Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 14 Issue 6 Version 1.0 Year 2014
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

An Wonderful Summation Formula Convoluted with Contiguous Relation

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GJSFR-F Classification : MSC 2010: 33C05, 33C20, 33D15, 33D50, 33D60

ANWONDERFULSUMMATIONFORMULACONVOLUTEDWITHCONTIGUOUSRELATION

Strictly as per the compliance and regulations of :

An Wonderful Summation Formula Convoluted with Contiguous Relation

Salahuddin [α], M. P. Chaudhary [σ] & Diriba Kejela Geleta [ρ]

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I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{matrix} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{matrix}\right] = \sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\cdots(a_A)_k z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k k!} \tag{1}$$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers and $\mid z \mid= 1$

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; & \\ c\ \ ; & z\end{matrix}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b\ ; & \\ c\ \ ; & z\end{matrix}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1\ ; & \\ c\ ; & z\end{matrix}\right] \tag{2}$$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$${}_2F_1\left[\begin{matrix} a,\ b\ ; & \\ \frac{a+b+1}{2}\ ; & \frac{1}{2}\end{matrix}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3}$$

$$= \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4}$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$${}_2F_1\left[\begin{matrix} a,\ b\ ; & \\ \frac{a+b-1}{2}\ ; & \frac{1}{2}\end{matrix}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{5}$$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

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$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\,\Gamma(\zeta) \tag{7} $$

II. Main Summation Formula

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+51}{2} & ; & \end{matrix}\right] = \frac{2^b\,\Gamma(\frac{a+b+51}{2})}{(a-b)\,\Gamma(b)\Big[\prod_{\varrho=1}^{25}\{a-b-(2\varrho-1)\}\Big]\Big[\prod_{\varsigma=1}^{25}\{a-b+(2\varsigma-1)\}\Big]} $$

$$ \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Big\{16777216(a^{26}+a^{25}(-625+1224b)+25a^{24}(7396-9775b+9163b^2) \right. $$

$$ +100a^{23}(-344425+752114b-262395b^2+156604b^3)+230a^{22}(19709459-33907350b+33726980b^2 $$

$$ -5313350b^3+2265165b^4)+17710a^{21}(-25320175+58459396b-30894950b^2+19064140b^3-1697875b^4 $$

$$ +549712b^5)+230a^{20}(150530498030-292095255025b+300323641639b^2-76706596750b^3 $$

$$ +33242331080b^4-1893929625b^5+483162491b^6)+2300a^{19}(-929947187675+2221532039250b $$

$$ -1430366003815b^2+898110785264b^3-134850725625b^4+43814588818b^5-1725580325b^6) $$

$$ +354976524b^7+2185a^{18}(49213453442791-102599357455300b+107537450188000b^2 $$

$$ -34831583707900b^3+15171737968030b^4-1494100037500b^5+378247207240b^6-10846504900b^7 $$

$$ +18241849 15b^8)+2185a^{17}(-2037048680061175+4974851454538832b-3608737921763100b^2 $$

$$ +2282171335907400b^3-445423671139650b^4+143879155669032b^5-9955177409100b^6 $$

$$ +2009515142040b^7-43458522975b^8+6009079720b^9)+1955a^{16}(7819664006993468 0 $$

$$ -170970567062484075b+180998915556065411b^2-67807865563445700b^3+29427554850189300b^4 $$

$$ -3827247742614450b^5+952626139804962b^6-48449597307300b^7+7889492452380b^8 $$

$$ -131448618875b^9+14872472307b^{10})+109480a^{15}(-3999070285605812 5+9903668635739917 0b $$

$$ -7779274205108247 5b^2+4914949684429462 8b^3-1127612677459125 0b^4+3589210801875180b^5 $$

$$ -330978283892550b^6+64822304809584b^7-2491392044625b^8+327096283330b^9-4215553375b^{10} $$

