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I. Introduction

Study of heat transfer in boundary layer find applications in extrusion of plastic sheets, polymer, spinning of fibers, cooling of elastic sheets etc. The quality of final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. Liquid metals have small Prandtl number of order 0.01~ 0.1(e.g. Pr = 0.01 is for Bismuth, Pr = 0.023 for Mercury etc.) and are generally used as coolants because of very large thermal conductivity.

Aim of the present paper is to investigate effects of variable thermal conductivity, heat source/sink and variable free stream on flow of a viscous incompressible electrically conducting fluid and heat transfer on a non-conducting stretching sheet. Linear stretching of the sheet is considered because of its simplicity in modelling of the flow and heat transfer over stretching surface and further it permits the similarity solution, which are useful in understanding the interaction of flow field with temperature field. The heat source and sink is included in the work to understand the effect of internal heat generation and absorption [Chaim (1998)].

The Homotopy Perturbation Method is a combination of the classical perturbation technique and homotopy technique, which has eliminated the limitations of the traditional perturbation methods. This technique can have full advantage of the traditional perturbation techniques. J. H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media. J. H. He, A coupling method of homotopy technique and perturbation technique for nonlinear problems. To illustrate the basic idea of the Homotopy Perturbation Method for solving nonlinear differential equations, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0 \tag{1}$$

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Subject to boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \tag{2}$$

Where A is a general differential operator, B is a boundary operator, f(r) is a known analytic function, and Γ is the boundary of the domain Ω.

The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N. Equation can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

By the homotopy technique, we construct a homotopy V(r,p) : Ω*(0,1)→R which satisfy

$$H(V,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{4}$$

$$H(V,p) = L(v) - L(u_0) + p\, L(u_0) + p[N(v) - f(r)] = 0 \tag{5}$$

Where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation of which satisfies the boundary conditions.

$$H(V,0) = L(v) - L(u_0)$$

$$H(V,1) = A(v) - f(r)] \tag{6}$$

Thus, the changing process of p from zero to unity is just that of $v(r, p)$ from $u_0(r)$ to $u(r)$. In Topology, this is called deformation and $L(v) - L(u_0)$, $A(v) - f(r)$ are called homotopic. According to the HPM, we can first use the embedding parameter p as a "small parameter," and assume that the solution of can be written as a power series in p:

$$V = V_0 + pV_1 + p^2 V_2 + \ldots\ldots\ldots\ldots\ldots \tag{7}$$

Setting p=1 results in the approximate solution of

$$u = \lim_{p\to 1} V = V_0 + V_1 + \ldots\ldots\ldots\ldots\ldots\ldots \tag{8}$$

The series is convergent for most cases; however, the convergent rate depends upon the Nonlinear operator $A(V)$. The second derivative of $N(V)$ with respect to V must be small because the parameter may be relatively large; that is, $p \to 1$.

In this paper is to investigate effects of variable thermal conductivity on flow of a viscous incompressible fluid in variable free stream near a stagnation point on a non-conducting stretching sheet.

Notes

II. Formulation of the Problem

Notes

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid of variable thermal conductivity in the vicinity of a stagnation point on a non-conducting stretching sheet It is assumed that external field is zero, the electric field owing to polarization of charges and Hall Effect are neglected. Stretching sheet is placed in the plane $y = 0$ and x-axis is taken along the sheet The fluid occupies the upper half plane i.e. $y > 0$. The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2}, \tag{10}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right) + Q\,(T - T_\infty)\ , \tag{11}$$

where ε -perturbation parameter, η-similarity parameter $\{ = (c/v)^{½}\, y\}$, η_∞-value of η at which boundary conditions is achived, κ-uniform thermal conductivity, κ*-variable thermal conductivity, v-kinematic viscosity, ρ-density of fluid, ψ-stream function, σ-electrical conductivity, θ-dimensionless temperature$\{ = (T - T_\infty) / (T_w - T_\infty)\}$, τ_w-shear stress, S-heat source/sink parameter $\{= Q/\rho\, C_p\, c\, \}$, T-fluid temperature.

The second derivatives of u and T with respect to x have been eliminated on the basis of magnitude analysis considering that Reynolds number is high. Hence the Navier-Stokes equation modifies into Prandtl's boundary layer equation.
The boundary conditions are.

$$\left.\begin{array}{ll} y = 0: & u = u_w(x) = c \quad v = 0, \quad T = T_w \\ y \to \infty: & u = U(x) = bx\,, \qquad T = T_\infty \end{array}\right\} \tag{12}$$

Introducing the stream function ψ (x, y) as defined by

$$u = \frac{\partial \psi}{\partial y}\,, \qquad\qquad v = -\frac{\partial \psi}{\partial x}, \tag{13}$$

the similarity variable $\eta = (c/\nu)^{1/2}\, y$ and

$$\Psi\,(x, y) = (c\,\nu)^{1/2}\ x\, f(\eta), \tag{14}$$

into the equations (3) and (5), we get

$$f''' + ff'' - (f')^2 + \lambda^2 = 0\,, \tag{15}$$

And

$$(1+\varepsilon)\,\theta'' + \varepsilon\,(\theta')^2 + \Pr\theta' f + \Pr S\theta = 0.$$

The governing boundary layer and thermal boundary layer equations (15) and (16) with the boundary conditions (17) are solved using Homotopy Perturbation Method.

