Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 14 Issue 3 Version 1.0 Year 2014
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Exact Solutions for Wick-type Stochastic Coupled KdV Equations

By Hossam A. Ghany & M. Zakarya
Taif University, Saudi Arabia

Abstract- Wick-type stochastic coupled KdV equations are researched. By means ofHermite transformation, white noise theory and F-expantion method, three typesof exact solutions for Wick-type stochastic coupled KdV equations are explicitlygiven. These solutions include the white noise functional solutions of Jacobi elibtic function (JEF) type, trigonometric type and hyperbolic type.

Keywords: coupled KdV equations; F-expantion method; hermite transform; wicktypeproduct; white noise theory.

GJSFR-F Classification : *MSC 2010: 39A50 , 37L55*

EXACTSOLUTIONSFORWICKTYPESTOCHASTICCOUPLEDKDVEQUATIONS

Strictly as per the compliance and regulations of :

Ref

[3] A. de Bouard and A. Debussche. *J. Funct. Anal.*, **154** (1998): 215-251.

Exact Solutions for Wick-type Stochastic Coupled KdV Equations

Hossam A. Ghany [α] & M. Zakarya [σ]

Abstract- Wick-type stochastic coupled KdV equations are researched. By means of Hermite transformation, white noise theory and F-expantion method, three types of exact solutions for Wick-type stochastic coupled KdV equations are explicitly given. These solutions include the white noise functional solutions of Jacobi elibtic function (JEF) type, trigonometric type and hyperbolic type.

Keywords: *coupled KdV equations; F-expantion method; hermite transform; wicktype product; white noise theory.*

PACS No. : 05.40.±a, 02.30.Jr.

I. Introduction

In this paper, we shall explore exact solutions for the following variable coefficients coupled KdV equations.

$$\begin{cases} u_t + h_1(t)uu_x + h_2(t)vv_x + h_3(t)u_{xxx} = 0, \\ v_x + h_4(t)uv_x + h_3(t)v_{xxx} = 0, \end{cases} \quad , (t,x) \in \mathbb{R}_+ \times \mathbb{R}, \qquad (1.1)$$

where $h_1(t)$, $h_2(t)$, $h_3(t)$ and $h_4(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. Random wave is an important subject of stochastic partial differential equations (PDEs). Many authors have studied this subject. Wadati first introduced and studied the stochastic KdV equations and gave the diffusion of soliton of the KdV equation under Gaussian noise in [28, 30] and others [3–6, 23] also researched stochastic KdV-type equations. Xie first introduced Wick-type stochastic KdV equations on white noise space and showed the auto- Backlund transformation and the exact white noise functional solutions in [35]. Furthermore, Xie [36–39], Ghany et al. [12–14, 16–19] researched some Wick-type stochastic wave equations using white noise analysis.

In this paper we use F-expantion method for finding new periodic wave solutions of nonlinear evolution equations in mathematical physics, and we obtain some new periodic wave solution for coupled KdV equations. This method is more powerful and will be used in further works to establish more entirely new solutions for other

Author α: *Department of Mathematics, Faculty of Science, Taif University, Taif, Saudi Arabia.*
Author α: *Department of Mathematics, Helwan University, Cairo, Egypt. e-mail: h.abdelghany@yahoo.com*
Author σ: *Department of Mathematics, Faculty of Science, Al-Azhar University, Assiut 71524, Egypt.*
e-mail: mohammed_zakarya@rocketmail.com

kinds of nonlinear (PDEs) arising in mathematical physics. The effort in finding exact solutions to nonlinear equations is important for the understanding of most nonlinear physical phenomena. For instance, the nonlinear wave phenomena observed in fluid dynamics, plasma, and optical fibers. Many effective methods have been presented, such as variational iteration method [7,8], tanh-function method [9,32,40], homotopy perturbation method [11,27,33], homotopy analysis method [1], tanh-coth method [29,31,32], exp-function method [21,22,34,41,42] , Jacobi elliptic function expantion method [10,24–26], the F-expansion method [43–46]. The main objective of this paper is using the F-expantion method to construct white noise functional solutions for wick-type stochastic coupled KdV equations via hermite transform, wick-type product, white noise theory. If equation (1.1) is considered in a random environment, we can get stochastic coupled KdV equations. In order to give the exact solutions of stochastic coupled KdV equations, we only consider this problem in white noise environment. We shall study the following Wick-type stochastic coupled KdV equations.

$$\begin{cases} U_t + H_1(t) \diamond U \diamond U_x + H_2(t) \diamond V \diamond V_x + H_3(t) \diamond U_{xxx} = 0, \\ V_x + H_4(t) \diamond U \diamond V_x + H_3(t) \diamond V_{xxx} = 0, \end{cases} \tag{1.2}$$

where "$\diamond$" is the Wick product on the Kondratiev distribution space (S_{-1}) which was defined in [20], $H_1(t), H_2(t), H_3(t)$ and $H_4(t)$ are (S_{-1})-valued functions.

II. Description of the F-expantion Method

In order to simultaneously obtain more periodic wave solutions expressed by various Jacobi elliptic functions to nonlinear wave equations, we introduce an F-expansion method which can be thought of as a succinctly over-all generalization of Jacobi elliptic function expansion. We briefly show what is F-expansion method and how to use it to obtain various periodic wave solutions to nonlinear wave equations. Suppose a nonlinear wave equation for u(t,x) is given by.

$$p(u, u_t, u_x, u_{xx}, u_{xxx}, ...) = 0, \tag{2.1}$$

where $u = u(t,x)$ is an unknown function, p is a polynomial in u and its various partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of a deformation F-expansion method.

