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Boundary and Sign Problems of Parameters along with its Solutions of the Augmented Dickey-Fuller Test

Shayla Naznin [α], Gowranga Kumar Paul [σ] & Ajit Kumar Majumder [ρ]

Abstract- Usual test of testing unit root such as Dickey-Fuller (DF), Augmented Dickey-Fuller (ADF), Phillips Perron etc ignores sign and boundary of parameters. In this paper, we demonstrated the ignorance of sign and boundary of parameters and consequences of this ignorance in estimation and testing by Monte Carlo Simulation. Our main objective is to develop a method to capture the non-stationarity keeping in mind the boundary and sign problem and to develop the restricted ADF test based on ESS using the constraint estimate of parameters. We compare the power properties of the usual ADF test and ADF test with restricted error sum of squares using the constraint estimate of parameters by Monte Carlo Simulation and we find that the proposed ADF test gives better result than the usual ADF test in terms of power properties.

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I. Introduction

In modeling time series econometrics, non-stationary test is very essential for analyzing the behavior of time series data and for advance research. Usually this can be tested by Dickey-Fuller, Augmented Dickey-Fuller, Phillips Perron etc. Almost all of the unit root test as well as the estimation of the model suffers from sign and boundary problem of the parameters. According to the assumption or analysis of the Dickey-Fuller test, $|\rho|<1$ or $-2<\delta<0$ of the time series models such as $Y_t = \rho Y_{t-1} + u_t$. Any estimated value of δ less than -2 or greater than 0 may results in invalid model. This invalid model can not be used for making decision regarding non-stationarity. To overcome this situation it is necessary to make suitable restrictions on the parameter. So our aim is to develop a model and estimates its parameters and check stationarity by using unit root test and to develop restricted testing approach based on error sum of square (ESS) of Augmented Dickey-Fuller test.

II. Problems And Motivations

In real world most of the time series data are non-stationary. But for our analysis purpose we assumed them stationary. Most of the financial time series data, such as share index, stock price, exchange rate, inflation rate, etc. often exhibit the phenomenon of non-stationarity. Non-stationary series leads to spurious regression and provides wrong results that misguide us (Brockwell and Davis,1996, Gujarati ,2003).In time series model, it is essential to identify correctly the non-stationarity of a time series with the aid of appropriate statistical tests. If we can not correctly identify the

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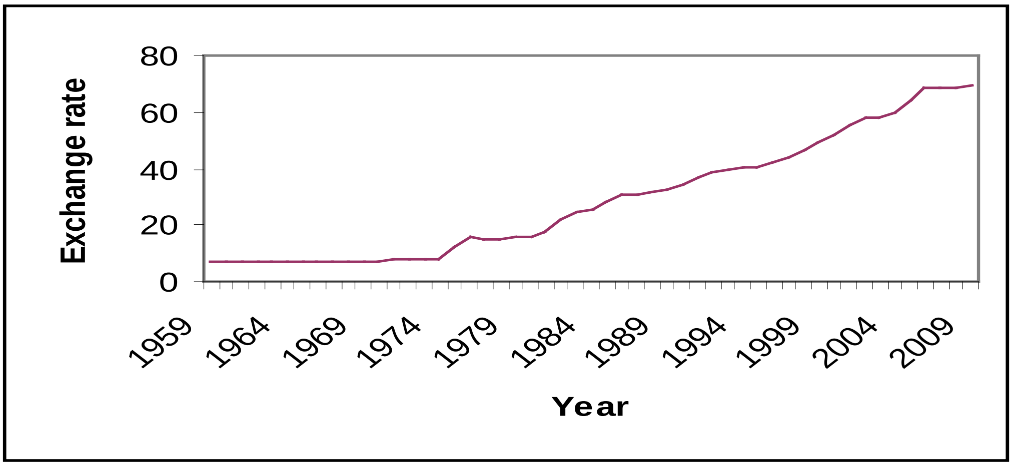
stationarity of the series, we will not be able to make better decision. If error terms are correlated, then Augmented Dickey-Fuller test is appropriate. The ADF test consists of estimating the following regression

$$\Delta y_t = \beta_0 + \beta_1 t + \delta y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta y_{t-i} + \varepsilon_t .$$

None of the tests such as Dickey-Fuller, Augmented Dickey-Fuller, Phillips-Perron unit root test etc can capture non-stationarity problem in all cases. For example, we consider some Bangladeshi time series data such as Exchange rate.

Notes

Figure 1 : Time series plot for exchange rate of Bangladesh



From the above graph we see that there is upward trend in exchange rate series. Since with the increase of time, mean Exchange rate is increasing and thus the graph guaranteed that the Exchange rate series is non-stationary.

