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mz-Compact Spaces

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Abstract- In this work we study mz-compact spaces and mz-Lindel of spaces, where m is an infinite cardinal number. Several new properties of them are given. It is proved that every mz-compact space is pseudocompact (a space on which every real valued continuous function is bounded). Characterizations of mz-compact and mz-Lindel of spaces by multifunctions are given.

Keywords: mz-compact space, mz-Lindel of space, m-compact space, pseudocompact space, realcompact space.

GJSFR-F Classification : AMS 2010: 54C, 54D

MZ-COMPACTSPACES

Strictly as per the compliance and regulations of :

Ref

[1] A.T. Al-Ani. Countably z-compact spaces. *ArchivumMathematicum*, 50 , 19-22 , 2014.

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I. Introduction

In this paper we study some properties of mz-compact spaces and mz-Lindelof spaces. Modifications of results about countably z-compact spaces [1] are proved. We relate mz-compact spaces to pseudocompact spaces (Theorem III b). Then we give some characterizations of mz-compact spaces. The collection of real valued continuous functions on a topological space X forms a ring denoted by $C(X)$ [2]. Characterizations of mz-compact spaces in terms of z-filter bases and z-ideals are given. No separation property is assumed unless otherwise is stated. Fordefinitions and notations not stated here see [2].

II. Preliminaries

a) Definition

A space X is called m-compact if every open cover of X of cardinality at most m, has a finite subcover. Recall that a *cozero set* in a space $X = (X, \tau)$ is an $f^{-1}[\mathrm{R} \therefore \{0\}]$ with a continuous function $f: X \to \mathrm{R}$. Cozero sets constitute a base of a topology $z\tau$ on X. (X, τ) is said to be mz-compact, (mz-Lindelof) if $zX = (X, z\tau)$ is m-compact (m-Lindelof). Filters and z -ideals here are modifications of their respective definitions [2] by taking z-closed set (closed in zX) instead of zero-set.

b) Definition

A multifunction α of a space X into a space Y is a set valued function on X into Y such that $\alpha(x) = \Phi$ for every $x \in X$. The class of all multifunctions on X into Y is denoted by $m(X, Y)$.

c) Definition

A multifunction α on X into Y is called closed graph if its graph $G(\alpha) = (x, y) \in X \times Y : y \in \alpha(x)$ is closed in $X \times Y$.

III. Some Properties of MZ-compact Spaces

We call a filter on X of cardinality at most m by m-filter. By mz-filter we mean a filter of z-closed sets, of cardinality at most m. The proof of the following theorem is straight forward.

a) Theorem

The following statements about a space X are equivalent.

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1. X is mz-compact.
2. Every family of subsets of X of cardinality at most m each is an intersection of zero sets, with the finite intersection property has a non empty intersection.
3. Every mz-filter on X is fixed.
4. Every mz-ideal in $C(X)$ is fixed.
5. zX is m-compact.

b) *Theorem*

Every mz-compact space is pseudo compact.

c) *Example ([3], [4])*

Let N be the set of positive integers. Topologize N by taking a subbase the collection $\beta = \{U_p(b) : b + np \in N, p$ prime $, b$ is not divisible by $p\}$ This space is a T_2 countably z-compact Lindel of space which is not countably compact. For $m = \chi_0$ in this example the space N is mz-compact but not m- compact.

IV. A Characterization of MZ -compactness in Terms of Multifunctions

A space X is said to be of character m if every point of X has a local base of cardinality at most m. We give here a characterization of mz-compact space X in terms of multifunctions. Equivalently a characterization of m-compactness of zX. It is to be noted that a space is m-compact if every family of closed sets with cardinality at most m, satisfying the finite intersection property has a non-empty intersection. We shall use this fact in the proof of the second part of the following theorem.

a) *Theorem*

A space X is mz-compact if for every space Y with character m and closed graph multifunction $\alpha \in m(zX, Y)$, the image of every closed set in zX is closed in Y.

Proof

Let zX be m-compact space, Y be a space of character m, $\alpha \in m(zX, Y)$ with closed graph.

Let K be closed in zX and $y \in Y - \alpha(K)$.

Let $\{B_\lambda : \lambda \in \Lambda\}$ be a local base of cardinality at most m at y.

For each $x \in K$, there exist open set V_x in zX and B_λ in Y such that

$$(x, y) \in V_x \times B_\lambda$$

$$\text{and } (V_x \times B_\lambda) \cap G(\alpha) = \Phi.$$

For each $\lambda \in \Lambda$, let

$W_\lambda = \cup\{V_x : x \in K, (x, y) \in V_x \times B_\lambda\}$

Then $\{W_\lambda\}$ is an open cover of K of cardinality at most m. So, it has a finite

subcover $\{W_{\lambda_i} : i = 1, 2, \ldots, n\}$

Now, let $W = \cup\{B_{\lambda_i} : i = 1, 2, \ldots, n\}$ .Then W is open in Y with $y \in W$ and $W \cap \alpha(K) = \Phi$;

So, $\alpha(K)$ is closed in Y.

To prove the converse, let $\{K_\lambda : \lambda \in \Lambda\}$ be a family of closed sets in zX of cardinality at most m, with the finite intersection property, let $y_0 \notin zX$. Topologize $zX \cup \{y_0\}$ by taking open sets all subsets of zX and sets containing $y_0 \cup \alpha(K_\lambda)$ for some $\lambda \in \Lambda$. Obviously, $zX \cup \{y_0\}$ has character m. Let β be the closure of the identity function of zX. Then β has a closed graph and so, by hypothesis, it maps closed sets in zX onto closed subsets in Y.

So, $\beta(K_\lambda)$ is closed in $zX \cup \{y_0\}$, for every $\lambda \in \Lambda$. So, $y_O \in \beta(K_\lambda)$ for every $\lambda \in \Lambda$.

Hence $\{K_\lambda : \lambda \in \Lambda\}$ has a non-empty intersection.

Therefore, zX is m-compact.

V. MZ-LINDEL OF SPACE

Notes

a) Definition

A space X is a P(m)-space if every intersection of at most m open sets in X is open.

The following result about mz-Lindelof space can be proved by the same technique of Theorems IV b.

b) Theorem

A space X is mz-Lindelof if for every P(m)-space Y and z-closed graph multi-function $\alpha \in m(X, Y)$ the image of every z-closed set in X is closed in Y.

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