Step 1. Look for traveling wave solution of Eq.(2.1)by taking

$$u(t,x) = u(\xi) \ \ ,\xi(t,x) = kx + s\int_0^t \delta(\tau)d\tau + c, \tag{2.2}$$

Hence, under the transformation (2.2). Eq.(2.1) can be transformed into ordinary differential equation (ODE) as following.

$$O(u, s\delta u', ku', k^2u'', k^3u''', ...) = 0, \tag{2.3}$$

Step 2. Suppose that $u(\xi)$ can be expressed by a finite power series of $F(\xi)$ of the form.

$$u(t,x) = u(\xi) = \sum_{i=1}^{N} a_i F^i(\xi), \tag{2.4}$$

Ref

[7] M. Dehghan, J. Manafian and A. Saadatmandi. *Math. Meth. Appl. Sci*, **33** (2010): 1384-1398.

where $a_0, a_1, ..., a_N$ are constants to be determined later, while $F'(\xi)$ in(2.4) satisfy

$$[F'(\xi)]^2 = PF^4(\xi) + QF^2(\xi) + R, \tag{2.5}$$

and hence holds for $F(\xi)$

$$\begin{cases} F'F'' = 2PF^3F' + QFF', \\ F'' = 2PF^3 + QF, \\ F''' = 6PF^2F' + QF', \\ \dots \end{cases} \tag{2.6}$$

where P, Q, and R are constants.

Step 3. The positive integer N can be determined by considering the homogeneous balance between the highest derivative term and the nonlinear terms appearing in (2.3). Therefore, we can get the value of N in (2.4).

Step 4. Substituting (2.4) into (2.3) with the condition (2.5), we obtain polynomial in $f^i(\xi)[f'(\xi)]^j$, $(i = 0 \pm 1, \pm 2, ..., j = 0, 1)$. Setting each coefficient of this polynomial to be zero yields a set of algebraic equations for $a_0, a_1, ..., a_N, s$ and δ.

Step 5. Solving the algebraic equations with the aid of Maple we have $a_0, a_1, ..., a_N, s$ and δ can be expressed by P, Q and R. Substituting these results into F-expansion (2.4), then a general form of traveling wave solution of Eq. (2.1) can be obtained.

Step 6. Since the general solutions of (2.4) have been well known for us Choose properly $(P, Q$ and $R.)$ in ODE (2.5) such that the corresponding solution $F(\xi)$ of it is one of Jacobi elliptic functions. (See Appendix A, B and C.)

III. New Exact Traveling Wave Solutions of Eq. (1.2)

Taking the Hermite transform, white noise theory, and F-expantion method to explore new exact wave solutions for Eq.(1.2). Applying Hermite transform to Eq.(1.2), we get the deterministic equations:

$$\begin{cases} \tilde{U}_t(t,x,z) + \tilde{H}_1(t,z)\tilde{U}(t,x,z)\tilde{U}_x(t,x,z) + \tilde{H}_2(t,z)\tilde{V}(t,x,z)\tilde{V}_x(t,x,z) \\ +\tilde{H}_3(t,z)\tilde{U}_{xxx}(t,x,z) = 0, \\ \\ \tilde{V}_t(t,x,z) + \tilde{H}_4\tilde{U}(t,x,z)\tilde{V}_x(t,x,z) + \tilde{H}_3\tilde{V}_{xxx}(t,x,z) = 0, \end{cases} \tag{3.1}$$

where $z = (z_1, z_2, ...) \in (\mathbb{C}^{\mathbb{N}})$ is a vector parameter. To look for the travelling wave solution of Eq.(3.1), we make the transformations $\tilde{H}_1(t,z) := h_1(t,z)$, $\tilde{H}_2(t,z) := h_2(t,z)$, $\tilde{H}_3(t,z) := h_3(t,z)$, $\tilde{H}_4(t,z) := h_4(t,z)$, $\tilde{U}(t,x,z) =: u(t,x,z) = u(\xi(t,x,z))$ and $\tilde{V}(t,x,z) =: v(t,x,z) = v(\xi(t,x,z))$ with.

$$\xi(t,x,z) = kx + s\int_0^t \delta(\tau,z)d\tau + c,$$

where k, s and c are arbitrary constants which satisfy $sk \neq 0$, $\delta(\tau)$ is a nonzero function of the indicated variables to be determined later. So, Eq.(3.1) can be transformed into the following (ODE).

$$\begin{cases} s\delta u' + kh_1uu' + kh_2vv' + k^3h_3u''' = 0, \\ s\delta v' + kh_4uv' + k^3h_3v''' = 0, \end{cases} \tag{3.2}$$

Notes

where the prime denote to the differential with respect to ξ. In view of F-expantion method, the solution of Eq. (3.1), can be expressed in the form.

$$\begin{cases} u(t,x,z) = u(\xi) = \Sigma_{i=1}^{N}(a_i F^i(\xi)), \\ v(t,x,z) = v(\xi) = \Sigma_{i=1}^{M}(b_i F^i(\xi)), \end{cases} \tag{3.3}$$

where a_i and b_i are constants to be determined later. considering homogeneous balance between $u^{'''}$ and $uu^{'}, vv^{'}$ and the order of $v^{'''}$ and $uv^{'}$ in (3.2), then we can obtain $N = M = 2$, so (3.3) can be rewritten as following.

$$\begin{cases} u(t,x,z) = a_0 + a_1 F(\xi) + a_2 F^2(\xi), \\ v(t,x,z) = b_0 + b_1 F(\xi) + b_2 F^2(\xi), \end{cases} \tag{3.4}$$

where a_0, a_1, a_2, b_0, b_1 and b_2 are constants to be determined later. Substituting (3.4) with the conditions (2.5),(2.6) into (3.2) and collecting all terms with the same power of $f^i(\xi)[f'(\xi)]^j$, $(i = 0 \pm 1, \pm 2, ..., j = 0, 1)$. as following.