$$ +380166268b^{11})+340a^{14}(308246102285629047091-696416403959283989450b $$

$$ +739141133551046804900b^2-305205676801283739250b^3+130862315493685966715b^4 $$

$$ -20189297381974528500b^5+4886797542957632400b^6-331816588273247700b^7 $$

$$ +51414459002198985b^8-1505891473581250b^9+155745474282860b^{10}-1496904680250b^{11} $$

$$ +96467190505b^{12})-340a^{13}(616880343249751686707 5-153909080045983474069 56b $$

$$ +12772321670861914650650b^2-7997813349418306546020b^3+2041950755934836024985b^4 $$

$$ -633657121834647845368b^5+69430174491959238300b^6-12968488981389722760b^7 $$

$$ +658096757009317605b^8-79151871484967260b^9+1708599193535850b^{10}-125088007991380b^{11} $$

$$ +6752703335 35b^{12})-340a^{12}(-10301782374950604047521 0+23791060178722728013562 5b $$

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$$
\begin{gathered}
-2512856661714001729250116^2+11083132007945233725489 0b^3-4646596443184698562014 0b^4\\
+79921660067640129252 15b^5-18495329291190086654 41b^6+14755978421361089822 0b^7\\
-20993248616914598570b^8+7734067499190254 95b^9-650786850489906 13b^{10}+7815904832908 90b^{11}\\
-6752703333535b^{13}+964671905 05b^{14})-680a^{11}(714914577355946689935125\\
-1785568469012540347426390b+1536661412180534230322445b^2-944266221187015176370984b^3\\
+258117062914597906943105b^4-76795443035578983296490b^5+9277161372998843308985b^6\\
-159491131053701191144 44b^7+90589991666702743215b^8-888467336318330769 0b^9\\
+1799565265779230 95b^{10}-3907952416454445b^{12}+625440039956 90b^{13}-7484523401 25b^{14}\\
+612067691148b^{15})-5a^{10}(-1114416507876347373186164291+2605251632506319846265006600b\\
-2713236530568155259987949280b^2+1245389268463706328299937880b^3\\
-502247168898897193562814492b^4+91487680589333895017187400b^5\\
-19539086080422267852518480b^6+1637272250391243481655 6640b^7-19039436677003398214959 4b^8\\
+585658719017221990140 0b^9-24474087614597540920b^{11}+4425350583331361684b^{12}\\
-1161847451604378 00b^{13}+10590692251234480b^{14}-923037566990 00b^{15}+5815136672037b^{16})\\
-35a^9(149313615080134446367938 7925-370390963276210959842514764 0b\\
+324655929843576997832796580 0b^2-192695353161330459397938712 0b^3\\
+54250769609827966946405782 0b^4-14941780558808567886964763 2b^5\\
+18213137091509715355964600b^6-256592291100414145138808 0b^7\\
+114192589173533471911310b^8-836655312881774570020 0b^{10}+172616511056132835120b^{11}\\
-751309414207053333 80b^{12}+768903894425396240b^{13}-1462866002907500 0b^{14}\\
+102315717425624 0b^{15}-73423442828 75b^{16}+375138262520b^{17})\\
-5a^8(-7920379643060620898668696938 0+1852887587366845439776244079 25b\\
-1874542513568301049374405726 25b^2+868123962754453633144717911 60b^3\\
-3253006075644016428548356692 0b^4+58164039944857373767160268 60b^5\\
-10209054876465928536291938 84b^6+644021632686715377554350 80b^7\\
-799348124214734303379170b^9+190394366770033982149594b^{10}-123202388666715730772 40b^{11}\\
+14275409059501927027 60b^{12}-447505794766335971 40b^{13}+3496183212149530980b^{14}\\
-54551520209109000b^{15}+308479154888058 0b^{16}-189913745400 75b^{17}+797168807855b^{18})\\
-20a^7(119162173363594180659738941625-289865238320982041975875921890b\\
+252318392792718541698360880915b^2-139829590206394802306612896492b^3\\
+37872700780492240721071991460b^4-860179087567859487702210636 0b^5\\
+7680502595166980294880378 20b^6-16100540817167884438858770b^8\\
+449036509425724753992914 0b^9-4093180625978108704139 10b^{10}+54226984558258404989096b^{11}\\
-25085163316313852697 40b^{12}+2204643126836252869 20b^{13}-5640882000645210900b^{14}\\
+3548372965276628 16b^{15}-4735948136788575b^{16}+219539529267870b^{17}-11849806603 25b^{18}
\end{gathered}
$$