The Dickey-Fuller test is estimated in three different forms such as random walk, random walk with drift, random walk with drift around a stochastic trend respectively. Consider the following model for Exchange rate series,

$\Delta Ex_t = 0.037741 Ex_{t-1}$	$\Delta Ex_t = 0.535546 + 0.025245 Ex_{t-1}$	$\Delta Ex_t = -0.38666 - 0.079t + 0.1518 Ex_{t-1}$
se =(0.005659)	se=(0.331992) (0.00954)	se=(0.42723) (0.03490) (0.0491)
τ =(2.6462)	τ =(2.6462)	τ =(3.091455)

From the above it is shown that the Exchange rate series is stationary. But from the graph we see that it is nonstationary. It happens because of Ignoring sign and boundary problem of parameter. This test suffers from a number of problems such as sign and boundary problem of parameter δ, $-2 < \delta < 0$ (Akter, 2009). Ignoring sign and boundary problem of parameter, the usual two-sided test is likely to be misleading. In this paper we will propose a method of tackle the boundary problem and compare the results with the existing tests in terms of power properties.

III. Objectives

Our main objective is to develop a method to capture the non-stationarity keeping in mind the boundary and sign problem and to develop the restricted ADF test based on ESS using the constraint estimate of parameters.

IV. Methodology

In the first stage we estimate the parameters by usual method such as least square method, maximum likelihood method. In the second stage we compare the power properties of the restricted ADF test based on ESS of constraint estimate of parameters by optimizing under the restriction.

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V. Data Generation

Monte Carlo Simulation required from generated observations. In this section we demonstrated how to generate sequence of observations of the model $\Delta y_t = \beta_0 + \beta_1 t + \delta y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta y_{t-i} + u_t$ and in the same we generated observations of the newly proposed restricted test based on error sum of squares.

Notes

VI. Proposed Augmented Dickey-Fuller Test

There are some problems in usual unit root tests. In some cases, time series may follow strictly upward or downward trend. The responses increase (or decreases) with rate β for every time. In case of β is strictly positive or negative for such case the following model will be more appropriate.

$$\Delta y_t = \beta_0 + \beta_1 t + \delta y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta y_{t-i} + u_t \qquad (1)$$

We impose some restrictions on the parameters. In case of upward or downward trended time series, the coefficient of time series is likely to be positive or negative respectively and ρ (autocorrelation coefficient) is bounded to lie between -1 to +1. Also the boundary problem of autocorrelation arises in testing co-integration. The autocorrelation is bounded, therefore the δ is bounded to be $-2<\delta<0$. In order to estimate the model we require constraint optimization subroutine. Appropriate modification is needed to overcome the testing problem. The most used least square method does not consider these restrictions on the parameters. For this reason the usual unrestricted test cannot capture the non-stationary for trend stationary series. We observe that the τ-statistic is positive, which indicate that δ is positive. But since $\delta = \rho - 1$, a positive δ would imply that $\rho > 1$ Hence, there is present uncertain unit root in the residual of two non-stationary series. If we want to estimate the model correctly we have to minimize the error sum of squares (ESS). Using equation (1) we get

Minimizing: $\text{ESS}=\sum_{t=0}^{T} \tilde{u}_t = \sum_{t=o}^{T} (\Delta y_t - \beta_0 - \beta_1 t - \delta y_{t-1} - \sum_{i=1}^{m} \alpha_i \Delta y_{t-i})^2$.

Subject to: $-2 < \tilde{\delta} < 0, \beta > 0 \, or \, \beta < 0.$

where, $\beta > 0$ means the series has an upward trend or $\beta < 0$ means the series has a downward trend.

Based on the optimized estimates, our proposed τ statistic is

$$\tilde{\tau} = \frac{\tilde{\delta}}{SE(\tilde{\delta}),}$$

where τ is the optimized τ statistic and $\tilde{\delta}, \tilde{\beta}$ optimized estimate of the parameters. The τ statistic follow the weighted mixture τ-distribution (Majumder, 1999).We expect this approach will give better result than all other methods with efficiency, consistency and in terms of power properties.

Table 1 : Estimated value of restricted parameter

Estimated value (before optimization)	τ-value	True value	Estimated value (after optimization)	τ-value
0.0326	2.2176		-0.0492	-3.5142
0.0591	1.8411		-0.0233	-4.2983
0.0261	2.8681		-0.0457	-3.6560
-0.0342	-2.2352		-0.5917	-43.1898
0.0229	1.4135		-0.2602	-27.6809