$$\begin{cases} 2k[12k^2 a_2 P h_3 + b_2^2 h_2 + a_2^2 h_1] F^3 F^{'} \\ +3k[2k^2 a_1 P h_3 + a_1 a_2 h_1 + b_1 b_2 h_2] F^2 F^{'} \\ +[2a_2 s\delta + k b_1^2 h_2 + 8k^3 a_2 Q h_3 + 2k a_0 a_2 h_1 + k a_1^2 h_1 + 2k b_0 b_2 h_2] F F^{'} \\ +[s a_1 h_3 + k a_0 a_1 h_1 + k b_0 b_1 h_2 + k^3 a_1 Q h_3] F^{'} = 0, \\ \\ 2k b_2 [12k^2 P h_3 + a_2 h_4] F^3 F^{'} \\ +k[6k^2 b_1 P h_3 + 2a_1 b_2 h_4 + a_2 b_1 h_4] F^2 F^{'} \\ +[2s b_2 \delta + 2k a_0 b_2 h_4 + k a_1 b_1 h_4 + 8k^3 b_2 Q h_3] F F^{'} \\ +b_1 [s\delta + k a_0 h_4 + k^3 Q h_3] F^{'} = 0. \end{cases} \tag{3.5}$$

Setting each coefficient of $f^i(\xi)[f'(\xi)]^j$ to be zero, we get a system of algebraic equations which can be expressed by.

$$\begin{aligned} & 2k[12k^2 a_2 P h_3 + b_2^2 h_2 + a_2^2 h_1] = 0, \\ & 3k[2k^2 a_1 P h_3 + a_1 a_2 h_1 + b_1 b_2 h_2] = 0, \\ & 2a_2 s\delta + k b_1^2 h_2 + 8k^3 a_2 Q h_3 + 2k a_0 a_2 h_1 + k a_1^2 h_1 + 2k b_0 b_2 h_2 = 0, \\ & s a_1 h_3 + k a_0 a_1 h_1 + k b_0 b_1 h_2 + k^3 a_1 Q h_3 = 0, \\ & 2k b_2 [12k^2 P h_3 + a_2 h_4] = 0, \\ & k[6k^2 b_1 P h_3 + 2a_1 b_2 h_4 + a_2 b_1 h_4] = 0, \\ & 2s b_2 \delta + 2k a_0 b_2 h_4 + k a_1 b_1 h_4 + 8k^3 b_2 Q h_3 = 0, \\ & b_1 [s\delta + k a_0 h_4 + k^3 Q h_3] = 0. \end{aligned} \tag{3.6}$$

with solving by Maple to get the following coefficient.

$$\begin{cases} a_1 = b_1 = 0, a_0 = \text{arbitrary constant}, \\ a_2 = -\frac{12k^2 P h_3(t,z)}{h_4(t,z)}, \\ b_2 = \pm i \frac{12k^2 P h_3(t,z)}{h_4(t,z)} \sqrt{\frac{h_1(t,z) - h_4(t,z)}{h_2(t,z)}} = \mp i a_2 \sqrt{\frac{h_1(t,z) - h_4(t,z)}{h_2(t,z)}}, \\ b_0 = \pm \frac{[3k^2 Q h_3(t,z) - a_0(h_1(t,z) - h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z) - h_4(t,z)]}}, \\ \delta = \frac{-k[a_0 h_4(t,z) + k^2 Q h_3(t,z)]}{S}. \end{cases} \tag{3.7}$$

Notes

Substituting by coefficient (3.7) into (3.4) yields general form solutions of eq. (1.2).

$$u(t,x,z) = a_0 - [\frac{12k^2Ph_3(t,z)}{h_4(t,z)}]F^2(\xi(t,x,z)), \tag{3.8}$$

$$\begin{aligned} v(t,x,z) &= \pm\tfrac{[3k^2Qh_3(t,z)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \\ &\pm\tfrac{12ik^2Ph_3(t,z)}{h_4(t,z)}\sqrt{\tfrac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}F^2(\xi(t,x,z)), \end{aligned} \tag{3.9}$$

with

$$\xi(t,x,z) = k\left[x - \int_0^t \{a_0h_4(\tau,z) + k^2Qh_3(\tau,z)\}d\tau\right].$$

From Appendix C, we give the special cases as following:
Case 1. if we take $P = 1, Q = (2-m^2)$ and $R = (1-m^2)$, we have $F(\xi) \to cs(\xi)$,

$$u_1(t,x,z) = a_0 - [\frac{12k^2h_3(t,z)}{h_4(t,z)}]\ cs^2(\xi_1(t,x,z)), \tag{3.10}$$

$$\begin{aligned} v_1(t,x,z) &= \pm\tfrac{[3k^2h_3(t,z)(2-m^2)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)(h_1(t,z)-h_4(t,z))}} \\ &\pm\tfrac{12ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\tfrac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\ cs^2(\xi_1(t,x,z)), \end{aligned} \tag{3.11}$$

with

$$\xi_1(t,x,z) = k\left[x - \int_0^t \{a_0h_4(\tau,z) + k^2(2-m^2)h_3(\tau,z)\}d\tau\right].$$

In the limit case when $m \to o$ we have $cs(\xi) \to \cot(\xi)$, thus (3.10),(3.11) become.

$$u_2(t,x,z) = a_0 - [\frac{12k^2h_3(t,z)}{h_4(t,z)}]\ \cot^2(\xi_2(t,x,z)), \tag{3.12}$$

$$\begin{aligned} v_2(t,x,z) &= \pm\tfrac{[6k^2h_3(t,z)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \\ &\pm\tfrac{12ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\tfrac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\ \cot^2(\xi_2(t,x,z)), \end{aligned} \tag{3.13}$$

with

$$\xi_2(t,x,z) = k\left[x - \int_0^t \{a_0h_4(\tau,z) + 2k^2h_3(\tau,z)\}d\tau\right].$$

In the limit case when $m \to 1$ we have $cs(\xi) \to \mathrm{csch}(\xi)$, thus (3.10).(3.11) become.

$$u_3(t,x,z) = a_0 - [\frac{12k^2h_3(t,z)}{h_4(t,z)}]\ \mathrm{csch}^2(\xi_3(t,x,z)), \tag{3.14}$$

$$v_3(t,x,z) = \pm\frac{[3k^2h_3(t,z)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \\ \pm\frac{12ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\ \operatorname{csch}^2(\xi_3(t,x,z)), \tag{3.15}$$

with

$$\xi_3(t,x,z) = k\Big[x - \int_0^t \{a_0h_4(\tau,z) + k^2h_3(\tau,z)\}d\tau\Big].$$