$$+408223002600b^{19}) - 2a^6(-5561476346804512129684103946537$$
$$+12770587331626002978218651480250b - 12150506205019695200672261741100b^2$$
$$+5332368515463920791664626497850b^3 - 1654856138217170933299827312475b^4$$
$$+212504641903276935470565517000b^5 - 7680502595166980294880378200b^7$$
$$+2552263719116482134072984710b^8 - 318729899101420018729380500b^9$$
$$+48847715201055669631296200b^{10} - 3154234866819606725054900b^{11}$$
$$+314420597950231473124970b^{12} - 118031296636330705110000b^{13} + 8307555823027975080000b^{14}$$
$$-1811775126027818700 0b^{15} + 93119205165935035 5b^{16} - 10876031319441750b^{17}$$
$$+413235073909700b^{18} - 19844173737750b^{19} + 555636864650b^{20})$$
$$-2a^5(19467371819284907086247501129625 - 45137539248279385834416152387628b$$
$$+36832256545029729132039727475250b^2 - 17007939334198378255326039759300b^3$$
$$+3260466076220605737745066939825b^4 - 212504641903276935470565517000b^6$$
$$+8601790875678594877022106360 0b^7 - 14541009986214343441790067150b^8$$
$$+2614811597791499380218833560b^9 - 228719201473334737542968500b^{10}$$
$$+26110450632096854320806600b^{11} - 1358668221149882197286550b^{12}$$
$$+107721710711890133712560b^{13} - 34321805549356698450000b^{14} + 1964733992946473532000b^{15}$$
$$-374113466840562487 5b^{16} + 1571879775684174600b^{17} - 16323042909687500b^{18} + 503867771407000b^{19}$$
$$-217801906875b^{20} + 4867699760b^{21}) - 10a^4(-9728985616542220949568597055050$$
$$+20860532420704340001307112417175b - 16706538508057279093839949494921b^2$$
$$+5136342421860407132560048428810b^3 - 652093215244121147549013387965b^5$$
$$+3309712276434341866599654624950b^6 - 7574540156098448144214398292 0b^7$$
$$+1626503037822008214274178346 0b^8 - 189877693634397884312420237 0b^9$$
$$+251123584449448596781407246b^{10} - 17551960278192657672131140b^{11}$$
$$+1579842790682797511084760b^{12} - 69426325701784424849490b^{13} + 4449318726785322868310b^{14}$$
$$-123451035928225005000b^{15} + 5753086973212008150b^{16} - 97325072144013525b^{17}$$
$$+3315024746014555b^{18} - 31015666893750b^{19} + 764573614840b^{20} - 3006936625b^{21} + 52098795b^{22})$$
$$-140a^3(11459167482684135008491961118 75 - 22882979875649136304614996928 50b$$
$$+13023807153846480781179848310 15b^2 - 36688160156145765232571774491 5b^4$$
$$+2429705619171196893618005679 90b^5 - 761766930780560113094946642 55b^6$$
$$+199756557437706860438018423 56b^7 - 31004427241230486898025639 70b^8$$
$$+481738382903326148494846780b^9 - 44478188159418083153569210b^{10}$$
$$+4586435931479787999516208b^{11} - 269161777335812819047590b^{12} + 19423260991444458754620b^{13}$$
$$-741213786517403366750b^{14} + 38434906532238399096b^{15} - 946888408403831025b^{16}$$
$$+35618174063983350b^{17} - 543621502869725b^{18} + 14754677186480b^{19} - 126017980375b^{20}$$
$$+2411613710b^{21} - 8729075b^{22} + 111860b^{23}) - 35a^2(-4326299607531226662947437702875$$