Equations (15) and (16) are non-linear coupled differential equation. To solve these equations, we introduce the following Homotopy.

$$D(f,p) = (1-p)\left[\left(\frac{d^3f}{d\eta^3}\right) - \left(\frac{d^3f_I}{d\eta^3}\right)\right] + p\left[\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} - \left(\frac{df}{dn}\right)^2 + \lambda^2\right] = 0 \quad (16)$$

$$D(\theta,p) = (1-p)\left[\left(\frac{\partial^2\theta}{\partial\eta^2} + P_rS\theta\right) - \left(\frac{\partial^2\theta_I}{\partial\eta^2} + P_rS\theta_I\right)\right]$$

$$+P\left[(1+\varepsilon\theta)\frac{\partial^2\theta}{\partial n^2} + \varepsilon\left(\frac{\partial\theta}{dn}\right)^2 + P_r\frac{\partial\theta}{\partial n}f + P_rS\theta\right] = 0 \quad (17)$$

With the following assumption

$$f = f_0 + pf_1 + p^2f_2 + \cdots \quad (18)$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 \quad (19)$$

Using equation (18),(19) into equation (10) and (11) and on comparing the like powers of p, we get the zeoth order equation,

$$\left[\left(\frac{d^3f_0}{d\eta^3}\right) - \left(\frac{d^3f_I}{d\eta^3}\right)\right] + \left[\frac{d^3f_0}{d\eta^3} + f_0\frac{d^2f_0}{d\eta^2} - \left(\frac{df_0}{dn}\right)^2 + \lambda^2\right] = 0, \quad (21)$$

$$\left[\left(\frac{\partial^2\theta_0}{\partial\eta^2} + P_rS\theta_0\right) - \left(\frac{\partial^2\theta_I}{\partial\eta^2} + P_rS\theta_I\right)\right] + \left[(1+\varepsilon\theta_0)\frac{\partial^2\theta_0}{\partial n^2} + \varepsilon\left(\frac{\partial\theta_0}{dn}\right)^2 + P_r\frac{\partial\theta_0}{\partial n}f + P_rS\theta_0\right]$$

$$= 0, \quad (21)$$

with the corresponding boundary conditions are of zeroth order equations are:

$$\eta = 0:\ f_0 = 0,\ f_0' = 1, \theta_0 = 1\,; \quad (22)$$

$$\eta = \infty: \quad f_0' = \lambda, \quad \theta_0 = 0 \quad (23)$$

$$\frac{d^3f_1}{d\eta^3} + (e^{-n} - \lambda e^{-\eta}) + (\lambda\eta - e^{-n} + \lambda e^{-n} - \lambda + 1)(\lambda e^{-\eta} - e^{-\eta}) - (\lambda + e^{-n} - \lambda e^{-n})(\lambda + e^{-n} - \lambda e^{-n}) + \lambda^2 = 0 \quad (24)$$

$$\frac{\partial^2\theta_I}{\partial\eta^2} + P_rS\theta_I + e^{-\eta}(1+\varepsilon e^{-\eta}) + \epsilon\, e^{-2\eta} - P_re^{-\eta}(\lambda\eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1) - P_rSe^{-\eta} = 0 \quad (25)$$

With the corresponding boundary conditions are of first order equations are:

$$\left.\begin{aligned} \boldsymbol{\eta} = \mathbf{0}:\ \boldsymbol{f_0} = \mathbf{0},\ \boldsymbol{f_0'} = \mathbf{0}, \boldsymbol{\theta_0} = \mathbf{0} \\ \boldsymbol{\eta} = \infty: \quad \boldsymbol{f_0'} = \mathbf{0}, \quad \boldsymbol{\theta_0} = \mathbf{0}\,; \end{aligned}\right\} \quad (26)$$

Solving equations with corresponding boundary conditions, the following functions can be obtained successively, by summing up the results, and $p \to 1$ we write the f($\boldsymbol{\eta}$) , $\boldsymbol{\theta(\eta)}$, profile as:

Notes

$$f(\eta) = \lambda\eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1 + (\eta e^{-\eta} + 4e^{-\eta} + 3\eta - 4) \quad (27)$$

$$\theta(\eta) = e^{-\eta} + \frac{A_2 e^{-\eta}}{(A_1)^2+1} + \frac{A_3 e^{-2\eta}}{(A_1)^2+4} + \frac{A_4 e^{-\eta}}{(A_1)^2}\left(\eta - \frac{\eta-2}{(A_1)^2}\right) - \left(\frac{2A_4}{(A_1)^4} + \frac{A_3}{(A_1)^2+4} + \frac{A_2}{(A_1)^2+1}\right) e^{-\eta} \quad (28)$$

Notes

where $A_1 = \sqrt{P_r S}$: $A_2 = P_r S + P_r - \lambda P_r - 1$: $A_3 = \lambda P_r - P_r - 2$: $A_4 = \lambda P_r$ (29)

Skin-Friction: Skin-friction coefficient at the sheet is given by

$$C_f = x f''(0) \quad (30)$$

Nusselt Number: The rate of heat transfer in terms of the Nusselt number at the sheet is given by

$$Nu = -\theta'(0) \quad (31)$$

III. Conclusion

It is observed from Table 1 as L increases, the numerical values of $f''(0)$ also increase. It is noted from Table 2 that the numerical values of $-\theta'(0)$ increase when λ increases and $-\theta'(0)$ decreases when ε increases. The skin-friction coefficient and Nusselt number are presented by equations (30) and (31) and they are directly proportional to $f''(0)$ and $-\theta'(0)$ respectively. The effects of ε, Pr and S on Nusselt number have been presented through Table 3 respectively.

Table 1

λ	f''(0)	λ	f''(0)
0.0	-1	0.1	-1.0800
0.01	-1.0098	1.0	0.0004
0.05	-1.0450	2.0	2.0175

Table 2

λ	$-\theta'(0)$	ε	$-\theta'(0)$
0.1	.81235	0.0	0.223558
0.5	.13629	0.05	0.215792
2.0	.24133	0.1	0.204672

Table 3

ε	S	P_r	Nu
0	0	0.5	2.010357
0.1	0	0.5	3.542315
0	-0.1	1	2.565423
0.1	-0.1	1	2.845633