Case 2. if we take $P=1, Q=(2m^2-1)$ and $R=-m^2(1-m^2)$, then $F(\xi)\to ds(\xi)$,

$$u_4(t,x,z) = a_0 - [\frac{12k^2h_3(t,z)}{h_4(t,z)}]\ ds^2(\xi_4(t,x,z)), \tag{3.16}$$

$$v_4(t,x,z) = \pm\frac{[3k^2h_3(t,z)(2m^2-1)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \\ \pm\frac{12ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\ ds^2(\xi_4(t,x,z)), \tag{3.17}$$

with

$$\xi_4(t,x,z) = k\Big[x - \int_0^t \{a_0h_4(\tau,z) + k^2(2m^2-1)h_3(\tau,z)\}d\tau\Big].$$

In the limit case when $m\to o$ we have $ds(\xi)\to \csc(\xi)$, thus (3.16),(3.17) become.

$$u_5(t,x,z) = a_0 - [\frac{12k^2h_3(t,z)}{h_4(t,z)}]\ \csc^2(\xi_5(t,x,z)), \tag{3.18}$$

$$v_5(t,x,z) = \pm\frac{[-3k^2h_3(t,z)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \\ \pm\frac{12ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\ \csc^2(\xi_5(t,x,z)), \tag{3.19}$$

with

$$\xi_5(t,x,z) = k\Big[x - \int_0^t \{a_0h_4(\tau,z) - k^2h_3(\tau,z)\}d\tau\Big].$$

Remark that. In the limit case when $m\to 1$ we have $ds(\xi)=cs(\xi)\to \operatorname{csch}(\xi)$, thus (3.16),(3.17) become the same solutions in case 1.

Case 3. if we take $P=\frac{1}{4}, Q=\frac{1-2m^2}{2}$ and $R=\frac{1}{4}$, then $F(\xi)\to ns(\xi)\pm cs(\xi)$,

$$u_6(t,x,z) = a_0 - \frac{3k^2h_3(t,z)}{h_4(t,z)}\Big[ns(\xi_6(t,x,z)) \pm cs(\xi_6(t,x,z))\Big]^2, \tag{3.20}$$

$$v_6(t,x,z) = \pm\frac{[3k^2h_3(t,z)(1-2m^2)+2a_0(h_1(t,z)-h_4(t,z))]}{2\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \tag{3.21}$$

Notes

Global Journal of Science Frontier Research (F) Volume XIV Issue III Version I Year 2014

$$\pm\frac{3ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\Big[ns(\xi_6(t,x,z))\pm cs(\xi_6(t,x,z))\Big]^2,$$

with

$$\xi_6(t,x,z)=k\Big[x-\frac{1}{2}\int_0^t\{2a_0h_4(\tau,z)+k^2(1-2m^2)h_3(\tau,z)\}d\tau\Big].$$

Notes

In the limit case when $m\to o$ we have $(ns(\xi)\pm cs(\xi))\to(\csc(\xi)\pm\cot(\xi))$, thus (3.22),(3.23) become.

$$u_7(t,x,z)=a_0-\frac{3k^2h_3(t,z)}{h_4(t,z)}\Big[\csc(\xi_7(t,x,z))\pm\cot(\xi_7(t,x,z))\Big]^2, \tag{3.22}$$

$$v_7(t,x,z)=\pm\frac{[3k^2h_3(t,z)+2a_0(h_1(t,z)-h_4(t,z))]}{2\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}}$$

$$\pm\frac{3ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\Big[\csc(\xi_7(t,x,z))\pm\cot(\xi_7(t,x,z))\Big]^2, \tag{3.23}$$

with

$$\xi_7(t,x,z)=k\Big[x-\frac{1}{2}\int_0^t\{2a_0h_4(\tau,z)+k^2h_3(\tau,z)\}d\tau\Big].$$

In the limit case when $m\to 1$ we have $(ns(\xi)\pm cs(\xi))\to(\coth(\xi)\pm csch(\xi))$, thus (3.22),(3.23) become.

$$u_8(t,x,z)=a_0-\frac{3k^2h_3(t,z)}{h_4(t,z)}\Big[\coth(\xi_8(t,x,z))\pm\operatorname{csch}(\xi_8(t,x,z))\Big]^2, \tag{3.24}$$

$$v_8(t,x,z)=\pm\frac{[-3k^2h_3(t,z)+2a_0(h_1(t,z)-h_4(t,z))]}{2\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}}$$

$$\pm\frac{3ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\Big[\coth(\xi_8(t,x,z))\pm\operatorname{csch}(\xi_8(t,x,z))\Big]^2, \tag{3.25}$$

with

$$\xi_8(t,x,z)=k\Big[x-\frac{1}{2}\int_0^t\{2a_0h_4(\tau,z)-k^2h_3(\tau,z)\}d\tau\Big].$$

Remark that: there are another solutions for Eq.(1.2). These solutions come from setting different values for the cofficients P,Q and R.(see Appendex C.). The above mentioned cases are just to clarify how far our technique is applicable.

IV. White Noise Functional Solutions of Eq.(1.2)

In this section, we employ the results of the Section 3 by using Hermite transform to obtain exact white noise functional solutions for Wick-type stochastic coupled KdV equations (1.2). The properties of exponential and trigonometric functions yield that there exists a bounded open set $\mathbf{D} \subset \mathbb{R}_+ \times \mathbb{R}$, $\rho < \infty$, $\lambda > 0$ such that the solution $u(t,x,z)$ of Eq.(3.1) and all its partial derivatives which are involved in Eq. (3.1) are uniformly bounded for $(t,x,z) \in \mathbf{D} \times K_\rho(\lambda)$, continuous with respect to $(t,x) \in \mathbf{D}$ for all $z \in K_\rho(\lambda)$ and analytic with respect to $z \in K_\rho(\lambda)$, for all $(t,x) \in \mathbf{D}$. From Theorem 4.1.1 in [20], there exists $U(t,x,z) \in (\mathcal{S})_{-1}$ such that $u(t,x,z) = \tilde{U}(t,x)(z)$ for all $(t,x,z) \in \mathbf{D} \times K_\rho(\lambda)$ and $U(t,x)$ solves Eq.(1.2) in $(\mathcal{S})_{-1}$. Hence, by applying the inverse Hermite transform to the results of Section 3, we get New exact white noise functional solutions of Eq.(1.2) as follows:

- New Wick-type stochastic solutions of (JEF):

$$U_1(t,x) = a_0 - [\frac{12k^2H_3(t)}{H_4(t)}] \diamond cs^{\diamond 2}(\Xi_1(t,x)), \tag{4.1}$$

$$\begin{aligned} V_1(t,x) &= \pm \tfrac{[3k^2H_3(t)(2-m^2)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \\ &\pm \tfrac{12ik^2H_3(t)}{H_4(t)} \diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond cs^{\diamond 2}(\Xi_1(t,x)), \end{aligned} \tag{4.2}$$

$$U_2(t,x) = a_0 - [\frac{12k^2H_3(t)}{H_4(t)}] \diamond ds^{\diamond 2}(\Xi_2(t,x)), \tag{4.3}$$

$$\begin{aligned} V_2(t,x) &= \pm \tfrac{[3k^2H_3(t)(2m^2-1)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \\ &\pm \tfrac{12ik^2H_3(t)}{H_4(t)} \diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond ds^{\diamond 2}(\Xi_2(t,x)), \end{aligned} \tag{4.4}$$

$$U_3(t,x) = a_0 - \frac{3k^2H_3(t)}{H_4(t)} \diamond \Big[ns^{\diamond}(\Xi_3(t,x)) \pm cs^{\diamond}(\Xi_3(t,x))\Big]^{\diamond 2}, \tag{4.5}$$

$$\begin{aligned} V_3(t,x) &= \pm \tfrac{[3k^2H_3(t)(1-2m^2)+2a_0(H_1(t)-H_4(t))]}{2\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \\ &\pm \tfrac{3ik^2H_3(t)}{H_4(t)} \diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \Big[ns^{\diamond}(\Xi_3(t,x)) \pm cs^{\diamond}(\Xi_3(t,x))\Big]^{\diamond 2}, \end{aligned} \tag{4.6}$$

with

$$\Xi_1(t,x) = k\Big[x - \int_0^t \{a_0H_4(\tau) + k^2(2-m^2)H_3(\tau)\}d\tau\Big],$$

$$\Xi_2(t,x) = k\Big[x - \int_0^t \{a_0H_4(\tau) + k^2(2m^2-1)H_3(\tau)\}d\tau\Big],$$

$$\Xi_3(t,x) = k\Big[x - \frac{1}{2}\int_0^t \{2a_0H_4(\tau) + k^2(1-2m^2)H_3(\tau)\}d\tau\Big].$$

Ref

[20] H. Holden, B. Øsendal, J. U b øe and T. Zhang. *Stochastic partial differential equations*, Bihkäuser: Basel, (1996).

Notes

- New Wick-type stochastic solutions of trigonometric functions:

$$U_4(t,x) = a_0 - \frac{12k^2H_3(t)}{H_4(t)} \diamond \cot^{\diamond 2}(\Xi_4(t,x)), \tag{4.7}$$

$$\begin{aligned} V_4(t,x) = &\pm \tfrac{[6k^2H_3(t)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond(H_1(t)-H_4(t))}} \\ &\pm \tfrac{12ik^2H_3(t)}{H_4(t)} \diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \cot^{\diamond 2}(\Xi_4(t,x)), \end{aligned} \tag{4.8}$$

$$U_5(t,x) = a_0 - [\frac{12k^2H_3(t)}{H_4(t)}] \diamond \csc^{\diamond 2}(\Xi_5(t,x)), \tag{4.9}$$

$$\begin{aligned} V_5(t,x) = &\pm \tfrac{[-3k^2H_3(t)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \pm \tfrac{12ik^2H_3(t)}{H_4(t)} \\ &\diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \csc^{\diamond 2}(\Xi_5(t,x)), \end{aligned} \tag{4.10}$$

$$U_6(t,x) = a_0 - \frac{3k^2H_3(t)}{H_4(t)} \diamond \Big[\csc^{\diamond}(\Xi_6(t,x)) \pm \cot^{\diamond}(\Xi_6(t,x))\Big]^{\diamond 2}, \tag{4.11}$$

$$\begin{aligned} V_6(t,x) = &\pm \tfrac{[3k^2H_3(t)+2a_0(H_1(t)-H_4(t))]}{2\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \pm \tfrac{3ik^2H_3(t)}{H_4(t)} \\ &\diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \Big[\csc^{\diamond}(\Xi_6(t,x)) \pm \cot^{\diamond}(\Xi_6(t,x))\Big]^{\diamond 2}, \end{aligned} \tag{4.12}$$

with

$$\Xi_4(t,x) = k\Big[x - \int_0^t \{a_0H_4(\tau) + 2k^2H_3(\tau)\}d\tau\Big],$$

$$\Xi_5(t,x) = k\Big[x - \int_0^t \{a_0H_4(\tau) - k^2H_3(\tau)\}d\tau\Big],$$

$$\Xi_6(t,x) = k\Big[x - \frac{1}{2}\int_0^t \{2a_0H_4(\tau) + k^2H_3(\tau)\}d\tau\Big].$$