Notes

$$+6557295780968170825294731733500b-5209522861538592312471939324060b^{3}$$
$$+4773296716587794026811414141406b^{4}-2104700374001698807545127284300b^{5}$$
$$+694314640286839725752700670920b^{6}-144181938738696309541920503380b^{7}$$
$$+26779178765261443562491510375b^{8}-3246559298435769978327965800b^{9}$$
$$+387605218652593608569707040b^{10}-29855136008078950760550360b^{11}$$
$$+2441060757093601679842964b^{12}-124073981945515742320600b^{13}+7180228154495883247600b^{14}$$
$$-243335697135785981800b^{15}+10110082283203082243b^{16}-225288353115782100b^{17}$$
$$+6713409390308000b^{18}-93995480250700b^{19}+1973555359342b^{20}-15632844700b^{21}+221634440b^{22}$$
$$-749700b^{23}+6545b^{24})+a(-584358414459472720534554743906 25$$
$$+22950535233388597888531561067 2500b^{2}-32036171825908790826460995699 9000b^{3}$$
$$+20860532420704340001307112417 1750b^{4}-90275078496558771668832304775256b^{5}$$
$$+25541174663252005956437302960500b^{6}-5797304766419640839517518437800b^{7}$$
$$+926443793683422719888122039625b^{8}-129636837146673835944880167400b^{9}$$
$$+13026258162531599231325033000b^{10}-1214186558928527436249945200b^{11}$$
$$+80889604607657275246112500b^{12}-5232908721563438118365040b^{13}$$
$$+236781577346156556413000b^{14}-10842536422408061131600b^{15}+334247458607156366625b^{16}$$
$$-10870050428167347920b^{17}+224179596039830500b^{18}-5109523690275000b^{19}+67181908655750b^{20}$$
$$-1035315903160b^{21}+77986905000b^{22}-75211400b^{23}+244375b^{24}-1224b^{25})$$
$$-b(-584358414459472720534554743906 25+1514204862635929332031603196006 25b$$
$$-16042834475757778901188874556625 00b^{2}+97289856165422209495685970550500b^{3}$$
$$-38934743638569814172495002259250b^{4}+11122952693609024259368207893074b^{5}$$
$$-2383243467271883613194778832500b^{6}+396018982153031044933434846900b^{7}$$
$$-52259765278047056228778577375b^{8}+5572082539381736865930821455b^{9}$$
$$-486141912602043749155885000b^{10}+35026060074832053761571400b^{11}$$
$$-2097393167049155734805500b^{12}+104803674777113876010940b^{13}-4378182148681243525000b^{14}$$
$$+152874431336722299400b^{15}-4450951365933667375b^{16}+107531395772498335b^{17}$$
$$-2138878531652500b^{18}+34622014546900b^{19}-448420299250b^{20}+4533175570b^{21}-34442500b^{22}$$
$$+184900b^{23}-625b^{24}+b^{25}))\Big\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Big\{67108864(25a^{25}+25a^{24}(-208+391b)+20a^{23}(101177$$
$$-58344b+52479b^{2})+460a^{22}(-455884+892959b-185640b^{2}+106267b^{3})+2530a^{21}(13346809$$
$$-11704160b+10766882b^{2}-1163344b^{3}+475405b^{4})+1610a^{20}(-1353942200+2704589143b$$
$$-912494856b^{2}+527319226b^{3}-34900320b^{4}+10822455b^{5})+460a^{19}(426815174659-459263696248b$$
$$+426282394643b^{2}-77700640640b^{3}+317269916 25b^{4}-1414134120b^{5}+345116065b^{6})$$
$$+8740a^{18}(-984715225276+1984309333429b-852036675616b^{2}+492261535759b^{3}-56362562620b^{4}$$
$$+17304624547b^{5}-552185704b^{6}+108465049b^{7})+2185a^{17}(222718961858047-272078729517312b$$

