

References Références Referencias

1. Chang, H.N. and J.S. Ha, J.K. Park, I.H. Kim and H.D. Shin, 1989. Velocity field of pulsatile flow in a porous tube. J. Biomech, 22: 1257-1262.
2. Dauenhauer, E.C. and J. Majdalani, 1999. Unsteady flows in semi-infinite expanding channels with wall injection. In: AIAA paper, 99: 3523.
3. Majdalani, J. and C. Zhou, 2003. Moderate-to-large injection and suction driven channel flows with expanding or contracting walls. ZAMM Z. Angew. Math. Mech., 83: 181-196.
4. Boutros, Y.Z. and M.B. Abd-el-Malek, N.A. Badran and H.S. Hassan, 2006. Lie-group method for unsteady flows in a semi-infinite expanding or contracting pipe with injection or suction through a porous wall. J. Comput. Appl. Math., 197 (2): 465-494.
5. Dauenhauer, E.C. and J. Majdalani, 1999. Unsteady flows in semi-infinite expanding channels with wall injection. In: AIAA paper, 99: 3523.
6. Majdalani, J. and C. Zhou, 2003. Moderate-to-large injection and suction driven channel flows with expanding or contracting walls. ZAMM Z. Angew. Math. Mech., 83: 181-196.
7. Boutros, Y.Z. and M.B. Abd-el-Malek, N.A. Badran and H.S. Hassan, 2006. Lie-group method for unsteady flows in a semi-infinite expanding or contracting pipe with injection or suction through a porous wall. J. Comput. Appl. Math., 197 (2): 465-494.
8. He, JH., 2000. A review on some new recently developed nonlinear analytical techniques. Int. J. Non-linear Sci. Numer Simul., 1: 51-70.
9. He, J.H., 2005. Homotopy perturbation method for bifurcation on nonlinear problems. Int. J. Non-linear Sci. Numer Simul., 8: 207-208.
10. Ganji, Z.Z. and D.D. Ganji, 2008. Approximate solutions of thermal boundary-layer problems in a semi-infinite flat plate by using He's homotopy perturbation method. Int. J. Nonlinear Sci. Numer. Simul., 9 (4): 415-422.
11. Ganji, D.D., H.R. Ashory Nezhad and A. Hasanpour, 2011. Effect of variable viscosity and viscous dissipation on the Hagen-Poiseuille flow and entropy generation. Numerical Methods for Partial Differential Equations, 27 (3): 529-540.
12. Ganji, D.D. and A. Sadighi, 2006 Application of He's homotopy-perturbation method to nonlinear coupled systems of reaction-diffusion equations. Int. J. Nonlinear Sci. Numer. Simul., 7 (4): 411-418.
13. Zhou, J.K., 1986. Differential transformation and its applications for electrical circuits. Wuhan. China: Huazhong University Press; [In Chinese]

Notes