- New Wick-type stochastic solutions of hyperbolic functions:

$$U_7(t,x) = a_0 - [\frac{12k^2H_3}{H_4}] \diamond \operatorname{csch}^{\diamond 2}(\Xi_7(t,x)), \tag{4.13}$$

$$\begin{aligned} V_7(t,x) = &\pm \tfrac{[3k^2H_3(t)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \pm \tfrac{12ik^2H_3(t)}{H_4(t)} \\ &\diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \operatorname{csch}^{\diamond 2}\Xi_7(t,x), \end{aligned} \tag{4.14}$$

$$U_8(t,x) = a_0 - \frac{3k^2 H_3(t)}{H_4(t)} \diamond \left[\coth^\diamond(\Xi_8(t,x)) \pm \operatorname{csch}^\diamond(\Xi_8(t,x))\right]^{\diamond 2}, \tag{4.15}$$

$$V_8(t,x) = \pm \tfrac{[-3k^2H_3(t)+2a_0(H_1(t)-H_4(t))]}{2\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \pm \tfrac{3ik^2H_3(t)}{H_4(t)} \diamond\sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \left[\coth^\diamond(\Xi_8(t,x)) \pm \operatorname{csch}^\diamond(\Xi_8(t,x))\right]^{\diamond 2}, \tag{4.16}$$

with

$$\Xi_7(t,x) = k\left[x - \int_0^t \{a_0H_4(\tau) + k^2H_3(\tau)\}d\tau\right],$$

$$\Xi_8(t,x) = k\left[x - \frac{1}{2}\int_0^t \{2a_0H_4(\tau) - k^2H_3(\tau)\}d\tau\right].$$

We observe that. For different forms of H_1, H_2, H_3 and H_4, we can get different exact white noise functional solutions of Eq.(1.2) from Eqs.(4.1)-(4.16).

V. EXAMPLE

It is well known that Wick version of function is usually difficult to evaluate. So, in this section, we give non-Wick version of solutions of Eq.(1.2). Let $W_t = \dot{B}_t$ be the Gaussian white noise, where B_t is the Brownian motion. We have the Hermite transform $\tilde{W}_t(z) = \sum_{i=1}^{\infty} z_i \int_0^t \eta_i(s)ds$ [20]. Since $\exp^\diamond(B_t) = \exp(B_t - \frac{t^2}{2})$, we have $\sin^\diamond(B_t) = \sin(B_t - \frac{t^2}{2})$, $\cos^\diamond(B_t) = \cos(B_t - \frac{t^2}{2})$, $\cot^\diamond(B_t) = \cot(B_t - \frac{t^2}{2})$, $\csc^\diamond(B_t) = \csc(B_t - \frac{t^2}{2})$, $\coth^\diamond(B_t) = \coth(B_t - \frac{t^2}{2})$ and $\operatorname{csch}^\diamond(B_t) = \operatorname{csch}(B_t - \frac{t^2}{2})$. Suppose that $H_1(t) = H_2(t) = \lambda_1 H_3(t), H_3(t) = \lambda_2 H_4(t)$ and $H_4(t) = \Gamma(t) + \lambda_3 W_t$ where λ_1, λ_2 and λ_3 are arbitrary constants and $\Gamma(t)$ is integrable or bounded measurable function on $\mathbb{R}_+$. Therefore, for $H_1(t)H_2(t)H_3(t)H_4(t) \neq 0$. thus exact white noise functional solutions of Eq.(1.2) are as follows:

$$U_9(t,x) = 3k^2\left\{\frac{a_0}{3k^2} - 4\lambda_2\ \cot^2(\Phi_1(t,x))\right\}, \tag{5.1}$$

$$V_9(t,x) = \pm 6k^2\lambda_2\left\{\frac{\left[1+\frac{a_0}{6k^2\lambda_2(\lambda_1\lambda_2-1)}\right]}{\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + 2i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\ \cot^2(\Phi_1(t,x))\right\}, \tag{5.2}$$

$$U_{10}(t,x) = 3k^2\left\{\frac{a_0}{3k^2} - 4\lambda_2\ \csc^2(\Phi_2(t,x))\right\}, \tag{5.3}$$

$$V_{10}(t,x) = \pm 3k^2\lambda_2\left\{\frac{\left[\frac{a_0}{3k^2\lambda_2(\lambda_1\lambda_2-1)}-1\right]}{\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} +\right.$$

Ref

[20] H. Holden, B. Øsendal, J. U b øe and T. Zhang. *Stochastic partial differential equations*, Birkäuser: Basel, (1996).

Notes

$$4i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\ \csc^2(\Phi_2(t,x))\Big\}, \tag{5.4}$$

$$U_{11}(t,x) = a_0 - 3k^2\lambda_2\Big[\csc(\Xi(t,x)) \pm \cot(\Phi_3(t,x))\Big]^2, \tag{5.5}$$

$$\begin{aligned} V_{11}(t,x) = \pm 3k^2\lambda_2\Big\{ & \frac{\left[1+\frac{2a_0}{3k^2\lambda_2(\lambda_1\lambda_2-1)}\right]}{2\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + \\ & i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\Big[\csc(\Phi(t,x)) \pm \cot(\Phi_3(t,x))\Big]^2\Big\}, \end{aligned} \tag{5.6}$$

$$_{12}(t,x) = a_0 - 12k^2\lambda_2\operatorname{csch}^2(\Phi_4(t,x)), \tag{5.7}$$

$$\begin{aligned} V_{12}(t,x) = \pm 3k^2\lambda_2\Big\{ & \frac{\left[1+\frac{a_0}{3k^2\lambda_2(\lambda_1\lambda_2-1)}\right]}{\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + \\ & 4i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\operatorname{csch}^2(\Phi_4(t,x)\Big\}, \end{aligned} \tag{5.8}$$

$$U_{13}(t,x) = a_0 - 3k^2\lambda_2\Big[\coth(\Phi_5(t,x)) \pm \operatorname{csch}(\Phi_5(t,x))\Big]^2, \tag{5.9}$$

$$\begin{aligned} V_{13}(t,x) = \pm 3k^2\lambda_2\Big\{ & \frac{\left[\frac{2a_0}{3k^2\lambda_2(\lambda_1\lambda_2-1)}-1\right]}{2\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + \\ & 3i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\Big[\coth(\Phi_5(t,x)) \pm \operatorname{csch}(\Phi_5(t,x))\Big]^{\diamond 2}\Big\}, \end{aligned} \tag{5.10}$$

with

$$\Phi_1(t,x) = k\Big[x - (a_0 + 2k^2\lambda_2)\Big\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Big\}\Big],$$

$$\Phi_2(t,x) = k\Big[x - (a_0 - k^2\lambda_2)\Big\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Big\}\Big],$$

$$\Phi_3(t,x) = k\Big[x - \frac{(2a_0 + k^2\lambda_2)}{2}\Big\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Big\}\Big],$$

$$\Phi_4(t,x) = k\Big[x - (a_0 + k^2\lambda_2)\Big\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Big\}\Big]$$

and

$$\Phi_5(t,x) = k\Big[x - \frac{(2a_0 - k^2\lambda_2)}{2}\Big\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Big\}\Big].$$