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$$+252683899863948b^2-600219604931152b^3+242934064133346b^4-19049883914888b^5+456089150748b^6$$
$$-108581054440b^7+1738340919b^8)+1955a^{16}(-7708080913032800+15557493409997523b$$
$$-7762351183361712b^2+4451249998293588b^3-674758641811632b^4+203539061251218b^5$$
$$-11548151961360b^6+220183585444484b^7-401749901288b^8+5257944755b^9)$$
$$+15640a^{15}(36085356535274431-48105532096658072b+44411429571288773b^2$$
$$-12447420898782912b^3+4957788393308982b^4-519828215919600b^5+121009816113354b^6$$
$$-5145294231168b^7+7879835002283b^8-111597194800b^9+1180354945b^{10})$$
$$+4760a^{14}(-263449356306945004+529516679574120898 3b-292784513762885560 8b^2$$
$$+1655174300820813575b^3-298864444151010600b^4+87812255490403086b^5-6688314593762592b^6$$
$$+122211140542474 2b^7-39651280435260b^8+48406049708075b^9-526101061 20b^{10}+427687051 5b^{11})$$
$$+68a^{13}(4678956711831316544015-6642118043129456790560b+6057457239184768540186b^2$$
$$-1899101580867991215632b^3+738204988653133947477b^4-92533200585371521920b^5$$
$$+20682696306956610108b^6-1174979311727291616b^7+168173670982052841b^8$$
$$-413181917915312 0b^9+383481740945226b^{10}-2905990952592b^{11}+135054066707b^{12})$$
$$-68a^{12}(7453455831561136614620 0-148336157581894868529005b+88182480614842847780456b^2$$
$$-48755503590552305307306b^3+9881925251367181441520b^4-2793490678146045928323b^5$$
$$+253095816833265752528b^6-43307286160470774988b^7+1831060446500146680b^8$$
$$-19654472755109273 9b^9+3331156177284936b^{10}-175207037850026b^{11}+135054066707b^{13})$$
$$-136a^{11}(-6414114704221140635687 65+952391263870199916976200b$$
$$-851530880817831240234369b^2+288041151657164042271680b^3-107958722843046786837217b^4$$
$$+15112252054344345746664b^5-3169716881851024760437b^6+208882234126009254720b^7$$
$$-26340965887457983671b^8+757967917191378712b^9-4560943230633689 9b^{10}+876035189250 13b^{12}$$
$$-14529954762 96b^{13}+1496904680 25b^{14})-136a^{10}(72075845672230776255669 40$$
$$-14104026856612135897961625b+8803543237219289085880080b^2-4704735450635305968636551b^3$$
$$+1023524341422403862628132b^4-272111704767122618436857b^5+26845238578197350230920b^6$$
$$-405565744913865730494 3b^7+178749985534955270436b^8-12465539301632788371b^9$$
$$+456094323063368 99b^{11}-166557808864246 8b^{12}+1917408704726 13b^{13}-1841353714200b^{14}$$
$$+1357408186 75b^{15})+a^9(112353003526097436195440702 95-171713980917547629593546508 80b$$
$$+148821920776395114850868845 04b^2-525886774109118655407362988 8b^3$$
$$+185821982882948503480394574 8b^4-272222373360422257212406848 0b^5$$
$$+50524473238981550170383112b^6-32577350013182071698240 64b^7+2674718055280627866049 54b^8$$
$$-1695313345022059218456b^{10}+1030836367380275048 32b^{11}-133650414734743062 52b^{12}$$
$$+28096370418241216 0b^{13}-230412796613650 00b^{14}+17453801266720 0b^{15}-102792819960 25b^{16})$$
$$-7a^8(1228564811375702148246572 4400-23386776574592838148542714215b$$
$$+149532409910297291387626883 52b^2-755485659743609105257540615 2b^3$$