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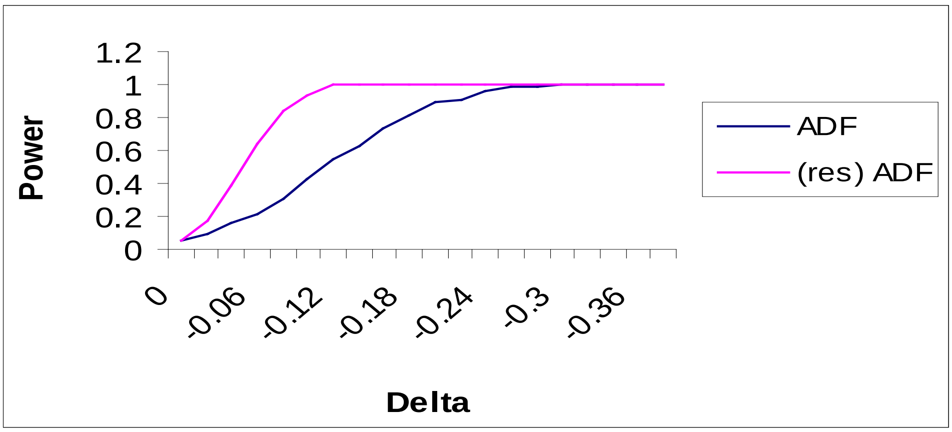
In ADF test we test $\delta = 0$. Before optimization the estimated δ coefficient is positive but after optimization the estimated δ coefficient is negative and it is happen because of estimation problem. For overcoming this problem we have to use constraint optimization. The computed τ value should be more negative than the critical τ value. Since in general δ is expected to be negative. A large negative τ value is generally an indication of stationarity and we capture stationarity properly.

VII. Power Comparison

This section compares power of the existing Augmented Dickey-Fuller tests and the newly proposed restricted test based on ESS using the constraint estimate of parameters of Augmented Dickey-Fuller tests.

Figure 2 : Power curve of usual Augmented Dickey-Fuller test and the restricted Augmented Dickey-Fuller test.



From the above figure it is obvious that power of the newly proposed restricted Augmented Dickey-Fuller test based on restricted ESS gives better result than the usual Augmented Dickey-Fuller test.

Table 2 : Estimated parameters of the unconstraint and constraint Augmented Dickey-Fuller test with corresponding standard error.

Unconstraint parameter			Constraint parameter		
β_1	δ	α_1	β_1	δ	α_1
-0.0160	-0.3263	-0.0011	-0.0160	-0.0263	-0.0011
(0.0011)	(0.0147)	(0.0216)	(0.0010)	(0.0124)	(0.0213)
0.0915	-0.1984	0.1102	0.0000	-0.0457	0.0279
(0.0058)	(0.0176)	(0.0368)	(0.0052)	(0.0125)	(0.0361)
0.1187	-0.2459	0.1506	0.0000	0.0000	0.0890
(0.0018)	(0.0104)	(0.0268)	(0.0012)	(0.0106)	(0.0264)
-0.0974	-0.5917	0.4465	-0.0974	-0.5917	0.3455
(0.0017)	(0.0121)	(0.0143)	(0.0007)	(0.0137)	(0.0143)
-0.0494	-0.2611	-0.1153	-0.0510	-0.2602	-0.1158
(0.0003)	(0.0091)	(0.0147)	(0.0002)	(0.0094)	(0.0124)
-0.0292	-0.3808	0.0391	-0.0107	-0.0400	-0.0592
(0.0002)	(0.0053)	(0.0136)	(0.0001)	(0.0088)	(0.0101)
-0.0159	-0.2298	-0.1054	-0.0159	-0.2298	-0.1054
(0.0005)	(0.0062)	(0.0020)	(0.0005)	(0.0062)	(0.0020)
-0.0342	-0.2086	-0.3393	-0.0213	-0.1563	-0.2254
(0.0001)	(0.0042)	(0.0033)	(0.0001)	(0.0039)	(0.0032)
0.0495	-0.1037	-0.2723	0.0000	0.0238	-0.1374
(0.0001)	(0.0025)	(0.0023)	(0.0001)	(0.0020)	(0.0030)

∗ Number of the parenthesis is the Standard Error.

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The above table shows the changes of the estimate of parameter of Augmented Dickey-Fuller with their corresponding standard errors when some prior information is given. We observe from the table that the restricted estimates are changed. The changes in the results of the parameters clearly demonstrate the potential of imposing this restriction in estimating parameters. It implies that parameters can be accurately estimated by using the obtained approach and is also proved effective when restriction is given.

We discuss the new approach of testing unit root for checking non-stationarity of the series considering the restriction on the parameter. We observe that this optimized method based on restricted ESS of Augmented Dickey-Fuller test. We found that this approach gives better result than all other methods with efficiency, consistency and in terms of power properties.

VIII. Applications

Any estimated value of δ less than -2 or greater than 0 may results in different solution. To overcome this situation it is necessary to impose suitable restrictions on the parameter during estimation. Our main focus is boundary problem ($-2 < \delta < 0$) of the Augmented Dickey-Fuller test. When the value of δ is close to zero then our proposed estimation technique is likely to be appropriate.

IX. Conclusions

Due to boundary condition on parameters, usual estimates can result in different solution. ADF test suffers from such conditions. To overcome this problem we propose a constraint based ADF test based on error sum of squares. Monte Carlo Simulation study indicates that the power of the proposed ADF test gives better result than the usual ADF test.

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