VI. Conclusion

We have discussed the solutions of stochastic (PDEs) driven by Gaussian white noise. There is a unitary mapping between the Gaussian white noise space and the Poisson white noise space. This connection was given by Benth and Gjerde [2]. We can see in the section 4.9 [20] clearly. Hence, by the aid of the connection, we can derive some stochastic exact soliton solutions if the coefficients , and are Poisson white noise functions in Eq. (1.2). In this paper, using Hermite transformation, white noise theory and F-expantion method, we study the white noise solutions of the Wick-type stochastic coupled KdV equations. This paper shows that the F-expantion method is sufficient to solve the stochastic nonlinear equations in mathematical physics. The method which we have proposed in this paper is powerful, direct and computerized method, which allows us to do complicated and tedious algebraic calculation. It is shown that the algorithm can be also applied to other NLPDEs in mathematical physics such as modified Hirota-Satsuma coupled KdV, (2+1)-dimensional coupled KdV, KdV-Burgers, schamel KdV, modified KdV Burgers, Sawada-Kotera, Zhiber-Shabat equations and Benjamin-Bona-Mahony equations. Since the equation (1.2) has other solutions if select other values of P, Q and R (see Appendix A, B and C). So there are many other of exact solutions for wick-type stochastic coupled KdV equations.

Appendix A.
the jacobi elliptic functions degenerate into trigonometric functions when $m \to 0$.

$$sn\xi \to \sin\xi, cn\xi \to \cos\xi, dn\xi \to 1, sc\xi \to \tan\xi, sd\xi \to \sin\xi, cd\xi \to \cos\xi,$$
$$ns\xi \to \csc\xi, nc\xi \to \sec\xi, nd\xi \to 1, cs\xi \to \cot\xi, ds\xi \to \csc\xi, dc\xi \to \sec\xi.$$

Appendix B.
the jacobi elliptic functions degenerate into hyperbolic functions when $m \to 1$.

$$sn\xi \to \tan\xi, cn\xi \to \operatorname{sech}\xi, dn\xi \to \operatorname{sech}\xi, sc\xi \to \sinh\xi, sd\xi \to \sinh\xi, cd\xi \to 1,$$
$$ns\xi \to \coth\xi, nc\xi \to \cosh\xi, nd\xi \to \cosh, cs\xi \to \operatorname{csch}\xi, ds\xi \to \operatorname{csch}\xi, dc\xi \to 1.$$

Appendix C. The ODE and Jacobi Elliptic Functions
Relation between values of $(P\ , Q\ , R)$ and corresponding $F(\xi)$ in ODE

$$(F')^2(\xi) = PF^4(\xi) + QF^2(\xi) + R,$$

P	Q	R	$F(\xi)$
m^2	$-1-m^2$	1	$\mathrm{sn}\xi$, $\mathrm{cd}\xi = \frac{cn\xi}{dn\xi}$
$-m^2$	$2m^2-1$	$1-m^2$	$\mathrm{cn}\xi$
-1	$2-m^2$	m^2-1	$\mathrm{dn}\xi$
1	$-1-m^2$	m^2	$\mathrm{ns}\xi = \frac{1}{\mathrm{sn}\xi}$, $\mathrm{dc}\xi = \frac{\mathrm{dn}\xi}{\mathrm{cn}\xi}$
$1-m^2$	$2m^2-1$	$-m^2$	$\mathrm{nc}\xi = \frac{1}{\mathrm{cn}\xi}$
m^2-1	$2-m^2$	-1	$\mathrm{nd}\xi = \frac{1}{\mathrm{dn}\xi}$
$1-m^2$	$2-m^2$	1	$\mathrm{sc}\xi = \frac{\mathrm{sn}\xi}{\mathrm{cn}\xi}$

Ref

[2] E. Benth and J. Gjerde. *Potential. Anal.*, **8** (1998): 179-193.

Notes

$-m^2(1-m^2)$	$2m^2-1$	1	$\mathrm{sd}\xi=\frac{\mathrm{sn}\xi}{\mathrm{dn}\xi}$
1	$2-m^2$	$1-m^2$	$\mathrm{cs}\xi=\frac{\mathrm{cn}\xi}{\mathrm{sn}\xi}$
1	$2m^2-1$	$-m^2(1-m^2)$	$\mathrm{ds}\xi=\frac{\mathrm{dn}\xi}{\mathrm{sn}\xi}$
$\frac{m^4}{4}$	$\frac{m^2-2}{2}$	$\frac{1}{4}$	$\frac{\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi},\ \frac{\mathrm{cn}\xi}{\sqrt{1-m^2}\pm\mathrm{dn}\xi}$
$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\mathrm{sn}\xi\pm i\mathrm{cn}\xi,\ \frac{\mathrm{dn}\xi}{i\sqrt{1-m^2}\mathrm{sn}\xi\pm\mathrm{cn}\xi},\ \frac{\mathrm{m\,sn}\xi}{1\pm\mathrm{dn}\xi}$
$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$\mathrm{ns}\xi\pm\mathrm{cs}\xi,\ \frac{\mathrm{cn}\xi}{\sqrt{1-m^2}\mathrm{sn}\xi\pm\mathrm{dn}\xi},\ \frac{\mathrm{sn}\xi}{1\pm\mathrm{cn}\xi},$
$\frac{m^2-1}{4}$	$\frac{m^2+1}{2}$	$\frac{m^2-1}{4}$	$\frac{\mathrm{dn}\xi}{1\pm m\mathrm{sn}\xi}$
$\frac{1-m^2}{4}$	$\frac{m^2+1}{2}$	$\frac{1-m^2}{4}$	$\mathrm{nc}\xi\pm i\mathrm{sc}\xi\ \frac{\mathrm{cn}\xi}{1\pm\mathrm{sn}\xi}$
$\frac{-1}{4}$	$\frac{m^2+1}{2}$	$\frac{-(1-m^2)^2}{4}$	$m\mathrm{cn}\xi\pm\mathrm{dn}\xi$
$\frac{1}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\mathrm{ns}\xi\pm\mathrm{ds}\xi$

References Références Referencias

[1] S. Abbasbandy. *Chem. Eng. J* , **136** (2008): 144-150.