Notes

Notes

$$
\begin{aligned}
&+167412553211361310684248144 0b^4 - 394794171821743986867539076b^5 \\
&+366257272399077089885050 56b^6 - 36105882199531283777714296b^7 + 382102579325803980864 22b^9 \\
&-3472856861821988111328b^{10} + 5117673372420408256 08b^{11} - 177874443374299963 20b^{12} \\
&+1633687089539941884b^{13} - 269628706959768 00b^{14} + 17605802777751 60b^{15} - 112203008143 20b^{16} \\
&+5426107011145b^{17}) - 4a^7(-157122336524884631192056543785 \\
&+242417207163302178448478320360b - 199298813033991870414852468503b^2 \\
&+70349588037621130322270847360b^3 - 22109669888156334494449903044b^4 \\
&+2964549734561995774480882848b^5 - 359887751391659698187588524b^6 \\
&+631852938491797466100001 8b^8 - 814433750329551792456016b^9 + 13789235327071434836806 2b^{10} \\
&-710199596028431466048 0b^{11} + 736223864728003174796b^{12} - 199746482993639574 72b^{13} \\
&+1454312572455442980b^{14} - 201181004438668 80b^{15} + 1076147273879055b^{16} - 59312400966 00b^{17} \\
&+2369961320 65b^{18}) - 4a^6(767941876522749251743430993460 - 1392869172915035250548561882505b \\
&+876902949819094417240613313400b^2 - 395331281118286748268970855877b^3 \\
&+78639113738400816511109906224b^4 - 12161965809626942191422387764b^5 \\
&+359887751391659698187588524b^7 - 64095022669838490729883848b^8 \\
&+12631118309745387542595778b^9 - 912738111658709907851280b^{10} + 107770373982934841854858b^{11} \\
&-4302628886165517792976b^{12} + 351605837218262371836b^{13} - 79590943665774844 80b^{14} \\
&+473148381003214140b^{15} - 56441592711147 00b^{16} + 249138698596095b^{17} - 12065257632 40b^{18} \\
&+396883347475b^{19}) - 2a^5(-6566105484008127386414285695785 \\
&+9871915481562597552130646417760b - 7272009082608192934087529171754b^2 \\
&+2271587750081918842847295429008b^3 - 469652076662064339904368905593b^4 \\
&+24323931619253884382844775528b^6 - 5929099469123991548961765696b^7 \\
&+1381779601376103954036386 766b^8 - 136111866802111286062034240b^9 \\
&+18503595924164338053706276b^{10} - 1027633139695415510773152b^{11} + 94978683056965561562982b^{12} \\
&-3146128819902631745280b^{13} + 208993168067159344680b^{14} - 4065056648491272000b^{15} \\
&+198959432373065595b^{16} - 2081199817700640b^{17} + 756212092703 90b^{18} - 3252508476 00b^{19} \\
&+8712076275b^{20}) - 2a^4(18077370164182354278691014838200 \\
&-29552020843171636838990239765155b + 16296662664952285424129230468104b^2 \\
&-4851985233921274473130646753514b^3 + 469652076662064339904368905593b^5 \\
&-157278227476801633022219812448b^6 + 44219339776312668988899806088b^7 \\
&-5859439362397645873948685040b^8 + 929109914414742517401972874b^9 \\
&-69599655216723462658712976b^{10} + 7341193153327181504930756b^{11} \\
&-335985458546484169011680b^{12} + 25098969614206554214218b^{13} - 711297377079405228000b^{14} \\
&+38769905235676239240b^{15} - 659576572370870280b^{16} + 26540546506580505b^{17} \\
&-246304398649400b^{18} + 7297208073750b^{19} - 28094757600b^{20} + 601387325b^{21})
\end{aligned}
$$