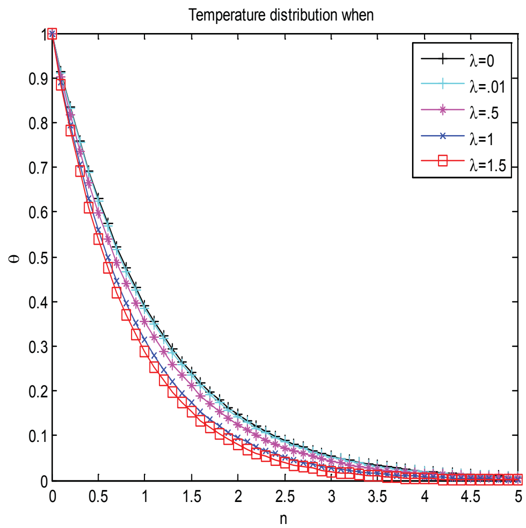

Figure 1

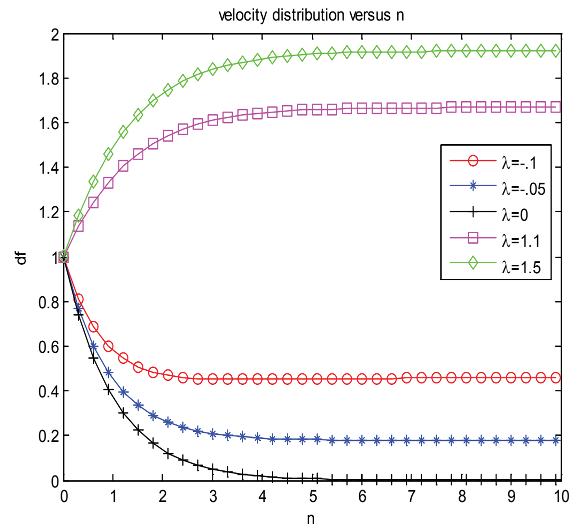

Figure 2

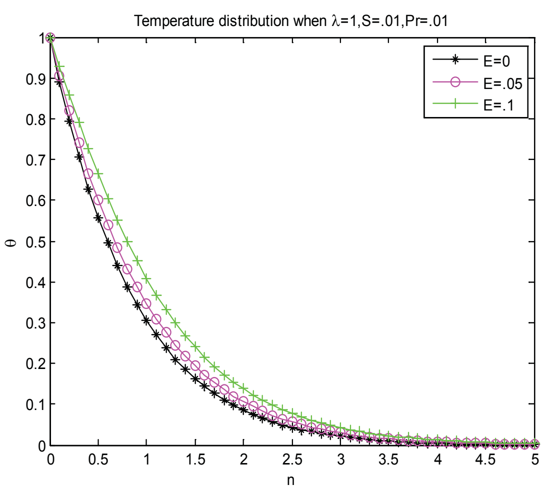

Figure 3

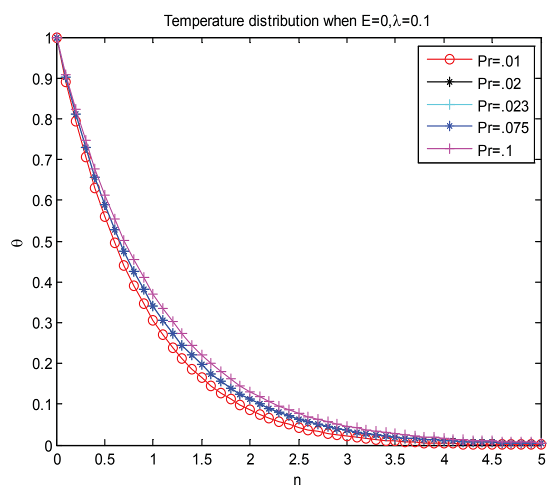

Figure 4

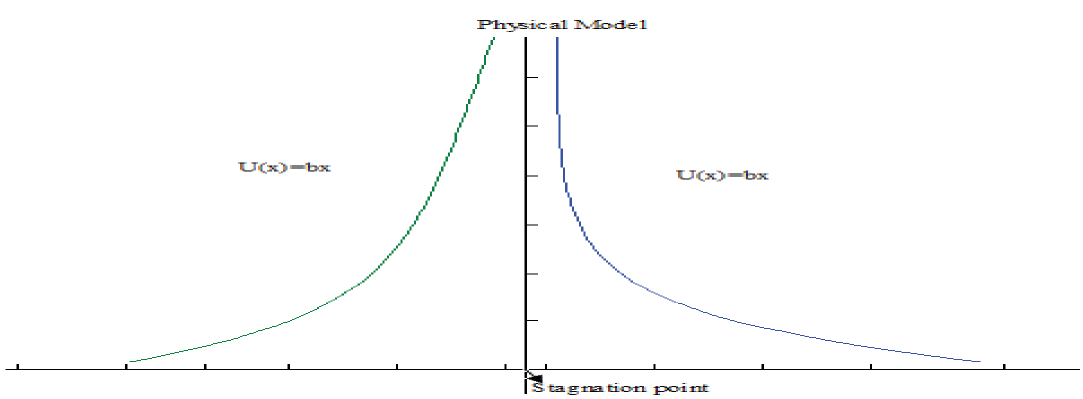

Figure 5

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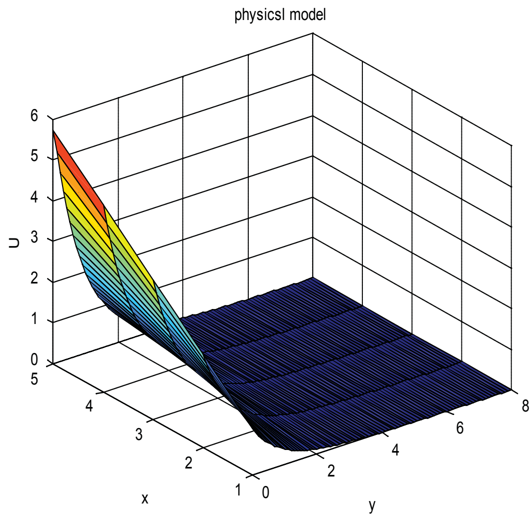

Figure 6

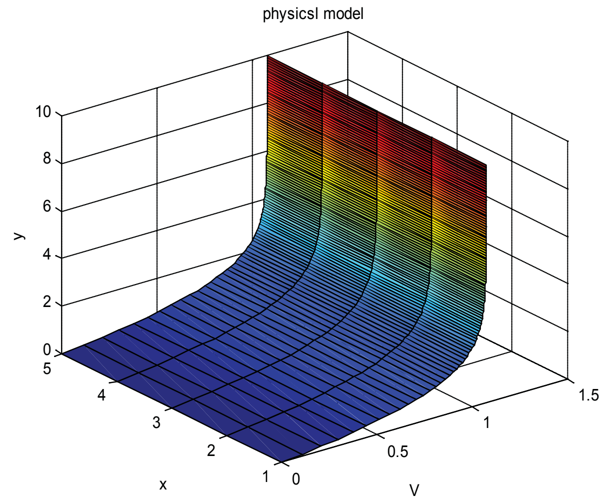

Figure 7

From figure 1, we observe that as λ increases, value of f' also increases. From figure 2 it is observed that when λ increase simultaneously θ also increases. In figure3, λ, S and Pr are constant but when ε increases θ will also increased.It is observed in figure 4, s, ε and λ are constant, when Pr increases, θ will also increase. Figure 5 is a physical model which becomes clearer from figure, 6 and 7.

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