[2] E. Benth and J. Gjerde. *Potential. Anal.*, **8** (1998): 179-193.

[3] A. de Bouard and A. Debussche. *J. Funct. Anal.*, **154** (1998): 215-251.

[4] A. de Bouard and A. Debussche. *J.Funct. Anal.* , **169** (1999): 532-558.

[5] A. Debussche and J. Printems. *Physica D*, **134** (1999): 200-226.

[6] A. Debussche and J. Printems. *J. Comput. Anal. Appl.*, **3** (2001): 183-206.

[7] M. Dehghan, J. Manafian and A. Saadatmandi. *Math. Meth. Appl. Sci*, **33** (2010): 1384-1398.

[8] M. Dehghan and M. Tatari. *Chaos Solitons and Fractals*, **36** (2008): 157-166.

[9] E. Fan. *Phys. Lett. A*, **277** (2000): 212-218.

[10] Z. T. Fu, S. K. Liu, S. D. Liu, Q. Zhao. *Phys. Lett. A*, **290** (2001): 72-76.

[11] D. D. Ganji and A. Sadighi. *Int J Non Sci Numer Simul*, **7** (2006): 411-418.

[12] H. A. Ghany. *Chin. J. Phys.*, **49** (2011): 926-940.

[13] H. A. Ghany. *International Journal of pure and applied mathematics*, **78** (2012): 17-27.

[14] H. A. Ghany and A. Hyder. *International Review of Physics*, **6** (2012): 153-157.

[15] H. A. Ghany and A. Hyder. *J. Comput. Anal. Appl.*, **15** (2013): 1332-1343.

[16] H. A. Ghany and A. Hyder. *Kuwait Journal of Science*, (2013), Accepted.

[17] H. A. Ghany and A. Hyder. *IAENG International Journal of Applied Mathematics*, (2013), Accepted.

[18] H. A. Ghany and M. S. Mohammed. *Chin. J. Phys.*, **50** (2012): 619-627.

[19] H. A. Ghany, A. S. Okb El Bab, A. M. Zabal and A. Hyder. *Chin. Phys. B*, (2013), Accepted.

[20] H. Holden, B. Øsendal, J. U b øe and T. Zhang. *Stochastic partial differential equations*, Bihkäuser: Basel, (1996).

[21] J .H. He and X. H. Wu. *Chaos Solitons and Fractals*, **30** (2006):700-708.

[22] J. H. He and M. A. Abdou *Chaos Solitons and Fractals*, **34** (2007): 1421-1429.

[23] V.V. Konotop, L. Vazquez. Nonlinear Random Waves, World Scientific, Singapore, (1994).

[24] S. K. Liu, Z. T. Fu, S. D. Liu, Q. Zhao. *Phys. Lett. A*, **289** (2001): 69-74.

[25] J. Liu, L. Yang, K. Yang *Chaos, Solitons and Fractals*, **20** (2004): 1157-1164.

[26] E. J. Parkes, B. R. Duffy, P. C. Abbott *Phys. Lett. A*, **295** (2002): 280-286.

[27] F. Shakeri and M. Dehghan. *Math. Comput. Model*, **48** (2008): 486-498.

[28] M. Wadati. *J. Phys. Soc. Jpn.*, **52** (1983): 2642-2648.

[29] A. M. Wazwaz. *Chaos Solitons Fractals*, **188** (2007): 1930-1940.

[30] M. Wadati and Y. Akutsu. *J. Phys. Soc. Jpn.*, **53** (1984): 3342-3350.

[31] A. M. Wazwaz. *Appl. Math. Comput*, **177** (2006): 755-760.

[32] A. M. Wazwaz. *Appl. Math. Comput*, **169** (2005): 321-338.

[33] X. Ma, L. Wei, and Z. Guo. *J. Sound Vibration*, **314** (2008): 217-227.

[34] X. H. Wu and J.H. He. *Chaos Solitons and Fractals*, **38** (2008): 903-910.

[35] Y. C. Xie . *Phys. Lett. A*, **310** (2003): 161-167.

[36] Y. C. Xie. *Solitons and Fractals*, **21** (2004): 473-480.

[37] Y. C. Xie. *Solitons and Fractals*, **20** (2004): 337-342.

[38] Y. C. Xie. *J. Phys. A: Math. Gen*, **37** (2004): 5229-5236.

[39] Y. C. Xie. *Phys. Lett. A*, **310** (2003): 161-167.

[40] S. Zhang, T. C. Xia. *Communications in Theoretical Physics*, **46** (2006): 985-990.

[41] S. D. Zhu. *Int J Non Sci Numer Simul*, **8** (2007): 461-464.

[42] S. D. Zhu. *Int J Non Sci Numer Simul*, **8** (2007): 465-468.

Notes

[43] Yubin Zhou. Mingliang Wang. Yueming Wang. *Physics Letters A*, **308** (2003): 3136.

[44] Sheng Zhang. Tiecheng Xia. *Applied Mathematics and Computation*, **189** (2007): 836843.

[45] Sheng Zhang. TieCheng Xia. *Applied Mathematics and Computation*, **183** (2006): 11901200.

[46] Sheng Zhang. Tiecheng Xia. *Communications in Nonlinear Science and Numerical Simulation*, **13** (2008): 12941301.

Notes

Notes

This page is intentionally left blank