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$$
\begin{aligned}
&-4a^3(-18767652058955009331731936861325+24566189373961227625674141811080b\\
&-12015540473174435277887487066981b^2+2425992616960637236565323376757b^4\\
&-1135793875040959421423647714504b^5+395331281118286748268970855877b^6\\
&-70349588037621130322270847360b^7+13220999045513159342006960766b^8\\
&-1314716935272796638518407472b^9+159961005321600402933642734b^{10}\\
&-9793399156343577437237120b^{11}+828843561039389190224202b^{12}-32284726874755850665744b^{13}\\
&+1969657417976768154250b^{14}-48669415714241185920b^{15}+2175548436665991135b^{16}\\
&-32786995919384280b^{17}+1075591455633415b^{18}-8935573673600b^{19}+212245988465b^{20}\\
&-735815080b^{21}+12220705b^{22})-28a^2(3057116769612200345160349480500\\
&-3347356545485788046647540587975b+1716505781882062182555355295283b^3\\
&-1164047333210877530294945033436b^4+519429220186299495291966369411b^5\\
&-125271849974156345320087616200b^6+28471259004855981487836066929b^7\\
&-3738310247757432284690672088b^8+531506859915696838753103018b^9\\
&-42760067152207975559988960b^{10}+4136007135400894595424078b^{11}\\
&-214157452921761201752536b^{12}+14710967580877295026166b^{13}-497733673396905453360b^{14}\\
&+24806955660534157490b^{15}-541978448695433820b^{16}+19718368614383085b^{17}\\
&-265957162317280b^{18}+7003210769135b^{19}-52468454220b^{20}+972864695b^{21}-3049800b^{22}+37485b^{23})\\
&-5b(9295905536910197181300544345125-17119853909828321932897957090800b\\
&+15014121647164007465385549489060b^2-7230948065672941711476405935280b^3\\
&+2626442193603250954565714278314b^4-614353501218199401394744794768b^5\\
&+125697869219907704953645235028b^6-17199907359259830075452014160b^7\\
&+2247060070521948723908814059b^8-196046300228467711415420768b^9\\
&+17446391995481502529070408b^{10}-1013669993092314579588320b^{11}\\
&+63633811280905904998604b^{12}-2508037872042116441888b^{13}+112874995242338420168b^{14}\\
&-3013859636995824800b^{15}+97328186331966539b^{16}-1721282213782448b^{17}+39266996068628b^{18}\\
&-435969388400b^{19}+6753485354b^{20}-41941328b^{21}+404708b^{22}-1040b^{23}+5b^{24})\\
&-5a(-9295905536910197181300544345125+18745196654720413061226227292660b^2\\
&-19652951499168982100539313448864b^3+11820808337268654735596095906062b^4\\
&-3948766192625039020852258567104b^5+1114295338332028200438849506004b^6\\
&-193933765730641742758782656288b^7+327414872044299734079597999901b^8\\
&-3434279618350952591870930176b^9+383629530499850096424556200b^{10}\\
&-25905042377269437741752640b^{11}+2017371743113770211994468b^{12}\\
&-90332805386560612351616b^{13}+5040998789545630951816b^{14}-150474104398346449216b^{15}\\
&+6082979923309031493b^{16}-118898404799065344b^{17}+3468572714833892b^{18}-42252260054816b^{19}\\
&+870877704046b^{20}-5922304960b^{21}+82152228b^{22}-233376b^{23}+1955b^{24}))\Big\}\Big]
\end{aligned}
\tag{8}
$$

Notes

III. Derivation of the Summation Formula

Substituting $c = \frac{a+b+51}{2}$ and $z = \frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+51}{2} & ; & \frac{1}{2} \end{matrix}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b & ; & \frac{1}{2} \\ \frac{a+b+51}{2} & ; & \frac{1}{2} \end{matrix}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1 & ; & \frac{1}{2} \\ \frac{a+b+51}{2} & ; & \frac{1}{2} \end{matrix}\right]$$

Notes

Now involving the derived formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

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Notes

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